King Abdulaziz University
Mechanical Engineering Department

CHAPTER 4

FLYWHEELS AND MOTOR SELECTION

MENG 364 Machine Dynamics

Prof. Hamza Diken

SPRING 2017

- Flywheels
- Example 1
- Example 2
- Example 3
- Example 4
- Example 5
- Motor selection
- lead screw drive
- Direct drive
- Gear drive
- Belt and pulley drive

- Motors and drivers
- Electric motors
- DC motors
- AC motors
- Servomotors
- Stepper motors
- Air and hydraulic motors
- Air and hydraulic cylinders
- Gyroscopic effects

FLYWHEEL

A **flywheel** is a rotating mechanical device that is used to store rotational energy.

Flywheels have a significant moment of inertia and thus resist changes in rotational speed. The amount of energy stored in a flywheel is proportional to the square of its rotational speed. Energy is transferred to a flywheel by applying torque to it, thereby increasing its rotational speed, and hence its stored energy. Conversely, a flywheel releases stored energy by applying torque to a mechanical load, thereby decreasing its rotational speed.

$$KE = \frac{1}{2} I\omega^2$$

Three common uses of a flywheel include:

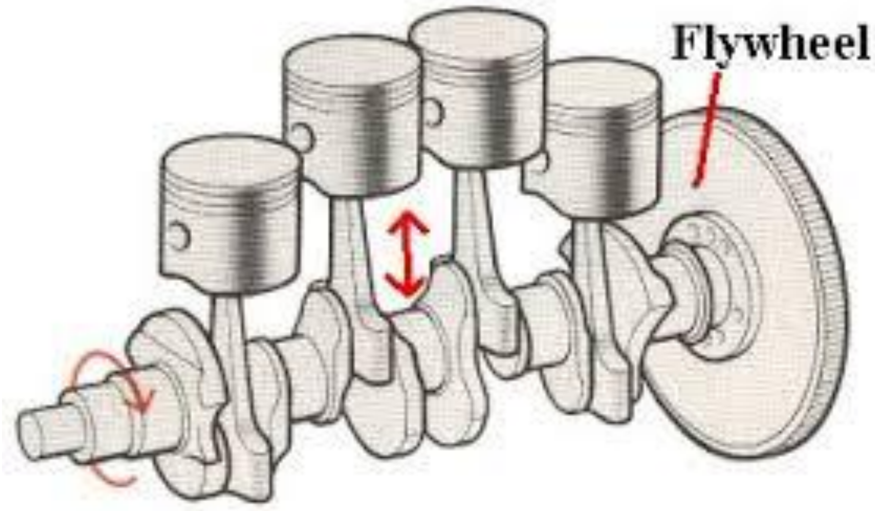
They provide continuous energy when the energy source is discontinuous. For example, flywheels are used in reciprocating engines because the energy source, torque from the engine, is intermittent.

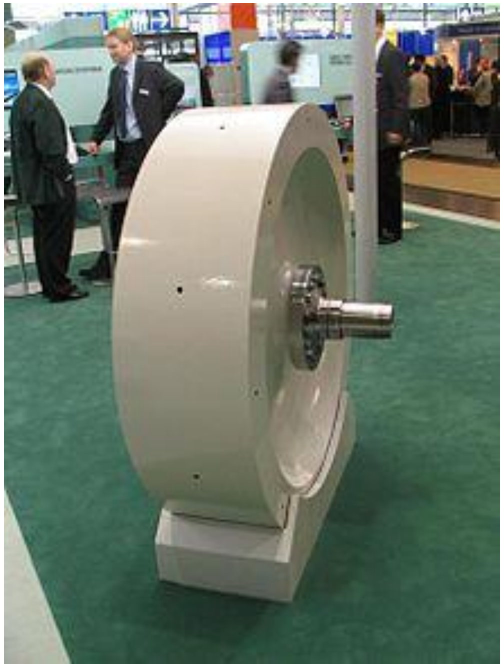
They deliver energy at rates beyond the ability of a continuous energy source. This is achieved by collecting energy in the flywheel over time and then releasing the energy quickly, at rates that exceed the abilities of the energy source.

They control the orientation of a mechanical system. In such applications, the angular momentum of a flywheel is purposely transferred to a load when energy is transferred to or from the flywheel.

Flywheels are typically made of steel and rotate on conventional bearings; these are generally limited to a revolution rate of a few thousand rpm. Some modern flywheels are made of carbon fiber materials and employ magnetic bearings, enabling them to revolve at speeds up to 60,000 rpm.

Punching press machines

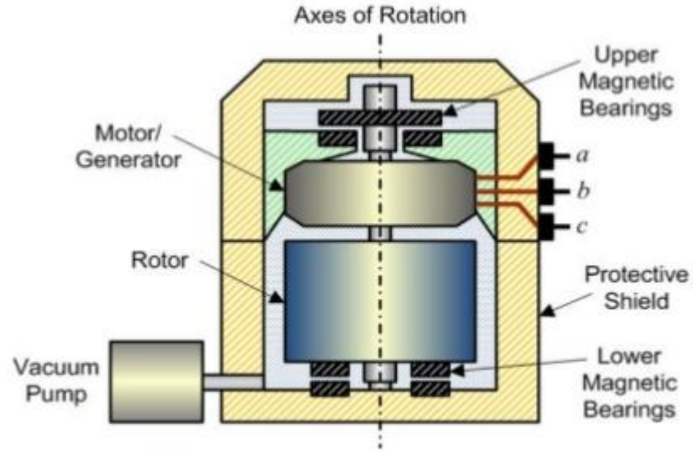


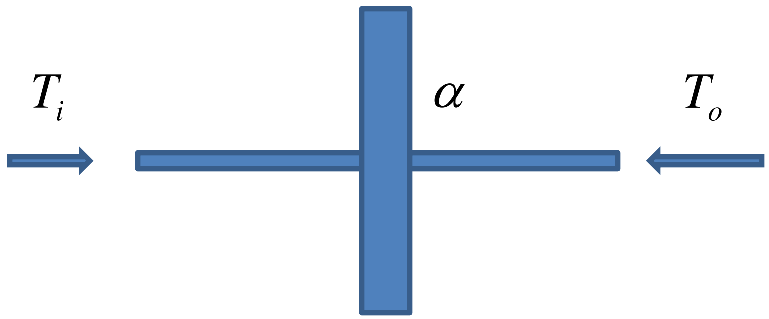


A flywheel mounted at the end of an automobile engine crankshaft.

An industrial flywheel.

Flywheel Energy Storage System





$$I\alpha = \sum T \qquad \text{(Newton's Law)}$$

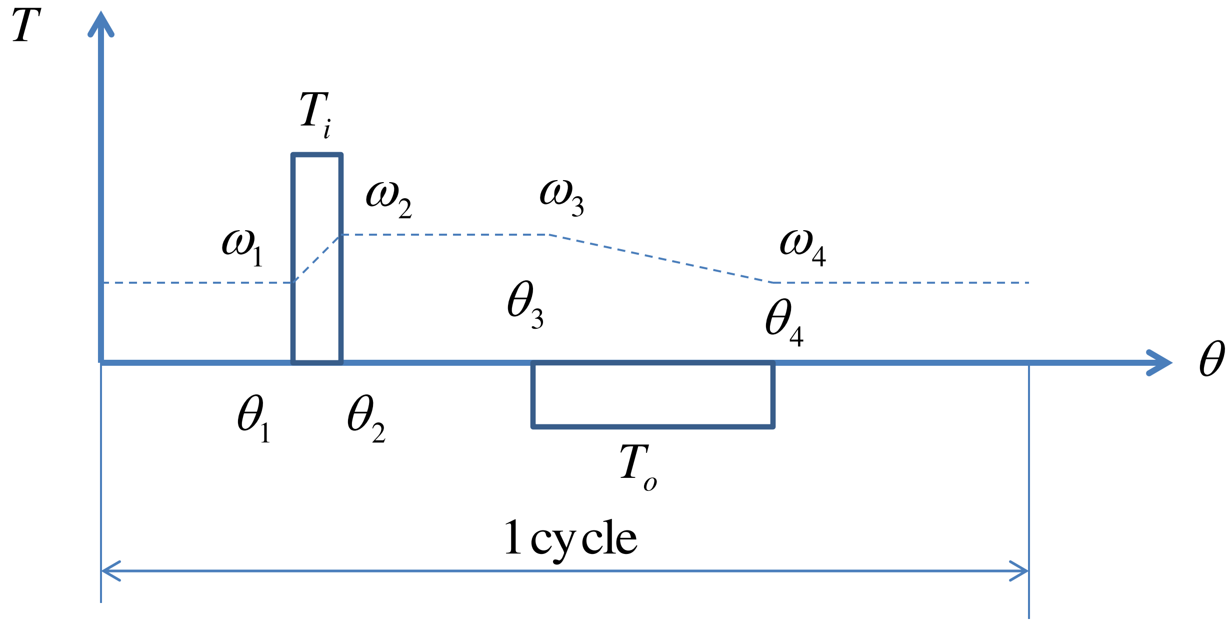
$$I\alpha = T_i(\theta,\omega) - T_o(\theta,\omega)$$

I : flywheel inertia, kgm^2

α : flywheel angular acceleration, rad/s^2

T_i : input torque, Nm

T_o : output torque, Nm



T_i is applied from θ_1 to θ_2 ω_1 is increased to ω_2

No torque is applied from θ_2 to θ_3 ω_2 and ω_3 are same

T_o is applied from θ_3 to θ_4 ω_3 is decreased to ω_4

The work input to the flywheel is

$$U_i = T_i(\theta_2 - \theta_1)$$

The work output from the flywheel is

$$U_o = T_o(\theta_4 - \theta_3)$$

If $U_o > U_i$ flywheel energy is decreased $\omega_4 < \omega_1$

If $U_o = U_i$ flywheel energy is not changed $\omega_4 = \omega_1$

If $U_o < U_i$ flywheel energy is increased $\omega_4 > \omega_1$

flywheel energy at $\theta = \theta_1$ is

$$U_1 = \frac{1}{2} I \omega_1^2$$

flywheel energy at $\theta = \theta_2$ is

$$U_2 = \frac{1}{2} I \omega_2^2$$

energy stored to the flywheel is

$$\Delta U = U_2 - U_1 = \frac{1}{2} I (\omega_2^2 - \omega_1^2)$$

$$\Delta U = U_2 - U_1 = \frac{1}{2} I (\omega_2^2 - \omega_1^2) = \frac{1}{2} I (\omega_2 - \omega_1)(\omega_2 + \omega_1)$$

$$U_2 - U_1 = \frac{1}{2} I \frac{(\omega_2 - \omega_1)}{\omega} \omega \frac{(\omega_2 + \omega_1)}{2} 2 = I C_s \omega^2$$

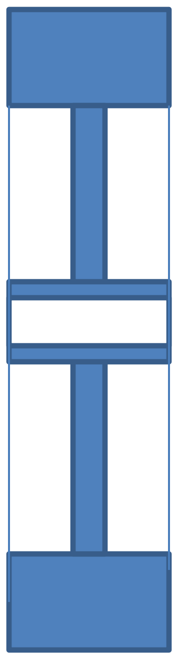
$$C_s = \frac{(\omega_2 - \omega_1)}{\omega} \quad \text{coefficient of speed fluctuation}$$

$$\omega = \frac{(\omega_2 + \omega_1)}{2} \quad \text{nominal(average) angular velocity, rad/s}$$

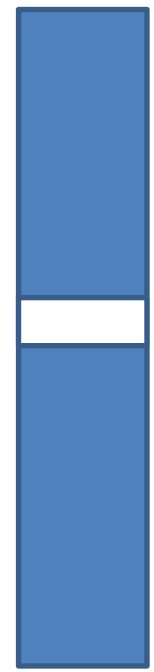
$$I = \frac{U_2 - U_1}{C_s \omega^2} \quad \text{flywheel inertia, kgm}^2$$

Application	Coefficient of speed fluctuation
Crushing and hammering machinery	0.2
Machine Tools	0.03
Pumps and shearing machinery	0.03-0.05
Textile machinery	0.025
Gearbox applications	0.02
Spinning machinery	0.01-0.02
Electrical machinery	0.003

Ref: Kent's Mechanical Engineering handbook, John Wiley and Sons, 1969.

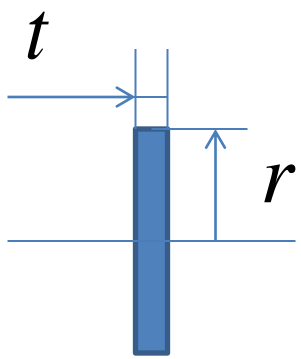


Rim type flywheel



Disk type flywheel

Flywheel Design

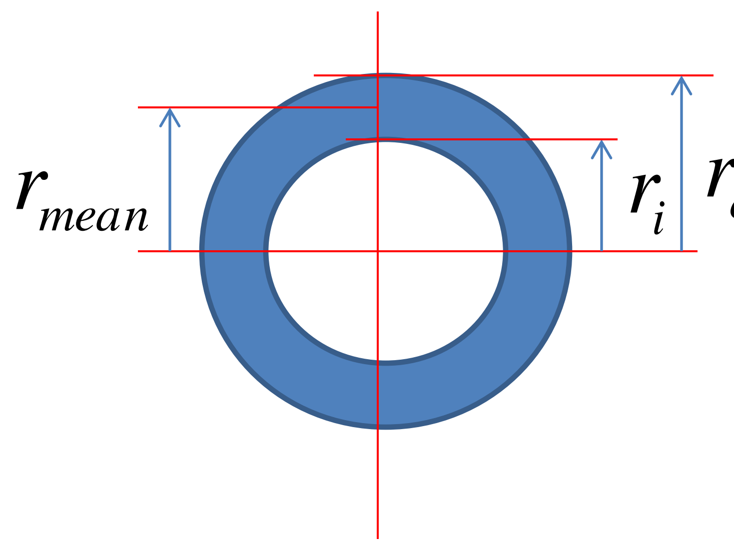


The flywheel inertia for a solid cylinder (disk) is

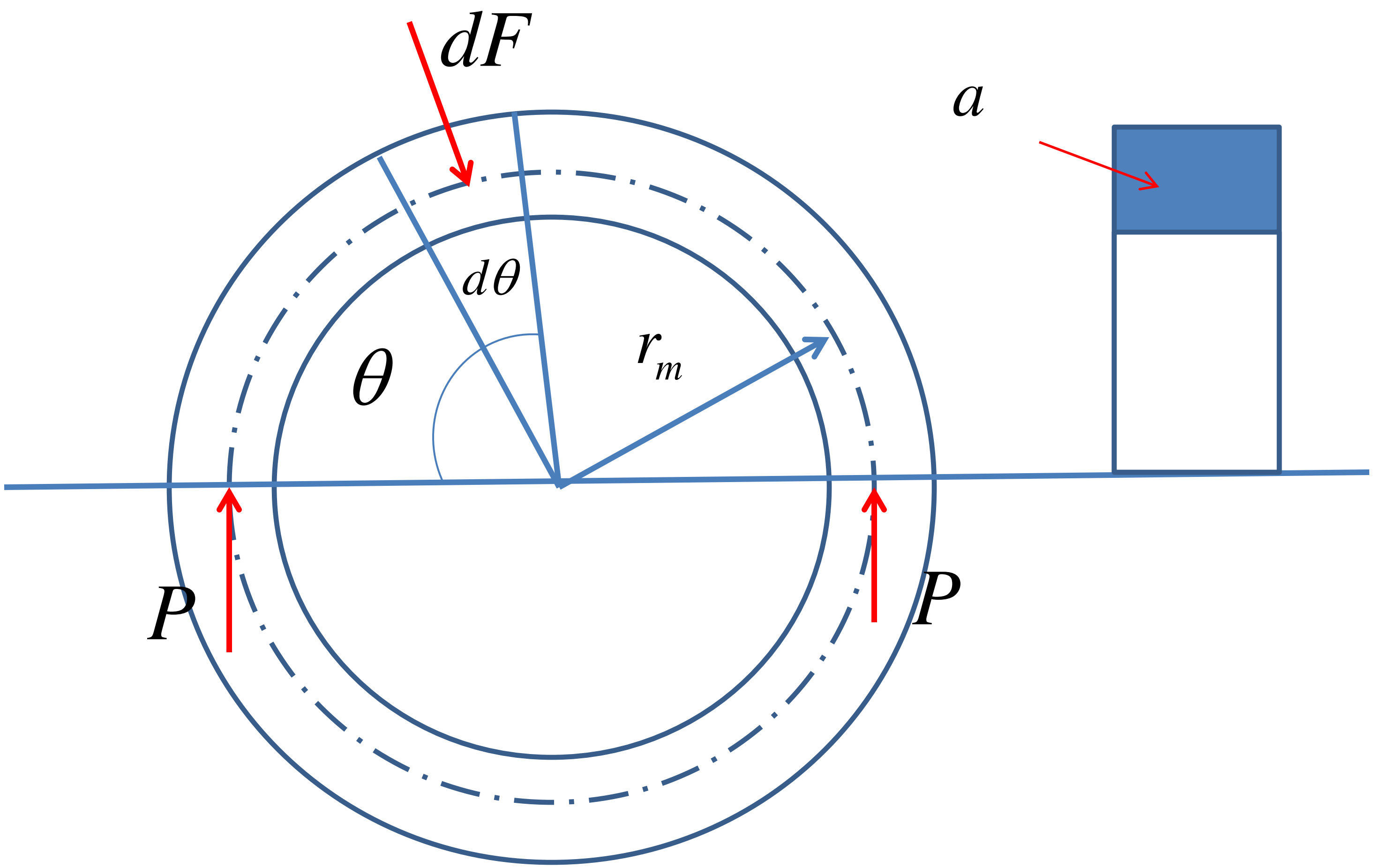
$$I = \frac{1}{2} m r^2 \qquad m = \rho \pi r^2 t$$

The flywheel inertia for a thick walled empty cylinder (rim) is

$$I = \frac{1}{2} m (r_e^2 + r_i^2) = m r_{mean}^2$$



The amount of energy that can safely be stored in the flywheel depends on the point at which the flywheel will warp or shatter. The hoop stress on the flywheel is a major consideration in the design of a flywheel energy storage system. This stress should not exceed the allowable material stress.



