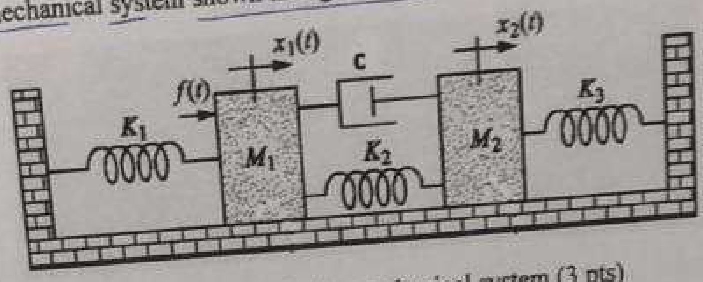
Question 1: mechanical system (6 pts).

Consider the mechanical system shown in Figure below.



Q1.1- Obtain the mathematical model for the mechanical system (3 pts)

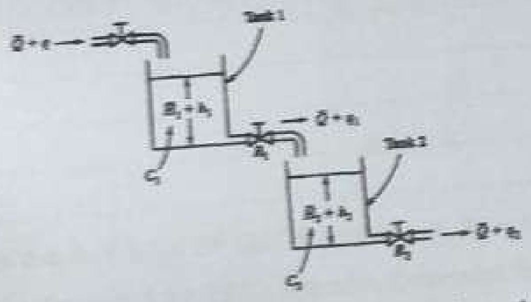
Q1.2- Using the force-voltage analogy shown in table 1, obtain and draw the electrical analogue of the mechanical system shown above (3 pts)

Table 1. Force-Voltage Analogy

Force, p (torque T)	Voltage, e
Mass, m (moment of inertia J)	Inductance, L
Viscous-friction coefficient, b	Resistance, R
Spring constant, k	Reciprocal of capacitance, 1/C
Displacement, x (angular displacement θ)	Charge, q
Velocity $\dot{x}$ (angular velocity $\dot{\theta}$)	Current, i

Q2: liquid level system (free tanks) (12 pts)

Consider the liquid-level system shown in figure below. q is the chang in the inflow rate (supposed to be small). (h_1 and h_2) and (q_1 and q_2) are the resulting changes in the heads and flow rates. Th capacitances of tanks 1 and 2 are C_1 and C_2, respectively. The resistance of the outflow valve of tank 1 is R_1 and that of tank 2 is R_2



Q2.1- Express the rate of change of fluid volume in the tank 1 (2 pts)

Q2.2-Express the rate of change of fluid volume in the tank 2 (2 pts)

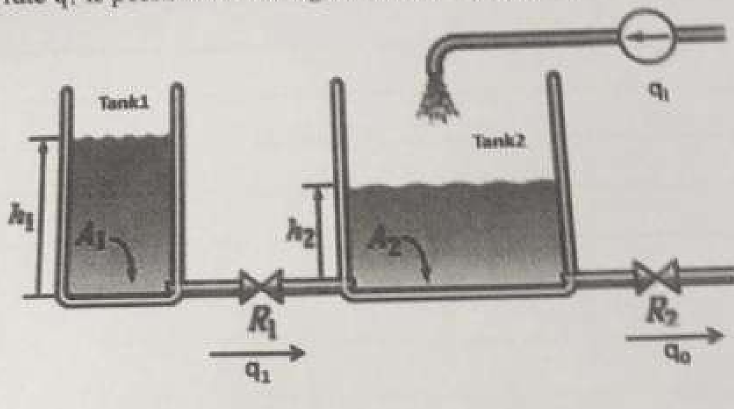
Q2.3-Derive the transfer function for the system when **q** is the input and **h_2** is the output. (3pts)

Q2.4- Suppose that $R_1 = 0.5 \times 10^4$ s/m^2, $C_1 = 0.02$ m^2, $R_2 = 0.6 \times 10^4$ s/m^2, $C_2 = 0.04$ m^2 and q is a unit step function $q = 0.5 \times 10^{-4} \times 1(t)$ m^3/s. Determine the change in head h_2 (3 pts)

Q2.5- Assuming that the steady-state head in tank2 is 3 m, Calculate the liquid head in this Tank after 3 min (2pts)

Q3: liquid level system (connected tanks) (7 pts).

The cylindrical tanks shown in figure 3 (liquid-level system) have bottom areas A_1 and A_2. The inflow rate $q_i(t)$ from the flow source is a given function of time. We Assume that $h_1 > h_2$ so that the flow rate q_1 is positive if flowing from tank 1 to tank 2.



3.1. Develop a model of the liquid heights h_1 (2pts)

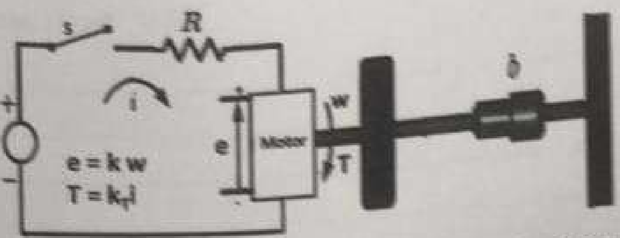
3.2. Develop a model of the liquid heights h_2 (2pts)

3.3. Suppose the resistances are equal: $R_1 = R_2 = R$, and the areas are $A_1 = A$ and $A_2 = 3A$.

Obtain the transfer function $H_1(s)/Qi(s)$ (3pts)

Question 4 (9 pts)

Considers the motor system shown below



The motor is driven by an armature voltage v via a switch S. The motor internal coil resistance is modeled by the resistor R. The motor is characterized by the relations $T = K_T\, i$, and $e = K\, w$, where T is the applied torque to the load, and ω is the angular speed. The load consists of a rotary inertia J, and a rotational damper b.

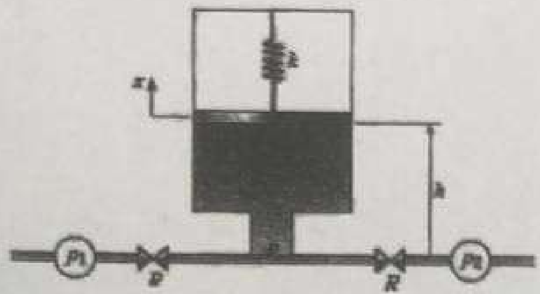
Q4.1. At $t = 0$, the switch S is closed. Determine the expression of $w(t)$ function of the system parameters. Clearly show the steps in your solution. (3 pts)

Q4.2. After the switch has been closed for a long time, the switch is opened at time t_1. Determine the expression of $w(t)$ function of the system parameters for $t > t_1$. Clearly show the steps in your solution. (3 pts)

Q4.3. Determine and write an expression for the coil current i as a function of time $i(t)$ throughout the motions determined above. (3 pts)

Q5: Fluid mechanical system (6 pts).

An example of a hydraulic accumulator is shown in figure below. This accumulator is supposed to reduce pressure fluctuation in hydraulic line or pipe. The fluid density is ρ, the plate mass is m, and the plate area is A.



Develop a dynamic model of the pressure p with the pressures p_1 and p_2 as the given input (Assume that $m\ddot{x}$ of the plate is small, and the hydrostatic pressure is small)

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