

# **Fundamentals of Engineering Economy**

Course # 8012101

Fall Semester – 2020/2021

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## **Lect. # 2**

### **Cash Flows: Estimation and Diagramming**

## TERMINOLOGY AND SYMBOLS

- P** = value or amount of money at a time designated as the present or time 0.  
Also, P is referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC); dollars
- F** = value or amount of money at some future time. Also, F is called future worth (FW) and future value (FV); dollars.
- A** = series of consecutive, equal, end-of-period amounts of money. Also, A is called the annual worth (AW) and equivalent uniform annual worth (EUAW); dollars per year, dollars per month.
- n** = number of interest periods; years, months, days.
- i** = interest rate or rate of return per time period; percent per year, percent month, percent per day
- t** = time, stated in periods; years, months, days.

**Example 1:**

A new college graduate has a job with Boeing Aerospace. She plans to borrow \$10,000 now to help in buying a car. She has arranged to repay the entire principal plus 8% per year interest after 5 years. Identify the engineering economy symbols involved and their values for the total owed after 5 years.

**Solution**

In this case, P and F are involved, since all amounts are single payments, as well as n and i. Time is expressed in years.

The future amount F is unknown.

$P = \$10,000$        $i = 8\% \text{ per year}$        $n = 5 \text{ years}$        $F = ?$

**Example 2:**

Assume you borrow \$2000 now at 7% per year for 10 years and must repay the loan in equal yearly payments. Determine the symbols involved and their values.

**Solution**

Time is in years.

$P = \$2000$

$A = ? \text{ per year for } 10 \text{ years}$

$n = 10 \text{ years}$

$i = 7\% \text{ per year}$

### **Example: 3**

You plan to make a lump-sum deposit of \$5000 now into an investment account that pays 6% per year, and you plan to withdraw an equal end-of-year amount of \$1000 for 5 years, starting next year. At the end of the sixth year, you plan to close your account by withdrawing the remaining money. Define the engineering economy symbols involved.

### **Solution**

Time is expressed in years.

$P = \$5000$

$A = \$1000$  per year for 5 years

$F = ?$  at end of year 6

$i = 6\%$  per year

$n = 5$  years for the A series and 6 for the F value

# Cash Flows

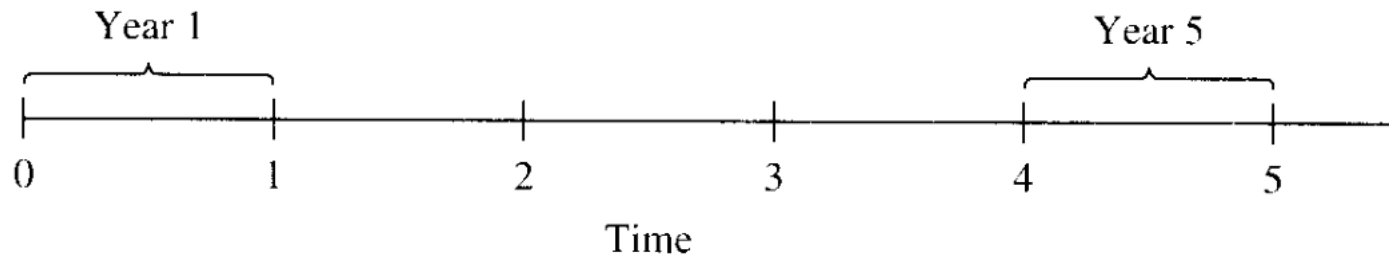
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- To financially analyze engineering projects, we need to model the projects in terms of cash flows.
- Cash flows represent the flow or movement of money at some specific time over some period of time.
- Outflows represent cash that is leaving an account such as a withdrawal (expenses or disbursements or losses or costs).
- Inflows represent cash that is entering an account such as a deposit (revenues or receipts or benefits or incomes).

## Cash Flows and Engineering Projects

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- An engineering project can be viewed as an account with outflows and inflows.
- Cash flow movements can be visually displayed through the use of a cash flow diagram.



## CASH FLOW DIAGRAM

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A cash flow diagram is a picture of a financial problem that shows all cash inflows and outflows plotted along a horizontal time line .