σ_t is the tensile stress on the rim of the flywheel, N/m^2

ρ is the density of the rim material kg/m^3

r_m is the mean radius of the rim, m

ω is the angular velocity of the rim, rad/s

V is the tangential velocity of the rim, m/s

a is the cross sectional area of the rim, m^2

Centripetal force on rim element ***dF***

$$dF = mr_m\omega^2 = \rho a r_m d\theta \, r_m\omega^2$$

Vertical component of ***dF***

$$dF = dF \sin\theta$$

Total bursting force

$$Q = \rho a r_m^2\omega^2 \int_0^{\pi} \sin\theta d\theta = 2\rho a r_m^2\omega^2$$

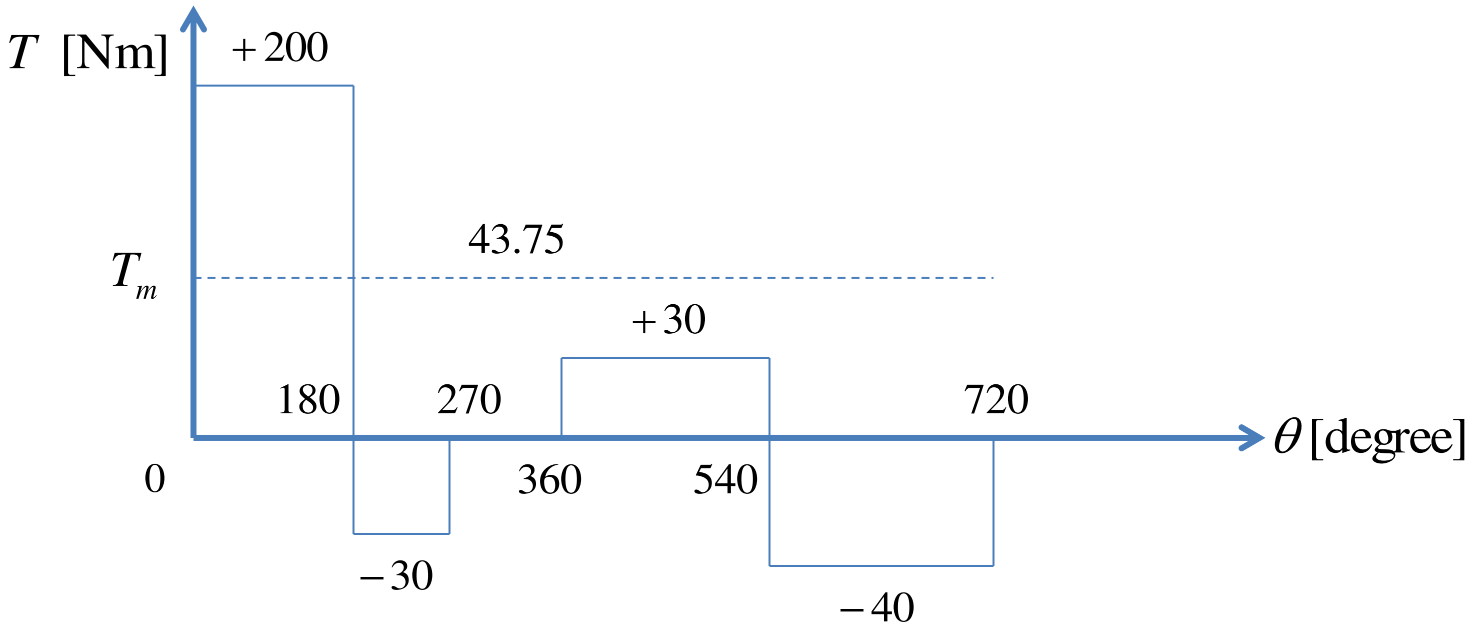
$$Q = 2P = 2\sigma_t a = 2\rho a r_m^2\omega^2$$

$$\sigma_t = \rho r_m^2\omega^2$$

Example #1

The plot shows torque-crank angle diagram for a one-cylinder four-cycle engine. The nominal speed of the engine is 250 rad/s.

a) Integrate the torque-displacement function for one cycle to find the energy that can be delivered to the load during the cycle.
b) Determine the mean torque Tm
c) Using a coefficient of speed fluctuation of 0.1, find a suitable value for the flywheel inertia.
d) Find ω_1 and ω_2



$$a)\qquad U = \int_0^{720} T(\theta)d\theta = \sum_i T_i\theta_i \quad \text{Total energy}$$

$$U = 200\times180 - 30\times90 + 30\times180 - 40\times180 = 31500\ \text{Nm}^\circ = 31500\times\frac{\pi}{180} = 549.5\ \text{Nm (J)}$$

$$b)\qquad T_m = \frac{\sum_i T_i\theta_i}{\sum_i \theta_i} = \frac{31500}{720} = \frac{549.5}{4\pi} = 43.75\ \text{Nm}$$

$c)$ Maximum speed change will be between 0° and 180°

$$U_2 - U_1 = (200 - 43.75)(180°) = 28125 \text{ Nm°} = 28125\frac{\pi}{180} = 490.6 \text{ Nm}$$

$$I = \frac{U_2 - U_1}{C_s \omega^2} = \frac{490.6}{0.1 \times 250^2} = 0.0785 \quad \text{kgm}^2$$

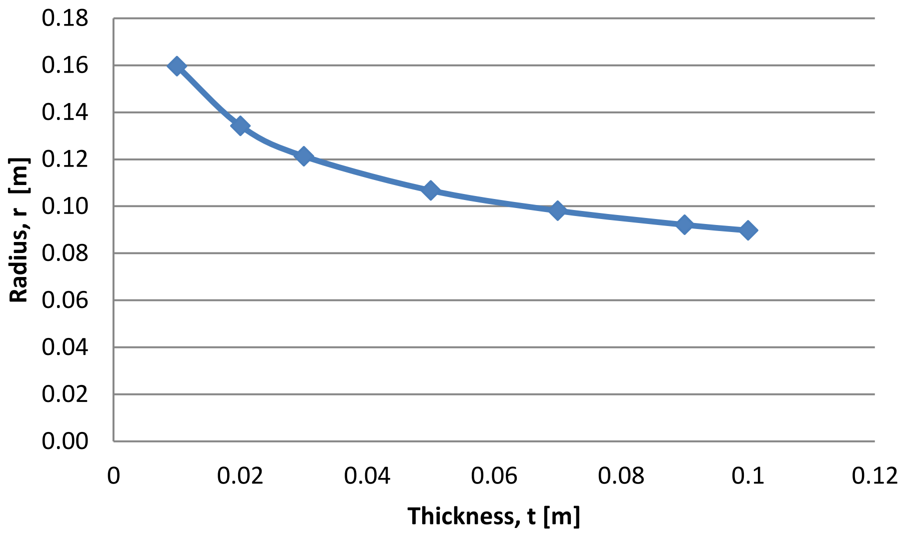
$$I = \frac{1}{2}mr^2 = \frac{1}{2}(\rho \pi r^2 t)r^2 = \frac{1}{2}\rho \pi t r^4 = 0.0785$$

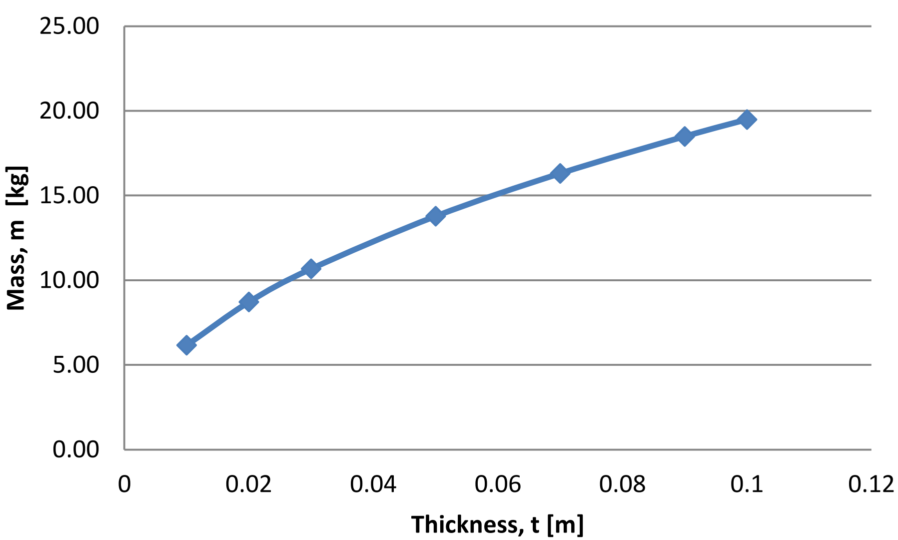
$$tr^4 = 6.4932 \times 10^{-6}$$

$$\sigma_t = \rho r_m^2 \omega^2$$

$$\omega = \frac{1}{r}\sqrt{\frac{\sigma_t}{\rho}} = \frac{1}{r}\sqrt{\frac{200MPa}{7700kg/m^3}} \times \frac{60}{2\pi} = 1539.79\frac{1}{r} \text{ rpm}$$

t [m]	r [m]	m [kg]	max speed [rpm]
0.01	0.16	6.16	9645.88
0.02	0.13	8.71	11470.95
0.03	0.12	10.67	12694.69
0.05	0.11	13.78	14423.95
0.07	0.10	16.30	15689.76
0.09	0.09	18.48	16707.15
0.1	0.09	19.48	17153.07





$d)$

$$\omega_2 - \omega_1 = C_s \omega$$

$$\omega_2 + \omega_1 = 2\omega$$

$$2\omega_2 = (2 + C_s)\omega$$

$$\omega_2 = \frac{(2 + C_s)}{2}\omega = \frac{(2 + 0.1)}{2} 250 = 262.5 \text{ rad/s at } \theta = 180°$$

$$2\omega_1 = (2 - C_s)\omega$$

$$\omega_1 = \frac{(2 - C_s)}{2}\omega = \frac{(2 - 0.1)}{2} 250 = 237.5 \text{ rad/s at } \theta = 0°$$

Example #2

A 2.2 kW, 960 rpm motor powers the punching press through a gear box of gear ratio 6:1. The torque needed to do the job is 600 Nm for the period of 60 degrees. Assume coefficient of speed fluctuation of 0.2, determine suitable inertia for the flywheel.

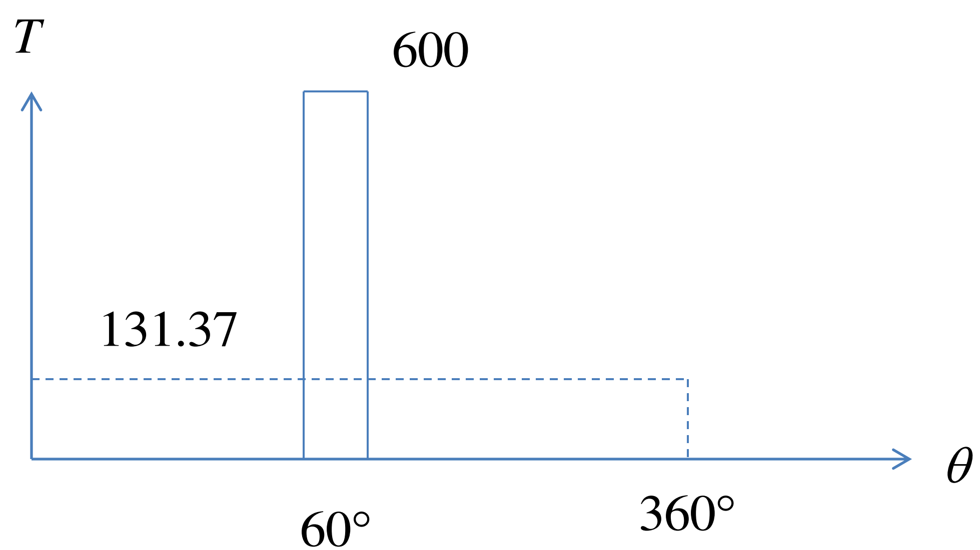
$$\omega_m = 960\frac{2\pi}{60} = 100.48\ \text{rad/s}$$

$$T_m = \frac{2200}{100.48} = 21.89\ \text{Nm} \quad (P = T_m\omega_m)$$

$$T_m\omega_m = T_f\omega_f$$

Torque after gear box

$$T_f = 21.89 \times 6 = 131.37\ \text{Nm}$$



Energy stored in the flywheel is

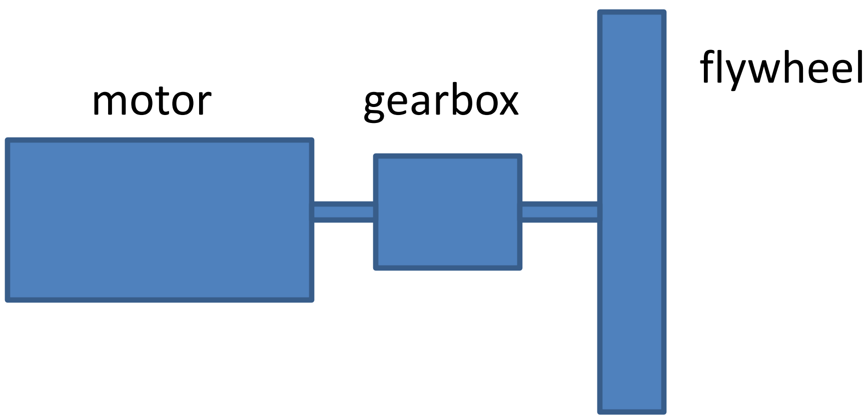
$$U_2 - U_1 = (600 - 131.37)(60\frac{\pi}{180}) = 490.5 \text{ Nm}$$

Speed after gearbox

$$\omega_f = \frac{100.48}{6} = 16.74 \text{ rad/s}$$

$$I = \frac{U_2 - U_1}{C_s \omega^2} = \frac{490.5}{0.2 \times 16.74^2} = 8.75 \quad \text{kgm}^2$$

Flywheel inertia



Example #3

A punching press machine is run by an electrical motor delivering 2.24 kW continuously. At the beginning of an operation a flywheel of inertia 60 kgm2 on the machine is rotating at the speed of 250 rpm. The pressing operation requires 5000 J of energy and occupies 0.75s.

a) Find the maximum number of pressings that can be made in one hour.
b) Find reduction in speed of the flywheel after each pressing.

