Umm Al-Qura Universtiy, Makkah
Department of Electrical Engineering
Electromagnetics (2) - (8022320)
Term 1: 2021/2022
Solution Homework 11

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Do not submit this homework. It will be a part of the makeup midterm on Monday, 22 Nov 2021.

Topics covered in this week:

- Smith chart:
  - Marking normalized impedance or admittance on the Smith chart
  - Finding reflection co-efficient and standing wave ratio (SWR)
  - Finding impedance along the transmission line
  - Location of maximum and minimum impedance

Q1. A transmission line has $\gamma = (0 + j0.3)\ m^{-1}$ and $Z_0 = 150\ \Omega$. Find Z_{in} at $5\ m$ from the load if the load is a short circuit.

Solution:

- Using Smith chart:

  Normalized load impedance $= z_L = 0$ (P1)

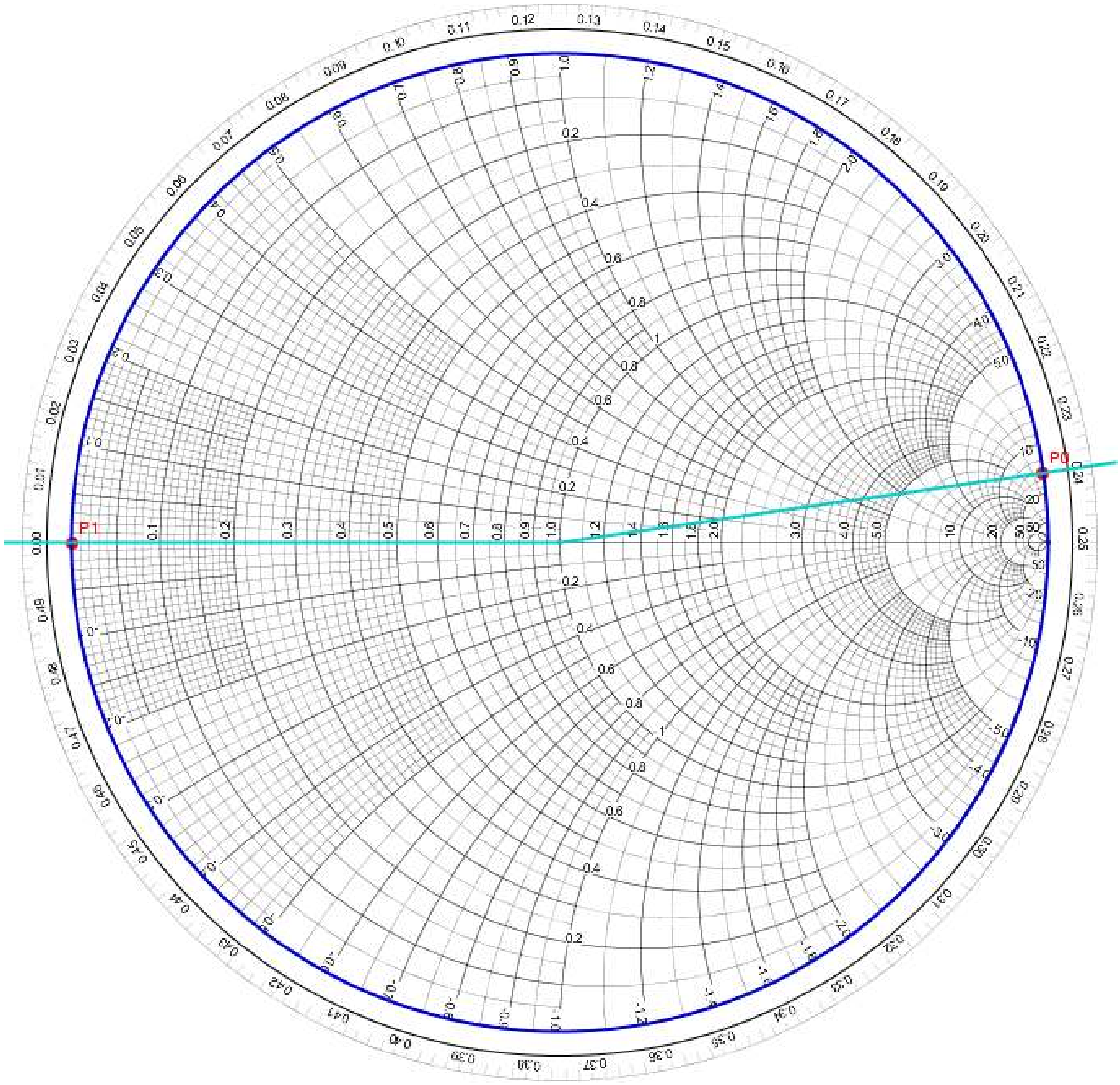
  Wavelength $= \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.3} = 20.943\ m$

