Taibah University

College of Engineering

Department of Civil Engineering

Course CE 341:

Water Resources

Lesson 1: Introduction to Open channel Flow

Introduction

Open Channel Flow :

- An open channel is a conduit in which a liquid flows with a free surface.

- The free surface is actually an interface between the moving liquid and an overlying fluid medium and will have constant pressure.

- In civil engineering applications; water is the most common liquid with air at atmospheric pressure as the overlying fluid.

Open Channel Flow : Few examples

- Flows in natural rivers, streams and rivulets;
- artificial, i.e. man-made canals for transmitting water from a source to a place of need, such as for irrigation, water supply and hydropower generation;
- sewers that carry domestic or industrial waste waters;
- navigation channels

Open Channel : Types

Prismatic Channel:

- ✓ cross-sectional shape, size and also bottom slope are constant.
- ✓ Example: Artificial channels

Non-prismatic Channels:

- ✓ All natural channels

  (generally have varying cross-sections and consequently are non-prismatic)

Open Channel : Types based on the nature of boundary

Rigid Boundary Channels / Rigid Channels: boundary is not deformable

(shape, planiform and roughness magnitudes are not functions of the flow parameters)

- ✓ can be considered to have only **one degree** of freedom; for a given channel geometry the only change that may take place is the depth of flow which may vary with space and time depending upon the nature of the flow.

- ✓ Examples: lined canals, sewers and non-erodible unlined canals.

Open Channel : Types based on the nature of boundary

Mobile Boundary Channels

- ✓ boundaries undergo deformation due to the continuous process of erosion and deposition due to the flow.
- ✓ The boundary of the channel is mobile in such cases and the flow carries considerable amounts of sediment through suspension and in contact with the bed.
- ✓ A general mobile-boundary channel can be considered to have **four degrees** of freedom. For a given channel not only the depth of flow but also the bed width, longitudinal slope and planiform (or layout) of the channel may undergo changes with space and time depending on the type of flow.

Open Channel Flow:

Steady Flow:

flow properties, such as the depth or discharge at a section do not change with time.

Unsteady Flows :

if the depth or discharge changes with time the flow is termed unsteady.

Examples: Flood flows , rapidly varying surges in canals

Open Channel Flow:

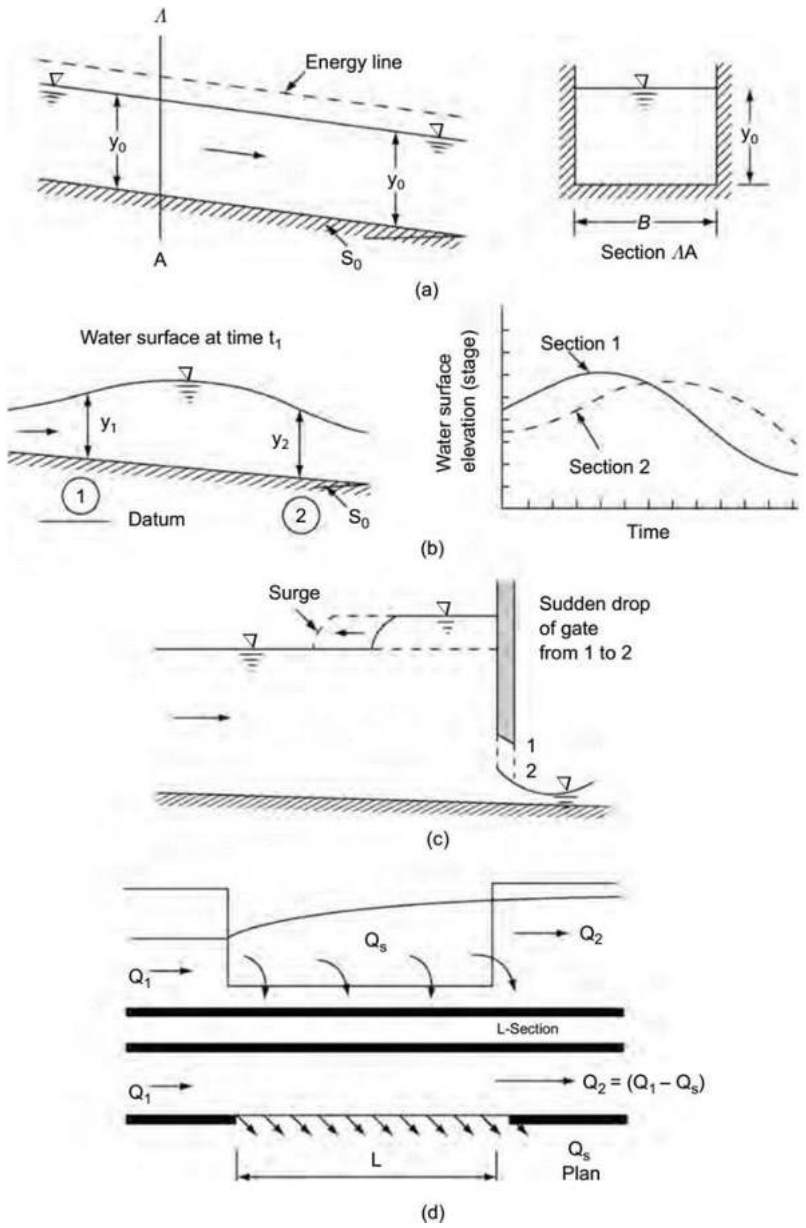
Uniform Flow:

Flow properties, say the depth of flow, in an open channel remain constant along the length of the channel, the flow is said to be uniform.
prismatic channel with constant discharge

Non-Uniform Flows :

flow properties vary along the channel is termed as non-uniform flow or varied flow.
gradually varied flow (GVF)
rapidly varied flow (RVF)
Spatially Varied Flows :
if some flow is added to or abstracted from the system.

Open Channel Flow:



Various types of open channel flows:

