Ch.3 Graphical Optimization

PART - 1

ME511 – Principle of Optimum Design

Lecturer :

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Introduction

- The ***graphical method*** is applicable and recommended to solve optimization problems having **one or two design variables ONLY**
- Graphical optimization method *helps enhance our understanding of the underlying problem* which may not be forthcoming in the case of mathematical and computational optimization methods
- The method involves plotting contours of the objective function over a feasible region enclosed by the constraint boundaries

Procedures of Graphical Optimization Method

- The technique is quite straight forward and simple
  1. Plot all constraint functions
  2. Identify set of feasible solutions (***feasible region***)
  3. Draw the objective function contours* and determine the optimum design by visual inspection

* A ***contour*** is a curve on the graph that connects all points having *the same objective function value*.

Procedures of Graphical Optimization Method

Consider the following example:

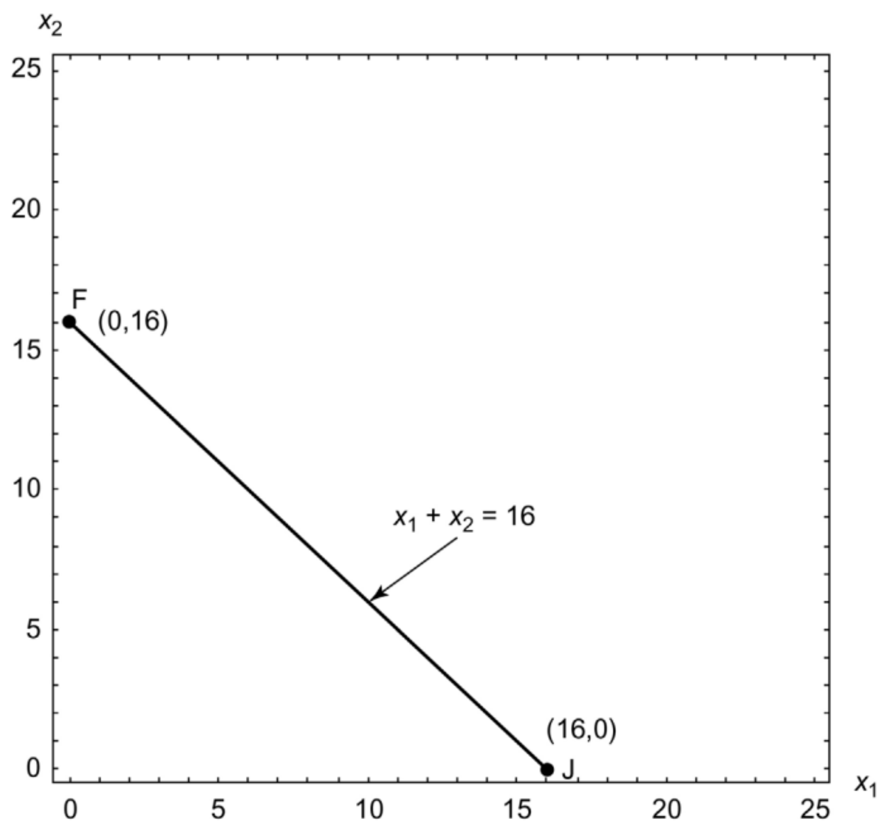
“***Profit Maximization Problem***”

- A company manufactures two machines, A and B. Using available resources, either 28 A or 14 B can be manufactured daily.
- The sales department can sell up to 14 A machines or 24 B machines.
- The shipping facility can handle no more than 16 machines per day.
- The company makes a profit of SR400 on each A machine and SR600 on each B machine.
- How many A and B machines should the company manufacture every day to maximize its profit?

Procedures of Graphical Optimization Method

Steps	Description	Mathematical Statement	Unit
1. Objective Function	Maximize the profit	$f(\boldsymbol{x}) = 400x_1 + 600x_2$	SR
2. Design Variables	Number of machines	x_1 & x_2	pcs
3. Constraints	shipping and handling constraint	$\boldsymbol{g_1}(\boldsymbol{x}) = x_1 + x_2 \leq 16$	N/A
	manufacturing constraint	$\boldsymbol{g_2}(\boldsymbol{x}) = \frac{x_1}{28} + \frac{x_2}{14} \leq 1$	N/A
	limitation on sale department	$\boldsymbol{g_3}(\boldsymbol{x}) = \frac{x_1}{14} + \frac{x_2}{24} \leq 1$	pcs
	Nonnegative constraint	$\boldsymbol{g_4}(\boldsymbol{x}) = x_1 \geq 0$ $\boldsymbol{g_5}(\boldsymbol{x}) = x_2 \geq 0$	pcs