  $l = 5\ m = \frac{5}{20.943} = 0.239\lambda$

  Moving from short circuit point (P1 on Smith chart) where wtg=0, in the clockwise direction by an amount of 0.239 we reached at wtg=0.239. This point is marked as P0. The normalised impedance here is 0+j14.1, which corresponds to $Z_{in} = 150(0 + j14.1) = j2{,}115\ \Omega$.

- Using Calculation:

  $$Z(l) = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = Z_0 \frac{0 + jZ_0 \tan\beta l}{Z_0 + j(0)\tan\beta l} = jZ_0 \tan\beta l = j(150)\tan\left(0.3 \times 5 \times \frac{180}{\pi}\right) = j2{,}115.2\ \Omega$$



Q2. A lossless line has $Z_0 = 100\ \Omega,\ \beta l = 0.9\pi,\ and\ Z_L = 125\ \Omega$. Find input impedance at the source.

Solution:

- Using Smith chart:

  Normalized load impedance $= z_L = \frac{Z_L}{Z_0} = \frac{125}{100} = 1.25 + j0$

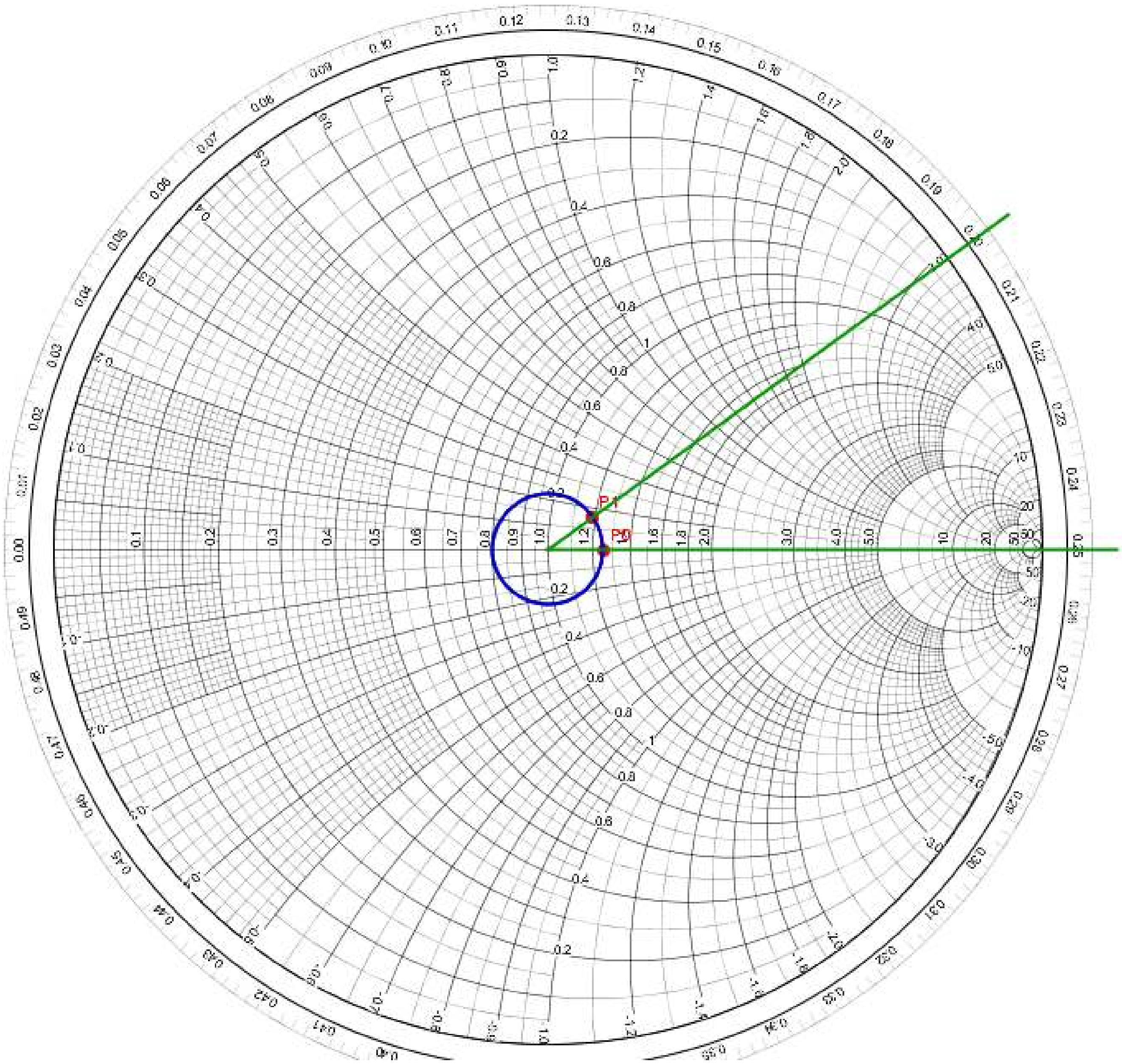
  $\beta l = 0.9\pi \quad \Rightarrow \quad l = \frac{0.9\pi}{\beta} = \frac{0.9\pi}{2\pi}\lambda = 0.45\lambda$

