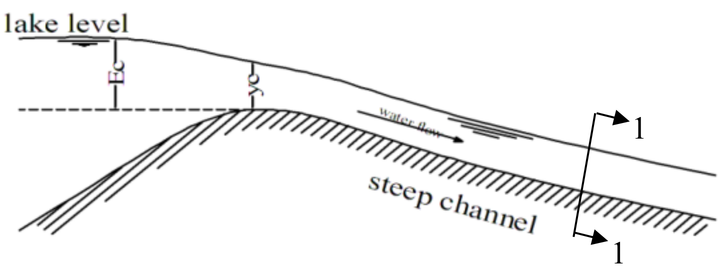
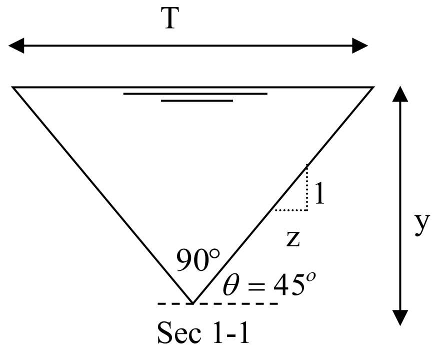
(1): A steep flume of triangular section and apex angle of 90^0 runs from a large tank in which the water surface level is 6.0m above the flume invert where it joins the tank. Find the discharge in the flume.

Sol:

Given: E_c= 6.0m , apex angle = $90^0 \Rightarrow \theta = 45^o \Rightarrow z = \frac{1}{\tan\theta} = \frac{1}{\tan 45} = 1.0$





A= $zy^2 = y^2$, T = 2zy = 2y

$$E_c = y_c + 0.5\frac{A_c}{T_c}$$

$$6 = y_c + 0.5\frac{y_c^2}{2y_c} \Rightarrow y_c = 0.48m.$$

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c}, \qquad \frac{Q^2}{9.81} = \frac{(y_c^2)^3}{2y_c}$$

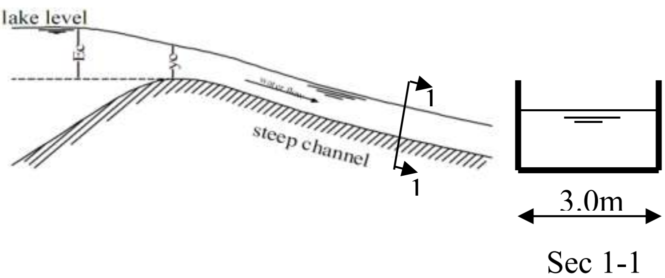
$$\frac{Q^2}{9.81} = \frac{(0.48^2)^3}{2*0.48} \Rightarrow Q = 0.35m^3/\text{sec.}$$

(2): Water flow from a lake into a steep rectangular channel 3.0m wide, and the lake level is 3.0m above the channel bed at the outfall. Find the discharge.

Sol:

Given: b = 3.0m, E_c = 3.0m.

$$E_c = 1.5y_c \Rightarrow y_c = \frac{2}{3}E_c = \frac{2}{3}*3 = 2.0m$$



$$y_c = \sqrt[3]{q^2/g}$$

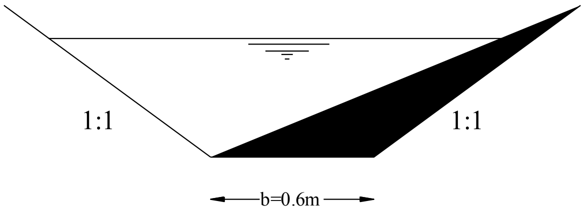
$$2.0 = \sqrt[3]{q^2/9.81}$$

$q = 8.86m^3/\text{sec}/m$ ⇨ $Q = q*b = 8.86*3 = 26.58m^3/\text{sec.}$

(3): A channel of trapezoidal cross section, width of base 0.6m and side slope 45^0 carries $0.34 m^3/sec$. Determine the critical depth.
Sol:

Given: b= 0.6m , Q = 0.34 m^3/sec

$$\Rightarrow \theta = 45^o \Rightarrow z = \frac{1}{\tan\theta} = \frac{1}{\tan 45} = 1.0$$



A = by + zy^2 = by + y^2 = 0.6y + y^2.