Energy supplied by motor in one hour (3600s)

$$a) \qquad U_m = 2.24kW \times 3600s = 8064kJ$$

$$\text{Work required per pressing is } \mathrm{U_p} = 5kJ$$

$$\text{Number of pressing per hour is } \frac{8064\text{kJ}}{5\text{kJ}} = 1612$$

$b)$ Motor supplies energy in 0.75 s $U_m = 2240W \times 0.75s = 1680J$

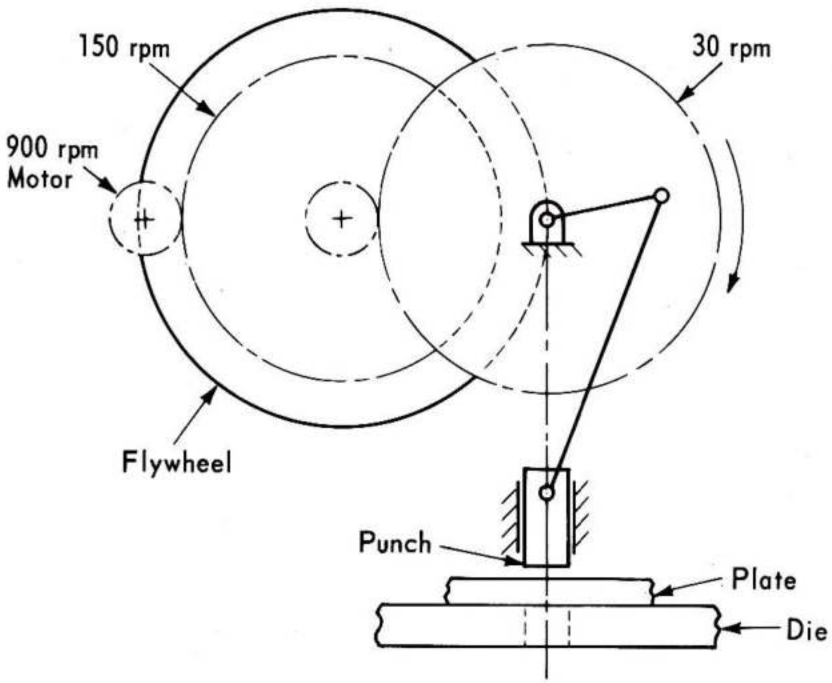
Energy taken from flywheel for each pressing is

$$U_p - U_m = 5000 - 1680 = 3320 = \frac{1}{2} I(\omega_1^2 - \omega_2^2) = \frac{1}{2} 60(250^2 - \omega_2^2)\left(\frac{2\pi}{60}\right)^2$$

$\omega_2 = 229$ rpm

$\Delta\omega = 250 - 229 = 21$ rpm

Example #4

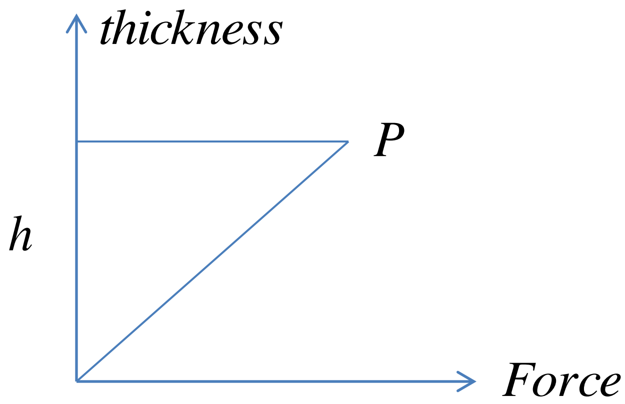


A diagram of the press is shown in the figure where the slider of a slider-crank mechanism serves as the punch to force a slug of metal in the plate through the hole in the die. The press is to punch 30 holes per minute. We will assume that the actual punching takes place 1/6 of the interval between punches. A hole 20 mm in diameter is to be punched in a plate of AISI 1080 steel 12 mm in thickness. Driving motor has 2.5 KW power and runs 900 rpm with a velocity reduction through gears to give the 30 punching operations per minute. Assume speed fluctuation coefficient as 0.03, design a suitable flywheel for the press.

Maximum force to shear the metal

$$P = \pi d h \tau = \pi \times 0.020m \times 0.012m \times 300 \times 10^6\,Pa = 226080\text{ N}$$

$$\tau = 300\text{ MPa for AISI 1080 steel}$$



Work needed in punching the hole

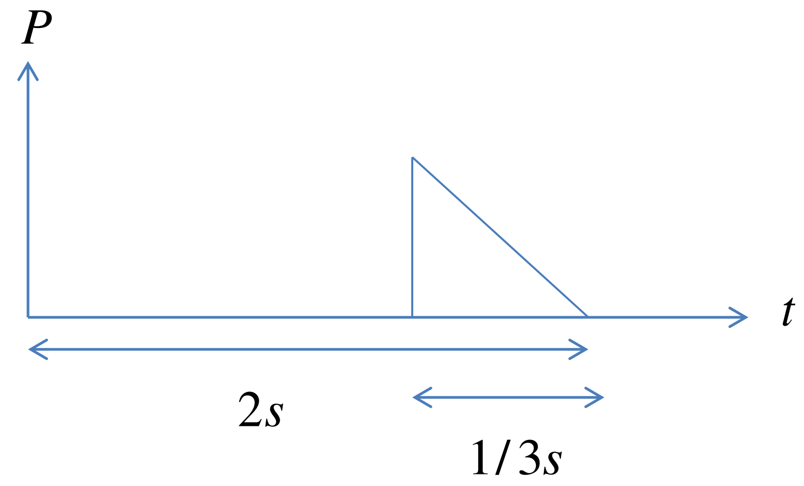
$$W_p = \frac{1}{2}Ph = \frac{1}{2} \times 226080\text{N} \times 0.012\text{m} = 1356.5\text{Nm}$$

Time for one cycle

$$\frac{60s}{30\,\text{punch/min}} = 2s$$

Time for punching operation

$$t = 2s \times \frac{1}{6} = \frac{1}{3}s$$



electric motor has 2.5kW power.

This motor will provide energy in 1/3 second

$$W_m = 2500\text{W} \times \frac{1}{3} s = 833.33\text{J}$$

Flywheel should store energy of

$$U_f = 1356.5 - 833.33 = 523.17\text{J}$$

Flywheel speed is

$$\omega_f = 30 \times \tfrac{2\pi}{60} = 3.14\text{rad/s}$$

coefficient of fluctuation of 0.03 then flywheel inertia is

$$I = \frac{U_f}{C_s \omega_f^2} = \frac{523.17}{0.03 \times 3.14^2} = 1768.73\text{kgm}^2$$

Assume cast iron rim type flywheel with thickness of 25 cm

$$I = m{r_m}^2 = \rho \times \pi \times {r_m}^2 \times w \times {r_m}^2 = \rho \pi w {r_m}^4$$

$$1768.73 = 7150\text{kg/m}^3 \times \pi \times 0.25\text{m} \times r_m{}^4$$

$$r_m = 0.75\text{m} \approx 75\text{cm}$$

The tensile stress in the rim of the flywheel because of the rotation

$$\sigma_t = \rho r_m{}^2 \omega^2 = 7150\text{kg/m}^3 \times (0.75\text{m})^2 \times (3.14\text{rad/s})^2 = 39654\text{Pa} = 0.04\text{MPa}$$

$$\sigma_t = 275\,\text{MPa} \text{ for cast iron}$$

$$0.04\text{MPa} << 275\text{MPa} \quad \text{safe}$$

Example #5

The crankshaft torque of a multi-cylinder engine is given by

$$T_m = 60 + 8\sin 3\theta \text{ Nm}$$

Where θ is the crankshaft angle measured from a convenient datum. The engine is coupled directly to a machine which requires a torque given by

$$T_L = 60 + 32\sin\theta \text{ Nm}$$

All the components rotating with the crankshaft together have the mass of 50 kg and radius of gyration of 0.15 m.
If the crankshaft speed is 300 rpm at $\theta = 0°$, find the speed when $\theta = 60°$.

$$\int_0^{60} (T_m - T_L)d\theta = \frac{1}{2} I(\omega_2^2 - \omega_1^2)$$

$$I = mk^2 = 50 \times 0.15^2 = 7.5 \text{ kgm}^2$$

$$\int_0^{60} [60 + 8\sin 3\theta - (60 + 32\sin\theta)]d\theta = \frac{1}{2} 7.5(\omega_2^2 - 300^2)\left(\frac{2\pi}{60}\right)^2$$

$$\int_0^{60} [8\sin 3\theta - 32\sin\theta]d\theta = \frac{1}{2}7.5(\omega_2^2 - 300^2)\left(\frac{2\pi}{60}\right)^2$$

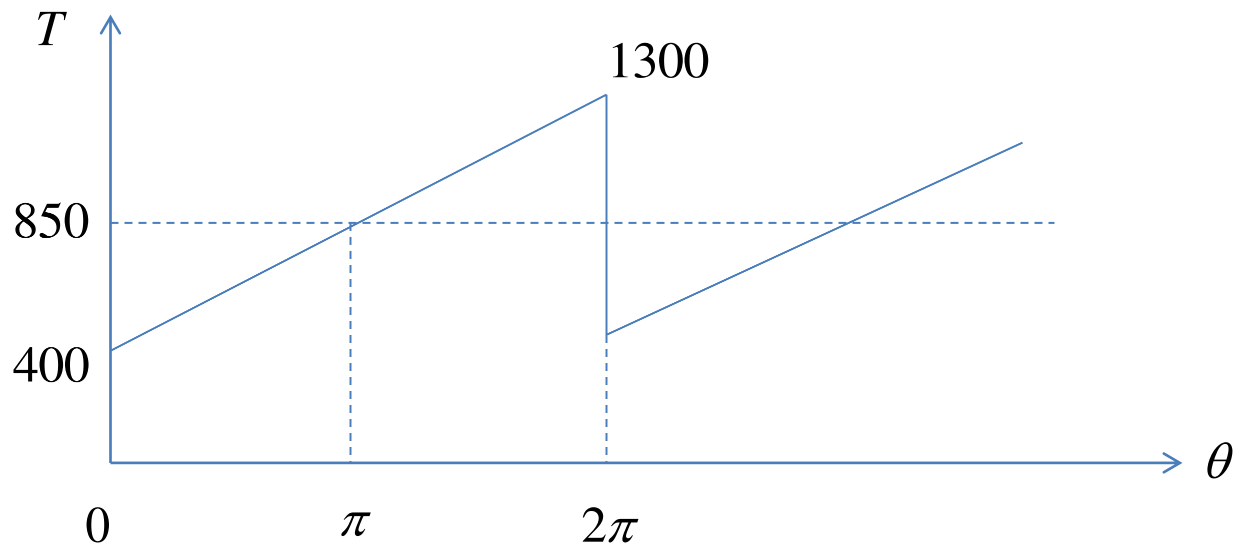
$$\left| 8(-\frac{1}{3})\cos 3\theta + 32\cos\theta \right|_0^{60} = \frac{1}{2}7.5(\omega_2^2 - 300^2)\left(\frac{2\pi}{60}\right)^2$$

$$18.66 - 29.33 = -10.67 = 6.16\times10^{-3}(\omega_2^2 - 300^2)$$

$$\omega_2 = 297\ \text{rpm}$$

Example #6

A machine is driven by a motor which exerts a constant torque of 850 Nm. The resisting torque (load torque) within the machine increases uniformly from 400 Nm to 1300 Nm throughout 360 deg Rotation of the driving shaft then drops suddenly to 400 Nm again at the beginning of the next revolution. The mean speed of the machine is 25 rad/s. The machine is fitted with a cast iron flywheel of 5 kgm2 inertia. Calculate the maximum and the minimum angular velocities of the machine.



between 0 and π load energy is less than motor energy, flywheel speed is increased

$$U_m - U_L = 850 \times \pi - \frac{1}{2}(400 + 850) \times \pi = 225\pi \text{ J}$$

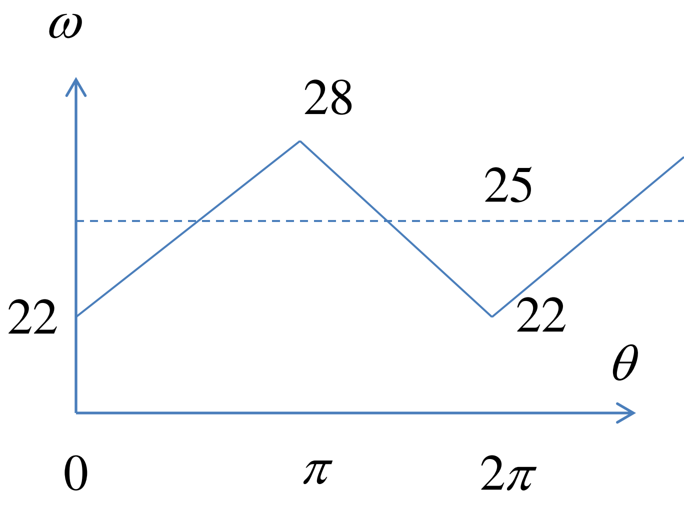
between π and 2π load energy is more than motor energy, flywheel speed is decreased

$$U_L - U_m = \frac{1}{2}(850 + 1300) \times \pi - 850 \times \pi = 225\pi \text{ J}$$

$$C_s = \frac{U_L - U_m}{I\omega^2} = \frac{225\pi}{5\times 25^2} = 0.226$$

$$\omega_2 = \frac{(2+C_s)}{2}\omega = \frac{(2+0.23)}{2}25 = 27.87 \ \text{rad/s} \ \text{at} \ \theta = \pi$$

$$\omega_1 = \frac{(2-C_s)}{2}\omega = \frac{(2-0.23)}{2}25 = 22 \ \text{rad/s} \ \text{at} \ \theta = 2\pi$$



Find the speed function

$$T_m - T_L = I\alpha$$

$$T_L = \frac{900}{2\pi}\theta + 400 \text{ Nm} \qquad T_m = 850 \text{ Nm} \qquad I = 5 \text{ kgm}^2$$

$$850 - (\frac{900}{2\pi}\theta + 400) = 5\alpha$$

$$\alpha = 90 - \frac{180}{2\pi}\theta \text{ rad/s}^2$$

$$\frac{d\omega}{dt} = \alpha \qquad \frac{d\theta}{dt} = \omega \qquad \omega d\omega = \alpha d\theta$$

$$\int_{\omega_0}^{\omega} \omega d\omega = \int_0^{\theta} \left(90 - \frac{180}{2\pi}\theta \right) d\theta$$