  Moving from point P0 (where wtg=0.25), in the clockwise direction by an amount of 0.45, we reached at wtg=0.2. This point is marked as P1. The normalised impedance here is 1.19+j0.157, which corresponds to $Z_{in} = 100(1.19 + j0.157) = (119 + j15.7)\ \Omega$.

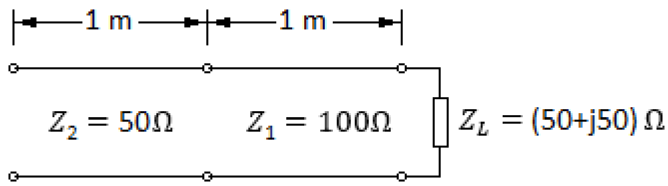
- Using Calculation:

$$Z(l) = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} = 100 \frac{125 + j100\tan\left(0.9\pi \times \frac{180}{\pi}\right)}{100 + j125\tan\left(0.9\pi \times \frac{180}{\pi}\right)} = 100 \frac{1.25 + j\tan 162°}{1 + j1.25\tan 162°} = 100 \frac{1.25 - j0.3249}{1 - j0.4061}$$

$$= 100 \frac{1.2915\angle -14.57°}{1.0793\angle -22.10°} = 119.66\angle 7.53° = (118.63 + j15.68)\ \Omega$$



Q3. A transmission line is made of two segments, each 1 m long. Calculate the input impedance of the combined line using a Smith chart if the speed of propagation on line-1 is 3×10^8 m/s and on line-2 is 1×10^8 m/s. The line operates at 300 MHz.