T = b + 2zy = b + 2y = 0.6+2y.

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

$$\frac{0.34^2}{9.81} = \frac{(0.6y_c + y_c^2)^3}{0.6 + 2y_c}$$

by using trial, we can get the critical depth (y_c) = 0.3m.

(4): A rectangular channel 4.0m wide and carries a discharge of $20m^3/sec$ at a depth 2.0m. At a certain location, it is proposed to build a hump. Calculate the water surface elevation at upstream of the hump and over the hump, if the hump height is:
_(a):_ 0.33m; and _(b):_ 2.0m. *(Assume no loss of energy at hump).*
Solution:

Given: b = 4.0m , Q = $20m^3/sec$, y_1 = 2.0m

$$q = \frac{Q}{b} = \frac{20}{4} = 5m^3/\sec/m.$$

$$f_{r1} = \frac{v_1}{\sqrt{gy_1}} = \frac{q}{\sqrt{gy_1^{\,3}}} = \frac{5}{\sqrt{9.81*2^3}} = 0.564 \prec 1.0 \Rightarrow$$ the flow is sub-critical.

Check of chocking br calculating Δz_c :

$$E_1 = E_c + \Delta z_c$$

$$E_1 = 1.5y_c + \Delta z_c$$

$$y_1 + \frac{q^2}{2gy_1^2} = 1.5y_c + \Delta z_c$$

$$y_c = \sqrt[3]{q^2/g} = \sqrt[3]{5^2/9.81} = 1.366m.$$

$$2+\frac{(5)^2}{2*9.81*2^2}=1.5*1.366+\Delta z_c$$

solving the equation and get $\Delta z_c = 0.269m.$

(a): for $\Delta z = 0.33m \succ \Delta z_c \Rightarrow$ chocking will occur.

$$E_1^{\backslash} = E_c + \Delta z_{given}$$

$$y_1^{\backslash} + \frac{q^2}{2gy_1^{\backslash 2}} = 1.5y_c + \Delta z_{given}$$

$$y_1^{\backslash} + \frac{(5)^2}{2*9.81*y_1^{\backslash 2}} = 1.5*1.366+0.33$$

يتم تحويل المعادلة إلي معادلة تكعيبية عن طريق ضرب الطرفين في $2*9.81*y_1^{\backslash 2}$

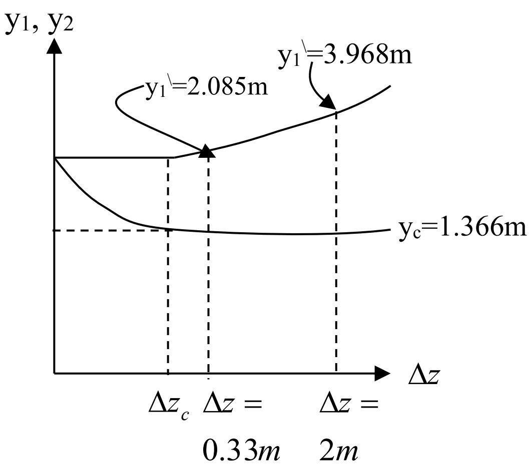
$$19.62y_1^{\backslash 3} - 46.676y_1^{\backslash 2} + 25 = 0.0$$

بعد حل هذه المعادلة سوف نأخذ الحل الموجب الأكبر و ذلك طبقا لحالة السريان المحددة سابقا و هي sub-critical flow و بالتالي يكون الحل الصحيح هو $y_1^{\backslash} = 2.085m.$

$\therefore at\ \Delta z = 0.33$

$u.s\ hump\ water\ depth \Rightarrow y_1^{\backslash} = 2.085m.$

$D.s\ hump\ water\ depth \Rightarrow y_2 = y_c = 1.366m.$



(b): for $\Delta z = 2.0m \succ \Delta z_c \Rightarrow$ chocking will occur.