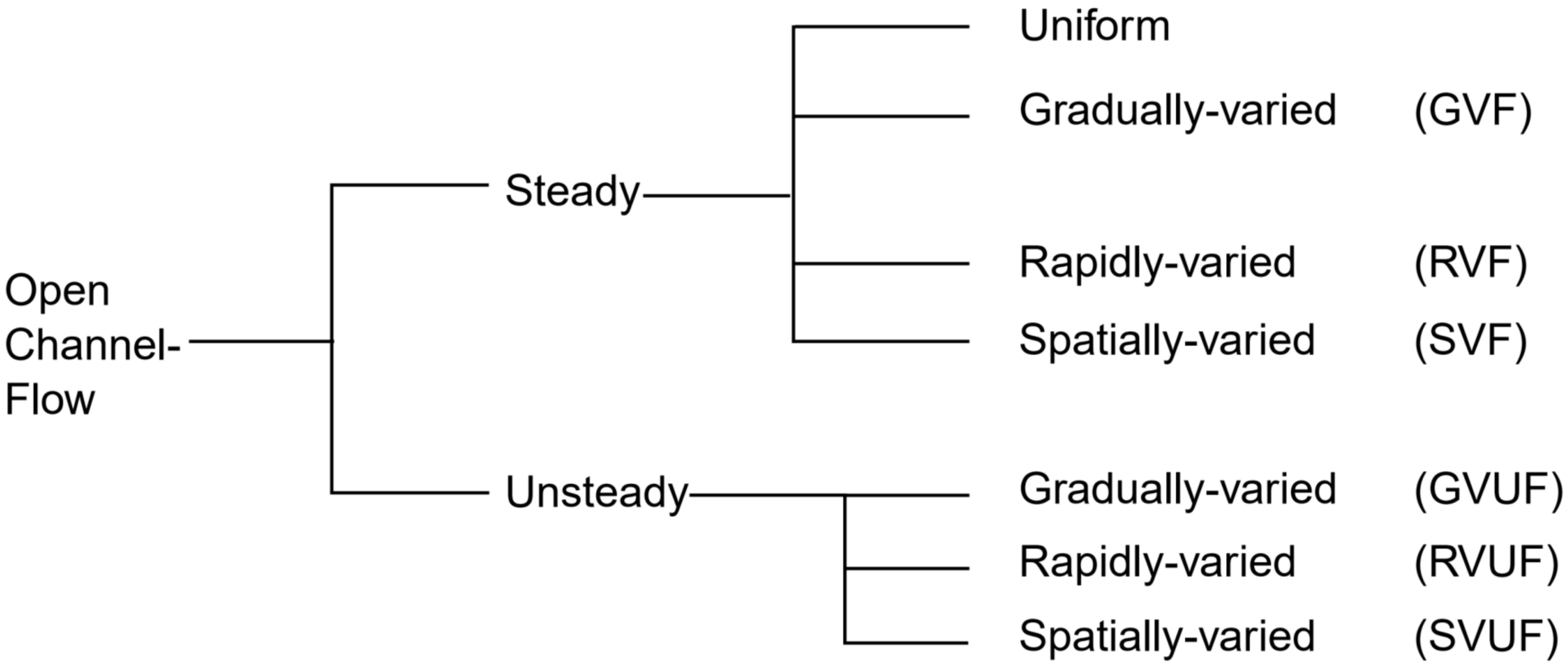
(a) Uniform flow,

(b) Gradually varied flow

(c) Rapidly varied flow and

(d) Side Weir: Spatially varied flow

Open Channel Flow:



Open Channel Flow:

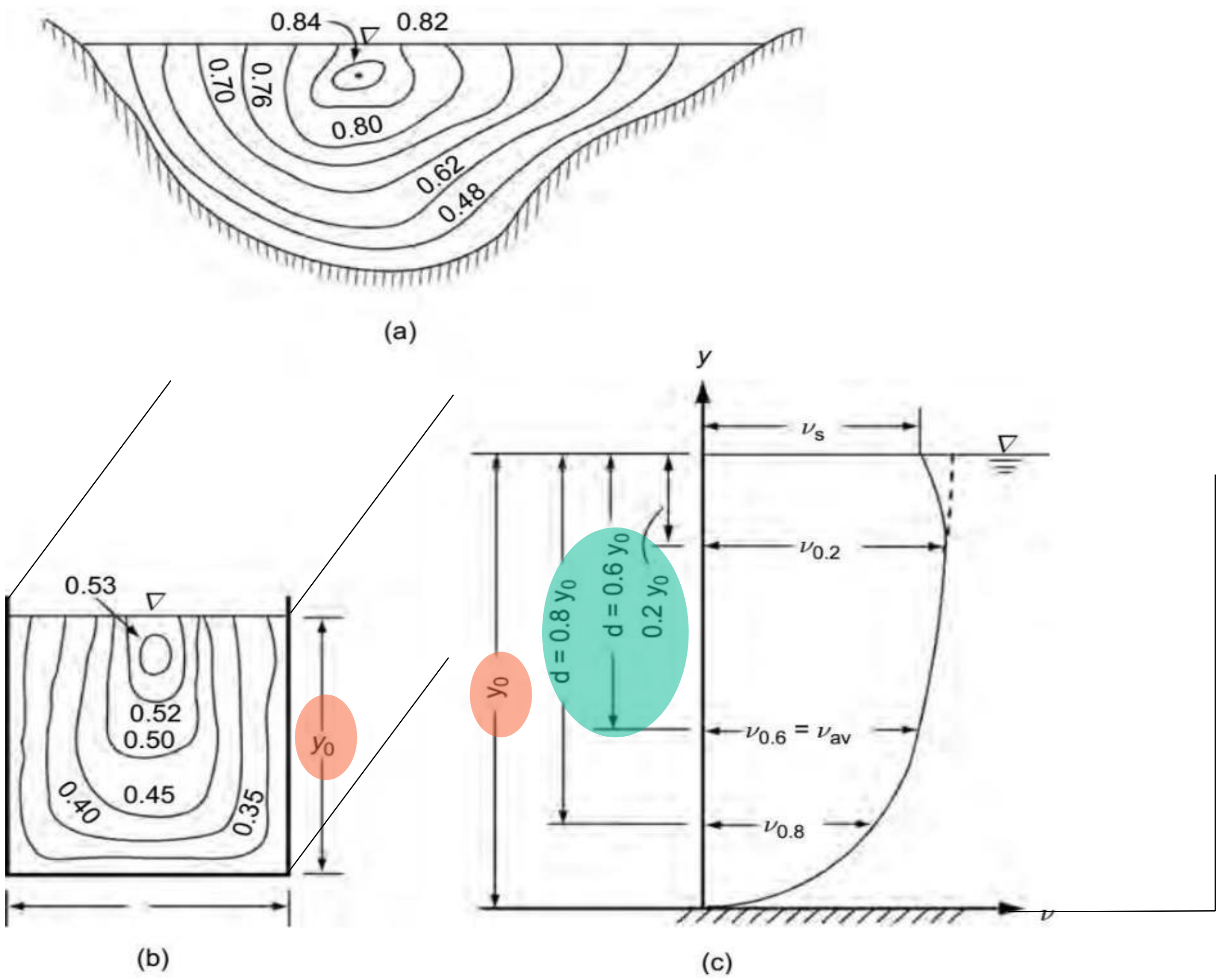


Fig. 1.2 *Velocity distribution in open channels: (a) Natural channel (b) Rectangular channel and (c) Typical velocity profile*

Open Channel Flow:

Field observations in rivers and canals have shown that the average velocity at any vertical v_{av} occurs at a level of 0.6 y_0 from the free surface, where $y_0 = depth\, of\, flow$.

$$V_{av} = \frac{V_{0.2} + V_{0.8}}{2}$$

$$V_{av} = kV_s$$

k = a coefficient with a value between 0.8 and 0.95.

ONE-DIMENSIONAL METHOD OF FLOW ANALYSIS

Mean velocity ***V*** for the entire cross-section is defined on the basis of the longitudinal component of the velocity **v** as

$$V = \frac{1}{A}\int_A v\,dA \qquad\qquad Q = \int v\,dA = VA$$

Assumptions for 1-D Flow Analysis:

- A single elevation represents the water surface perpendicular to the flow.
- Velocities in directions other than the direction of the main axis of flow are not considered.

Kinetic Energy correction factor (α)

For an elemental area dA, the flux of kinetic energy through it is equal to:

$$\left(\frac{\text{mass}}{\text{time}}\right)\left(\frac{\text{KE}}{\text{mass}}\right)=(\rho v\,dA)\frac{v^2}{2}$$

For the total area, the kinetic energy flux

$$=\int_A \frac{\rho}{2}\,v^3\,dA=\alpha\frac{\rho}{2}\,V^3 A$$

from which

$$\alpha=\frac{\int v^3\,dA}{V^3 A}$$

or for discrete values of *v*,

$$\alpha=\frac{\sum v^3 \Delta A}{V^3 A}$$

Open Channel Flow

Momentum Similarly, the flux of momentum at a section is also expressed in terms of V and a correction factor β. Considering an elemental area dA, the flux of momentum in the longitudinal direction through this elemental area

$$= \left(\frac{\text{mass}}{\text{time}} \times \text{velocity} \right) = (\rho v \, dA)(v)$$

For the total area, the momentum flux

$$= \int \rho v^2 \, dA = \beta \rho V^2 A$$

which gives

$$\beta = \frac{\int_A v^2 \, dA}{V^2 A} = \frac{\Sigma v^2 \Delta A}{V^2 A}$$

β is known as the momentum correction factor and is equal to or greater than unity