Solution:

The normalized load impedance = $z_L = \frac{Z_L}{Z_0} = \frac{50+j50}{100} = 0.5 + j0.5$. The wavelength on line-1 = $\lambda_1 = \frac{3\times10^8\ m/s}{3\times10^8\ s^{-1}} = 1\ m$. Hence the impedance at a point between both transmission lines is same as load impedance (i.e. $0.5 + j0.5$).

Wavelength on line-2 = $\lambda_2 = \frac{1\times10^8\ m/s}{3\times10^8\ s^{-1}} = 0.333\ m$. Hence 1m is equivalent to 3λ and the impedance at the start of line-1 would also be the same as load impedance (i.e. $(50 + j50)\Omega$).

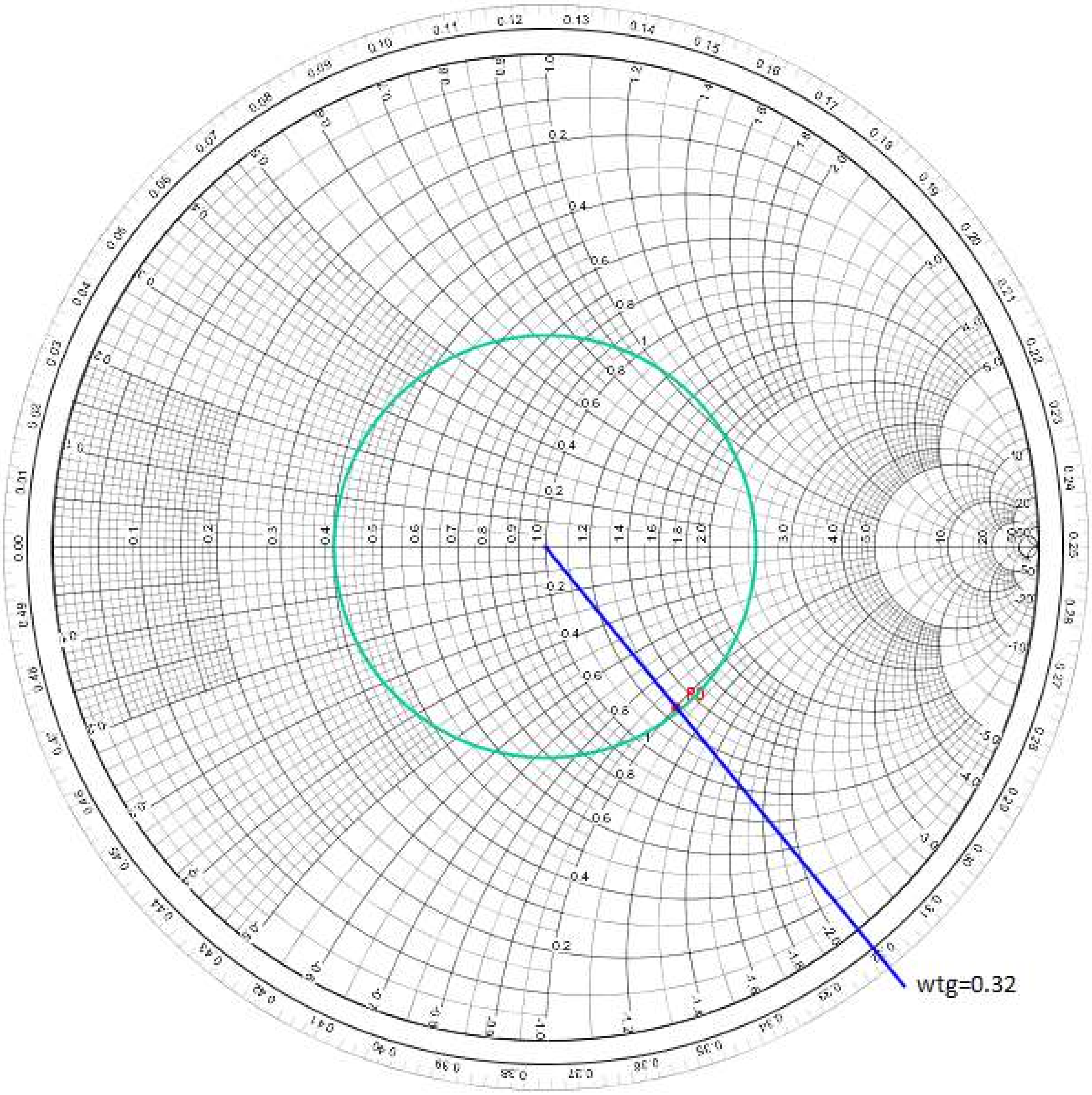
Q4. An unknown load is connected to a $75\,\Omega$ lossless transmission line. To find the load, two measurements are performed (1) The location of the first voltage minimum is found at 0.18λ from the load. (2) The SWR is measured as 2.5. Find, using the Smith chart