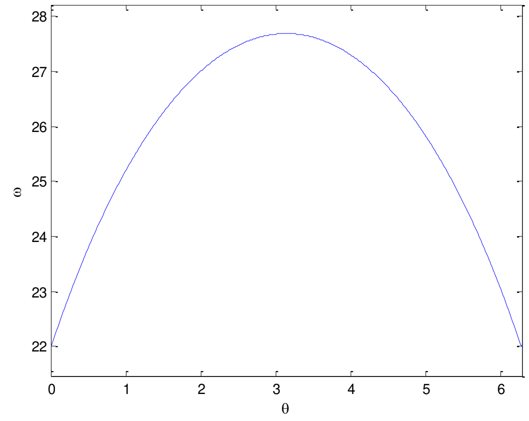
$$\frac{1}{2}(\omega^2 - \omega_0^2) = 90\theta - \frac{180}{2\pi}\frac{\theta^2}{2}$$

$$\omega^2 = \omega_0^2 + 180\theta - \frac{180}{2\pi}\theta^2$$

$$\omega_0 = 22 \text{ rad/s}$$

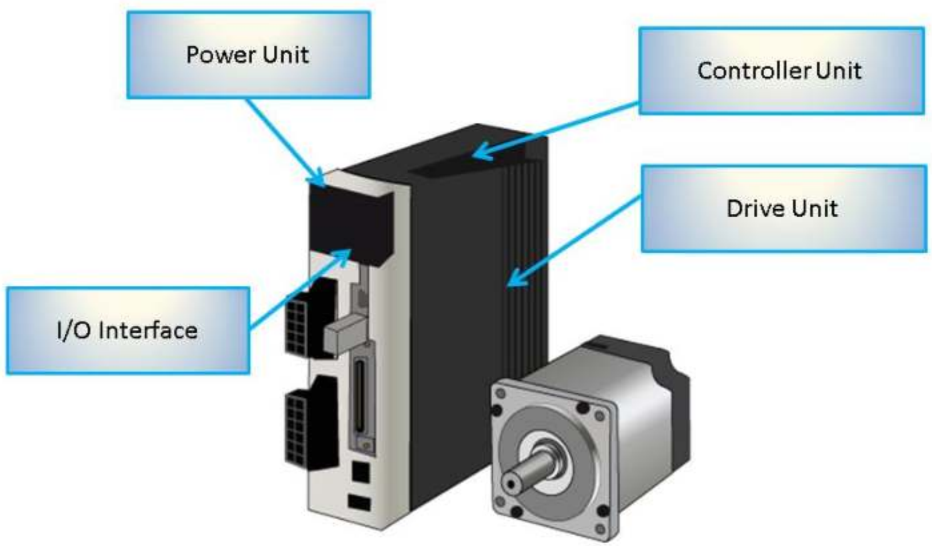
$$\omega^2 = 484 + 180\theta - \frac{180}{2\pi}\theta^2$$

$$\omega = \sqrt{484 + 180\theta - \frac{180}{2\pi}\theta^2}$$

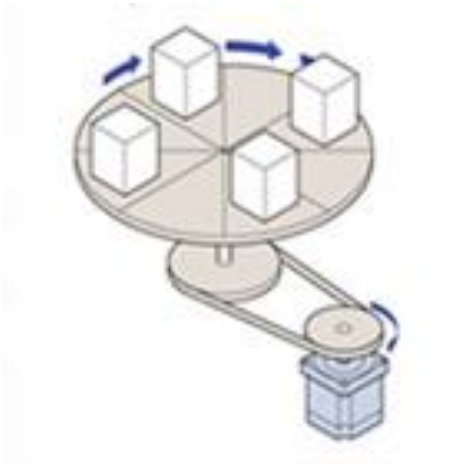


SERVOMOTORS AND SERVO SYSTEMS

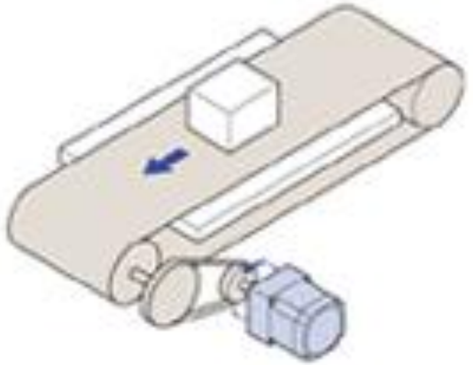
Servomotor



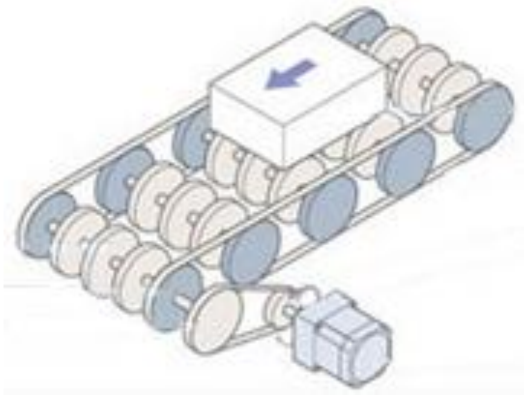
Application examples



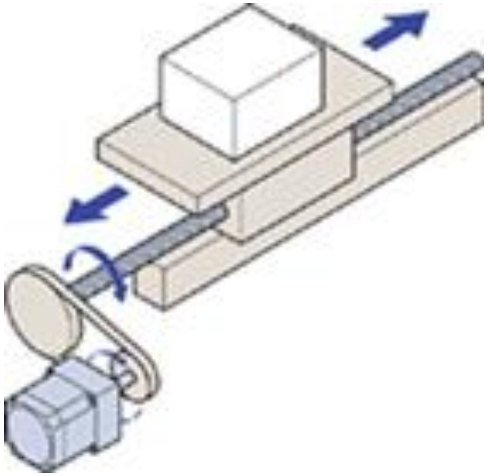
Index table



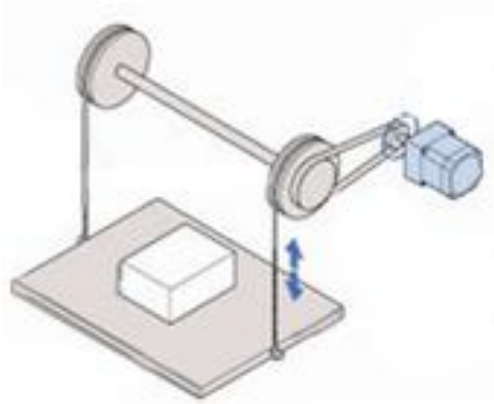
Belt conveyor



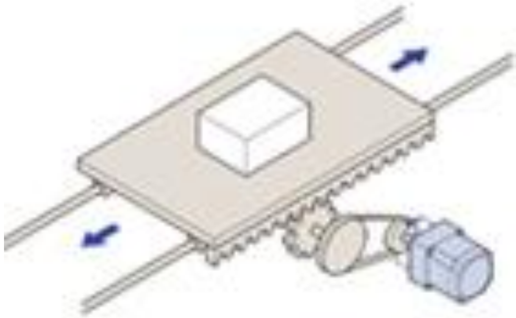
Roller conveyor



Ball-lead screw

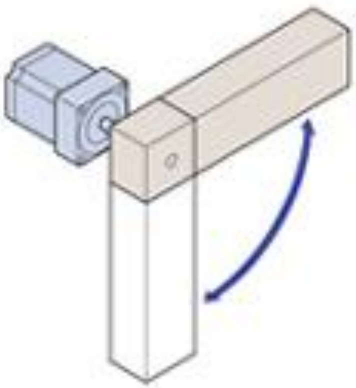


lifter

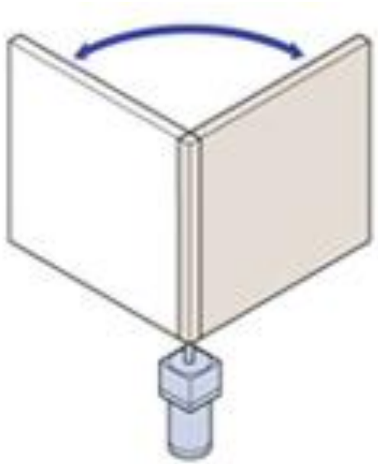


Rack and pinion

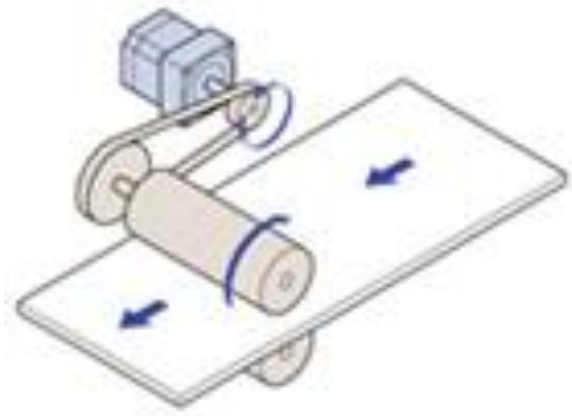
Rotary device



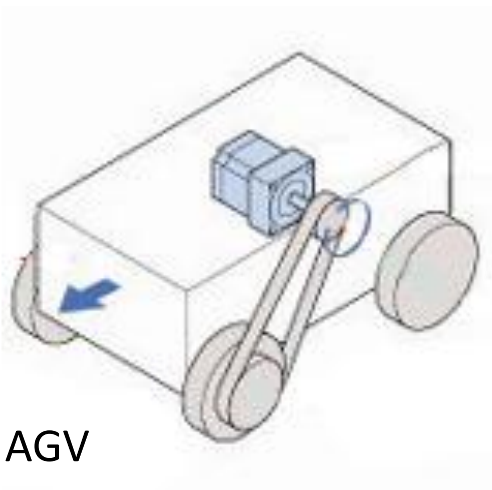
arm



Door or gate



Roller feeder



AGV

SERVOMOTOR SELECTION

TRAPEZOIDAL MOTION PROFILE

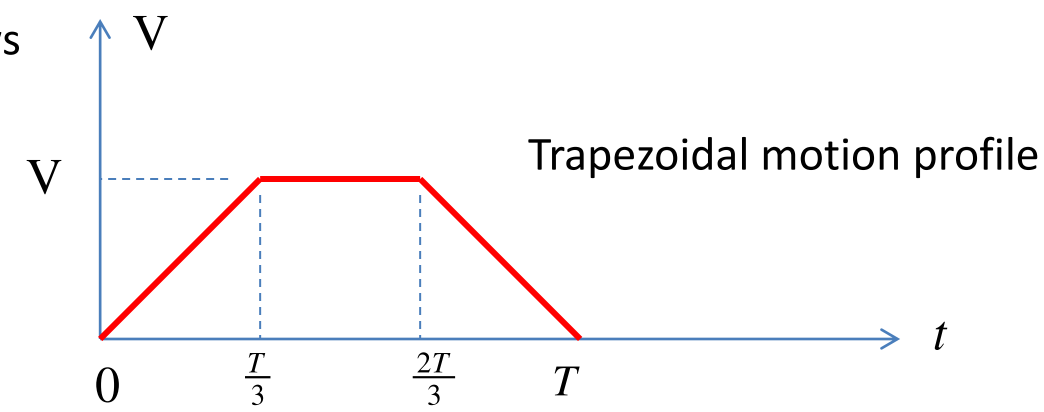
Before calculating torque requirements of an application you need to know the velocities and accelerations needed. For those positioning applications where only a distance X and time T to move that distance is known, the trapezoidal motion profile is a good starting point for determining the requirements.

Move distance X in time T

Assume that

1. Distance X/4 is moved in time T/3 (acceleration)
2. Distance X/2 is moved in time T/3 (run)
3. Distance X/4 is moved in time T/3 (deceleration)

The graph would appear as follows



For a constant acceleration motion

$$V = at \quad S = \tfrac{1}{2} a t^2$$

$$a = -d = \frac{2S}{t^2} = \frac{2\left(\frac{X}{4}\right)}{\left(\frac{T}{3}\right)^2} = \frac{18}{4}\frac{X}{T^2} = 4.5\frac{X}{T^2}$$

$$V = at = \frac{18}{4}\frac{X}{T^2}\left(\frac{T}{3}\right) = \frac{18}{12}\frac{X}{T} = 1.5\frac{X}{T}$$

Example

You need to move 15 cm in 2 seconds with trapezoidal motion profile, what should be the acceleration and velocity.

$$a = -d = 4.5\frac{X}{T^2} = 4.5\frac{15cm}{(2s)^2} = 16.87 cm/s^2$$

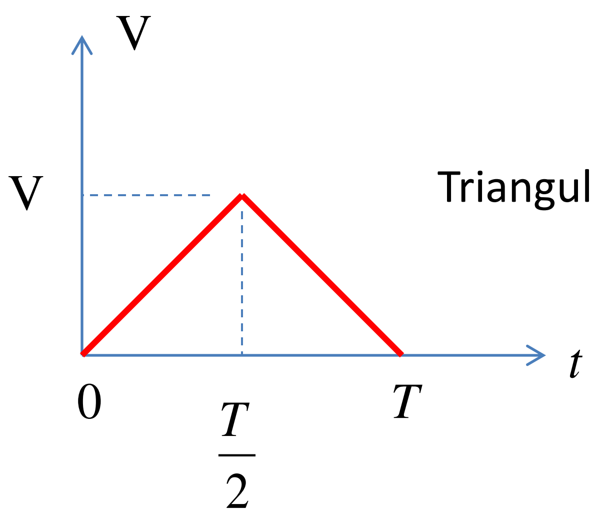
$$V = at = 1.5\frac{X}{T} = 1.5\frac{15cm}{2s} = 11.25 cm/s$$

TRIANGULAR MOTION PROFILE

Move distance X in time T

Assume that
1. Distance X/2 is moved in time T/2 (acceleration)
2. Distance X/2 is moved in time T/2 (deceleration)

The graph would appear as follows



Triangular motion profile

For constant acceleration motion

$$V = at \quad S = \tfrac{1}{2}at^2$$

$$a = -d = \frac{2S}{t^2} = \frac{2\left(\frac{X}{2}\right)}{\left(\frac{T}{2}\right)^2} = 4\frac{X}{T^2}$$

$$V = at = 4\frac{X}{T^2}\left(\frac{T}{2}\right) = 2\frac{X}{T}$$

Example

You need to move 15 cm in 2 seconds with triangular motion profile, what should be the acceleration and velocity.

$$a = -d = 4\frac{X}{T^2} = 4\frac{15cm}{(2s)^2} = 15cm/s^2$$

$$V = at = 2\frac{X}{T} = 2\frac{15cm}{2s} = 15cm/s$$

Motor selection based on continuous torque requirements:

Having calculated the torque requirements for an application, you can select the motor suited to your needs. Motors have speed-torque curves based on continuous duty cycle. To chose a motor simply plot total torque vs. velocity on the speed-torque curve. This point should fall under the curve and allow a 50% margin for safety.

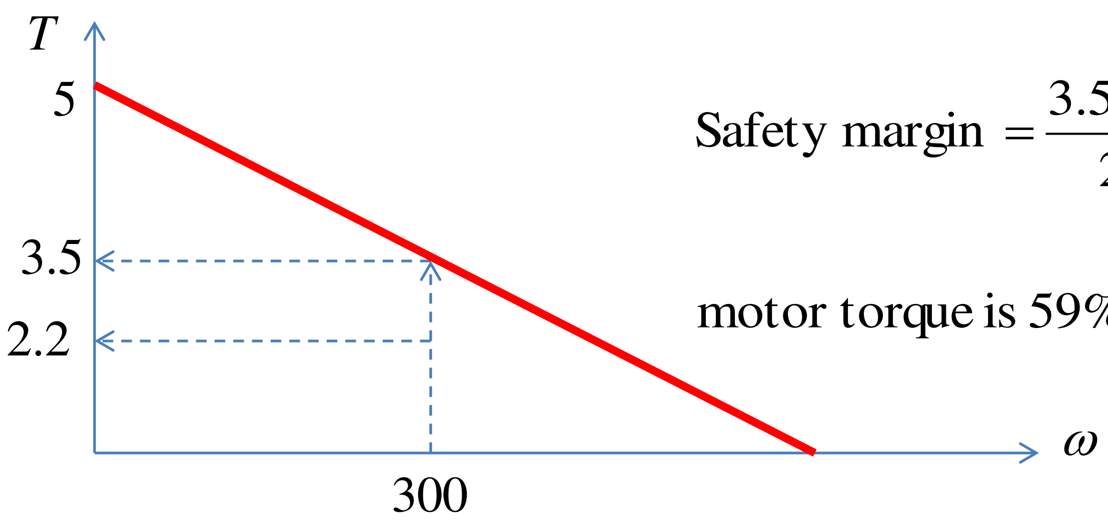
Example

T_F = 0.2 Nm Friction torque

T_A = 2 Nm Acceleration torque

T_T = 2.2 Nm total torque

ω = 300 rpm maximum velocity



$$\text{Safety margin} = \frac{3.5 - 2.2}{2.2} = 0.59$$

motor torque is 59% more than the required torque

Motor selection based on peak torque requirements:

Servomotors have two speed-torque curves. One for continuous duty cycle (continuous torque curve)and another for intermittent duty cycle (peak torque curve). A servomotor can be selected according to the total torque and maximum velocity indicated by the continuous torque curve. However, by calculating the root mean square (rms) torque based on the duty cycle, you may be able to take advantage of the higher peak torque available in the peak torque range.

