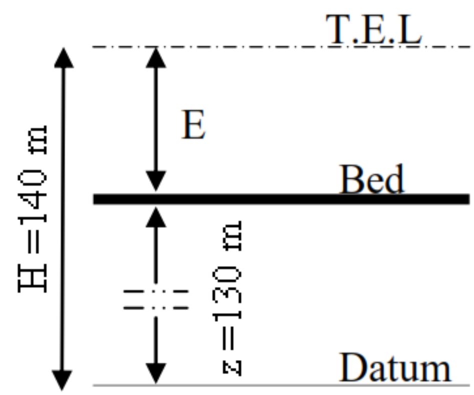
Example

In a horizontal rectangular channel of bed width 30 m, the energy line is at an elevation of 140 m. While the bed elevation is 130 m. Calculate the two possible depths if the discharge is 450 m^3/sec. If a hydraulic jump occurred in this channel, find the water depth and the elevation of energy line just after the jump.

Solution

Given: b = 30 m, H = 140 m, z = 130 m, and Q = 450 m^3/sec

E = H – z = 140 – 130 = 10 m



$$q = \frac{Q}{b} = \frac{450}{30} = 15 m^3/\sec/m$$

$$E = y + \frac{V^2}{2g} \Rightarrow E = y + \frac{q^2}{2gy^2}$$

$$\therefore 10 = y + \frac{(15)^2}{2*9.81*y^2}$$

$$\therefore y^3 - 10y^2 + 11.46 = 0.0$$

<u>by trial and error:</u>

<u>*y_1 = 1.14 m.*</u>

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{450/(30*1.14)}{\sqrt{9.81*1.14}} = 3.93 > 1.0$$

then, the flow is supercritical.

<u>*y_2 = 9.883m.*</u>

$$Fr_2 = \frac{v_2}{\sqrt{gy_2}} = \frac{450/(30*9.88)}{\sqrt{9.81*9.883}} = 0.154 < 1.0$$

then, the flow is subcritical.

Hydraulic jump occurs when ***Fr_1* is greater than 1.0**. Then, y_1 is taken equal to 1.14 m and y_2 is calculated as follows.

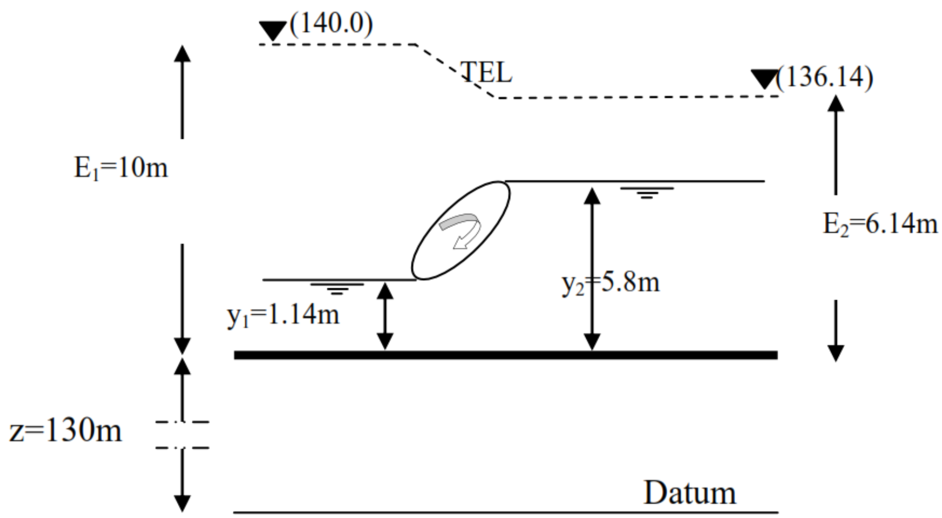
$$\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{8Fr_1^2 + 1} - 1\right]$$

$$\frac{y_2}{1.14} = \frac{1}{2}\left[\sqrt{8*3.934^2 + 1} - 1\right] \Rightarrow get\ y_2 = 5.8\ \ m$$