The Values of α ***and*** β The coefficients α and β are both unity in the case of uniform velocity distribution. For any other velocity distribution $\alpha > \beta > 1.0$

Channels	*Values of α*		*Values of β*	
	Range	*Average*	*Range*	*Average*
Natural channels and torrents	*1.15 – 1.50*	*1.30*	*1.05 – 1.17*	*1.10*
River valleys, overflooded	*1.50 – 2.00*	*1.75*	*1.17 – 1.33*	*1.25*

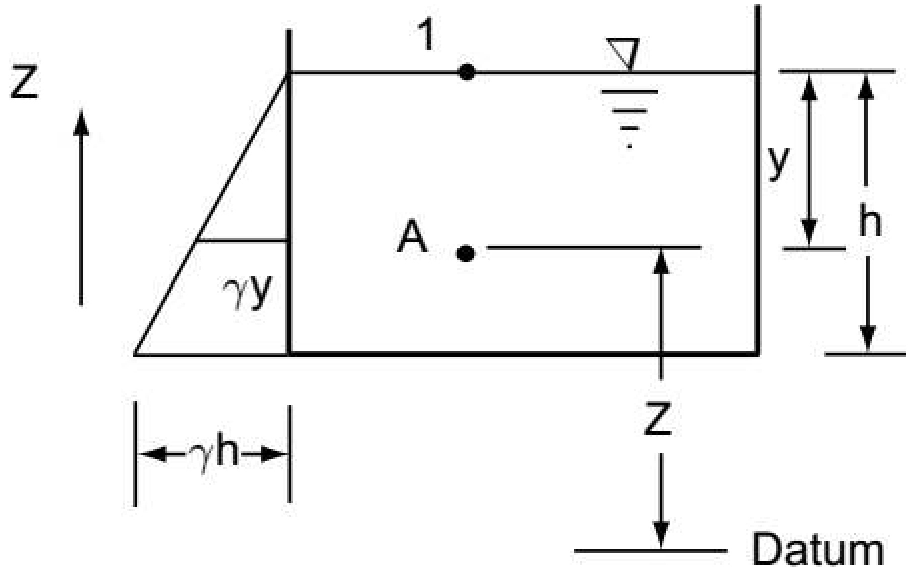
Open Channel Flow - *Pressure distribution*

The normal acceleration a_n will be zero

(i) if $v = 0$, i.e., when there is no motion, or

(ii) if $r \rightarrow \infty$, i.e., when the streamlines are straight lines.

Hydrostatic Pressure Distribution



Pressure distribution in still water

$$\frac{p_A}{\gamma} = (Z_1 - Z_A) = y$$

$$p_A = \gamma y$$

Open Channel Flow:

Channels with Large Slope Figure 1.5 shows a uniform free-surface flow in a channel with a large value of inclination θ. The flow is uniform, i.e. the water surface is parallel to the bed. An element of length ΔL and unit width is considered at the Section $0-1$

At any point *A* at a depth y measured normal to the water surface, the weight of column $A1\,1'A' = \gamma \Delta L y$ and acts vertically downwards. The pressure at *AA′* supports the normal component of the column $A1\,1'A'$. Thus

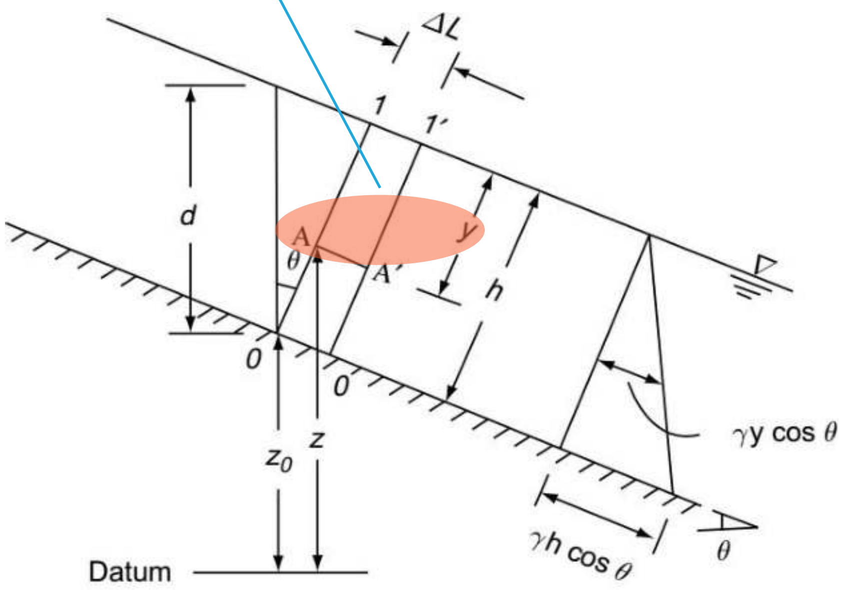


Fig. 1.5 *Pressure distribution in a channel with large slope*

$$p_A \Delta L = \gamma y \Delta L \cos\theta$$

$$p_A = \gamma y \cos\theta$$

Open Channel Flow:

Channels with Large Slope

The pressure p_A varies linearly with the depth *y* but the constant of proportionality is γ cos θ. If *h* = normal depth of flow, the pressure on the bed at point 0, $p0 = \gamma h \cos\theta$. If *d* = vertical depth to water surface measured at the point *O*, then $h = d\cos\theta$ and the pressure head at point *O*, on the bed is given by

$$\frac{p_0}{\gamma} = h\cos\theta = d\cos^2\theta$$

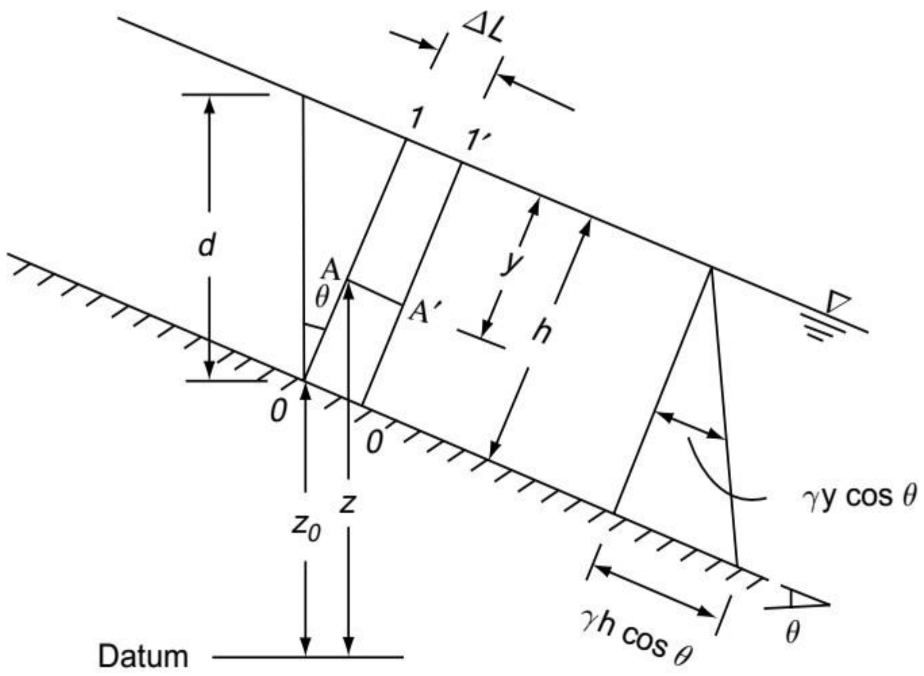


Fig. 1.5 *Pressure distribution in a channel with large slope*

Open Channel Flow:

EQUATION OF CONTINUITY

Steady Flow

$$Q = VA = V_1A_1 = V_2A_2 = \ldots$$

Unsteady Flow In the unsteady flow of incompressible fluids, if we consider a reach of the channel, the continuity equation states that the net discharge going out of all the boudary surfaces of the reach is equal to the rate of depletion of the storage within it.