$$T_{rms} = \sqrt{\frac{\sum T_i^2 t_i}{\sum t_i}}$$

Example

Assume the following results from load calculations

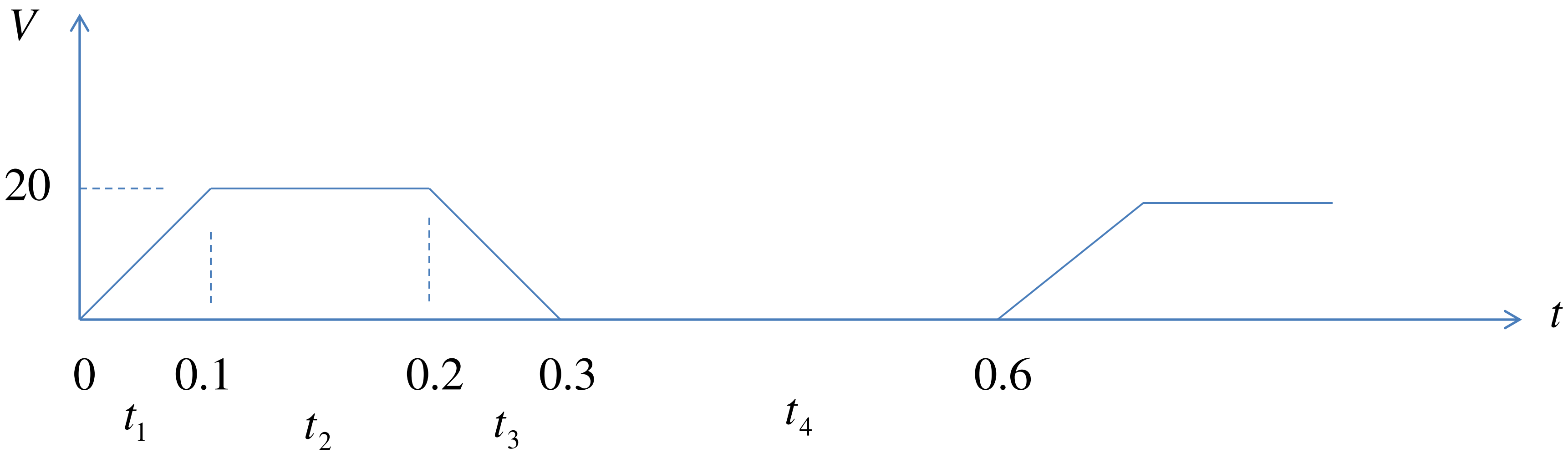
T_F=0.2 Nm Friction torque

T_A=5.5 Nm Acceleration torque

T_T=5.7 Nm Total torque

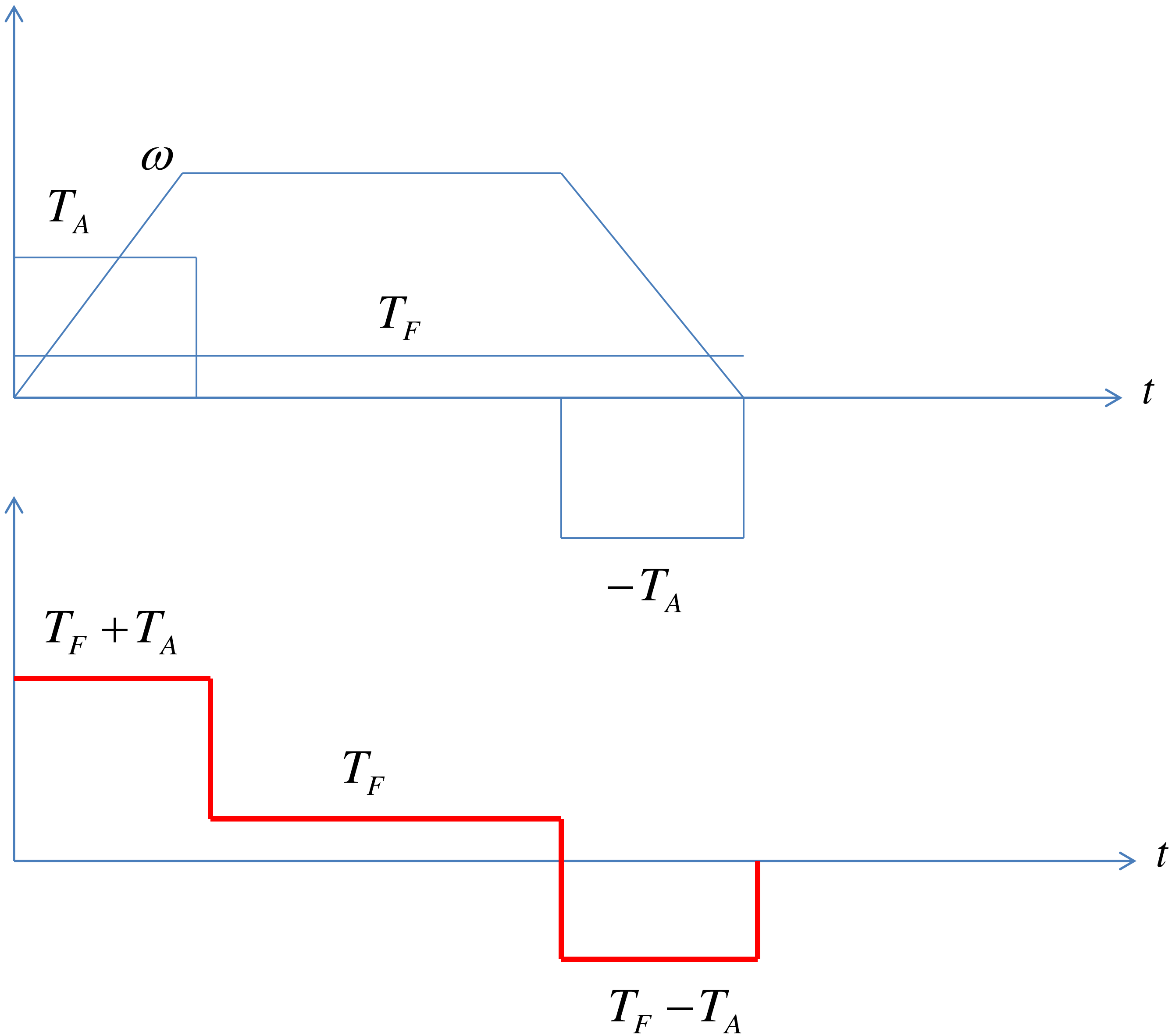
ω_M=1200 rpm Maximum velocity

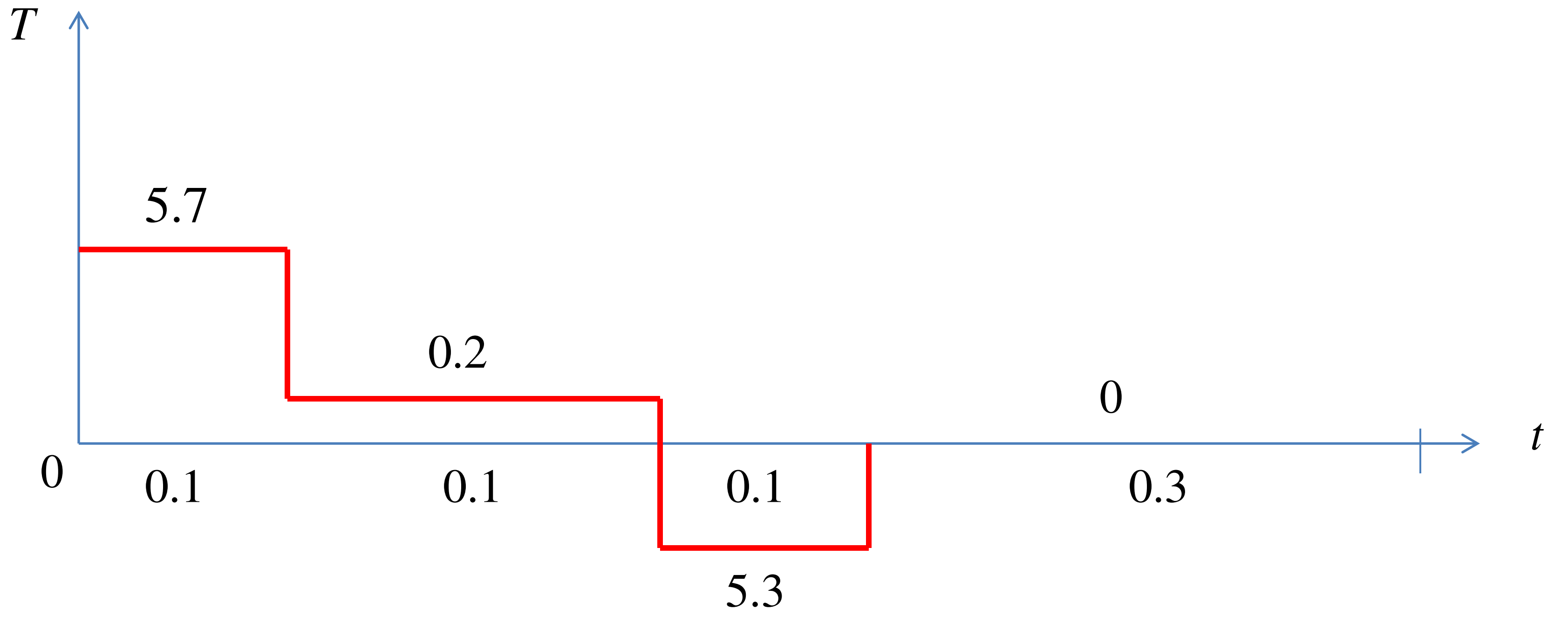
Motion profile



Duty cycle is to rotate 4 revolutions in 0.3 seconds and dwell 0.3 seconds then repeat

t_1= Torque required to accelerate the load from zero speed to maximum speed ,$T_1=|T_F+T_A|$
t_2= Torque required to keep the motor moving ($T_2=T_F$)
t_3= Torque required to decelerate the load from maximum speed to zero speed ,$T_3=|T_F-T_A|$
t_4= Torque required while motor is sitting still at zero speed ($T_4=0$)


ω
T_A
T_F
t
$-T_A$
$T_F + T_A$
T_F
t
$T_F - T_A$



$$T_1 = |T_F + T_A| = 0.2 + 5.5 = 5.7\,Nm$$

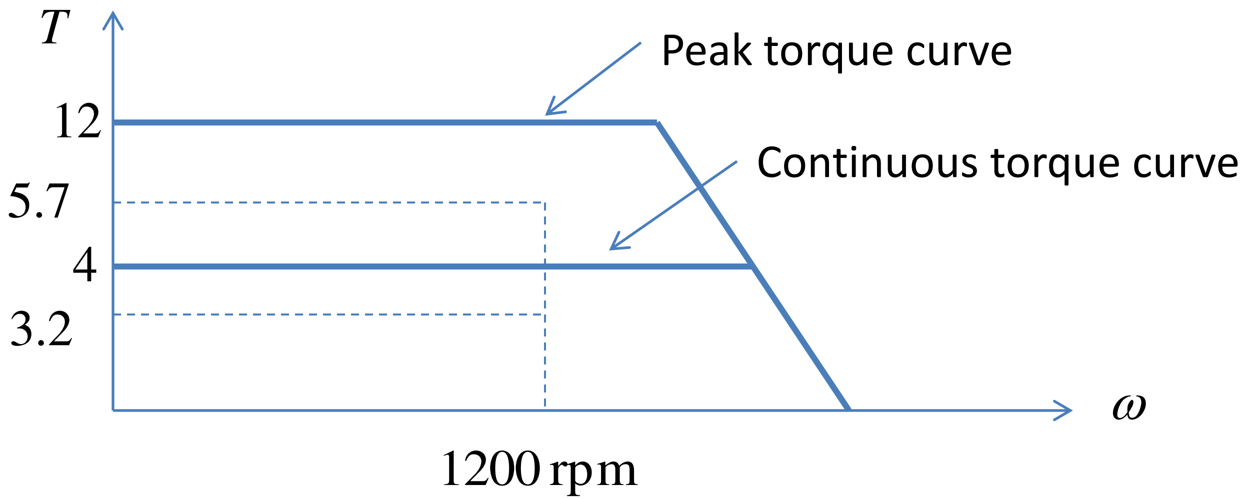
$$T_2 = T_F = 0.2\,Nm$$

$$T_3 = |T_F - T_A| = 0.2 - 5.5 = 5.3\,Nm$$

$$T_4 = 0\,Nm$$

$$T_{rms} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3 + T_4^2 t_4}{t_1 + t_2 + t_3 + t_4}}$$

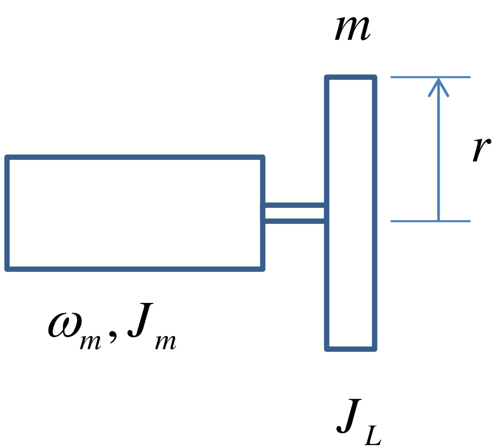
$$T_{rms} = \sqrt{\frac{5.7^2 \times 0.1 + 0.2^2 \times 0.1 + 5.3^2 \times 0.1 + 0^2 \times 0.3}{0.1 + 0.1 + 0.1 + 0.3}} = 3.2Nm$$



This motor meets the requirements. rms torque value falls within the continuous torque curve, maximum torque value falls within the peak torque curve.

Motor torque calculations for some mechanical Systems

Direct Drive:



$$KE_e = \sum_i KE_i$$

$$\frac{1}{2} J_{eq} \omega_m^2 = \frac{1}{2} J_m \omega_m^2 + \frac{1}{2} J_L \omega_m^2$$

$$J_{eq} = J_m + J_L$$

$$T_A = J_{eq} \alpha = J_{eq} \frac{\omega_m}{t}$$

$$J_L = \frac{1}{2} m r^2$$

J_{eq} is the equvalent mass moment of inertia of the mechanical system, kgm^2

J_m is the motor inertia, kgm^2

J_L is the load inertia, kgm^2

Example

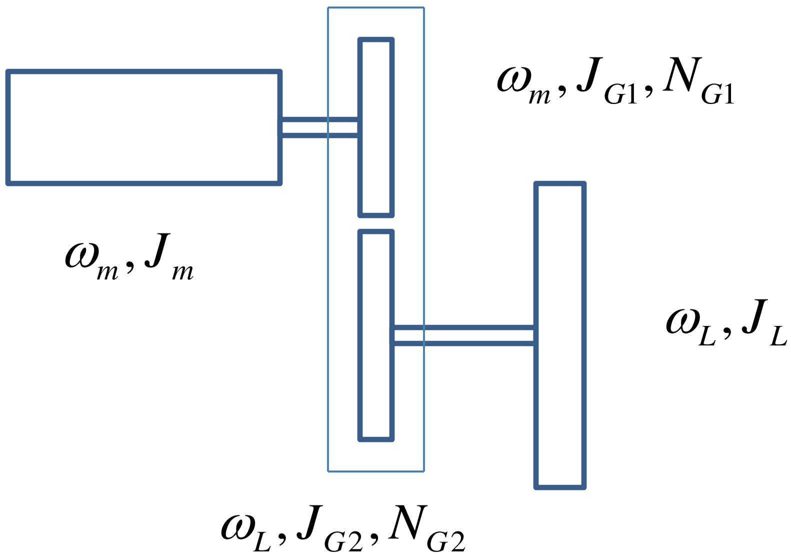
Calculate the motor torque required to accelerate a solid cylinder of aluminum 13 cm in radius and 0.8 kg in mass from rest to 2 rad/s in 0.25 seconds. Motor inertia is $70\times10^{-5}\ \mathrm{kgm}^2$

$$J_L = \frac{1}{2}mr^2 = \frac{1}{2}(0.8kg)(0.13m)^2 = 676\times10^{-5}\,kgm^2$$

$$J_{eq} = J_m + J_L = 70\times10^{-5} + 676\times10^{-5} = 746\times10^{-5}\,\mathrm{kgm}^2$$

$$T_A = J_{eq}\times\alpha = 746\times10^{-5}\times\frac{2rad/s}{0.25s} = 59.68\times10^{-3}\,Nm$$

Gear drive formulas:



J_{G1} is the inertia of first gear
J_{G2} is the inertia of second gear
N_{G1} is the teeth number of first gear
N_{G2} is the teeth number of second gear

Equivalent inertia with respect to the motor shaft

$$\frac{1}{2}J_{eq}\omega_m^2 = \frac{1}{2}J_m\omega_m^2 + \frac{1}{2}J_{G1}\omega_m^2 + \frac{1}{2}J_{G2}\omega_L^2 + \frac{1}{2}J_L\omega_L^2$$

$$J_{eq} = J_m + J_{G1} + (J_{G2} + J_L)\frac{\omega_L^2}{\omega_m^2}$$

$$T_A = J_{eq}\alpha$$

$$\text{Speed ratio} = \frac{\omega_L}{\omega_m}$$

$$\omega_m N_{G1} = \omega_L N_{G2}$$

$$\frac{\omega_L}{\omega_m} = \frac{N_{G1}}{N_{G2}}$$

$$\text{Gear ratio} = \frac{N_{G2}}{N_{G1}} = \frac{1}{\text{speed ratio}}$$

Example

Motor is coupled to a gearbox and flywheel. The inertias are given. If the motor has starting torque of 5 Nm, what is the angular acceleration of the flywheel? How long it will take to reach the speed of 1200 rpm.

