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Department of Electrical Engineering
Electromagnetics (2) - (8022320)
Term 1: 2021/2022
Solution Homework 8
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Do not submit this homework. There will be a quiz from this homework on Wednesday, 03 Nov 2021.

Topics covered in this week:

- Transmission line theory – distributed parameters (Recall that circuit theory – lumped parameters).

$$\frac{dV_s}{dz} = -(R + j\omega L)I_s$$

$$\frac{dI_s}{dz} = -(G + j\omega C)V_s$$

Propagation constant $= \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

Characteristic impedance $= Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$

$$\frac{d^2V_s}{dz^2} = -(R + j\omega L)\frac{dI_s}{dz} = (R + j\omega L)(G + j\omega C)V_s = \gamma^2 V_s$$

$V_s(z) = V_0 e^{-\gamma z}$ $\qquad$ $V(z,t) = V_0 e^{-\alpha z}\cos(\omega t - \beta z)$

$I_s(z) = \frac{V_0}{Z_0} e^{-\gamma z}$ $\qquad$ $I(z,t) = \frac{V_0}{Z_0} e^{-\alpha z}\cos(\omega t - \beta z)$

$\lambda = \frac{2\pi}{\beta}, \quad v = \frac{\omega}{\beta}$

Q1. At a frequency of 100 MHz, the following are the values of a transmission line: $R = 0.15\ \Omega/m,\ L = 0.25\ \mu H/m,\ C = 80\ pF/m,\ G = 8\ \mu S/m.$ Calculate the values of (a) attenuation constant, (b) phase constant, (c) wavelength, (d) phase velocity, and (e) characteristic impedance.

Solution:

a. $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$= \sqrt{\left(0.15 + j2\pi 10^8(0.25 \times 10^{-6})\right)(8 \times 10^{-6} + j2\pi 10^8(80 \times 10^{-12}))}$$

$$= \sqrt{(0.15 + j157.08)(0.000008 + j0.0503)} = \sqrt{-7.8957 + j0.0088}$$

$$= 0.0016 + j2.8099$$

Attenuation constant = $\alpha = 0.0016\ m^{-1}$

b. Phase constant = $\beta = 2.8099\ rad/m.$

c. Wavelength = $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2.8099} = 2.24\ m.$

d. Phase velocity = $v_p = \frac{\omega}{\beta} = \frac{2\pi 10^8}{2.8099} = 2.24 \times 10^8\ m/s.$

e. Characteristic impedance = $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{0.15 + j157.08}{0.000008 + j0.0503}} = \sqrt{3125 - j2.4868}$

$$= 55.9 - j0.0222 = (55.9\angle - 0.0227°)\ \Omega$$

Q2. A transmission line operating at 500 Mrad/sec has $R = 25\ \Omega/m,\ L = 0.5\ \mu H/m,\ C = 32\ pF/m,\ G = 100\ \mu S/m$. Calculate (a) $\gamma,\ \alpha,\ \beta,\ v_p,\ \lambda,\ and\ Z_0$, (b) What distance down the line can a voltage wave travel before it is reduced to 10% of its original amplitude, and (c) what distance must it travel to undergo a 90° phase shift.

Solution:

a. $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$
$$= \sqrt{[25 + j(5 \times 10^8)(0.5 \times 10^{-6})][100 \times 10^{-6} + j(5 \times 10^8)(32 \times 10^{-12})]}$$
$$= \sqrt{(2.5 + j250)(0.0001 + j0.016)} = \sqrt{-3.9975 + j0.425}$$
$$= 0.1061 + j2.0022$$
$$\alpha = 0.1061\, m^{-1}$$
$$\beta = 2.0022\, rad/m$$
$$v_p = \frac{\omega}{\beta} = \frac{5 \times 10^8}{2.0022} = 2.497 \times 10^8 \ m/s$$
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2.0022} = 3.138\, m$$
$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{2.5 + j250}{0.0001 + j0.016}} = \sqrt{15634 - j1464.8} = 125.17 - j5.851$$
$$= (125.3\angle - 2.68°)\ \Omega$$

b. $V(z) = V_0 e^{-\alpha z} \quad \Rightarrow \quad z = -\frac{\ln[V(z)/V_0]}{\alpha} = -\frac{\ln[10/100]}{0.1061} = 21.7\, m$

c. $\beta z = \frac{\pi}{2} \quad \Rightarrow \quad z = \frac{\pi}{2\beta} = \frac{\pi}{2(2.0022)} = 0.785\, m$

Q3. At a frequency of 80 MHz, a lossless transmission line has a characteristic impedance of 300 Ω and a wavelength of 2.5 m. Find (a) L, (b) C.