$$E_1^{\backslash} = E_c + \Delta z_{given}$$

$$y_1^{\backslash} + \frac{q^2}{2gy_1^{\backslash 2}} = 1.5y_c + \Delta z_{given}$$

$$y_1^{\backslash} + \frac{(5)^2}{2*9.81*y_1^{\backslash 2}} = 1.5*1.366+2.0$$

يتم تحويل المعادلة إلي معادلة تكعيبية عن طريق ضرب الطرفين في $2*9.81*y_1^{\backslash 2}$

$$19.62y_1^{\backslash 3} - 79.44y_1^{\backslash 2} + 25 = 0.0$$

بعد حل هذه المعادلة سوف نأخذ الحل الموجب الأكبر و ذلك طبقا لحالة السريان المحددة سابقا و هي sub-critical flow و بالتالي يكون الحل الصحيح هو $y_1^{\backslash} = 3.968m.$

$\therefore at\ \Delta z = 2.0m$

$u.s\ \ hump\ \ water\ depth \Rightarrow y_1^{\backslash} = 3.968m.$

$D.s\ \ hump\ \ water\ depth \Rightarrow y_2 = y_c = 1.366m.$

(5): A uniform flow of 12m^3/sec occurs in a long rectangular channel of 5.0m width and 1.5m water depth. A flat hump is to be build at a certain section. Assuming head loss equal to the upstream velocity head, compute the minimum height at hump provide critical flow. What will happen: *(a):* if the height of hump is higher than that causing critical flow. *(b):* if the energy loss is less than the assumed value.

Sol:

Given: b = 5.0m , Q = 12m^3/sec , y1 = 1.5m, $h_L = \frac{v_1^2}{2g} = \frac{q^2}{2gy_1^2}$

$q = \frac{Q}{b} = \frac{12}{5} = 2.4m^3/\sec/m.$

from figure, $E_1 = E_c + \Delta z_c + h_L$

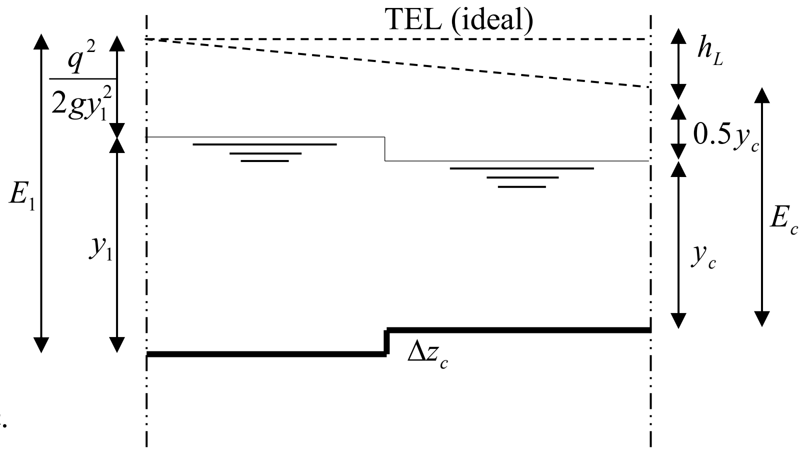
$E_1 = 1.5y_c + \Delta z_c + h_L$

$y_1 + \frac{q^2}{2gy_1^2} = 1.5y_c + \Delta z_c + \frac{q^2}{2gy_1^2}$

$y_1 = 1.5y_c + \Delta z_c$

$y_c = \sqrt[3]{q^2/g} = \sqrt[3]{2.4^2/9.81} = 0.837m.$

$1.5 = 1.5 * 0.837 + \Delta z_c \Rightarrow \Delta z = 0.244m.$



(a): if $\Delta z \succ \Delta z_c \Rightarrow$ chocking will occur.