$$J_m = 9.8 \times 10^{-3}\, kgm^2$$

$$J_{G1} = 0.4 \times 10^{-3}\, kgm^2$$

$$J_{G2} = 19.4 \times 10^{-3}\, kgm^2$$

$$J_f = 308 \times 10^{-3}\, kgm^2$$

$$N_{G1} = 50T$$

$$N_{G2} = 230T$$

$$\frac{\omega_L}{\omega_m} = \frac{N_{G1}}{N_{G2}} = \frac{50}{230}$$

$$J_{eq} = J_m + J_{G1} + (J_{G2} + J_L)\frac{\omega_L^2}{\omega_m^2}$$

$$= 9.8\times10^{-3} + 0.4\times10^{-3} + (19.4\times10^{-3} + 308\times10^{-3})\left(\frac{50}{230}\right)^2 = 0.02567 kgm^2$$

$$T_A = J_{eq}\alpha = 0.02567\alpha = 5Nm$$

$$\alpha = \frac{5Nm}{0.02567 kgm^2} = 194.78 rad/s^2$$

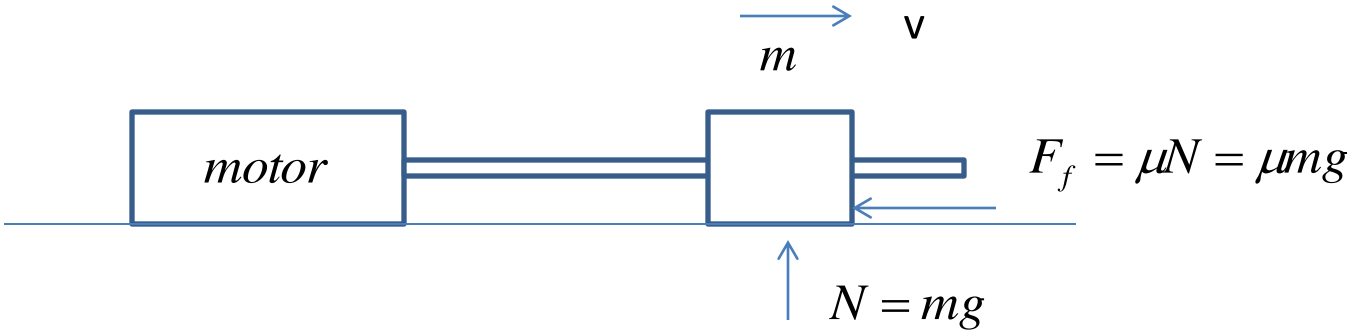
$$\omega_m = \alpha t$$

$$1200\left(\frac{2\pi}{60}\right) = 194.78t$$

$$t = 0.64s$$

Lead-screw drives

Lead-screw drives:



$$T_{motor} = T_{Friction} + T_{Acceleration}$$

$$T_f = \frac{p}{2\pi\eta} F_f = \frac{p}{2\pi\eta} \mu mg$$

F_f : Frictional force, N

p : pitch or lead, m/rev

η : leadscrew efficiency

Energy equation

$$\frac{1}{2}J_{eq}\omega_m^2 = \frac{1}{2}J_m\omega_m^2 + \frac{1}{2}J_{LS}\omega_m^2 + \frac{1}{2}mv^2$$

$$J_{eq} = J_m + J_{LS} + m\frac{v^2}{\omega_m^2} \qquad \text{equvalent inertia}$$

$$\theta_m = \frac{2\pi}{p}s \qquad \omega_m = \frac{2\pi}{p}v \qquad \alpha_m = \frac{2\pi}{p}a$$

$$J_{eq} = J_m + J_{LS} + \left(\frac{p}{2\pi}\right)^2 m$$

$$T_A = J_{eq}\alpha_m$$

Example

Find the torque required to accelerate a 90 kg load sliding on a steel table to a speed of 50 cm/s in 0.2 seconds using steel leadscrew having pitch of 2 cm/rev. Assume that leadscrew has an Acme thread and uses a plastic nut (efficiency is 65%). Friction coefficient is 0.15 (steel on steel lubricated). Leadscrew inertia is $150\times10^{-5}\ \mathrm{kgm}^2$ motor inertia is $12\times10^{-5}\ \mathrm{kgm}^2$

$$F_f = \mu mg = 0.15\times90\times9.8 = 132.3\ \mathrm{N}$$

$$T_f = \frac{p}{2\pi\eta}F_f = \frac{0.02\mathrm{m/rev}}{2\pi\times0.65}\times132.3\mathrm{N} = 0.65 Nm$$

$$J_{eq} = J_m + J_{LS} + \left(\frac{p}{2\pi}\right)^2 m = 12\times10^{-5} + 150\times10^{-5} + \left(\frac{0.02}{2\pi}\right)^2 90 = 253\times10^{-5} kgm^2$$

$$\omega_m = \frac{2\pi}{p} v = \frac{2\pi}{0.02}\times0.5 = 157 rad/s$$

$$T_A = J_{eq}\alpha_m = J_{eq}\frac{\omega_m}{t} = 253\times10^{-5}\frac{157 rad/s}{0.2s} = 1.98Nm$$

$$T_m = T_f + T_A = 0.65 + 1.98 = 2.63Nm$$

$$P = T_m\omega_m = 2.63\times157 = 413\text{ W (motor power needed)}$$

If the same load is moved 60 cm in 0.8 seconds by using a triangular motion profile, what is the required torque?

$$a = 4\frac{X}{T^2} = 4\frac{60cm}{(0.8s)^2} = 375cm/s^2$$

$$\alpha = \frac{2\pi}{p}a = \frac{2\pi}{0.02}\times 3.75 = 1177.5rad/s^2$$

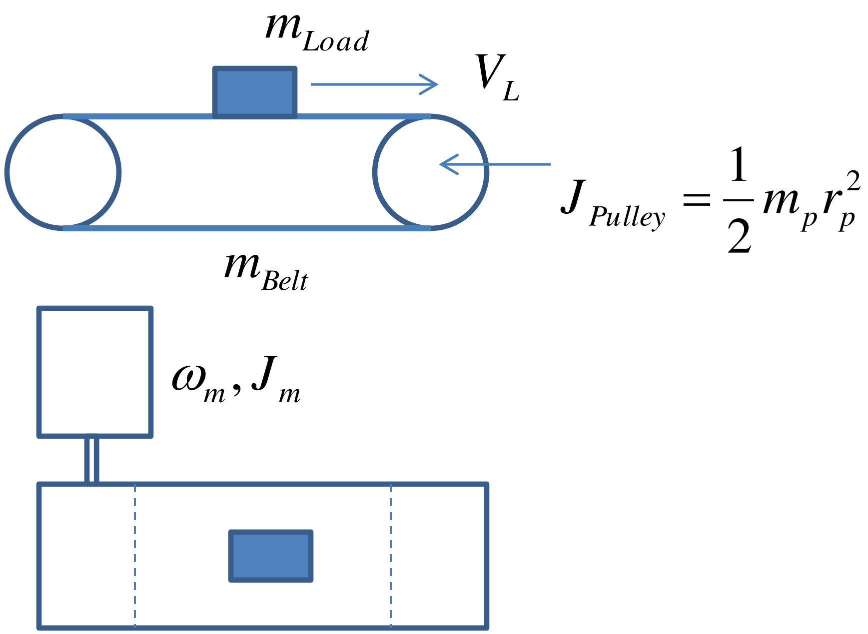
$$T_A = J_{eq}\alpha_m = 253\times 10^{-5}\times 1177.5 = 2.98Nm$$

$$T_m = T_f + T_A = 0.65 + 2.98 = 3.63Nm$$

$$P = T_m\omega_m = 3.63\times 157 = 570\text{ W (motor power needed)}$$

Belt and Pulley Drives

Belt and pulley derives:



Equivalent inertia with respect to the motor shaft

$$\frac{1}{2} J_{eq}\omega_m^2 = \frac{1}{2} J_m \omega_m^2 + 2\left(\frac{1}{2} J_P \omega_m^2\right) + \frac{1}{2} m_B V_B^2 + \frac{1}{2} m_L V_L^2$$

$$V_B = r_p \omega_m \qquad J_{eq} = J_m + 2J_P + (m_B + m_L) r_p^2 \qquad T_A = J_{eq}\alpha = J_{eq}\frac{\omega_m}{t}$$

$$V_L = r_p \omega_m$$

Example

What torque is required to accelerate a 2 kg mass load to a velocity of 50 cm/s in 10 milliseconds using a flat timing belt? The motor drives a 10 cm diameter steel pulley which has mass of 0.1 kg. Friction torque is 0.2 Nm, belt mass is 0.8 kg, motor inertia is $19\times10^{-5}\ \mathrm{kgm}^2$

$$J_{Pulley} = \frac{1}{2}m_p r_p^2 = \frac{1}{2}(0.1kg)(0.05m)^2 = 12.5\times10^{-5}kgm^2$$

$$J_{eq} = J_m + 2J_P + (m_B + m_L)r_p^2$$

$$= 19\times10^{-5} + 2\times12.5\times10^{-5} + (0.8+2)(0.05)^2 = 744\times10^{-5}kgm^2$$

$$V_L = r_p\omega_m$$

$$50\,cm/s = (5cm)\omega_m$$

$$\omega_m = 10rad/s$$

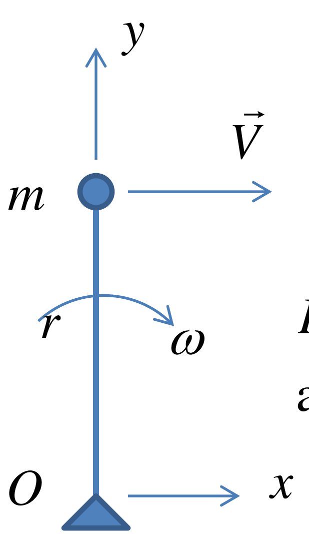
$$T_A = J_{eq}\frac{\omega_m}{t} = 744\times10^{-5}\frac{10rad/s}{10\times10^{-3}s} = 7.44Nm$$

$$T_m = T_A + T_f = 7.44 + 0.2 = 7.64Nm$$

GYROSCOPIC EFFECTS

A body rotating about an axis of symmetry offers a resistance to a change in the direction of these axis. This is known as the “gyroscopic effect”. Important applications of the gyroscopic effect are the gyrocompass used on airplanes and ships and in inertial guidance control systems for missiles and space travel, where gyroscopes are used to sense the angular motion of the body.

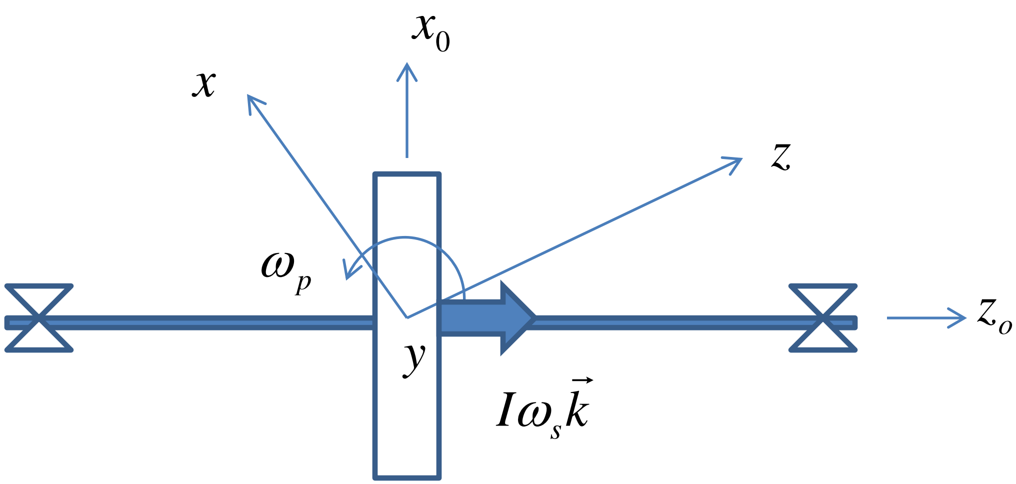
Forces due to gyroscopic effects must often be taken into account in the design of machines. These forces are encountered in the bearings of an automobile engine as the automobile engine makes a turn, in marine turbines as the ship pitches in a heavy sea, and in a jet airplane engine shaft as the airplane changes direction.



$$\vec{L} = m\vec{V} = \text{ linear momentum}$$

$$\vec{H}_o = \vec{r} \times m\vec{V} = r\vec{j} \times mV\vec{i} = -rmV\vec{k} = -rm\omega r\vec{k} = -mr^2\omega\vec{k} = -I\omega\vec{k}$$

angular momentum(moment of linear momentum)



$\vec{H} = I\omega_s\vec{k}$ I is mass moment of inertia, ω_s is spin of the disk

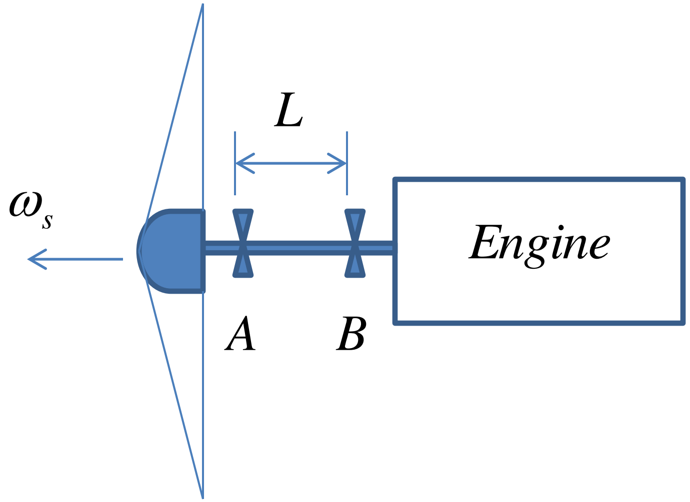
$$\vec{M} = \dot{\vec{H}} = \omega_p\vec{j} \times I\omega_s\vec{k} = I\omega_s\omega_p\vec{i}$$

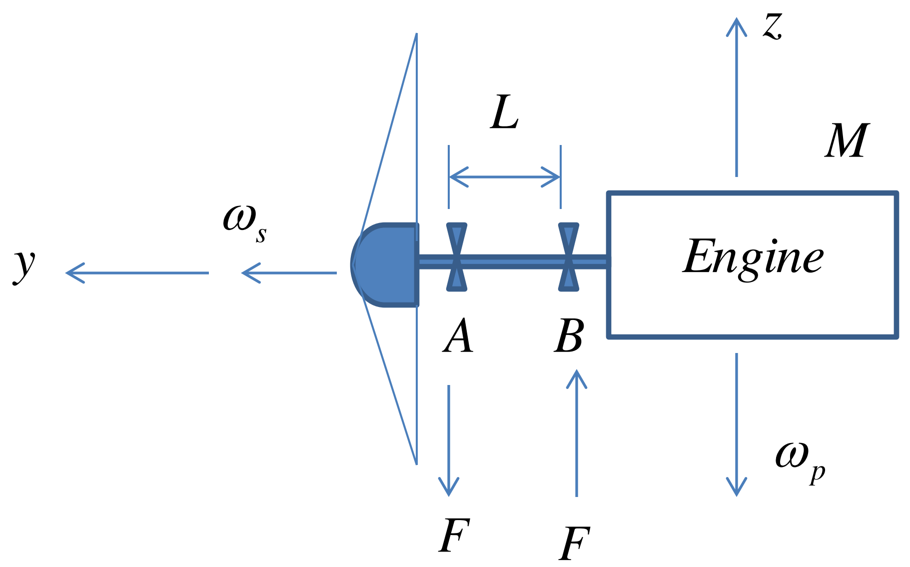
ω_p is precession of the disk, M is the gyroscopic moment

If there is a precession in the direction of y, it will create a moment M in the direction of x or if a moment is applied in the direction of x it will cause a precession in the direction of y.