Open Channel Flow: ENERGY EQUATION

The term $(Z + y \cos \theta) = h$ represents the elevation of the hydraulic grade line above the datum.

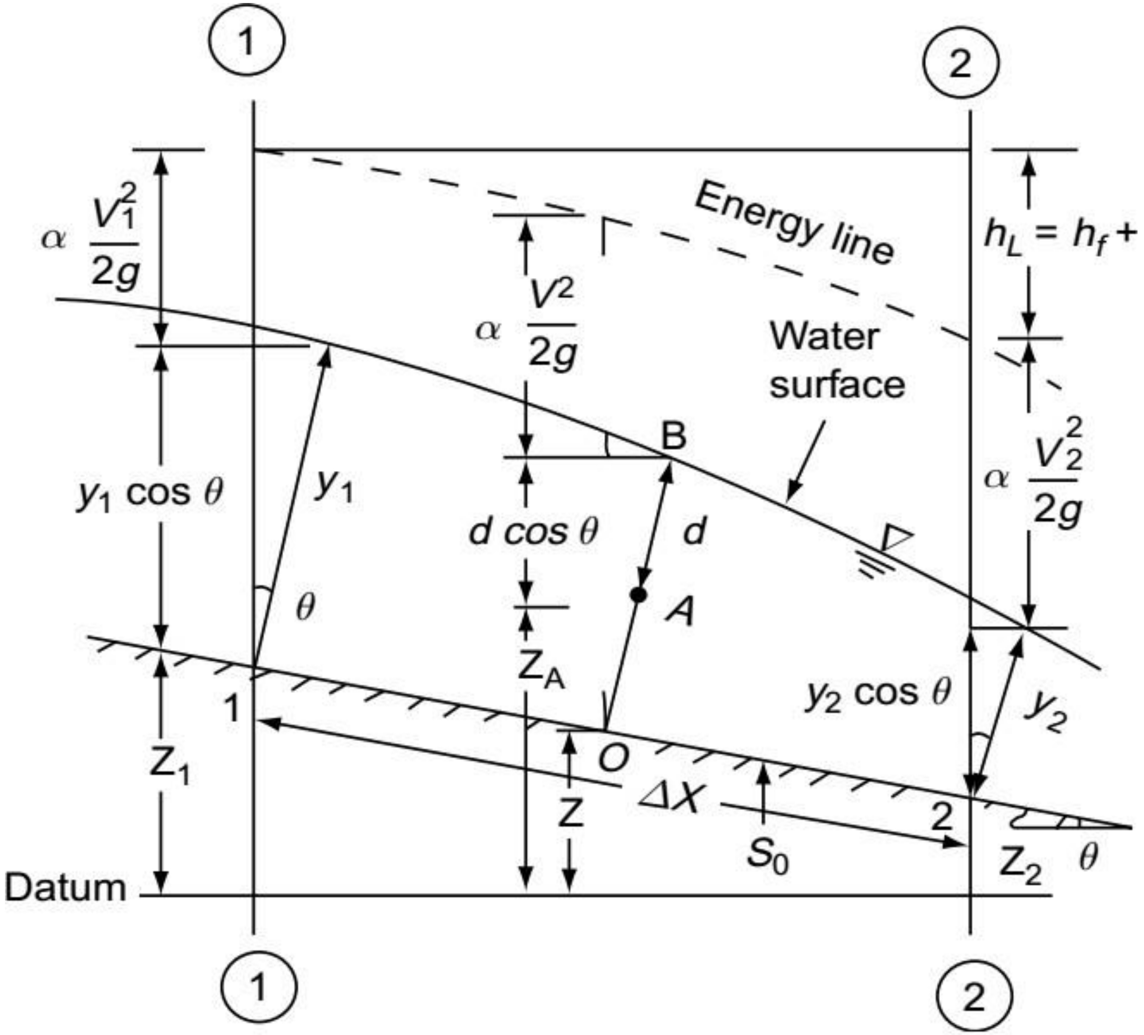


Fig.1.14 *Definition sketch for the energy equation*

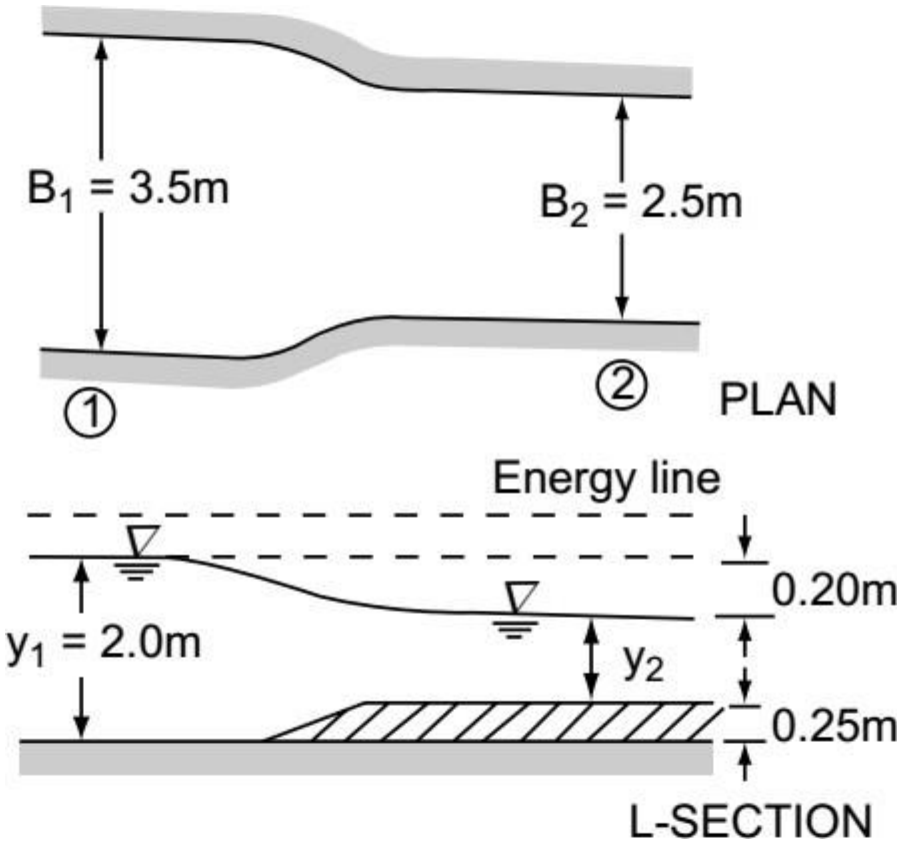
$$H = Z + y \cos \theta + \alpha V^2/2g$$

$$H_1 = Z_1 + y_1 \cos\theta + \alpha_1 \frac{V_1^2}{2g}$$

$$H_2 = Z_2 + y_2 \cos\theta + \alpha_2 \frac{V_2^2}{2g}$$

Open Channel Flow:

Example 1.7 *The width of a horizontal rectangular channel is reduced from 3.5 m to 2.5 m and the floor is raised by 0.25 m in elevation at a given section. At the upstream section, the depth of flow is 2.0 m and the kinetic energy correction factor α is 1.15. If the drop in the water surface elevation at the contraction is 0.20 m, calculate the discharge if (a) the energy loss is neglected, and (b) the energy loss is one-tenth of the upstream velocity head. [The kinetic energy correction factor at the contracted section may be assumed to be unity].*