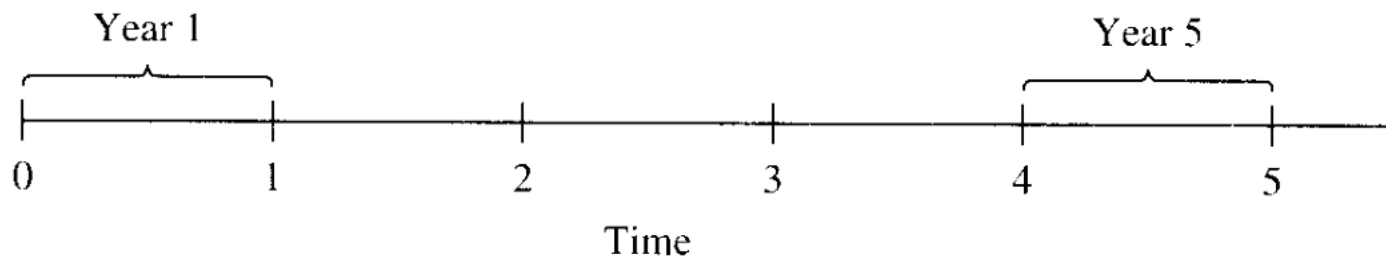
The cash flows over time are represented by arrows at relevant periods: upward arrows denote positive flows and downward arrows denote negative flows.

Arrows represent net cash flows since two or more values at the same time are summed and shown as a single arrow.

$$\begin{aligned} \text{Net cash flows} &= \text{receipts} - \text{disbursement} \\ &= \text{cash inflows} - \text{cash outflows} \end{aligned}$$

## CASH FLOW DIAGRAM

- Generally, the start of the diagram represents the *beginning* of the interest period.
- When  $t = 0$ , this is the present
- When  $t = 1$ , this is the *end of the first year* or (*beginning of the second year*).



A typical cash flow time scale for 5 years

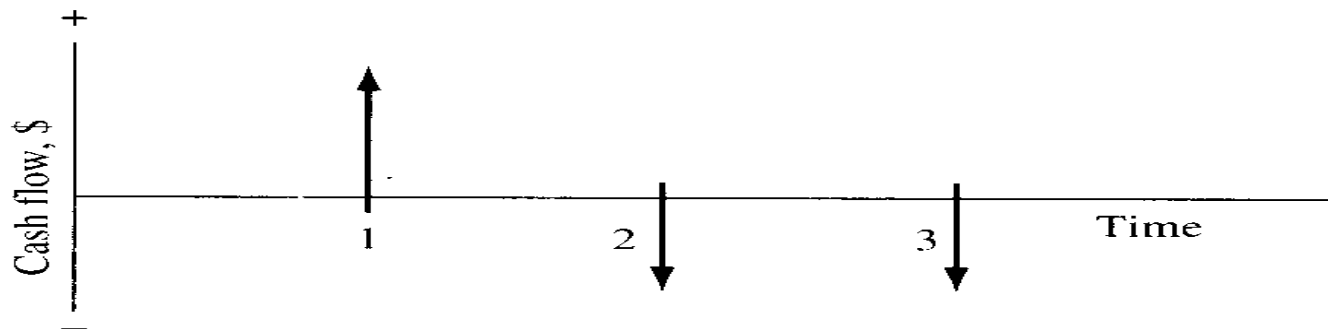


# CASH FLOW DIAGRAM

Cash inflows are the receipts, revenues, incomes, and savings generated by project and business activity. A **plus sign** indicates a cash inflow.

Cash outflows are costs, disbursements, expenses, and taxes caused by projects and business activity. A **negative or minus sign** indicates a cash outflow. When a project involves only costs, the minus sign may be omitted for some techniques, such as benefit/cost analysis.