Example 1

The propeller shaft of an airplane is shown. The propeller rotates at 2000 rpm cw when viewed from the rear and is driven by the engine through reduction gears. Suppose the airplane is flying horizontally and is making a turn to the right at a 0.2 rad/s when viewed from above. The propeller mass is 30 kg and radius of gyration is 1 m. It is desired to find the gyroscopic forces which the propeller shaft exerts against bearings A and B which are 15cm apart.





$$\omega_s = 2000\frac{2\pi}{60} = 209 rad/s$$

$$\omega_p = 0.2 rad/s \quad \text{precession speed}$$

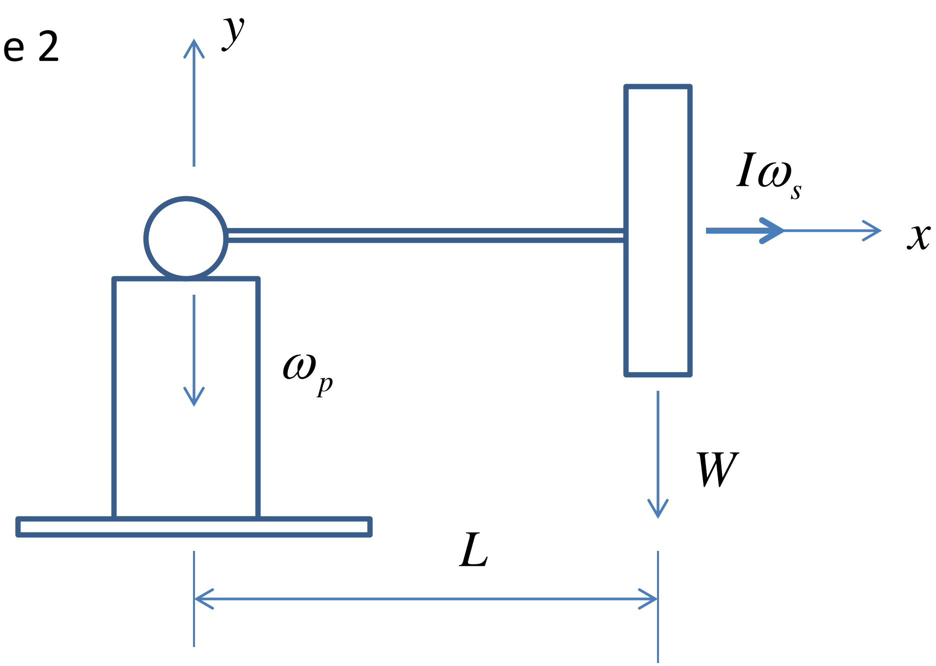
$$\vec{M} = \dot{\vec{H}} = -\omega_p \vec{k} \times I\omega_s \vec{j} = I\omega_s\omega_p \vec{i} = (30kg)(1m)^2(209\text{rad/s})(0.2\text{rad/s})\vec{i} = 1254\vec{i}\ \text{Nm}$$

$$I = mk^2$$

$$M = FL$$

$$F = \frac{M}{L} = \frac{1254Nm}{0.15m} = 8360N$$

Example 2



A toy gyroscopic top is shown. If the disk of weight W is given a high angular velocity of ω_s about its axis and one end of the shaft placed on a pedestal, the shaft and disk will not fall but will precess around the axis y because of the torque WL acting on the top.

$$\vec{M} = -\omega_p \vec{j} \times I\omega_s \vec{i} = I\omega_s\omega_p \vec{k}$$

$$\vec{M} = -WL\vec{k}$$

$$I\omega_s\omega_p = WL$$

$$\omega_p = \frac{WL}{I\omega_s} \quad \text{precession of the top}$$

If the shaft were prevented from precessing by some means such as holding one's finger in its path it would immediately fall. Friction on the pedestal dissipates some of the energy and the top gradually drops lower. Also friction between the disk and its shaft causes a gradual decrease in ω_s and because of this the angular velocity of precession increases.

MOTORS AND DRIVERS

- Unless manually operated, a mechanism will require some type of driver device to provide the input motion and energy. There are many possibilities. If the design requires a continuous rotary input motion, such as for a four-bar linkage, a slider-crank, or a cam follower, then a motor or engine is the logical choice. Motors come in a wide variety of types. The most common energy source for a motor is electricity, but compressed air and pressurized hydraulic fluid are also used to power air and hydraulic motors. Gasoline or diesel engines are another possibility. If the input motion is translation, as is common in earth-moving equipment, then a hydraulic or pneumatic cylinder is usually needed.

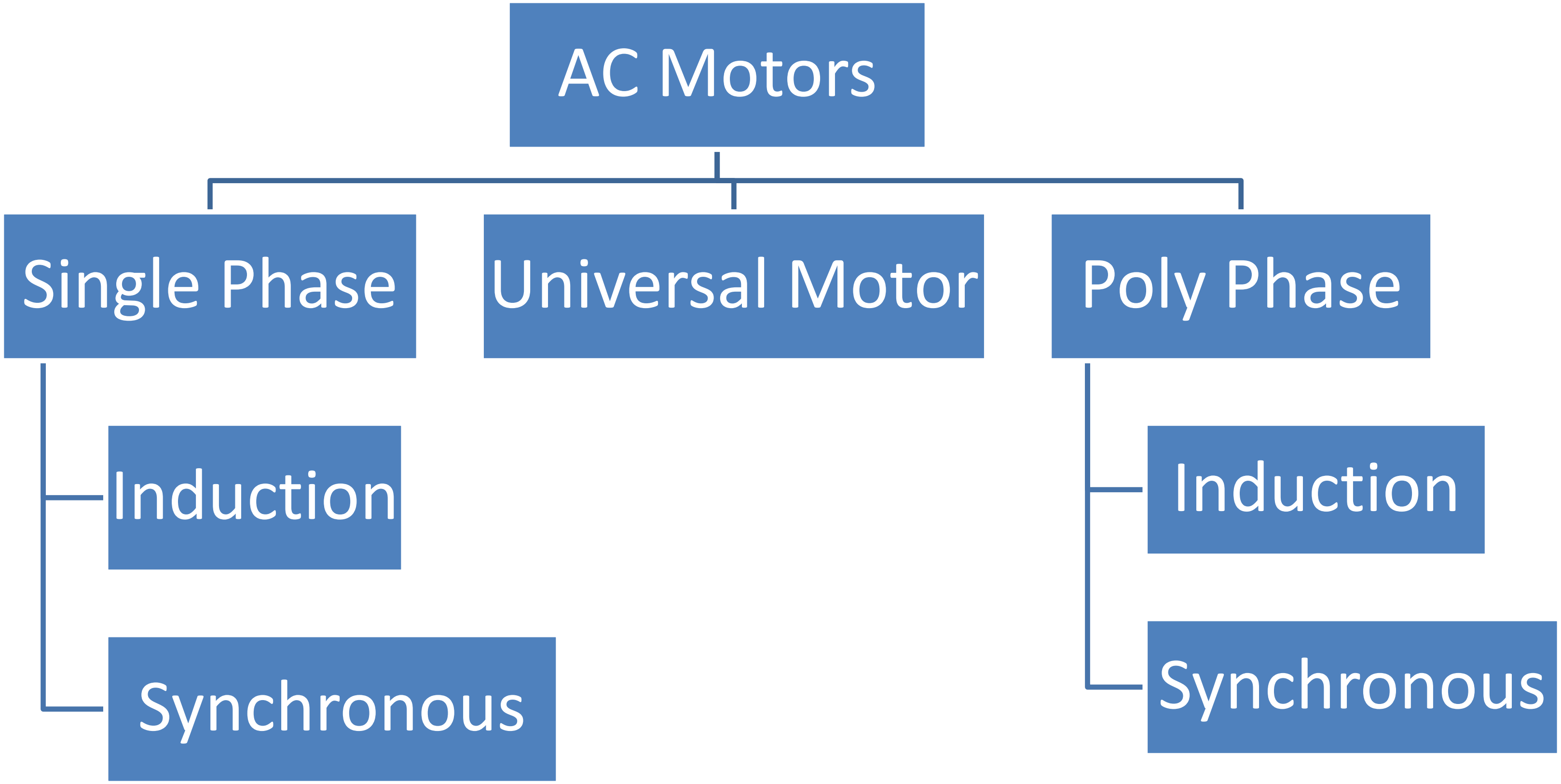
- Single Phase Induction
  - Squirrel Cage
    - Split Phase
    - Shaded Pole
    - Capacitor Start
    - Two-Value Capacitor
    - Permanent Split Capacitor
  - Wound Rotor
    - Repulsion
    - Repulsion Start
    - Repulsion Induction
- Single Phase Synchronous
  - Shaded Pole
  - Hysteresis
  - Reluctance
  - Permanent Magnet
- Poly phase Induction
  - Wound rotor
  - Squirrel cage

Electric Motors

Electric motors are classified both by their function or application and by their electrical configuration. Some functional classifications are Gear motors, Servomotors, and Stepping motors. The main electrical configuration division is between AC and DC motors, though one type, the universal motor, is designed to run on either AC or DC.

- AC and DC refer to alternating current and direct current respectively. AC is typically supplied by the power companies and in the United States will be alternating at 60 Hz at about 120, 240 or 480 Volts. Many other countries supply AC at 50 Hz. Single-Phase AC provides a single sinusoid varying with time, and 3-phase AC provides three sinusoids at 120 degree phase angles.

- DC is constant with time, supplied from generators or battery sources and is most often used vehicles such as ships, automobiles, aircraft, etc. Batteries are made in multiple of 1.5 V, with 6, 12, and 24 V being the most common. Electric motors are also classed by their rated power ;
- Subfractional Class <1/20 hp
- Fractional Class 1/20-1 hp
- Integral Class >1 hp

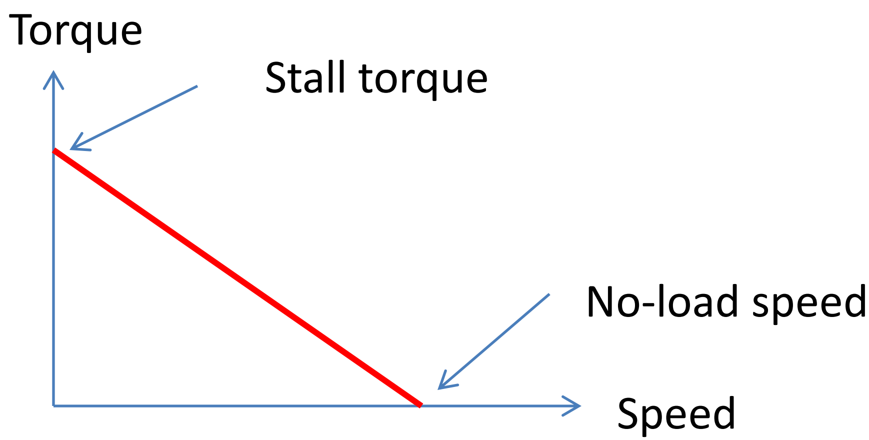

AC Motors
Single Phase
Universal Motor
Poly Phase
Induction
Synchronous
Induction
Synchronous

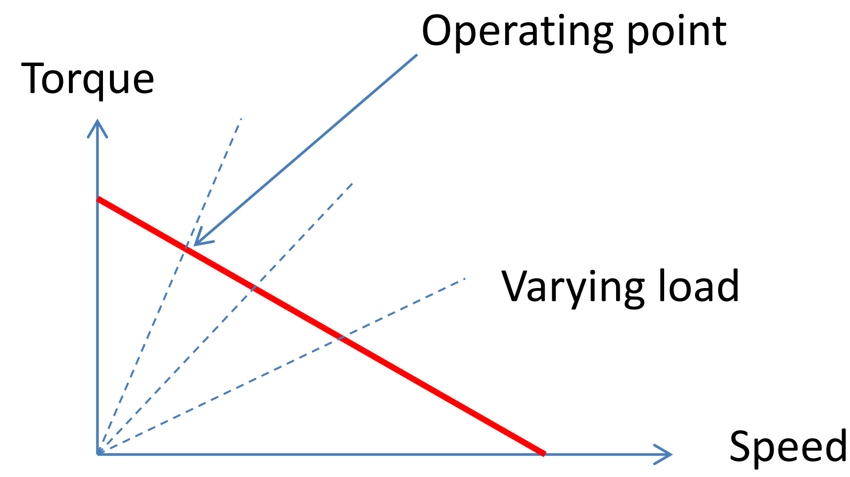
DC Motors

- DC motors are made in different electrical configurations, such as permanent magnet, shunt-wound, series-wound, and compound-wound. The names refer to the manner in which the rotating armature coils are electrically connected to the stationary field coils in parallel (shunt), in series, or in combined series-parallel (compound). Permanent magnets replace the field coils in a PM motor.

- Each configuration provides different torque-speed characteristics. The torque-speed curve of a motor describes how it will respond to an applied load and is of great interest to the mechanical designer as it predicts how the mechanical-electrical system will behave when the load varies dynamically with time.

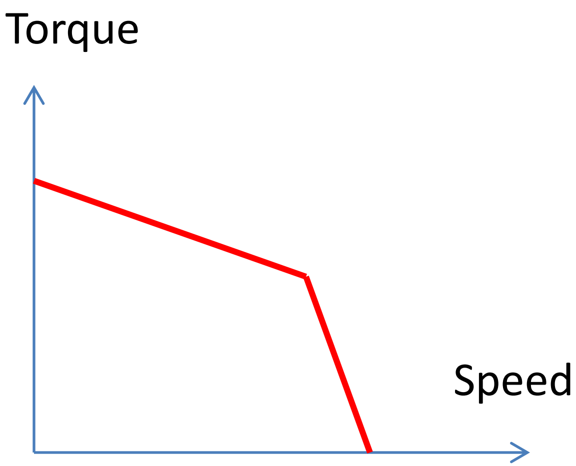
Permanent Magnet DC motors



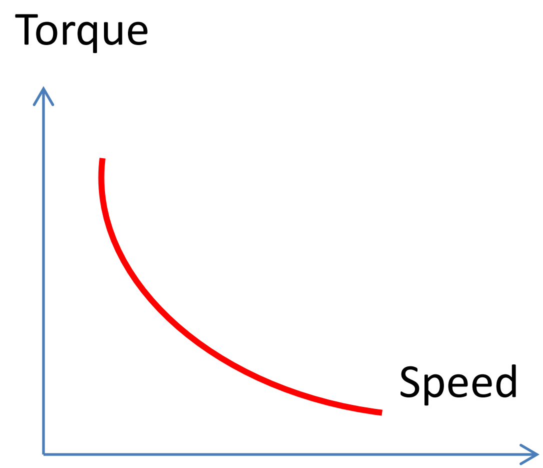


In PM(permanent magnet) motors, torque varies greatly with speed, ranging from a maximum (stall) torque at zero speed to zero torque at maximum (no-load) speed. The relationship comes from the fact that power = torque x angular velocity. Since the power is limited to some finite value, an increase in torque requires a decrease in angular velocity and vice versa.

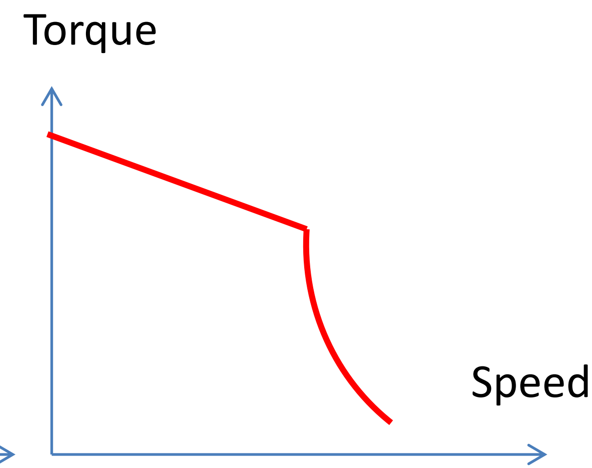
PM motor's torque is maximum at stall, which is typical for all electric motors. This is an advantage when starting heavy loads. An electric motor powered vehicle needs no clutch, unlike one powered by an internal combustion engine that cannot start from stall under load. An engine's torque increases rather than decreases with increasing angular velocity.