Open Channel Flow:

$$y_2 = 2.0 - 0.25 - 0.20 = 1.55 \text{ m}$$

By continuity

$$B_1 y_1 V_1 = B_2 y_2 V_2$$

$$V_1 = \frac{2.5 \times 1.55}{3.5 \times 2.0} V_2 = 0.5536 V_2$$

(a) *When there is no energy loss*

By energy equation applied to Sections 1 and 2,

$$Z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = (Z_1 + \Delta Z) + y_2 + \alpha_2 \frac{V_2^2}{2g}$$

$$\alpha_1 = 1.15 \text{ and } \alpha_2 = 1.0$$

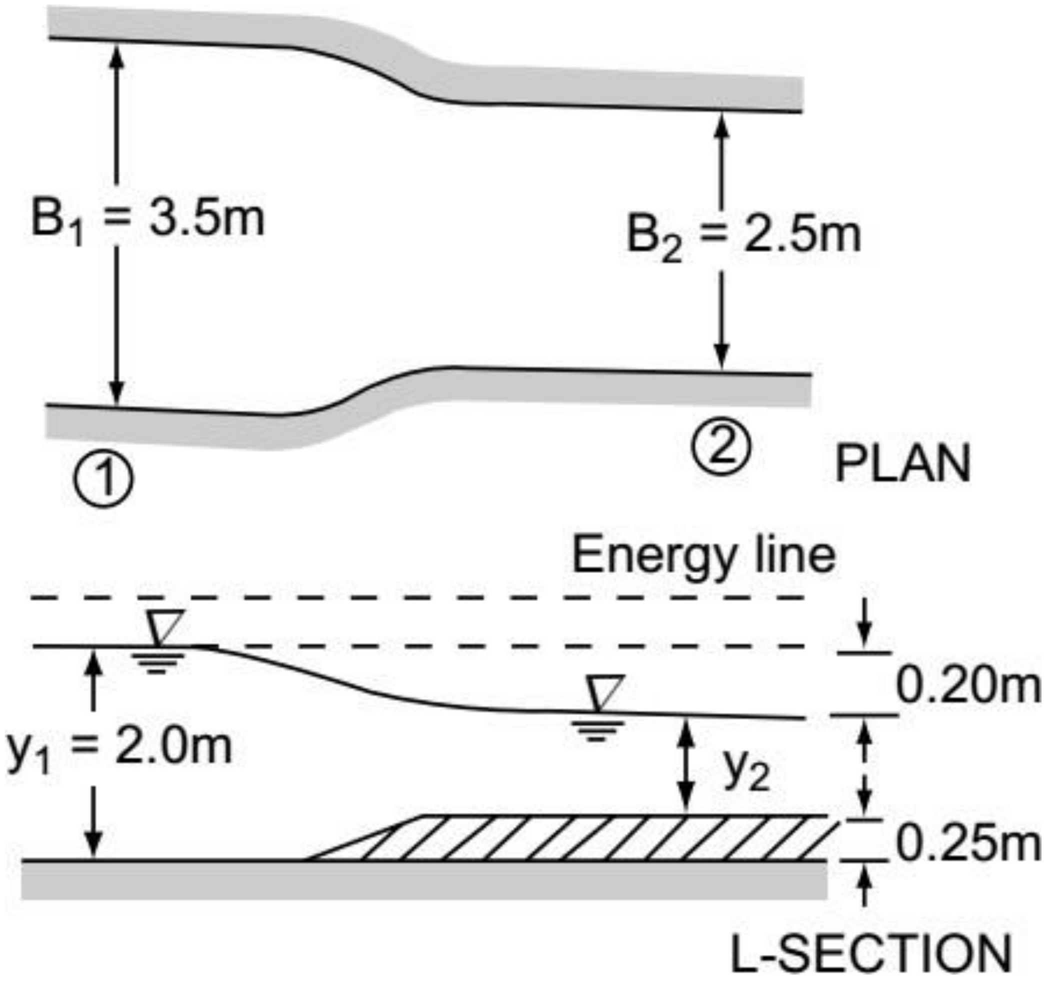
$$\frac{V_2^2 - (1.15 V_1^2)}{2g} = y_1 - y_2 - \Delta Z$$

$$\frac{V_2^2}{2g}\left[1 - (1.15)(0.5536)^2\right] = 2.00 - 1.55 - 0.25$$

$$0.6476 \frac{V_2^2}{2 \times 9.81} = 0.2$$

$$V_2 = 2.462 \text{ m/s}$$

Discharge $Q = 2.5 \times 1.55 \times 2.462 = 9.54 \text{ m}^3/\text{s}$



Open Channel Flow:

(b) *When there is an energy loss*

$$H_L = 0.1\left[\alpha_1 \frac{V_1^2}{2g}\right] = 0.115\frac{V_1^2}{2g}$$

By energy equation,

$$Z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = (Z_1 + \Delta Z) + y_2 + \alpha_2 \frac{V_2^2}{2g} + H_L$$

$$\left[\alpha_2 \frac{V_2^2}{2g} - \alpha_1 \frac{V_1^2}{2g} + H_L\right] = y_1 - y_2 - \Delta Z$$

Substituting $\alpha_2 = 1.0$, $\alpha_1 = 1.15$ and $H_L = 0.115\frac{V_1^2}{2g}$

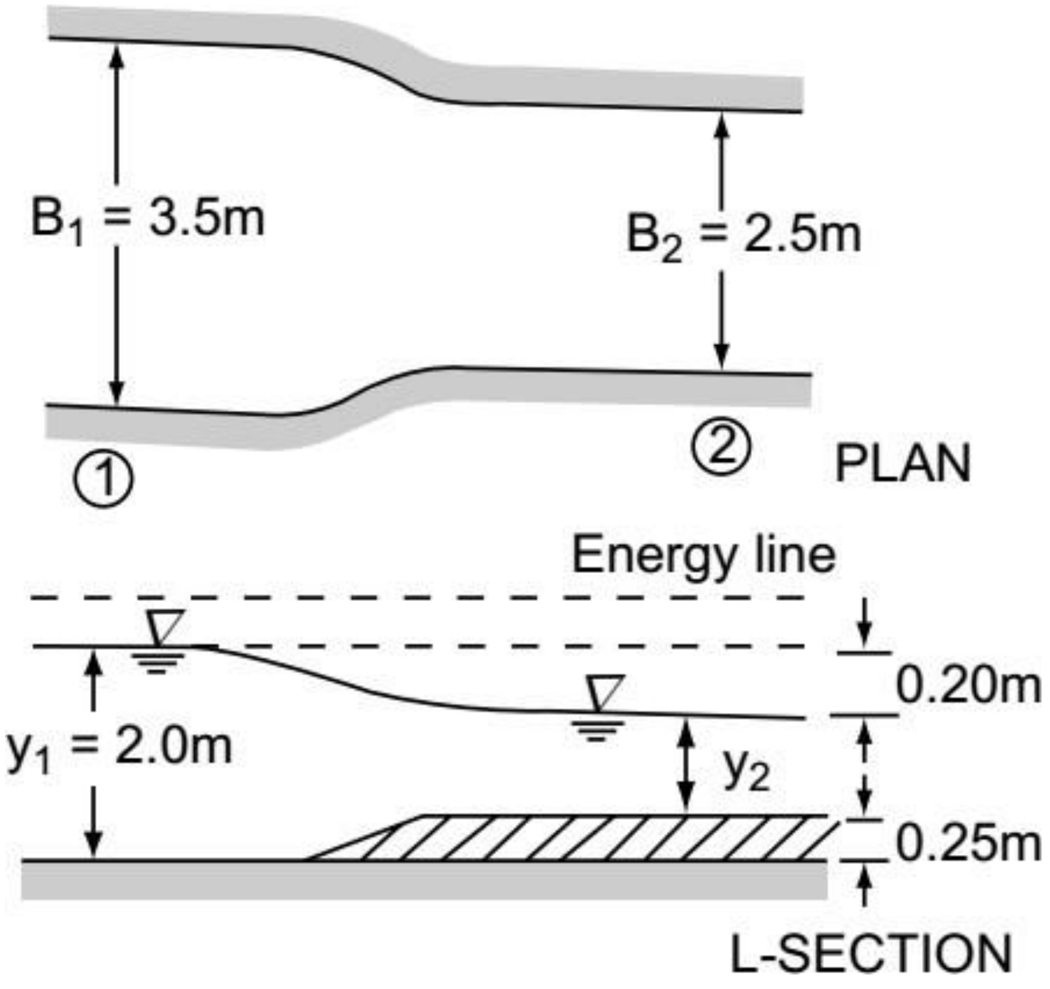
$$\frac{V_2^2}{2g} - 1.15\frac{V_1^2}{2g} - 0.115\frac{V_1^2}{2g} = 2.00 - 1.55 - 0.25$$