Solution:

For lossless transmission line: $Z_0 = \sqrt{\frac{L}{C}} = 300\ \Omega$

and $\lambda = \frac{2\pi}{\omega\sqrt{LC}} \quad \Rightarrow \quad \sqrt{LC} = \frac{2\pi}{\omega\lambda} = \frac{2\pi}{2\pi f \lambda} = \frac{1}{\lambda f} = \frac{1}{2.5(80 \times 10^6)} = 5 \times 10^{-9}$

a. $L = \sqrt{LC}\,\sqrt{\frac{L}{C}} = (5 \times 10^{-9})(300) = 1.5 \times 10^{-6} = 1.5\ \mu H/m$

b. $C = \sqrt{LC}\ \div \sqrt{\frac{L}{C}} = \frac{5 \times 10^{-9}}{300} = 16.67 \times 10^{-12} = 16.67\ pF/m$

Q4. A 75-ohm coaxial line has a current $I(z,t) = 1.8\cos(3.77 \times 10^9 t - 18.13z)\, mA$. Determine (a) the frequency, (b) the phase velocity, (c) the wavelength, (d) the phasor form of the current, and (e) the time domain voltage on the line.

Solution:

a. $f = \frac{\omega}{2\pi} = \frac{3.77 \times 10^9}{2\pi} = 600 \times 10^6\, Hz = 600\, MHz$

b. Phase velocity $= v_p = \frac{\omega}{\beta} = \frac{3.77 \times 10^9}{18.13} = 2.08 \times 10^8$ m/s

c. $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{18.13} = 0.3466\, m$

d. $I_s(z) = 1.8 e^{-j18.13z}$

e. $V(z,t) = (75)0.0018\cos(3.77 \times 10^9 t - 18.13z) = 0.135\cos(3.77 \times 10^9 t - 18.13z)$ volt

Q5. An antenna cable has a characteristic impedance of 300-ohm and phase velocity "c". Find the phase constant when receiving (a) VHF channel 3 (63 MHz), and (b) UHF channel 69 (803 MHz).

Solution:

a. $\beta = \frac{\omega}{v_p} = \frac{\omega}{c} = \frac{2\pi(63\times10^6)}{3\times10^8} = 1.32$ rad/m
b. $\beta = \frac{\omega}{v_p} = \frac{\omega}{c} = \frac{2\pi(803\times10^6)}{3\times10^8} = 16.82$ rad/m

Q6. A low loss antenna cable has an attenuation of 1 dB/km. The phase velocity is 1.353×10^8 m/s. The frequency of the signal is 80 MHz (approximately middle frequency of VHF channel 5).

a. Find the propagation constant.
b. This cable is used to connect a cable TV distribution center to a TV, 20 km away. If the TV requires a signal of at least 100 mV, what must be the signal amplitude at the generator.

Solution:

a. $V(z) = V_0 e^{-\alpha z} \quad \Rightarrow \quad V(1\,km) = V_0 e^{-\alpha(1000\,m)} \quad \Rightarrow \quad \frac{V_0}{V(1\,km)} = e^{1000\alpha}$

   $\Rightarrow \quad 20\log\frac{V_0}{V(1\,km)} = 20\log e^{1000\alpha}$

   The quantity on the left is given as 1 dB. Hence

   $20\log e^{1000\alpha} = 1 \quad \Rightarrow \quad e^{1000\alpha} = 10^{0.05} \quad \Rightarrow \quad \alpha = \frac{\ln 10^{0.05}}{1000} = 1.151 \times 10^{-4}$ Np/m

   $\beta = \frac{\omega}{v_p} = \frac{2\pi(80\times10^6)}{1.353\times10^8} = 3.715$ rad/m

   $\gamma = \alpha + j\beta = (1.151 \times 10^{-4} + j3.715)\, m^{-1}$

b. $V(z) = V_0 e^{-\alpha z}$

   $\Rightarrow \quad V_0 = V(z)e^{\alpha z} = V(20{,}000\,m)e^{\alpha(20{,}000)} = 0.1e^{(1.151\times10^{-4})(20{,}000)} = 0.9994\, volt$

Q7. A TV cable has an attenuation of 10 dB/km. What is the required voltage amplitude at the generator to produce a signal of 10 mV at the TV 10 km away?

Solution:

Attenuation in 10 km = (10 dB/km) x (10 km) = 100 dB.

Hence, $20\log\frac{V_0}{V(10\,km)} = 100\,dB \quad \Rightarrow \quad \log\frac{V_0}{V(10\,km)} = \frac{100}{20} = 5 \quad \Rightarrow \quad \frac{V_0}{V(10\,km)} = 10^5$

$\Rightarrow \quad V_0 = V(10\,km)10^5 = (0.01)10^5 = 1{,}000\, volt$