$$E_2 = y_2 + \frac{q^2}{2gy_2{}^2}$$

$$E_2 = 5.8 + \frac{15^2}{2*9.81*5.8^2} = 6.14\ \ m$$

Energy line elevation after the jump = E_2 + z = 6.14 + 130 = 136.14 m

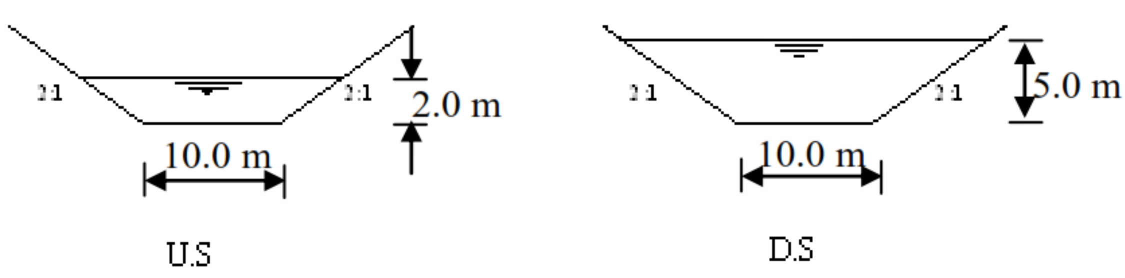


Example

A hydraulic jump is formed in a horizontal open channel of trapezoidal section. The bed width is 10.0 m and the side slopes are 2:1. The two conjugate depths are 2.0 m and 5.0 m. calculate the discharge passing, the power dissipated by the jump in horse power.

Solution

Given: b = 10.0 m, y_1 = 2.0 m, y_2 = 5.0 m.



(a): $M_1 = M_2$

$$\frac{Q^2}{gA_1} + A_1 y_1' = \frac{Q^2}{gA_2} + A_2 y_2'$$

$$\frac{Q^2}{9.81*(10*2+2*2^2)} + \left[10*2*1+0.5*4*2*\frac{2}{3}*2\right] =$$

$$\frac{Q^2}{9.81*(10*5+2*5^2)} + \left[10*5*2.5+0.5*10*5*\frac{5}{3}*2\right]$$

$$\frac{Q^2}{274.68} + 25.333 = \frac{Q^2}{981} + 208.333$$

by trial and error: $Q = 264.2 \quad m^3/\sec$

(b): $E_1 = y_1 + \dfrac{V_1^2}{2g}$, $v_1 = \dfrac{Q}{A_1} = \dfrac{264.2}{10*2+2*2^2} = 9.4 \quad m/s$

$$\therefore E_1 = 2 + \frac{9.4^2}{2*9.81} = 6.54 \quad m$$

$$E_2 = y_2 + \frac{V_2^{\,2}}{2g}\ ,\ v_2 = \frac{Q}{A_2} = \frac{264.2}{10*5+2*5^2} = 2.6 \quad m/s$$

$$\therefore E_2 = 5 + \frac{2.6^2}{2*9.81} = 5.34m$$

$$\Delta E = E_1 - E_2 = \left(y_1 + \frac{v_1^{\,2}}{2g}\right) - \left(y_2 + \frac{v_2^2}{2g}\right) = 6.54 - 5.34 = 1.2 \quad m$$

$$\therefore \text{ in horse power } = \frac{\gamma Q \Delta E}{75} = \frac{1000*264.2*1.2}{75} = 4227.6 \quad h.p$$

Example

A hydraulic jump is formed in a horizontal smooth rectangular open channel, the bed width is 10 m and the two conjugate depths are 2 m and 5 m. Calculate the discharge passing, the power dissipated by the jump in HP, the relative sequent depth, the overall efficiency of the hydraulic jump, and estimation for the length of hydraulic jump.

Solution

Given: b = 10 m, y_1 = 2.0 m, and y_2 = 5.0 m

(a): $M_1 = M_2 \Rightarrow \frac{y_1^2}{2} + \frac{q^2}{gy_1} = \frac{y_2^2}{2} + \frac{q^2}{gy_2}$

$$\frac{2^2}{2} + \frac{q^2}{9.81*2} = \frac{5^2}{2} + \frac{q^2}{9.81*5} \Rightarrow q = 18.53 m^3/\sec/m$$

$$\therefore Q = q*b = 18.53*10 = 185.3 m^3 /\sec.$$

(b): $E_1 = y_1 + \frac{q^2}{2gy_1^{\,2}} = 2 + \frac{18.53^2}{2*9.81*2^2} = 6.38 \quad m$

$$E_2 = y_2 + \frac{q^2}{2gy_2^{\,2}} = 5 + \frac{18.53^2}{2*9.81*5^2} = 5.7m$$

$$\therefore \Delta E = E_1 - E_2 = 6.38 - 5.7 = 0.68m$$

$$p = \frac{\gamma Q \Delta E}{75} = \frac{1000*185.3*0.68}{75} = 1680 \quad h.p$$

