Industrial Engineering Program

Operations Research 1

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- **Course # 8053102-3**
- **Fall Semester 2020-2021**

The Computer Solution and the Sensitivity Analysis

The Computer and Sensitivity Analysis

- If an LP has more than two decision variables, the range of values for a rhs (or objective function coefficient) for which the basis remains optimal cannot be determined graphically.

- These ranges can be computed by hand but this is often tedious, so they are usually determined by a packaged computer program. LINDO will be used and the interpretation of its sensitivity analysis discussed.

The Computer and Sensitivity Analysis

- Consider the following maximization problem. **Winco** sells four types of products. The resources needed to produce one unit of each are:

	Product 1	Product 2	Product 3	Product 4	Available
Raw material	2	3	4	7	4600
Hours of labor	3	4	5	6	5000
Sales price	$4	$6	$7	$8	

To meet customer demand, exactly 950 total units must be produced. Customers demand that at least 400 units of product 4 be produced. Formulate an LP to maximize profit.

Let x_i = number of units of product i produced by Winco.

The Computer and Sensitivity Analysis

- The Winco LP formulation:

$$\max z = 4x_1 + 6x_2 + 7x_3 + 8x_4$$

$$\text{s.t.} \quad x_1 + x_2 + x_3 + x_4 = 950$$

$$x_4 \geq 400$$

$$2x_1 + 3x_2 + 4x_3 + 7x_4 \leq 4600$$

$$3x_1 + 4x_2 + 5x_3 + 6x_4 \leq 5000$$

$$x_1, x_2, x_3, x_4 \geq 0$$

The Computer and Sensitivity Analysis

LINDO output and sensitivity analysis example(s).

Reduced cost is the amount the objective function coefficient for variable i would have to be increased for there to be an alternative optimal solution.

```
MAX    4 X1 + 6 X2 + 7 X3 + 8 X4
 SUBJECT TO
        2)   X1 + X2 + X3 + X4 =    950
        3)   X4 >=   400
        4)   2 X1 + 3 X2 + 4 X3 + 7 X4 <=   4600
        5)   3 X1 + 4 X2 + 5 X3 + 6 X4 <=   5000
 END

 LP OPTIMUM FOUND AT STEP      4

        OBJECTIVE FUNCTION VALUE
        1)      6650.000

      VARIABLE          VALUE              REDUCED COST
            X1          0.000000             1.000000
            X2        400.000000             0.000000
            X3        150.000000             0.000000
            X4        400.000000             0.000000

           ROW     SLACK OR SURPLUS        DUAL PRICES
            2)          0.000000             3.000000
            3)          0.000000            -2.000000
            4)          0.000000             1.000000
            5)        250.000000             0.000000

 NO. ITERATIONS=      4
```

The Computer and Sensitivity Analysis

LINDO sensitivity analysis example(s).

Allowable range (w/o changing basis) for the x_2 coefficient (c_2) is:

$5.50 \leq c_2 \leq 6.667$

Allowable range (w/o changing basis) for the rhs (b_1) of the first constraint is:

$850 \leq b_1 \leq 1000$

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	4.000000	1.000000	INFINITY
X2	6.000000	0.666667	0.500000
X3	7.000000	1.000000	0.500000
X4	8.000000	2.000000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	950.000000	50.000000	100.000000
3	400.000000	37.500000	125.000000
4	4600.000000	250.000000	150.000000
5	5000.000000	INFINITY	250.000000

The Computer and Sensitivity Analysis

Shadow prices are shown in the Dual Prices section of LINDO output.

Shadow prices are the amount the optimal z-value improves if the RHS of a constraint is increased by one unit (assuming no change in basis).

```
MAX     4 X1 + 6 X2 + 7 X3 + 8 X4
 SUBJECT TO
       2)  X1 + X2 + X3 + X4 =    950
       3)  X4 >=   400
       4)  2 X1 + 3 X2 + 4 X3 + 7 X4 <=   4600
       5)  3 X1 + 4 X2 + 5 X3 + 6 X4 <=   5000
 END

 LP OPTIMUM FOUND AT STEP      4

       OBJECTIVE FUNCTION VALUE
       1)      6650.000

   VARIABLE          VALUE            REDUCED COST
         X1         0.000000            1.000000
         X2       400.000000            0.000000
         X3       150.000000            0.000000
         X4       400.000000            0.000000

        ROW    SLACK OR SURPLUS     DUAL PRICES
         2)         0.000000            3.000000
         3)         0.000000           -2.000000
         4)         0.000000            1.000000
         5)       250.000000            0.000000

 NO. ITERATIONS=      4
```

The Computer and Sensitivity Analysis

- Interpretation of shadow prices for the Winco LP

ROW	SLACK OR SURPLUS	DUAL PRICES	
2)	0.000000	3.000000	(overall demand)
3)	0.000000	-2.000000	(product 4 demand)
4)	0.000000	1.000000	(raw material availability)
5)	250.000000	0.000000	(labor availability)

Assuming the allowable range of the RHS is not violated, shadow (Dual) prices show: $3 for constraint 1 implies that each one-unit increase in total demand will increase net sales by $3. The -$2 for constraint 2 implies that each unit increase in the requirement for product 4 will decrease revenue by $2. The $1 shadow price for constraint 3 implies an additional unit of raw material (at no cost) increases total revenue by $1. Finally, constraint 4 implies any additional labor (at no cost) will not improve total revenue.

The Computer and Sensitivity Analysis

- Shadow price signs

1. Constraints with ≥ symbols will always have nonpositive shadow prices.
2. Constraints with ≤ will always have nonnegative shadow prices.
3. Equality constraints may have a positive, a negative, or a zero shadow price.

The Computer and Sensitivity Analysis

- Sensitivity Analysis and Slack/Excess Variables

For any inequality constraint, the product of the values of the constraint's slack/excess variable and the constraint's shadow price must equal zero. This implies that any constraint whose slack or excess variable > 0 will have a zero shadow price. Similarly, any constraint with a nonzero shadow price must be binding (have slack or excess equaling zero). For constraints with nonzero slack or excess, relationships are detailed in the table below:

Type of Constraint	Allowable Increase for rhs	Allowable Decrease for rhs
≤	∞	= value of slack
≥	= value of excess	∞