Step 1 – Constraint Boundary Plot



Plot the constraint by locate both of *x*-intercept and *y*-intercept

- *x*-intercept when $x_2 = 0$

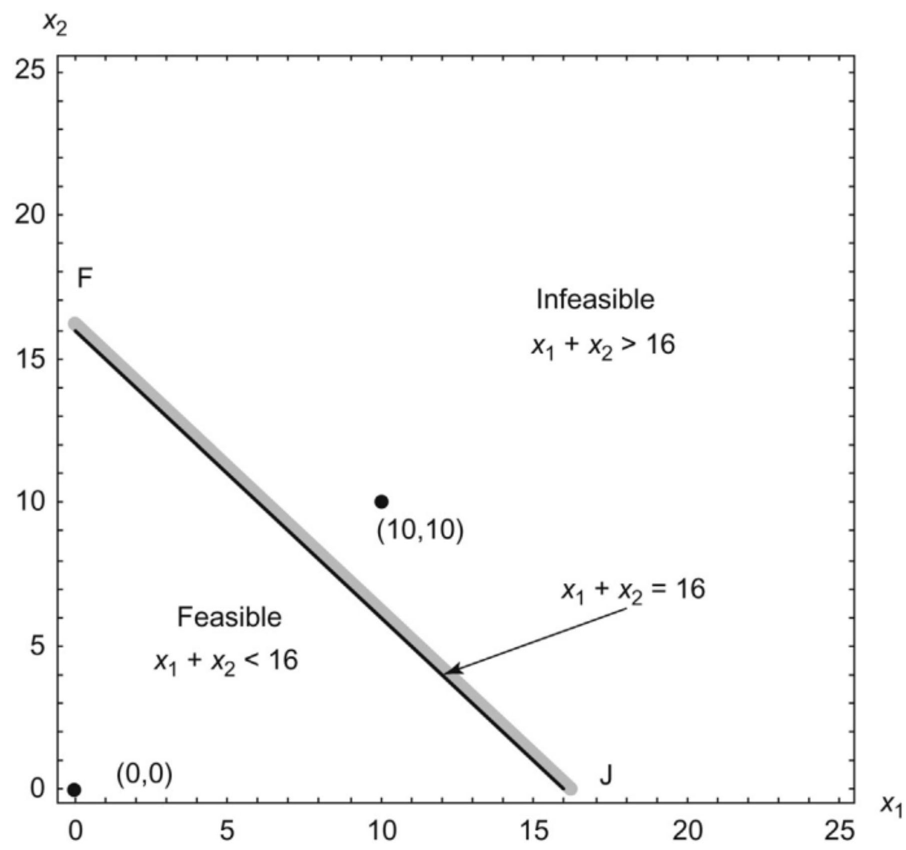
  $x_1 + x_2 = 16 \rightarrow x_1 = 16$

- *y*-intercept when $x_1 = 0$

  $x_1 + x_2 = 16 \rightarrow x_2 = 16$

Constraint boundary for the inequality $x_1 + x_2 \leq 16$

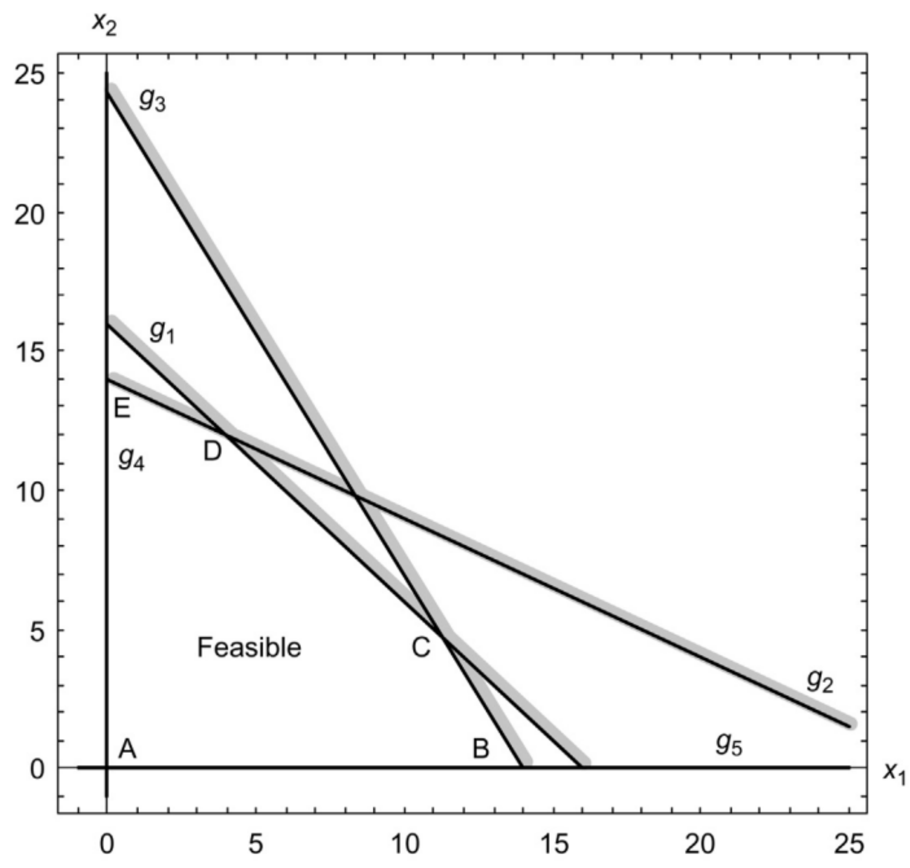
Step 2 – Identify Feasible Region



- Choose arbitrary point on the left or right side of the constraint, i.e point (0,0) and (10,10)
- Point (0,0) has a value of 0 → ***satisfied*** (represent **feasible region**)
- Point (10,10) has a value of 20 → ***violated*** (represent **infeasible region**)

Feasible/infeasible side for the inequality $x_1 + x_2 \leq 16$

Step 2 – Identify Feasible Region (con't)