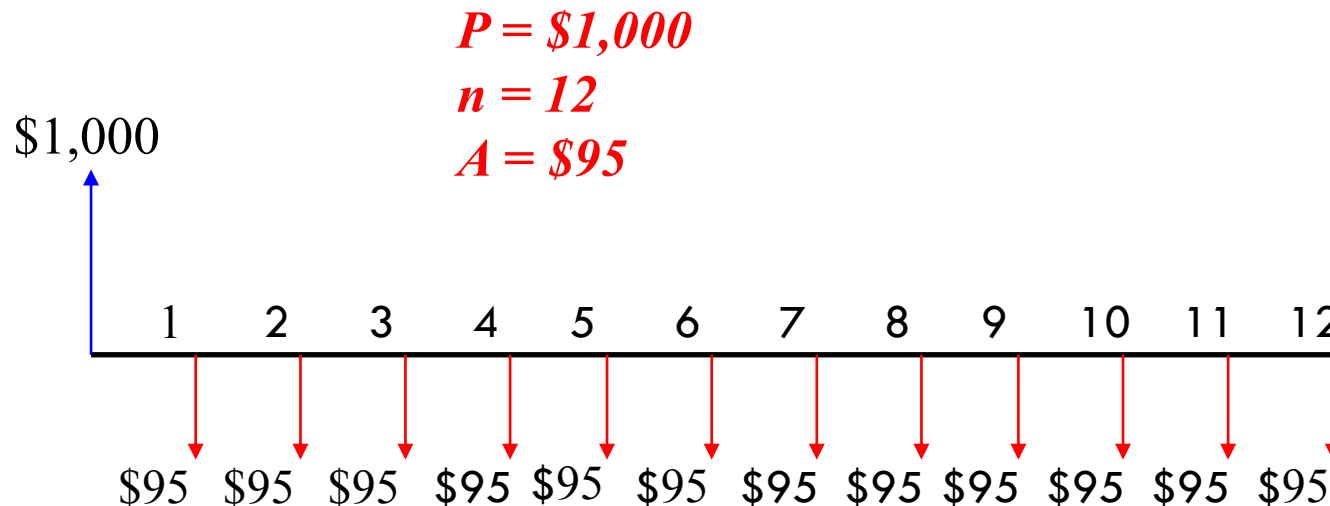
The figure illustrates a **receipt** (cash *inflow*) at the end of year 1 and equal **disbursements** (cash *outflows*) at the end of years 2 and 3.



Example of positive and negative cash flows

## CASH FLOW DIAGRAM – EXAMPLE [1]

- You borrowed \$1,000 from a bank to purchase a laptop. The bank requires you to make 12 equal monthly payments of \$95 to pay off the loan.
- Draw the cash flow diagram for this scenario



## CASH FLOW DIAGRAM – EXAMPLE [2]

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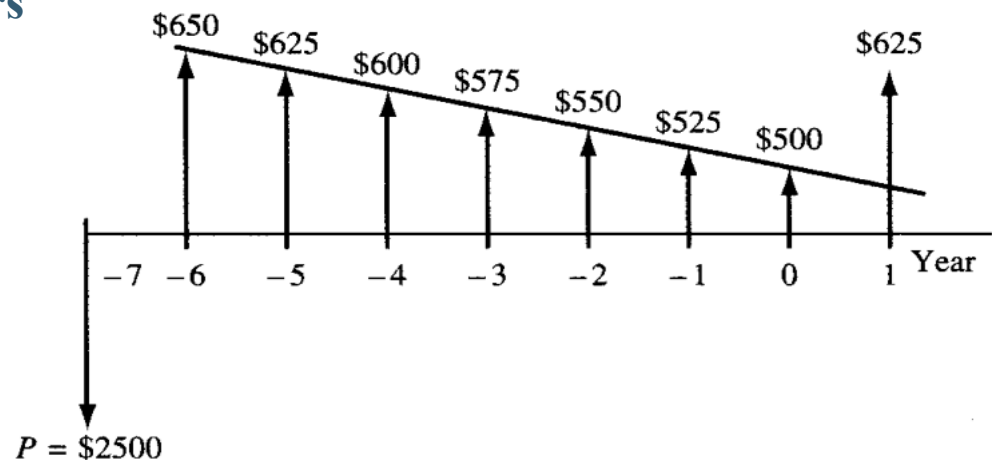
- A company **spent** \$2,500 on a new compressor 7 years ago.
- The annual *income* from the compressor has been **\$750**.
- Additionally, the \$100 spent on *maintenance* during the first year has increased each year by \$25.
- The company plans to sell the compressor at the end of next year for \$150.
- Construct the cash flow diagram from the *company's perspective*.