(b): $h_L \prec h_{Lgiven} \Rightarrow \Delta z_c$ *will increased.*

(6): A uniform flow of 20m^3/sec occurs in a rectangular channel of 5.0 m width and 2.5m water depth. A smooth hump of height 0.5m is placed in the bottom of the channel. *Determine:*

(a): The difference in water levels before and at the hump;

(b): The height of hump to produce critical depth on it, and the drop in water level;

(c): Draw a relationship between y_1 (u.s. the hump), y_2 (at the hump) versus Δz (the hump size);
(d): What the effect of increasing the height of hump to 1.0m on the water levels; and

Sol:

Given: $b_1 = 5.0$ m , $Q = 20$ m^3/sec, $y_1 = 2.5$ m,

$\Delta z = 0.5$ m.

$$Fr_1 = \frac{v_1}{\sqrt{g y_1}},\ v_1 = \frac{Q}{b.y} = \frac{20}{5*2.5} = 1.6 m/\text{sec.}$$

$$Fr_1 = \frac{1.6}{\sqrt{9.81*2.5}} = 0.32 \prec 1$$

$\therefore$ The flow is sub-critical.

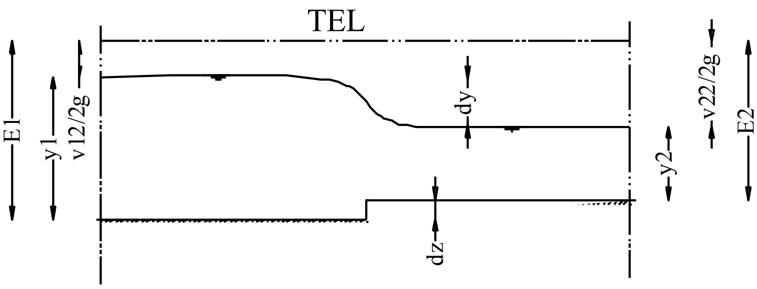
$\therefore$ according to both specific energy and specific discharge diagrams y_2 decreases.

(a): $E_1 = E_2 + \Delta z$

$$y_1 + \frac{v_1^2}{2g} = y_2 + \frac{v_2^2}{2g} + \Delta z$$

$$y_1 + \frac{v_1^2}{2g} = y_2 + \frac{q^2}{2g y_2^2} + \Delta z$$

$$2.5 + \frac{(1.6)^2}{2*9.81} = y_2 + \frac{\left(20/5\right)^2}{2*9.81*y_2^2} + 0.5$$



$$19.62 y_2^3 - 41.8 y_2^2 + 16 = 0.0$$

by trial and error, $y_2 \cong 1.9 m.$

$$\therefore y_1 = \Delta y + y_2 + \Delta z$$

$$\therefore 2.5 = \Delta y + 1.9 + 0.5$$

$$\boxed{\therefore \Delta y = 0.1 m.}$$

$\Rightarrow$ *(b): For critical flow occurs,* $E_1 = E_c + \Delta z_c$

$$E_1 = 1.5 y_c + \Delta z_c$$

$$y_c = \sqrt[3]{q^2/g} = \sqrt[3]{(20/5)^2/9.81} = 1.178m.$$

$$2.5 + \frac{(1.6)^2}{2*9.81} = 1.5*1.178 + \Delta z_c$$

$$\therefore \Delta z_c = 0.865m$$

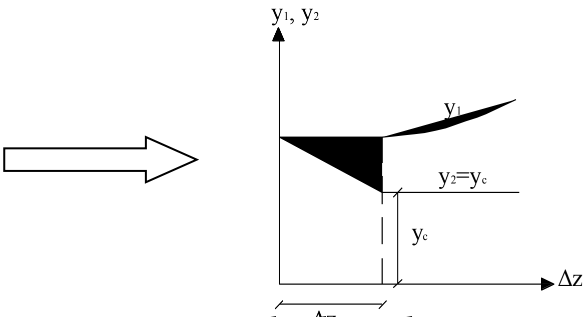
$$\because y_1 = \Delta y + y_c + \Delta z_c$$

$$2.5 = \Delta y + 1.178 + 0.865$$

$$\boxed{\therefore \Delta y = 0.458m.}$$

⇒ *(c): The relationship between y_1&y_2 versus Δz:*

For $Fr \prec 1$



⇒ *(d): For $\Delta z_{given} = 1.0m \succ \Delta z_c$, chocking phenomenon occurs.*