a. The load impedance
b. The load reflection coefficient.

Solution:

First draw a circle corresponding to SWR=2.5. Since minimum voltage is on real axis towards the left, we move 0.18λ towards the load (counter-clockwise) from the point of minimum voltage. This will take us to wtg=0.5-0.18 = 0.32. Draw a line from origin to this point. The intersection of this line and circle (point P0) will give the normalized load impedance.

a. Load impedance = $75(1.278 - j1.025)\Omega = (95.85 - j76.875)\Omega$.
b. The reflection coefficient at the load can be found at the same point of load impedance: $\Gamma_L = 0.425\angle - 50.6°$.



Q5. A transmission line with characteristic impedance $Z_0 = 100\ \Omega$ has a load of $Z_L = 50\ \Omega$. Calculate:

a. Location of voltage maxima and minima on the line
b. The maximum and minimum impedance on the line. Where do these occur?

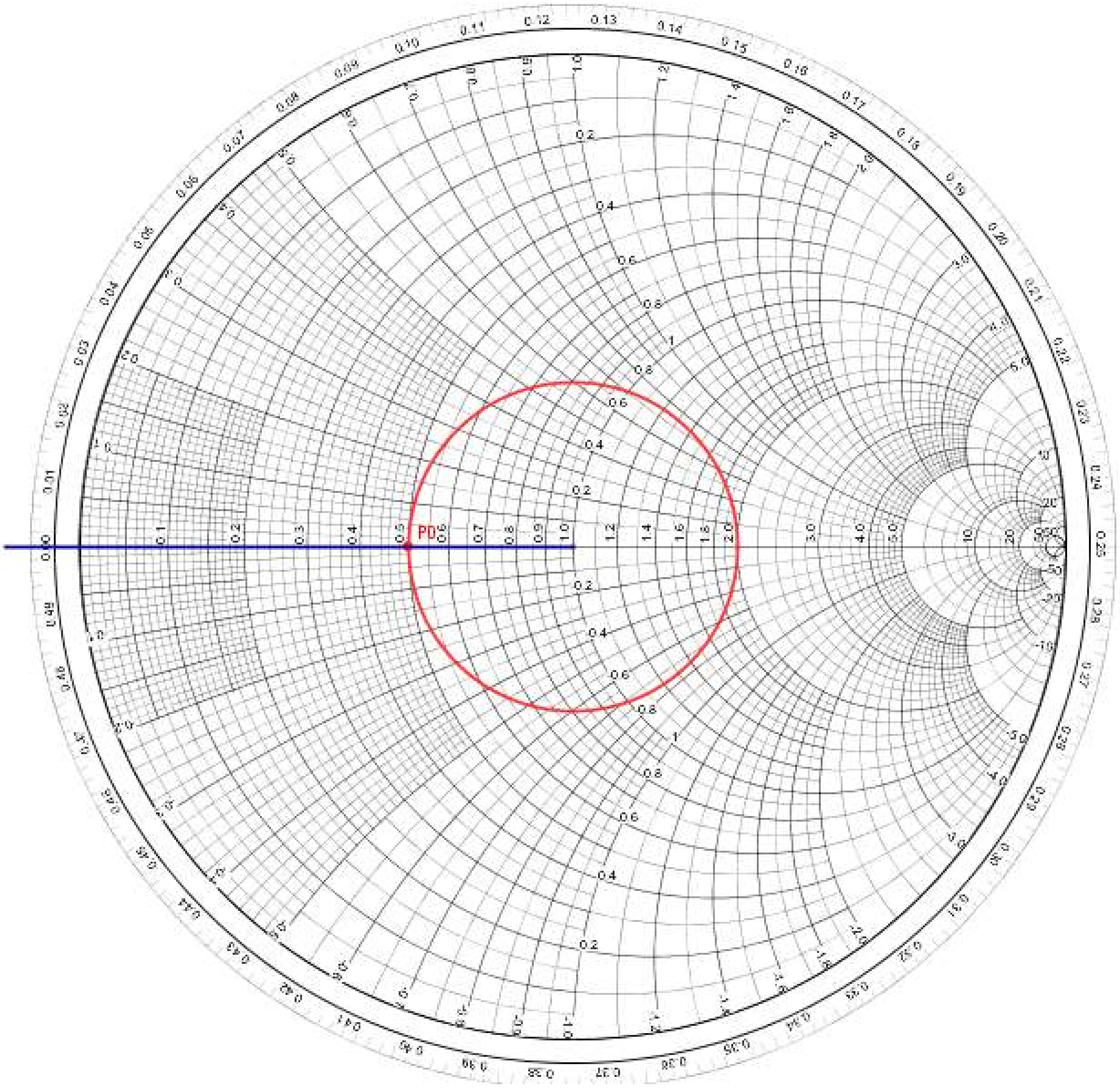
Solution:

The normalized impedance $z_L = \frac{Z_L}{Z_0} = \frac{50}{100} = 0.5 + j0$ is marked on the Smith chart at P0.

From the Smith chart, $\Gamma = -\frac{1}{3} = 0.3333\angle 180° \quad and \quad SWR = 2$

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50-100}{50+100} = \frac{-50}{150} = -\frac{1}{3} = 0.3333\angle 180° \qquad SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.333}{1-0.333} = 2$

a. Voltage maxima occurs at the positive real axis: wtg = 0.25; $l = 0.25\lambda, 0.75\lambda, 1.25\lambda, 1.75\lambda, \ldots$
   Voltage minima occurs at the negative real axis: wtg = 0; $l = 0, 0.5\lambda, \lambda, 1.5\lambda, 2\lambda, \ldots$
b. Maximum impedance (on the positive real axis): 2 (normalized). Or $200\ \Omega$
   Minimum impedance (on the negative real axis): 0.5 (normalized). Or $50\ \Omega$



Q6. A transmission line with characteristic impedance $Z_0 = 50\ \Omega$ has a load of $Z_L = 100\ \Omega$. Calculate:

a. Location of voltage maxima and minima on the line
b. The maximum and minimum impedance on the line. Where do these occur?

Solution:

The normalized impedance $z_L = \frac{Z_L}{Z_0} = \frac{100}{50} = 2 + j0$ is marked on the Smith chart at P0.

From the Smith chart, $\Gamma = \frac{1}{3} = 0.3333\angle 0° \quad and \quad SWR = 2$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100-50}{100+50} = \frac{50}{150} = \frac{1}{3} = 0.3333\angle 0° \qquad SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.333}{1-0.333} = 2$$

a. Voltage maxima occurs at the positive real axis: wtg = 0.25; $l = 0, 0.5\lambda, \lambda, 1.5\lambda, 2\lambda, \ldots$
   Voltage minima occurs at the negative real axis: wtg = 0; $l = 0.25\lambda, 0.75\lambda, 1.25\lambda, 1.75\lambda, \ldots$
b. Maximum impedance (on the positive real axis): 2 (normalized). Or $100\ \Omega$
   Minimum impedance (on the negative real axis): 0.5 (normalized). Or $25\ \Omega$