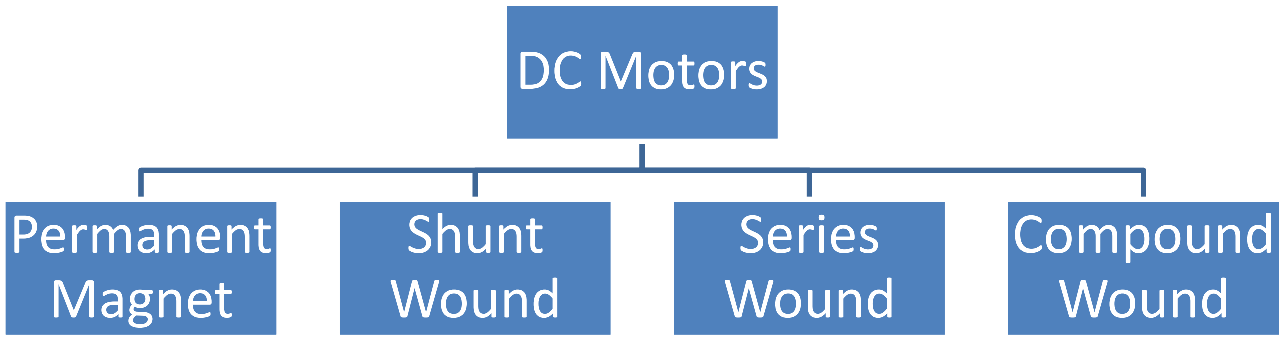
Shunt-wound motor



Series-wound motor



Compound-wound motor


DC Motors
Permanent Magnet
Shunt Wound
Series Wound
Compound Wound

Shunt-Wound DC motor

The shunt-wound motor is less speed sensitive to load variations in its operating range,

But stalls very quickly when the load exceeds its maximum overload capacity . Shunt-wound motors are typically used on fans and blowers

Series-Wound DC motor

This type is more speed sensitive than the shunt-wound motor. It does not have any theoretical maximum no-load speed, which makes it tend to run away if the load is removed. Overspeed detectors are sometimes fitted to limit its unloaded speed. Series-wound motors are used in sewing machines and portable grinders. They are also used in heavy-duty applications such as vehicle traction drives where their high starting torque is an advantage.

Compound-Wound DC motor

Their field and armature coils connected in a combination of series and parallel. As a result their torque-speed characteristics has aspects of both the shunt-wound and series-wound motors. Their speed sensitivity is grater than a shunt-wound but less than a series-wound motor and it will not run away when unloaded. This feature plus its high starting torque and soft-start capability make it good choice for cranes and hoists that experience high inertial loads.

Speed-Controlled DC motors

If precise speed control is needed, as is often the case in production machinery, another solution is to use a speed-controlled DC motor that operates from a controller that increases and decreases the current to the motor in the face of changing load to try to maintain constant speed.

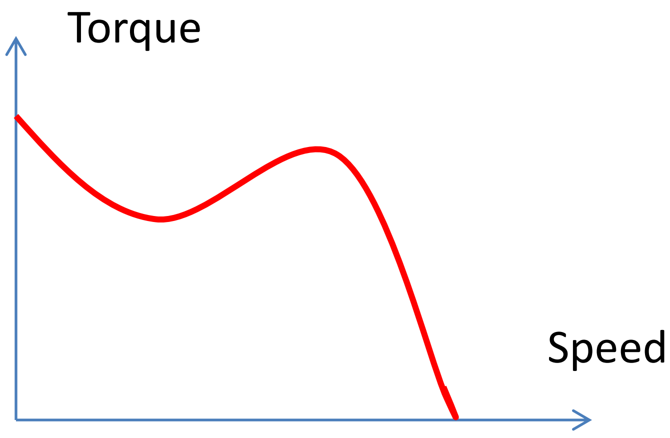
AC motors

AC motors are the least expensive way to get continuous rotary motion, and they can be had with a variety of torque-speed curves to suit various load applications. They are limited to a few standard speeds that are function of the AC line frequency (60 Hz in US, 50 Hz elsewhere). The synchronous motor speed n is a function of line frequency f and the number of magnetic poles p present in the rotor.

$$n=120f/p$$

Synchronous motors lock on to the AC line frequency and run exactly at synchronous speed. These motors are used for clocks and timers. Non synchronous AC motors have small amount of slip that makes them lag the line frequency by about 3 to 10 %.

Poles	Synchronous (rpm)	Asynchronous (rpm)
2	3600	3450
4	1800	1725
6	1200	1140
8	900	850
10	720	690
12	600	575



Typical torque-speed curve for AC motors

AC motors

Gear-motors

Gear motors

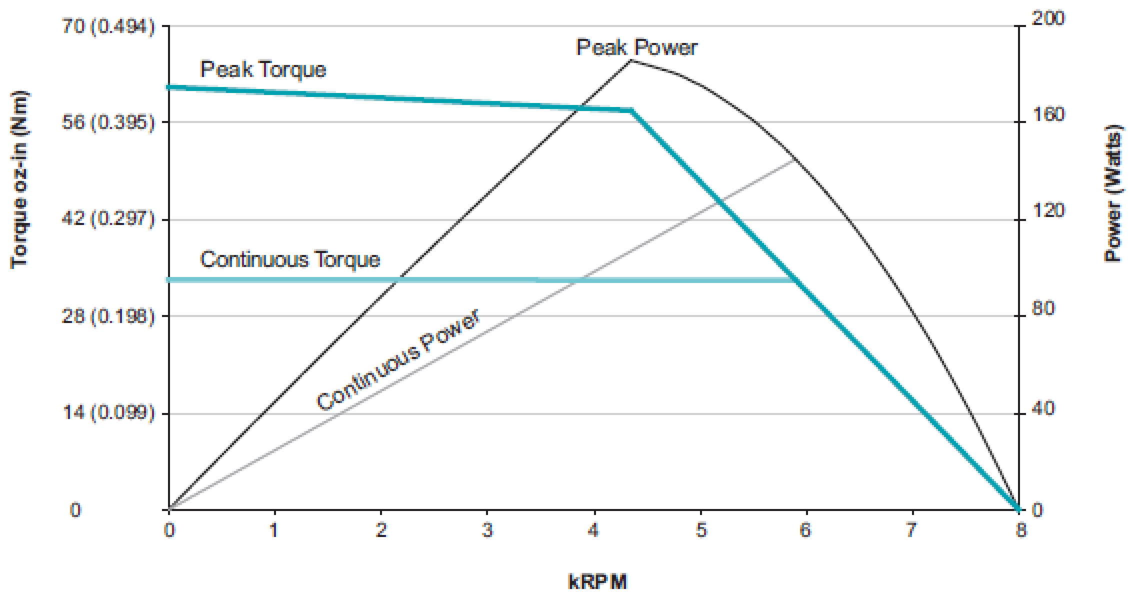
If different single output speeds than the standard ones needed, a gearbox speed reducer can be attached to the motor's output shaft or gear motor can be purchased that has an integral gearbox.

Servomotors

These are fast response closed-loop controlled motors capable of providing a programmed function of acceleration or velocity, providing position control and of holding a fixed position against a load. Closed-loop means that sensors (shaft encoders) on the motor or the output device being moved feed back information on its position and velocity. Circuitry in the motor controller responds to the feedback information by reducing or increasing the current flow to the motor. Precise positioning of the output device is than possible as is control of the speed and shape of the motor's response to changes in the load or input commands. These are relatively expensive devices that are commonly used as moving the flight control surfaces in aircraft and guided missiles, in numerically controlled machining centers, automated manufacturing machinery, and in controlling robots. They have high torque capability and a flat torque-speed curve.

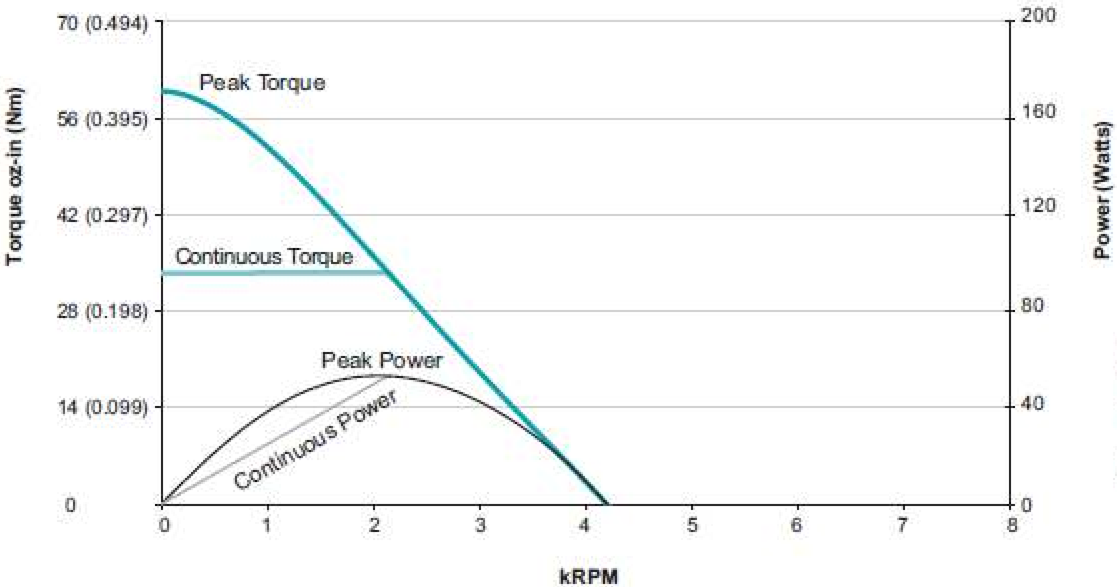
Servomotor and its characteristics

SM17205D		
Continuous Torque	2.08	in-lb
	33	oz-in
	0.24	N-m
Peak Torque	3.82	in-lb
	61	oz-in
	0.43	N-m
Nominal Continuous Power	145	Watt
No Load Speed	7,900	RPM
Continuous Current @ Nominal Power	4.5	Amps
Voltage Constant	5.5	V/kRPM
Winding Resistance	1.8	ohms
Encoder Resolution	4,000	Counts/Rev
Rotor Inertia	0.00026	oz-in-sec^2
	0.184	10^{-5} Kg-m^2
Weight	1.2	lb
	0.55	kg
Shaft Diameter	0.197	in
	5.00	mm
Shaft, Radial Load	7	lb
	3.18	kg
Shaft, Axial Thrust Load	3	lb
	1.36	kg
DeviceNet Available	Yes	
ProfiBus Available		
CANopen Available	Yes	



SM17205D
at 48 VDC
at rise to 85°C

Servomotor torque-speed curve



SM17205D
at 24 VDC
at rise to 85°C

All Torque curves based on 25°C ambient.

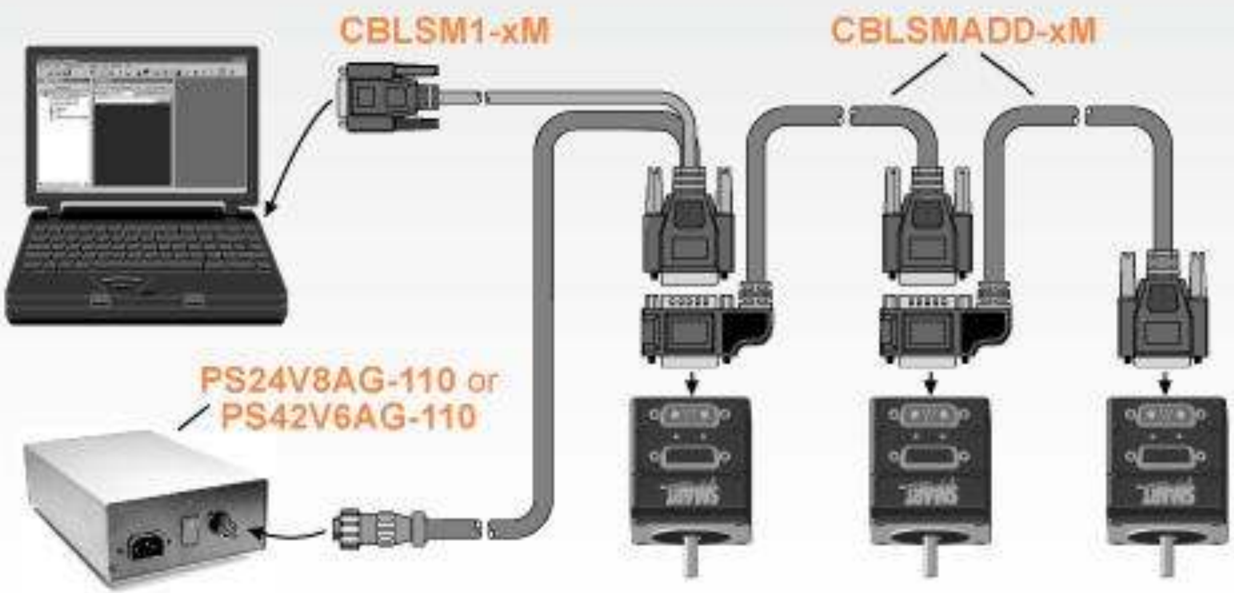
Motors were operated in Trap-Commutation Mode.

Please consult factory for Sine-Commutation Torque Curves.

Servomotor torque-speed curve

RS232 Communications using USB Adapter

RS232 Multidrop using Add-A-Motor Cable

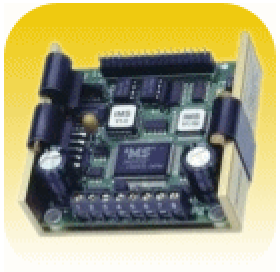


Stepper motors

- These are brushless permanent magnet, variable reluctance (resistance to magnetic flux), or hybrid type motors designed to position an output device. Unlike servomotors, they typically run open-loop, meaning they receive no feedback as to whether the output device has responded as requested. Thus they can get out of phase with the desired program.

- Their internal construction consists of a number of magnetic strips arranged around the circumference of both the rotor and stator. When energized the motor will move one step to the next magnet, for each pulse received. This are intermittent motion devices and do not provide continuous rotary motion like other motors. The number of magnetic strips and controller type determine the resolution (typically 200 steps/rev) but a micro-stepper drive can increase this to 2000 steps/rev. They are moderately expensive and require special controller.

Motor controller/drives



Stepper motors

Linear actuators

Air and hydraulic motors

- These have more limited application than electric motors, simply because they require the availability of a compressed air or hydraulic source. Both of these devices are less energy efficient than the electric motors. Hydraulic motors are most often found within machines or systems such as cranes, aircrafts, and ships. Hydraulic and pneumatic systems are generally expensive.

Air (pneumatic) motors

Hydraulic motors

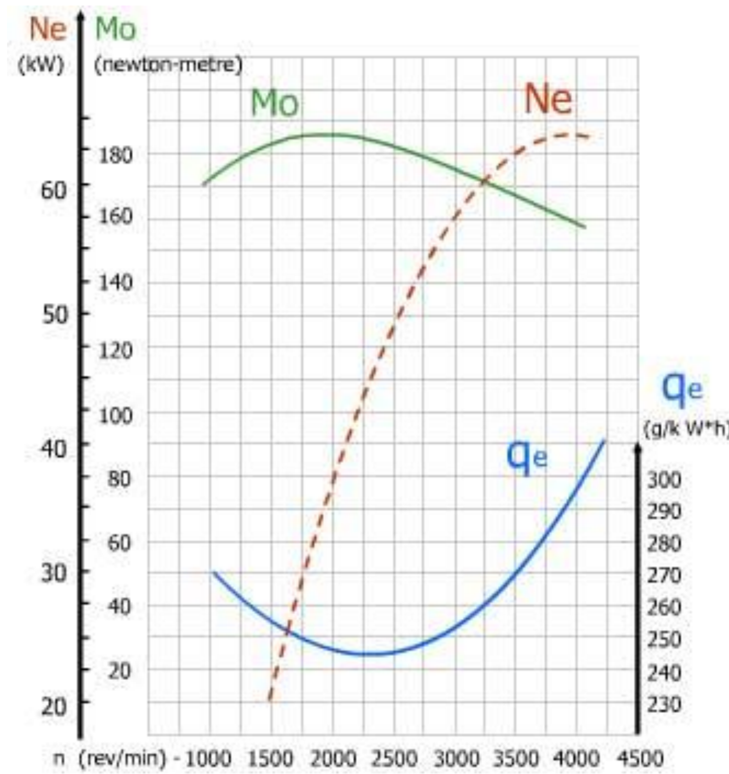
Air and hydraulic cylinders

- These are linear actuators (pistons) that provide a limited stroke, straight line output from a pressurized fluid flow input of either compressed air or hydraulic fluid.
- Another problem is that of control. Most motors will tend to run at a constant speed. A linear actuator, when subjected to a constant pressure fluid source, typical of most compressors, will respond with more nearly constant acceleration, which means its velocity will increase linearly with time. This can result in severe impact loads on the driven mechanism when the actuator come to the end of its stroke at maximum velocity.
- The most common application of fluid power is tractors, bulldozers.

Pneumatic Cylinders

Hydraulic cylinders

Engine


Ne
(kW)
Mo
(newton-metre)
Mo
Ne
qe
(g/k W*h)
qe
180
160
140
120
100
80
60
40
20
60
50
40
30
20
300
290
280
270
260
250
240
230
n (rev/min) - 1000 1500 2000 2500 3000 3500 4000 4500