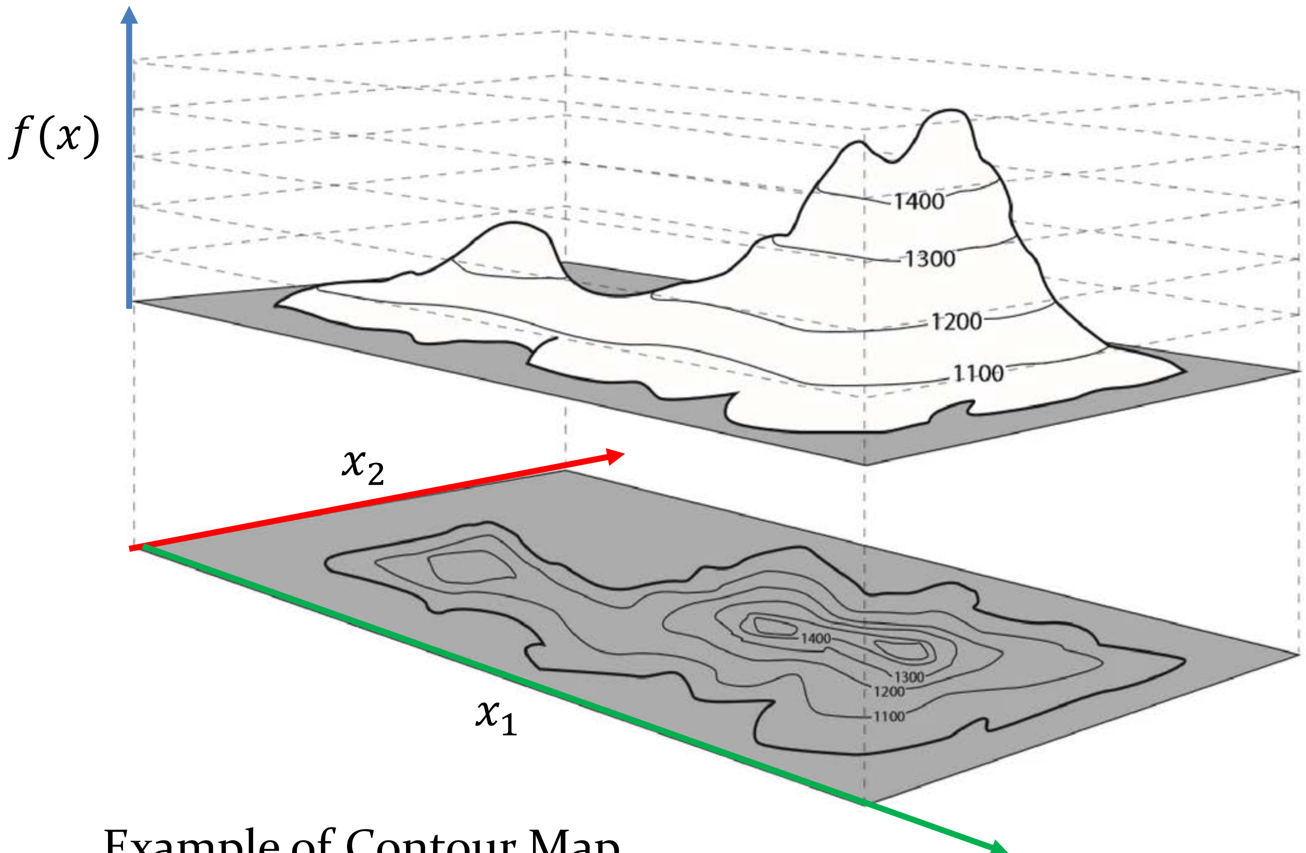


Feasible region for the profit maximization problem

- Repeat the same process for other constraints

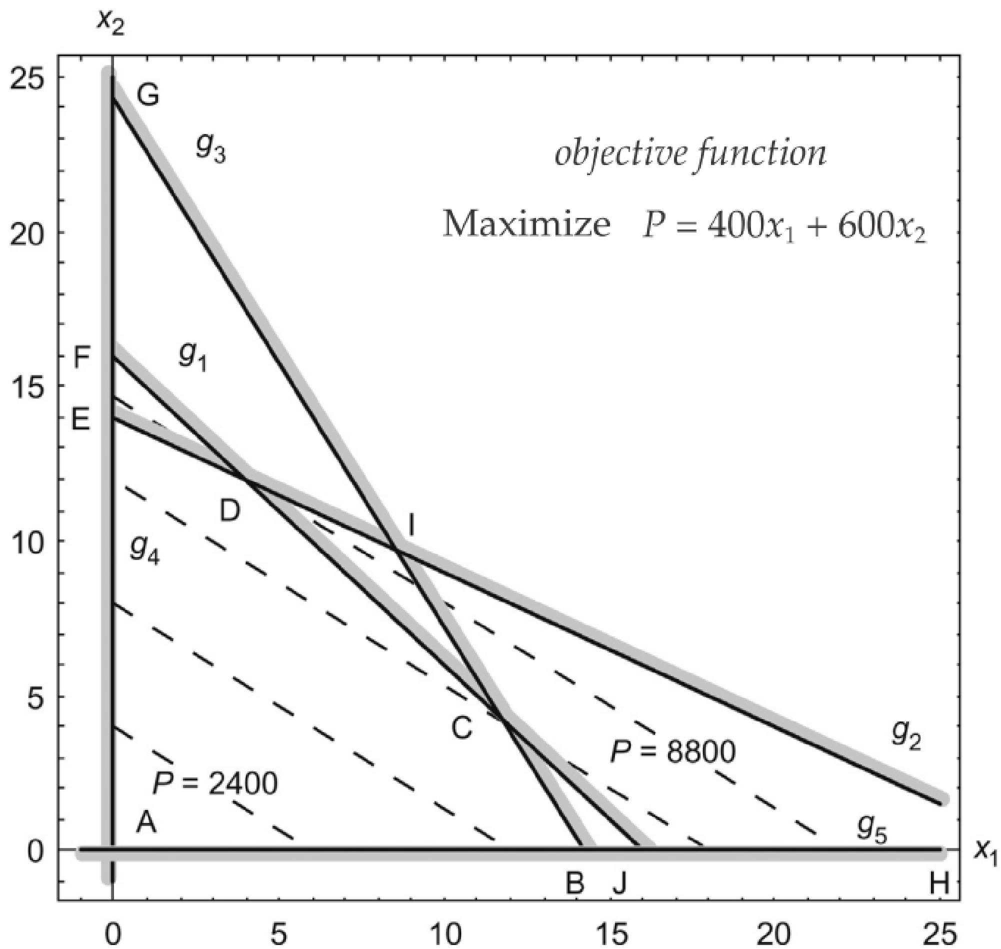
Constraints
$g_1(x) = x_1 + x_2 \le 16$
$g_2(x) = \frac{x_1}{28} + \frac{x_2}{14} \le 1$
$g_3(x) = \frac{x_1}{14} + \frac{x_2}{24} \le 1$
$g_4(x) = x_1 \ge 0$
$g_5(x) = x_2 \ge 0$

Step 3 – Plot the Objective Function Contours



Example of Contour Map

Step 3 – Plot the Objective Function Contours



Select any point in the feasible region and evaluate the profit function there.

For example, at point (3,2), the profit function is calculated as $P = (3 \times 400) + (2 \times 600) = 2400$.

To plot the $P = 2400$ contour, we plot the function

$$400x_1 + 600x_2 = 2400$$

Repeat the same process for the 2nd point (i.e point(6,4))and then see the trend. From the observation we found point D gives the max profit *P = 8800 SR*

END OF THE SLIDES