## CASH FLOW DIAGRAM – EXAMPLE [2]

End of year	Income	Cost	Net Cash Flow
-7	\$ 0	\$2500	\$-2500
-6	750	100	650
-5	750	125	625
-4	750	150	600
-3	750	175	575
-2	750	200	550
-1	750	225	525
0	750	250	500
1	750 + 150	275	625

- Use now as time  $t = 0$

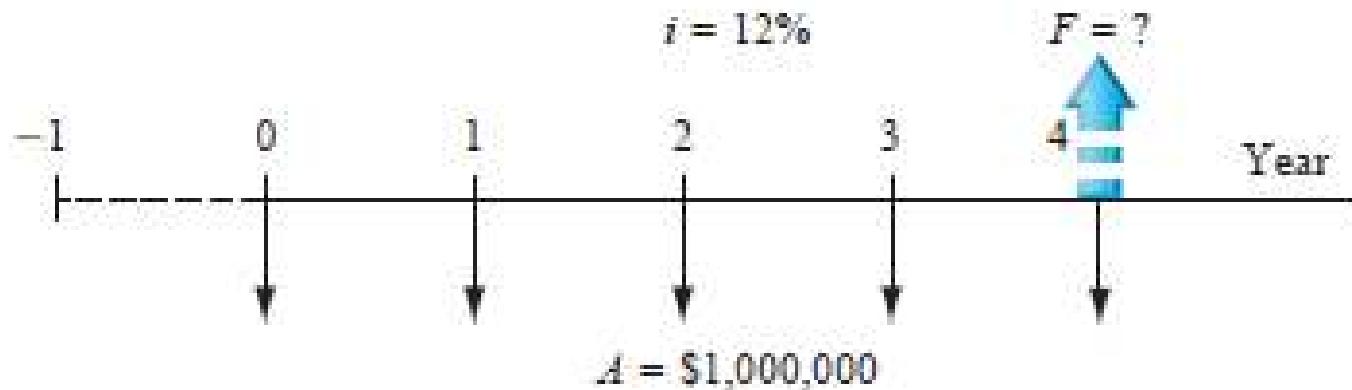
- The incomes and costs for years -7 through 1 (next year) are tabulated



## CASH FLOW DIAGRAM – EXAMPLE [3]

Each year a company expends large amounts of funds for mechanical safety features throughout its worldwide operations. The company plans expenditures of \$1 million *now* and each of the next 4 years just for the improvement of field-based pressure-release valves. **Construct the cash flow diagram** to find the equivalent value of these expenditures at the end of year 4, using a cost of capital estimate for safety-related funds of 12% per year.

Solution



Cash flow diagram, Example 3

## CASH FLOW DIAGRAM – EXAMPLE [4]

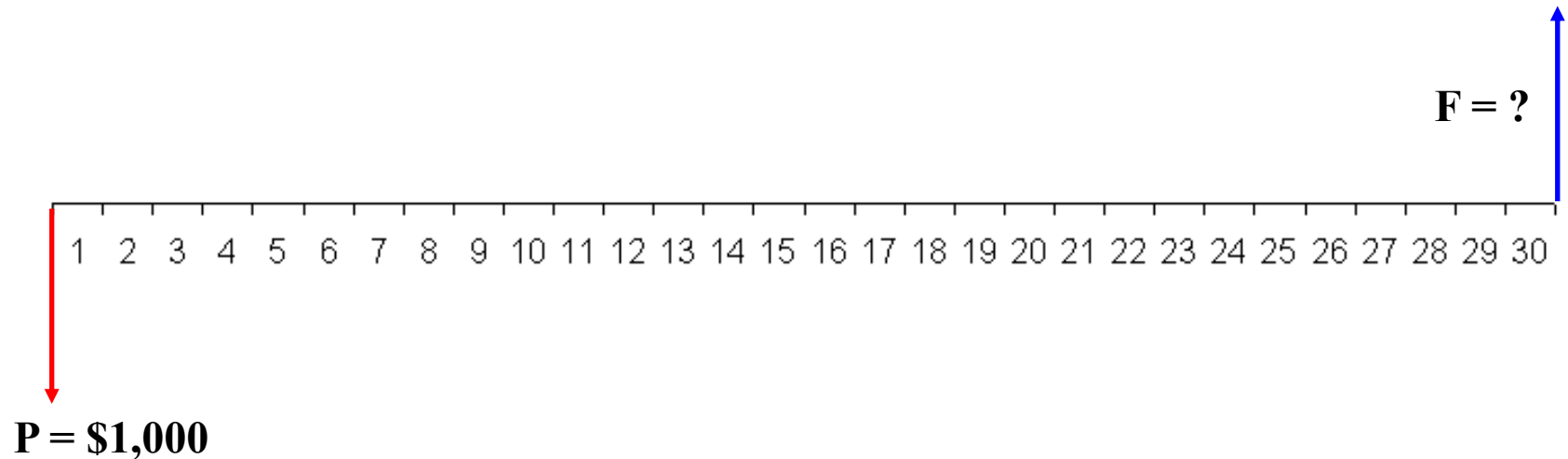
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- You *deposited* a \$1,000 in your account in a bank that gives a daily interest of 0.003% where interest is paid monthly. Assume simple interest.
- [1] For this scenario, what is your balance after 30 days?
- [2] If you deposit another \$2,000 on the 11<sup>th</sup> day and withdraw \$500 on the 26<sup>th</sup> day, what is your balance at the end of the 30<sup>th</sup> day?
- In both cases, draw the cash flow diagram.