Since $V_1 = 0.5536\,V_2$

$$\frac{V_2^2}{2g}\left[1 - (0.9)(1.15)(0.5536)^2\right] = 0.2$$

$$\frac{0.6826\,V_2^2}{2 \times 9.81} = 0.2$$

$V_2 = 2.397$ m/s and discharge $Q = 2.5 \times 1.55 \times 2.397 = 9.289\ \text{m}^3/\text{s}$



Open Channel Flow:

Example 1.8 *A sluice gate in a 2.0-m wide horizontal rectangular channel is discharging freely as shown in Fig. 1.16. If the depths a small distance upstream (y_1) and downstream (y_2) are 2.5 m and 0.20 m respectively, estimate the discharge in the channel (i) by neglecting energy losses at the gate, and (ii) by assuming the energy loss at the gate to be 10% of the upstream depth y_1.*

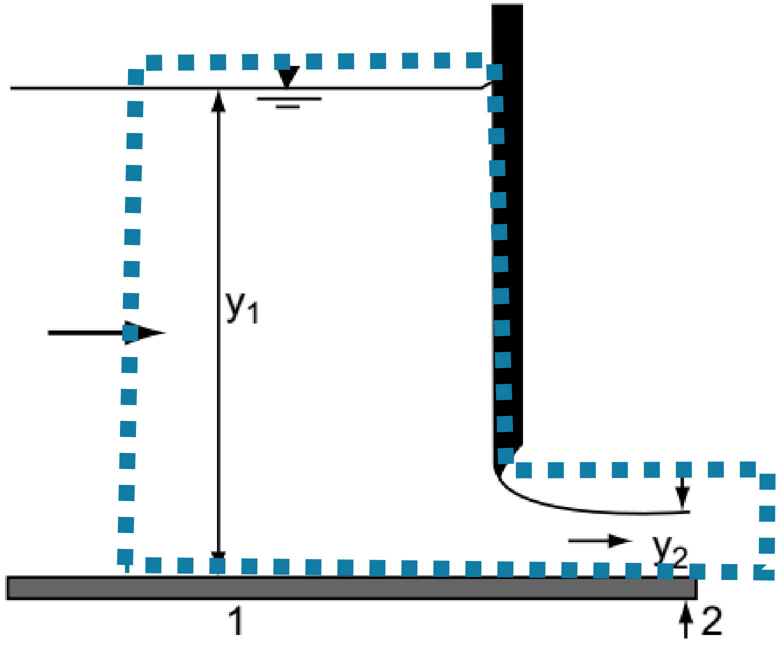


Fig. 1.16 *Free flow from a sluice gate – Example 1.8*

Solution Referring to Fig. 1.16, $y_1 = 2.5$ m and $y_2 = 0.20$ m

$$B\,y_1 V_1 = B\,y_2 V_2$$

$$V_2 = \frac{y_1}{y_2} V_1 = \frac{2.5}{0.20} \times V_1 = 12.5\,V_1$$

(i) *When there is no energy loss*

$$Z_1 + y_1 + \frac{V_1^2}{2g} = Z_2 + y_2 + \frac{V_2^2}{2g}$$

Open Channel Flow

$$\frac{V_1^2}{2g}\left[(12.5)^2 - 1\right] = 2.50 - 0.20 = 2.30$$

$$\frac{V_1^2}{2g} = \frac{2.30}{155.25} = 0.01481 \text{ and } V_1 = 0.539 \text{ m/s.}$$

Discharge $Q = By_1 V_1 = 2.0 \times 2.5 \times 0.539 = 2.696 \text{ m}^3\text{/s.}$

(ii) *When there is energy loss*

H_L = Energy loss $= 0.10\, y_1 = 0.25$ m

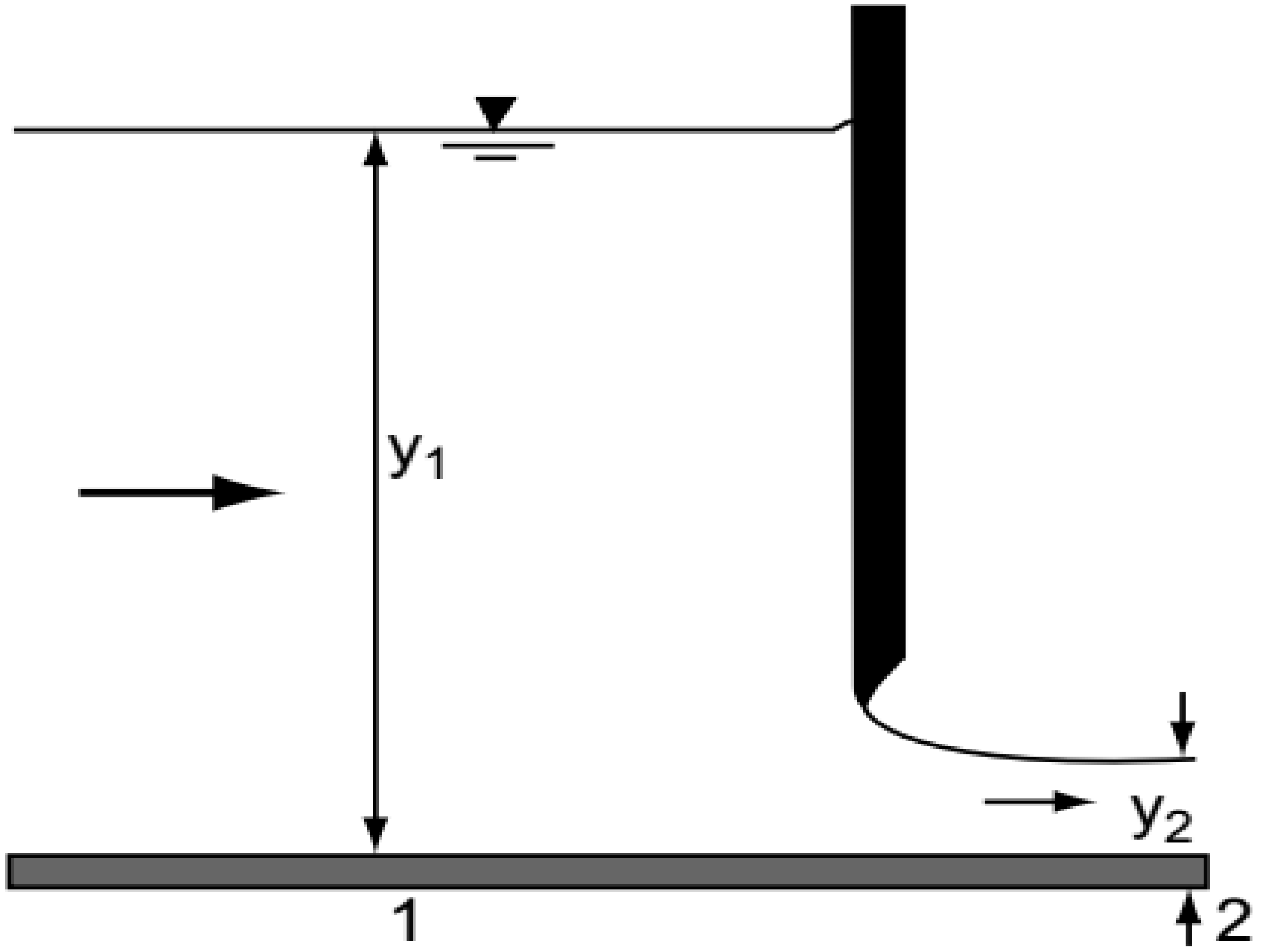
$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + H_L$$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = (y_1 - y_2 - H_L)$$