$$E_1^{\backslash} = E_c + \Delta z_{given}$$

$$y_1^{\backslash} + \frac{v_1^{\backslash 2}}{2g} = 1.5y_c + \Delta z_{given}$$

$$y_1^{\backslash} + \frac{q^2}{2gy_1^{\backslash 2}} = 1.5y_c + \Delta z_{given}$$

$$y_1^{\backslash} + \frac{\left(20/5\right)^2}{2*9.81*y_1^{\backslash 2}} = 1.5*1.178 + 1.0$$

$$19.62y_1^{\backslash 3} - 54.26y_1^{\backslash 2} + 16 = 0.0$$

by trial and error, $\boxed{y_1^{\backslash} \cong 2.65m.}$

(7): A uniform flow of 20m^3/sec occurs in a rectangular channel of 5.0m width and 2.5m water depth. The channel bed is gradually contracted to a width of 3.0m. *Find:*

(a): The difference in water levels just before and after the contraction;
(b): The width of contraction to produce critical depth on it, and the drop in water levels;

sol:

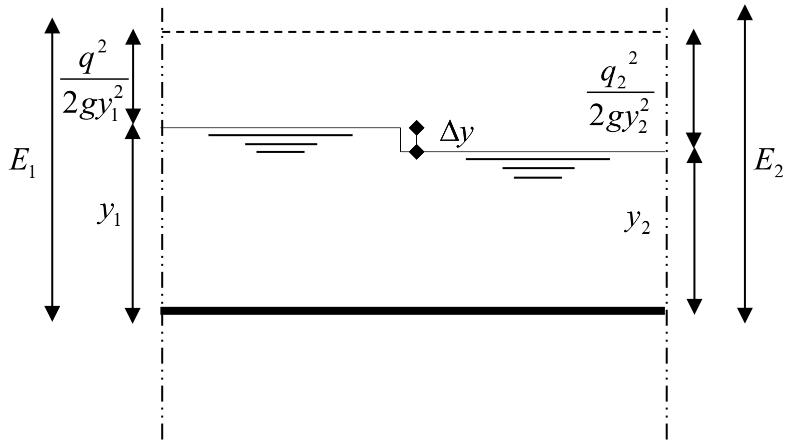
Given: Q = 20m³/sec , b_1 = 5.0m , b_2 = 3.0m , y_1 = 2.5m.

$$q_1 = \frac{Q}{b_1} = \frac{20}{5} = 4.0 m^3/\sec/m.$$

$$q_2 = \frac{Q}{b_2} = \frac{20}{3} = 6.67 m^3/\sec/m.$$

$$f_{r1} = \frac{v_1}{\sqrt{gy_1}} = \frac{q}{\sqrt{gy_1^{\,3}}} = \frac{4}{\sqrt{9.81*2.5^3}} = 0.32 \prec 1.0 \Rightarrow \therefore$$ the flow is sub-critical.

(a): $E_1 = E_2$



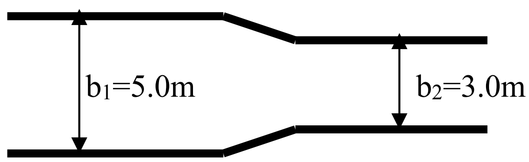
$$y_1 + \frac{v_1^{\,2}}{2g} = y_2 + \frac{v_2^2}{2g}$$