## CASH FLOW DIAGRAM – EXAMPLE [4]

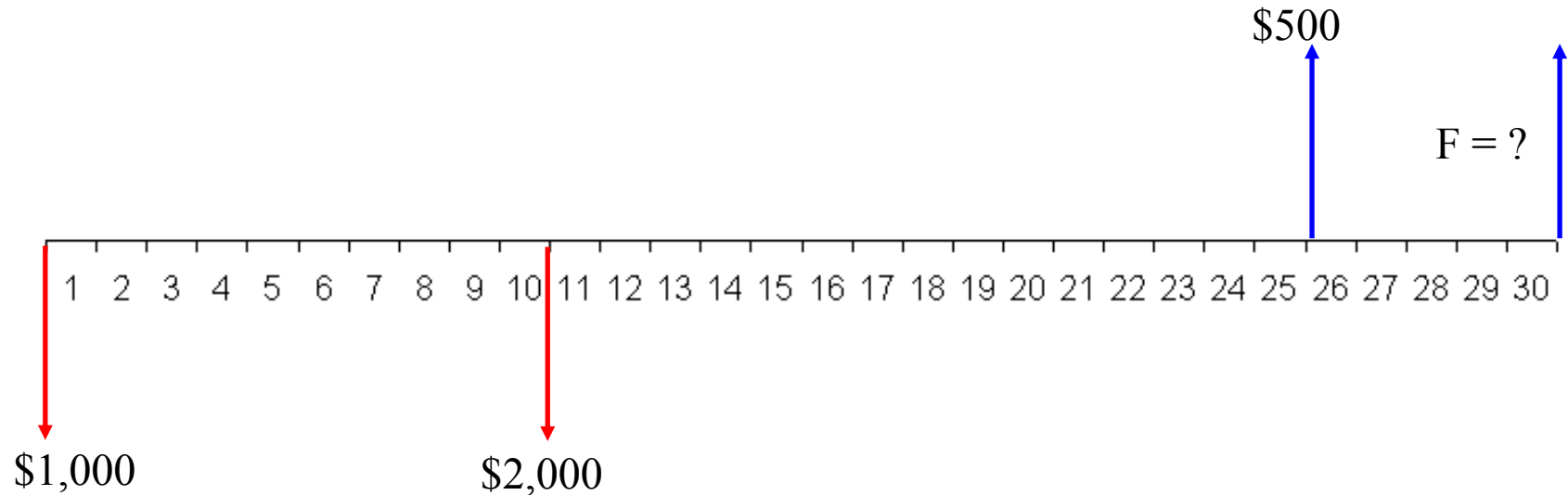
- [1] Since we have simple interest, then  $F = P(1+ni)$

$$\rightarrow F = \$1,000 \times (1 + 30 \times 0.003\%) = \$1,000.9$$



## CASH FLOW DIAGRAM – EXAMPLE [4]

○ [2]  $F = \$1,000 \times (1 + \underline{30} \times 0.003\%) +$   
 $\$2,000 \times (1 + \underline{20} \times 0.003\%) - \$500 \times (1 + \underline{5} \times 0.003\%) = \$2,502.03$



