Q1

Given $f(x) = e^x$, for x = 0, 1, 2, 3 Determine $f'(3)$ and $f''(2)$ by using a difference formula and determine the error at each case

x	f	$f(x_0,x_1)$	$f(x_0,x_1,x_2)$	$f(x_0,x_1,x_2,x_3)$
0	1			
		$\frac{2.718-1}{1-0}=1.718$		
1	2.718		$\frac{4.671-1.1718}{2-0}=2.953$	
		$\frac{7.389-2.718}{2-1}=4.671$		
2	7.389			$\frac{4.013-2.953}{3-0}=0.3533$
			$\frac{12.697-4.671}{3-1}=4.013$	
		$\frac{20.086-7.389}{3-2}=12.697$		
3	20.086			

$$f(x) = y_0 + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2)$$
$$+(x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_2)+.....$$

$$f(x) = 1+(x-0)(1.718)+(x-0)(x-1)(2.953)+(x-0)(x-1)(x-2)(0.3533)$$
$$f'(x) = 1.718+(x+x-1)(2.953)+\left[(x-1)(x-2)+x(x-2)+x(x-1)\right](0.3533)$$
$$f'(x) = 1.718+2.953(2x-1)+(0.3533)(3x^2-6x+2)$$

$$f''(x) = 5.906+(0.3533)(6x-6)$$

$$f'(3) = 1.718+2.953(2(3)-1)+(0.3533)(3(3)^2-6(3)+2) = 20.3693$$

$$EROR = \left|e^3-20.363\right| \approx 0.277$$

$$f''(2) = 5.906+(0.3533)(6(2)-6) \approx 8.0258$$

$$EROR = \left|e^2-8.0258\right| \approx 0.363$$

Q2

Determine the solution of $\int_0^1 \frac{1}{1+x^3}\, dx$; by Trapizoidal, take h= 0.1

x_k	$f(x_k)$	m	$mf(x_k)$
0	1	1	1
0.1	0.999	2	1.998
0.2	0.992	2	1.9841
0.3	0.9737	2	1.9474
0.4	0.9398	2	1.8796
0.5	0.8888	2	1.7777
0.6	0.8223	2	1.6447
0.7	0.7446	2	1.4892
0.8	0.6613	2	1.3227
0.9	0.5783	2	1.1567
1	0.5	1	0.5
			16.7001

$$\int_a^b f(x)dx \approx \frac{h}{2}[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)]$$

$$\int_0^1 \frac{1}{1+x^3}dx \approx \frac{0.1}{2}(16.7001)$$

$$\int_0^1 \frac{1}{1+x^3}dx \approx 0.835005$$

Q3

Determine the solution of $\int_0^1 \frac{1}{1+x^3}\, dx$; by Simpson's method take , h = 0.1

x_k	$f(x_k)$	m	$mf(x_k)$
0	1	1	1
0.1	0.999	4	3.996
0.2	0.992	2	1.984
0.3	0.9737	4	3.8948
0.4	0.9398	2	1.8796
0.5	0.8888	4	3.5552
0.6	0.8223	2	1.6447
0.7	0.7446	4	2.9784
0.8	0.6613	2	1.3227
0.9	0.5783	4	2.3132
1	0.5	1	0.5
			25.0686

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\ldots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)]$$

$$\int_0^1 \frac{1}{1+x^3}dx = \frac{0.1}{3}(\ 25.0686\ )$$

$$\int_0^1 \frac{1}{1+x^3}dx = (\ 0.83562\ )$$

Q4

Evaluate $f'(1.2)$ and $f''(1.2)$

x	0.6	0.8	1.00	1.2
y	0.329287	0.359463	0.367879	0.361433

Using Backward differential formula

$$f'(x) = \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$$

$$f'(1.2) = \frac{3(0.361433) - 4(0.367879) + (0.359463)}{2(0.2)}$$

$$f'(1.2) = -0.069385$$

$$f''(x) = \frac{2f(x) - 5f(x-h) + 4f(x-2h) - f(x-3h)}{h^2}$$

$$f''(x) = \frac{2(0.361433) - 5(0.367879) + 4(0.359463) - (0.329287)}{(0.2)^2} \approx -0.1991$$