$$y_1 + \frac{q_1^{\,2}}{2gy_1^2} = y_2 + \frac{q_2^{\,2}}{2gy_2^2}$$

$$2.5 + \frac{(4)^2}{2*9.81*2.5^2} = y_2 + \frac{(6.67)^2}{2*9.81*y_2^2}$$

$$19.62y_2^3 - 51.61y_2^2 + 44.44 = 0.0$$

by trial and error, $y_2 \cong 2.13m.$



$$\therefore y_1 = \Delta y + y_2$$

$$\therefore 2.5 = \Delta y + 2.13$$

$$\boxed{\therefore \Delta y = 0.37m.}$$

(b): *For critical flow occurs,* $E_1 = E_c$

$$E_1 = 1.5y_c$$

$$2.5 + \frac{(4)^2}{2*9.81*2.5^2} = 1.5y_c \Rightarrow y_c = 1.753m.$$

$$y_c = \sqrt[3]{q_c^{\,2}/g}$$

$$1.753 = \sqrt[3]{q_c^{\,2}/9.81} \Rightarrow q_c = 7.27m^3/\sec/m.$$

$$b_c = \frac{Q}{q_c} = \frac{20}{7.27} = 2.75m.$$

$$\because y_1 = \Delta y + y_c$$

$$2.5 = \Delta y + 1.753 \Rightarrow \Delta y = 0.747m.$$

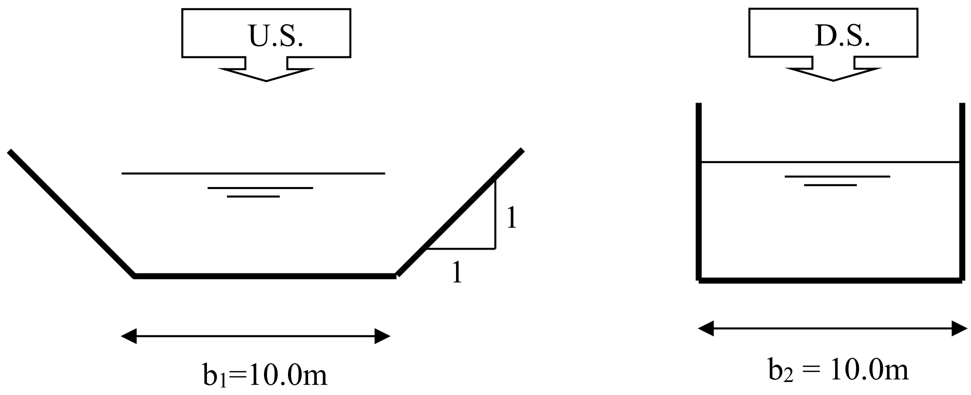
(8): A trapezoidal canal 10.0m bed width, 1:1 side slope, 3.0m water depth and carries a discharge of $25m^3$/sec. The canal is constricted by raising the sides to vertical positions. *Calculate:*

(a): The depth of water in the **constriction**; and

(b): The minimum height of **hump** which may be installed in the constriction to produce critical depth there.

Sol:

Given: Q = $25m^3$/sec , y_1 = 3.0m.



$$A_1 = b_1 y_1 + z y_1^2 = 10*3 + 1*3^2 = 39m^2$$

$$T_1 = b_1 + 2zy_1 = 10 + 2*1*3 = 16m.$$

$$Fr_1 = \frac{v_1}{\sqrt{gD_1}},\ D_1 = \frac{A_1}{T_1} = \frac{39}{16} = 2.4375m$$

$$Fr_1 = \frac{25/39}{\sqrt{9.81*2.4375}} = 0.13 \prec 1 \Rightarrow$$ the flow is sub-critical.

(a): $E_1 = E_2$

$$y_1 + \frac{v_1^2}{2g} = y_2 + \frac{v_2^2}{2g}$$

$$y_1 + \frac{Q^2}{2gA_1^2} = y_2 + \frac{q_2{}^2}{2gy_2^2}$$

$$q_2 = \frac{Q}{b_2} = \frac{25}{10} = 2.5m^3/\sec/m$$

$$3 + \frac{(25)^2}{2*9.81*39^2} = y_2 + \frac{(2.5)^2}{2*9.81*y_2^2}$$

$$19.62y_2^3 - 59.27y_2^2 + 6.25 = 0.0$$

by trial and error, $y_2 \cong 2.985m.$

(b): $E_1 = E_c + \Delta z_c$

$$y_1 + \frac{Q^2}{2gA_1^2} = 1.5y_c + \Delta z_c$$

$$y_c = \sqrt[3]{q_2{}^2/g} = \sqrt[3]{2.5^2/9.81} = 0.86m$$

$$3 + \frac{25^2}{2*9.81*39^2} = 1.5*0.86 + \Delta z_c$$

$$\therefore \Delta z_c = 1.73m.$$

(9): A 50 wide rectangular channel is carrying a flow of 250m^3/sec at a flow depth of 5m. To produce a **critical flow** in this channel, *determine:*

(a): The height of step in the channel bottom if the width remains constant;

(b): The reduction in the channel width if the channel bottom level remain unchanged;

(c): A combination of the width reduction and the bottom step; and

(d): what is the minimum channel width without affecting the upstream water level.