**Just keep in mind that the day is represented by its beginning**



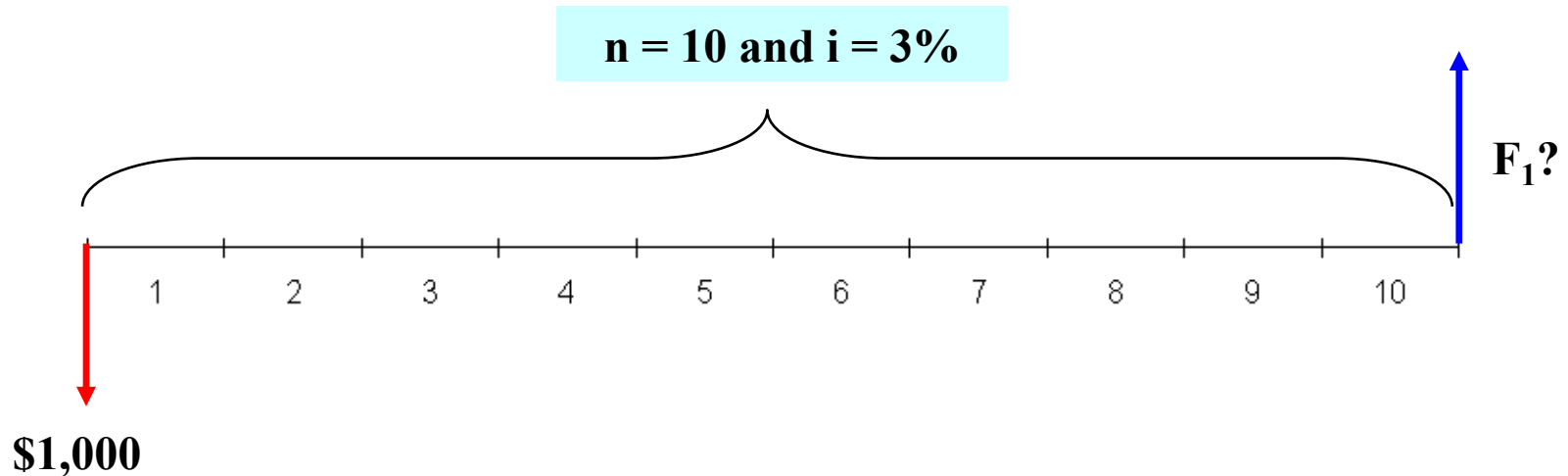
## CASH FLOW DIAGRAM – EXAMPLE [5]

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- You have deposited \$1,000 with an interest rate of 3% every 6 months where the interest is computed every 6 months.
- How much you will have after 5 years?
- Two years later after the initial deposit of the money, you deposited additional \$1,000 with an interest rate of 2% every 6 months (*applies only to this deposit*). How much will you have after 5 years?

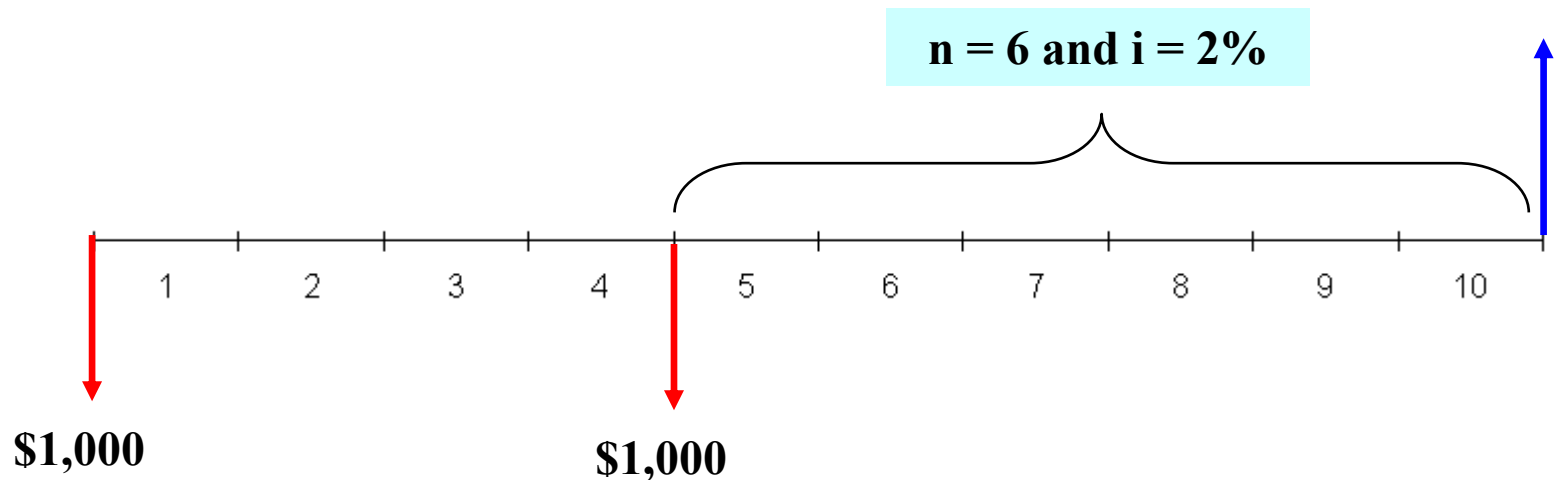
## CASH FLOW DIAGRAM – EXAMPLE [5]