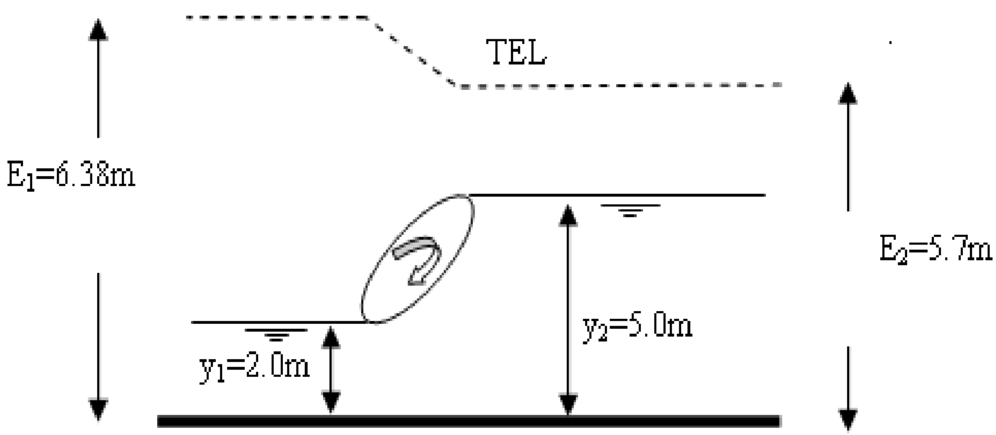
(c): $\text{Re}\,lative\ loss = \frac{\Delta E}{E_1} = \frac{0.68}{6.38} = 0.104$

(d): $\text{Re}\,lative\ sequent\ depth = \frac{y_2}{E_1} = \frac{5}{6.38} = 0.785$

(e): $efficiency\ of\ h.J = \frac{E_2}{E_1} = \frac{5.7}{6.38} = 0.896$

(f): $L_j = 6.91(y_2 - y_1) = 6.91(5-2) = 20.7 \quad m$

or $L_j = 6.1*y_2 = 6.1*5 = 30.5\ m$ (take the largest in design)

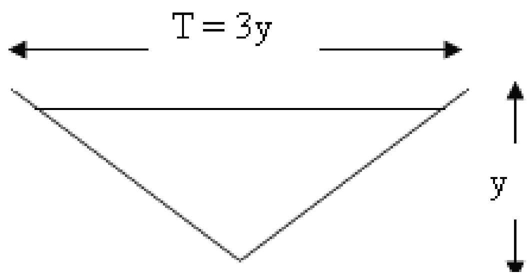


Example: A triangular channel whose top width is three times the water depth, Manning's n = 0.02, conveys a discharge of 3.32 m^3/sec, Find the critical depth and critical slope. If the discharge passes at depth of 0.6 m, find the conjugate depth if the hydraulic jump takes place. Calculate also the horsepower lost through the jump.

Solution:

Given: n = 0.02, Q = 3.32 m^3/sec, and y_1 = 0.6 m

$T = 2zy = 3y \Rightarrow z = 1.5.$



$A_c = zy_c^2 = 1.5y_c^2$

$T_c = 3y_c$

$p_c = 2y_c\sqrt{z^2+1} = 2y_c\sqrt{1.5^2+}$

$$R_c = \frac{A_c}{p_c} = \frac{1.5y_c^2}{3.6y_c} = 0.417y_c$$

$$D_c = \frac{A_c}{T_c} = \frac{1.5y_c^2}{3y_c} = 0.5y_c$$

(a): $\frac{Q^2}{g} = \frac{A_c^3}{T_c} \Rightarrow \frac{3.32^2}{9.81} = \frac{\left(1.5y_c^2\right)^3}{3y_c} \Rightarrow y_c \cong 1.0 \quad m$

(b): $v_c = \frac{1}{n} * R_c^{2/3} * s_c^{1/2}$

$$V_c = \sqrt{g * D_c} = \sqrt{g * 0.5y_c} = \sqrt{9.81*0.5*1} = 2.21 \quad m/\sec$$

$$2.21 = \frac{1}{0.02} * (0.417*1.0)^{2/3} * s_c^{1/2} \Rightarrow s_c = 0.0063$$

(c): $M_1 = M_2$

$$\frac{Q^2}{gA_1} + A_1 y_1' = \frac{Q^2}{gA_2} + A_2 y_2'$$

$$\frac{Q^2}{gA_1} + \left[\frac{zy_1^3}{3}\right] = \frac{Q^2}{gA_2} + \left[\frac{zy_2^3}{3}\right]$$

$$\frac{3.32^2}{9.81*(1.5*0.6^2)} + \left[\frac{1.5*0.6^3}{3}\right] = \frac{3.32^2}{9.81*(1.5y_2^2)} + \left[\frac{1.5y_2^3}{3}\right]$$

by trial and error get $y_2 = 1.5$ m

(d): $E_1 = y_1 + \frac{Q^2}{2gA_1^2} = 0.6 + \frac{3.32^2}{2*9.81*(1.5*0.6^2)^2} = 2.53 \quad m$

$$E_2 = y_2 + \frac{Q^2}{2gA_2^2} = 1.5 + \frac{3.32^2}{2*9.81*(1.5*1.5^2)^2} = 1.55 \quad m$$

$$\therefore \Delta E = E_1 - E_2 = 2.53 - 1.55 = 0.98 \quad m$$

$$p = \frac{\gamma Q \Delta E}{75} = \frac{1000*3.32*0.98}{75} = 43.38 \quad h.p$$