$$\frac{V_1^2}{2g}\left[(12.5)^2 - 1\right] == 2.50 - 0.20 - 0.25 = 2.05$$

$$\frac{V_1^2}{2g} = \frac{2.05}{155.25} = 0.0132 \text{ and } V_1 = 0.509 \text{ m/s}$$

Discharge $Q = By_1 V_1 = 2.0 \times 2.5 \times 0.509 = 2.545 \text{ m}^3\text{/s.}$



MOMENTUM EQUATION

- The momentum equation commonly used in most of the open channel flow problems is the *linear-momentum equation*.
- This equation states that the algebraic sum of all external forces, acting in a given direction on a fluid mass equals the time rate of change of linear momentum of the fluid mass in the direction.
- In a steady flow the rate of change of momentum in a given direction will be equal to the net flux of momentum in that direction.

MOMENTUM EQUATION

Figure 1.17 shows a *control volume* (a volume fixed in space) bounded by Sections 1 and 2, the boundary and a surface lying above the free surface. The various forces acting on the control volume in the longitudinal direction are as follows:

(i) Pressure forces acting on the control surfaces, F_1 and F_2.
(ii) Tangential force on the bed, F_3,
(iii) Body force, i.e., the component of the weight of the fluid in the longitudinal direction, F_4

$$\Sigma F_1 = F_1 - F_2 - F_3 + F_4 = M_2 - M_1$$

in which $M_1 = \beta_1 \rho Q V_1$ = momentum flux entering the control volume, $M_2 = \beta_2 \rho Q V_2$ = momentum flux leaving the control volume.

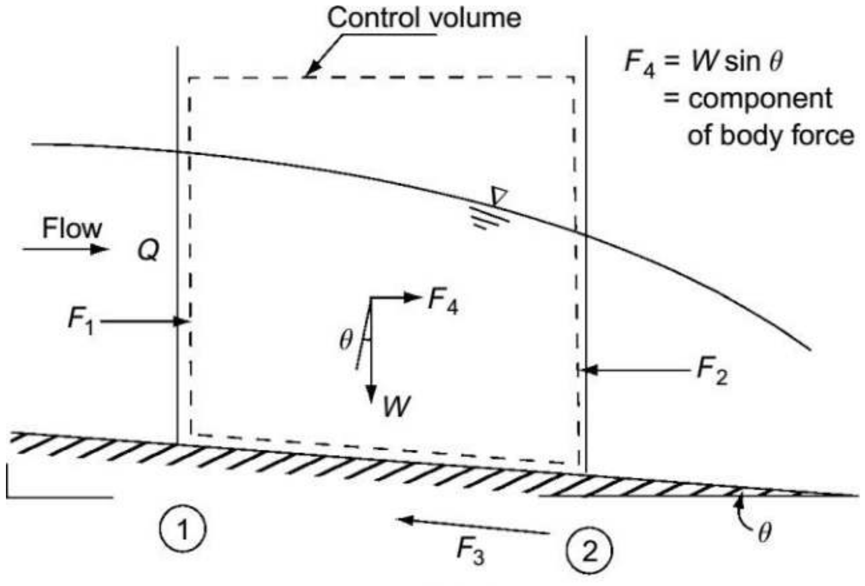


Fig.1.17 *Definition sketch for the momentum equation*

Example 1.9

Estimate the force, on a sluice gate shown in Fig. 1.18.

The frictional force on the bed between Sections 1 and 2 is neglected.

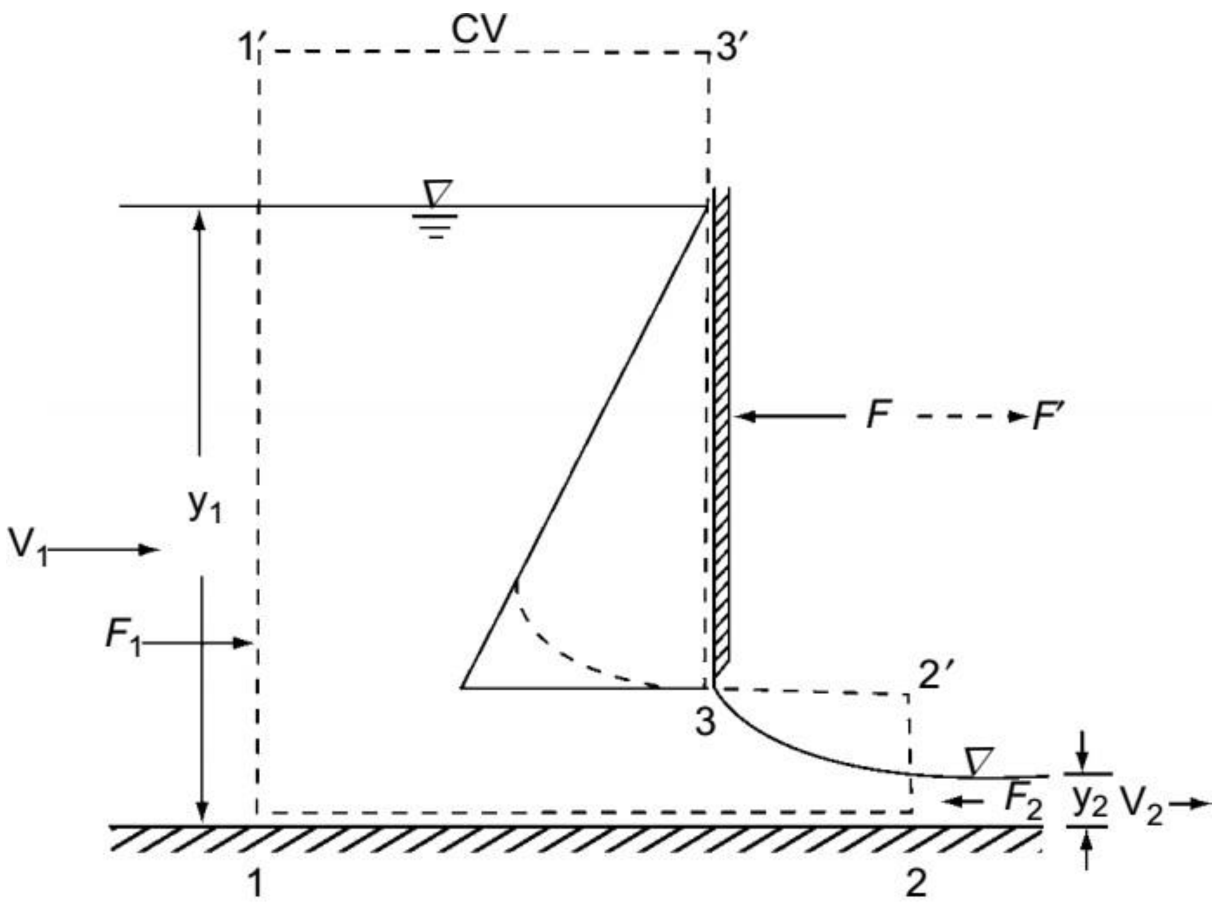
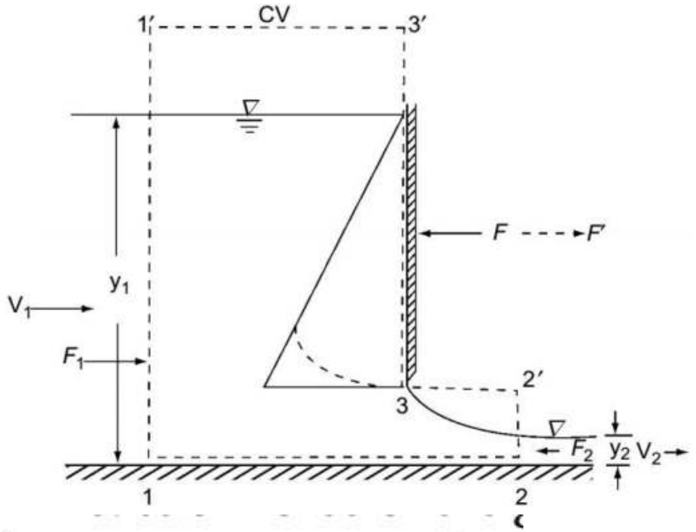


