



Industrial Engineering Program

Operations Research 1

Prepared and animated by:

Dr. Najeh Yousfi for "Female section"

Dr. Wafik Hachicha for "Male Section"

- Course # 8053102-3
- Fall Semester 2020-2021

The Computer Solution and the Sensitivity Analysis

- If an LP has more than two decision variables, the range of values for a rhs (or objective function coefficient) for which the basis remains optimal cannot be determined graphically.
- These ranges can be computed by hand but this is often tedious, so they are usually determined by a packaged computer program. LINDO will be used and the interpretation of its sensitivity analysis discussed.

Consider the following maximization problem.
 Winco sells four types of products. The resources needed to produce one unit of each are:

	Product 1	Product 2	Product 3	Product 4	Available
Raw material	2	3	4	7	4600
Hours of labor	3	4	5	6	5000
Sales price	\$4	\$6	\$7	\$8	

To meet customer demand, exactly 950 total units must be produced. Customers demand that at least 400 units of product 4 be produced. Formulate an LP to maximize profit.

Let x_i = number of units of product i produced by Winco.

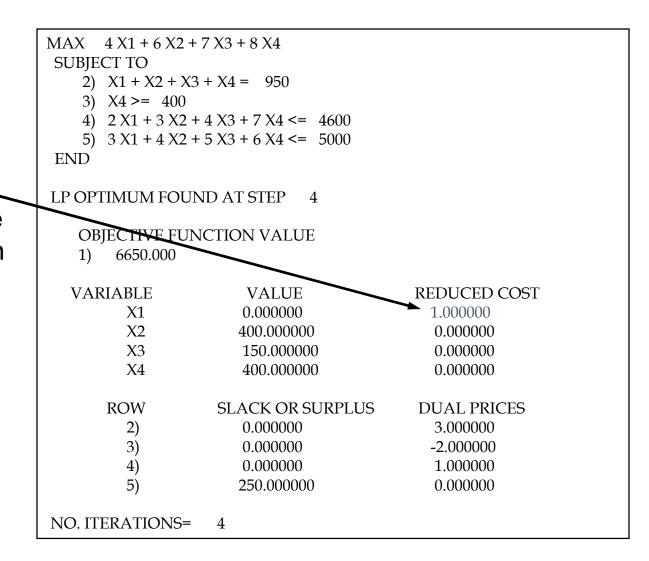
The Winco LP formulation:

max
$$z = 4x_1 + 6x_2 + 7x_3 + 8x_4$$

s.t. $x_1 + x_2 + x_3 + x_4 = 950$
 $x_4 \ge 400$
 $2x_1 + 3x_2 + 4x_3 + 7x_4 \le 4600$
 $3x_1 + 4x_2 + 5x_3 + 6x_4 \le 5000$
 $x_1, x_2, x_3, x_4 \ge 0$

LINDO output and sensitivity analysis example(s).

Reduced cost is the amount the objective function coefficient for variable i would have to be increased for there to be an alternative optimal solution.



LINDO sensitivity analysis example(s).

Allowable range (w/o changing basis) for the x₂ coefficient (c₂) is:

 $5.50 \le c_2 \le 6.667$

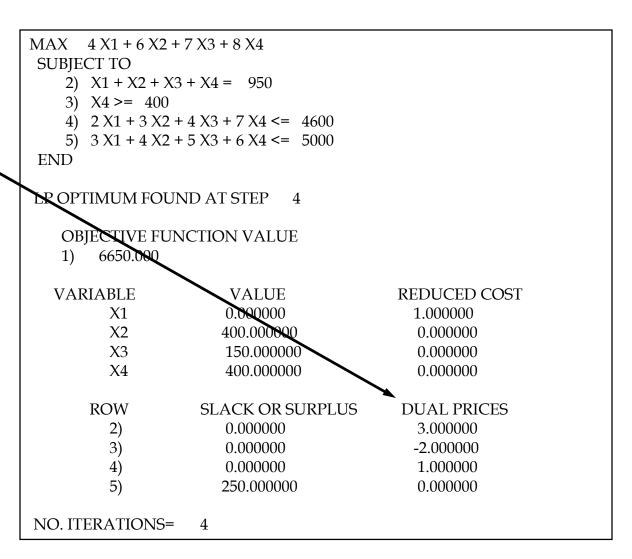
Allowable range (w/o changing basis) for the rhs (b₁) of the first constraint is:

 $850 \le b_1 \le 1000$

RANGES IN WHICH THE BASIS IS UNCHANGED:						
OBJ COEFFICIENT RANGES						
VARIABLE	CURRENT	ALLOWABLE	ALLOWABLE			
	COEF	INCREASE	DECREASE			
X1	4.000000	1.000000	INFINITY			
X2	6.000000	0.666667	0.500000			
Х3	7.000000	1.000000	0.500000			
X4	8.000000	2.000000	INFINITY			
RIGHTHAND SIDE RANGES						
ROW	CURRENT	ALLOWABLE	ALLOWABLE			
	RHS	INCREASE	DECREASE			
2	950.000000	50.000000	100.000000			
3	400.000000	37.500000	125.000000			
4	4600.000000	250.000000	150.000000			
5	5000.000000	INFINITY	250.000000			

Shadow prices are shown in the Dual Prices section of LINDO output.

Shadow prices are the amount the optimal z-value improves if the RHS of a constraint is increased by one unit (assuming no change in basis).



Interpretation of shadow prices for the Winco LP

ROW	SLACK OR SURPLUS	DUAL PRICES	3
2)	0.000000	3.000000	(overall demand)
3)	0.000000	-2.000000	(product 4 demand)
4)	0.000000	1.000000	(raw material availability)
5)	250.000000	0.000000	(labor availability)

Assuming the allowable range of the RHS is not violated, shadow (Dual) prices show: \$3 for constraint 1 implies that each one-unit increase in total demand will increase net sales by \$3. The -\$2 for constraint 2 implies that each unit increase in the requirement for product 4 will decrease revenue by \$2. The \$1 shadow price for constraint 3 implies an additional unit of raw material (at no cost) increases total revenue by \$1. Finally, constraint 4 implies any additional labor (at no cost) will not improve total revenue.

- Shadow price signs
 - 1. Constraints with ≥ symbols will always have nonpositive shadow prices.
 - 2. Constraints with ≤ will always have nonnegative shadow prices.
 - 3. Equality constraints may have a positive, a negative, or a zero shadow price.

Sensitivity Analysis and Slack/Excess Variables

For any inequality constraint, the product of the values of the constraint's slack/excess variable and the constraint's shadow price must equal zero. This implies that any constraint whose slack or excess variable > 0 will have a zero shadow price. Similarly, any constraint with a nonzero shadow price must be binding (have slack or excess equaling zero). For constraints with nonzero slack or excess, relationships are detailed in the table below:

Type of Constraint	Allowable Increase for rhs	Allowable Decrease for rhs
≤	∞	= value of slack
2	= value of excess	∞