- We have a total of 10 periods each period of 6 months for the \$1,000
- $F_1 = P(1+i)^n = \$1,000 \times (1+3\%)^{10} = \$1,343.92$



## CASH FLOW DIAGRAM – EXAMPLE [5]

- We have a total of 10 periods each period of 6 months for the first \$1,000 [F1].
- We have a total of 6 periods each period of 6 months for the second \$1,000 [F2].
- $F = F1 + F2 = \$1,000 \times (1 + 3\%)^{10} + \$1,000 \times (1 + 2\%)^6 = \$2,470.08$



## EXAMPLE [6]

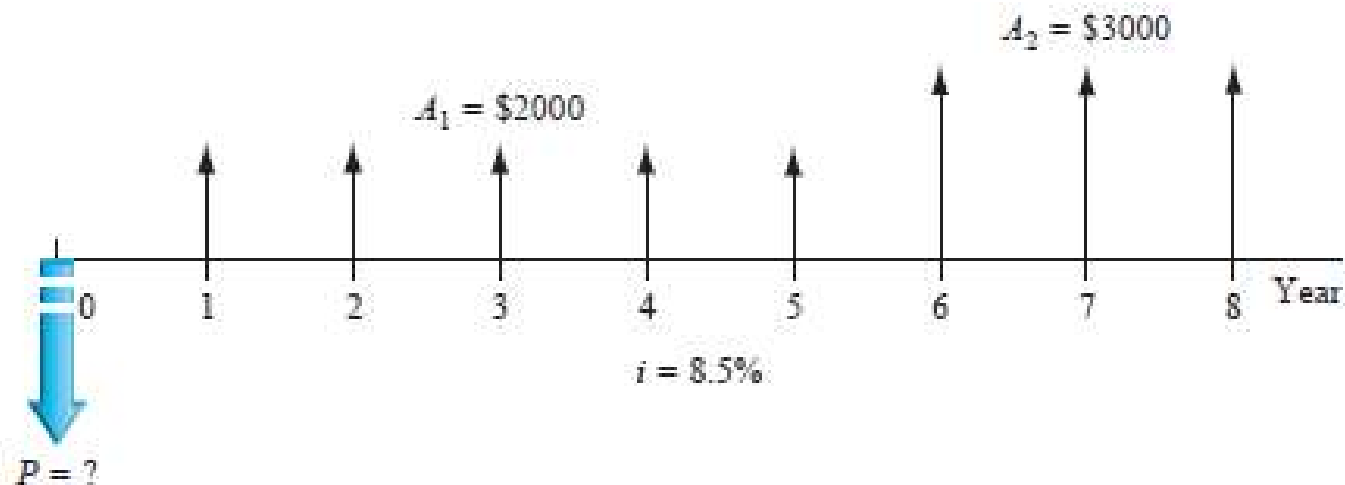
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- What would be the future worth after two years of a deposit of \$1,000 now if the interest rate for the first year is 10% and for the second year is 5%?
- By the end of the first year, the total amount becomes:  
 $1,000(1+10\%)^1 = \$1,100$
- By the end of the second year, the total amount becomes:  
 $1,100(1+5\%)^1 = \$1,155$

## EXAMPLE [7]

An electrical engineer wants to deposit an amount  $P$  now such that he can withdraw an equal annual amount of  $A_1 = \$2000$  per year for the first 5 years, starting 1 year after the deposit, and a different annual withdrawal of  $A_2 = \$3000$  per year for the following 3 years. How would the cash flow diagram appear if  $i = 8.5\%$  per year?

### Solution



Cash flow diagram with two different  $A$  series, Example 7