Sol:

Given: Q = 250m^3/sec , y_1 = 5m , b_1 = 50m.

$$q_1 = \frac{Q}{b_1} = \frac{250}{50} = 5m^3/\sec/m$$

(a): For critical flow occurs, $E_1 = E_c + \Delta z_c$

$$E_1 = 1.5y_c + \Delta z_c$$

$$y_c = \sqrt[3]{q^2/g} = \sqrt[3]{5^2/9.81} = 1.366m.$$

$$y_1 + \frac{q_1^{\ 2}}{2*9.81*y_1^2} = 1.5y_c + \Delta z_c$$

$$5 + \frac{5^2}{2*9.81*5^2} = 1.5*1.366 + \Delta z_c$$

$$\therefore \Delta z_c = 3.0m$$

(b): *For critical flow occurs,* $E_1 = E_c$

$$E_1 = 1.5y_c$$

$$5 + \frac{(5)^2}{2*9.81*5^2} = 1.5y_c \Rightarrow y_c = 3.367m.$$

$$y_c = \sqrt[3]{q_c^{\ 2}/g}$$

$$3.367 = \sqrt[3]{q_c^{\ 2}/9.81} \Rightarrow q_c = 19.35m^3/\sec/m.$$

$$b_c = \frac{Q}{q_c} = \frac{250}{19.35} = 12.92m.$$

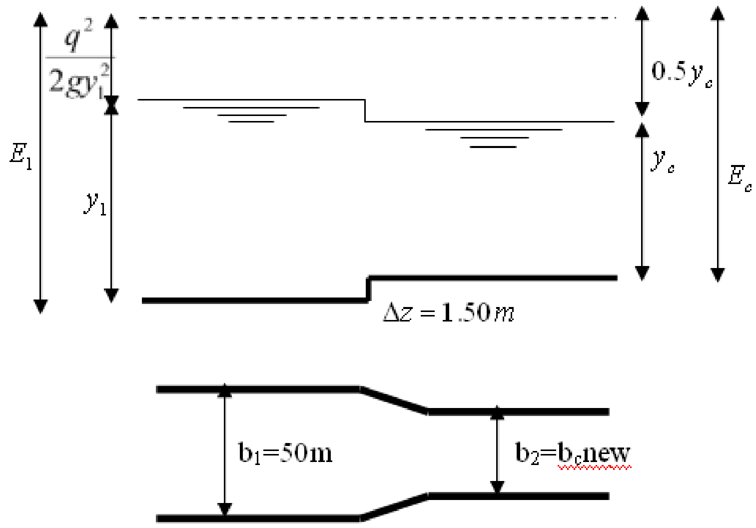
(c): assume $\Delta z = 1.50m$ then we can calculate the value of ($b_c$new) which produce critical flow with the assumed hump:

$$E_1 = E_c + \Delta z_c$$

$$E_1 = 1.5y_c + 1.5$$

$$5 + \frac{(5)^2}{2*9.81*5^2} = 1.5y_c + 1.5$$

$$\Rightarrow y_c = 2.367m.$$



$$y_c = \sqrt[3]{q_c^{\ 2}/g}$$

$$2.367 = \sqrt[3]{q_c^{\ 2}/9.81}$$

$$\Rightarrow q_c = 11.408m^3/\sec/m.$$

$$b_c new = \frac{Q}{q_c} = \frac{250}{11.408} = 21.91m.$$

$\therefore$ to produce critical flow, we use b_2 = 21.91m and $\Delta z = 1.50m.$

(d): the minimum width required for no change in upstream = b_c = 12.92m

وهي نفسها المحسوبة من المطلوب رقم 2 في المسألة.