Fig. 1.18 *Forces in a sluice gate glow-Example 1.9*

MOMENTUM EQUATION



F_1 = pressure force on the control surface at Section 11' $= \frac{1}{2}\gamma y_1^2$

F_2 = pressure force on the control surface at Section 22' $= \frac{1}{2}\gamma y_2^2$ acting in a direction opposing F_1.

F = reaction force of the gate on the Section 33'.

By the momentum equation,

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 - F = \rho q (V_2 - V_1)$$

in which q = discharge per unit width $= V_1 y_1 = V_2 y_2$. Simplifying

$$F = \frac{1}{2}\gamma \frac{(y_1 - y_2)}{y_1 y_2}\left(y_1 y_2 (y_1 + y_2) - \frac{2q^2}{g} \right)$$

If the loss of energy between Sections 1 and 2 is assumed to be negligible, by the energy equation with $\alpha_1 = \alpha_2 = 1.0$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

Substituting $V_1 = \frac{q}{y_1}$ and $V_2 = \frac{q}{y_2}$

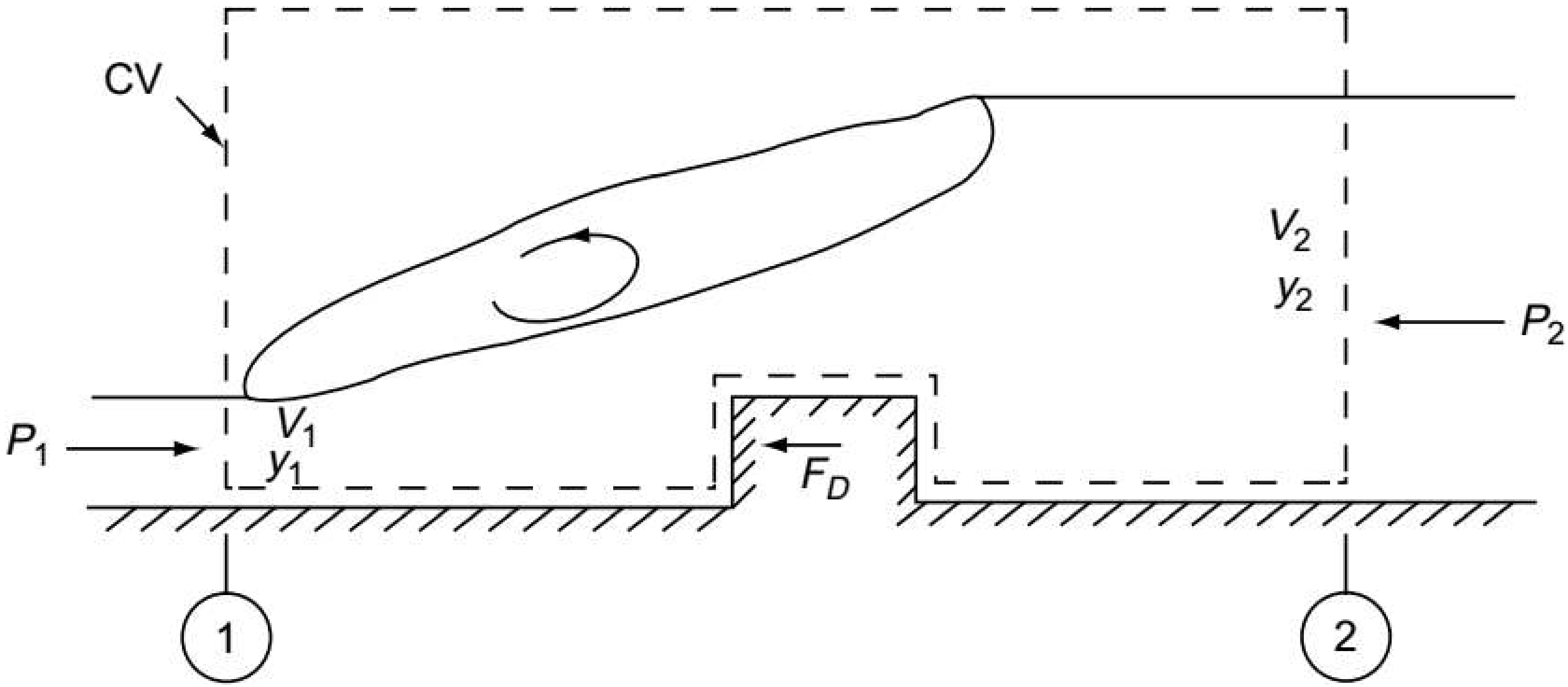
$$\frac{q^2}{g} = \frac{2 y_1^2 y_2^2}{(y_1 + y_2)}$$

$$F = \frac{1}{2}\gamma \frac{(y_1 - y_2)^3}{(y_1 + y_2)}$$

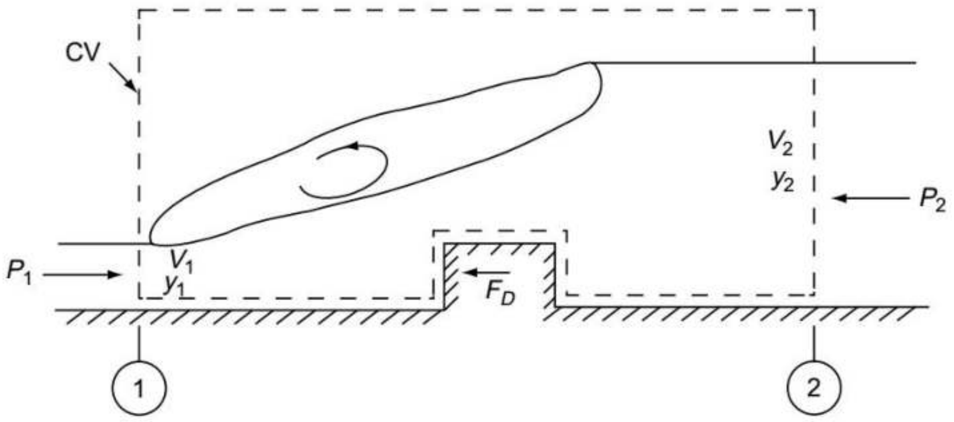
MOMENTUM EQUATION

Example 1.10

Figure 1.19 shows a hydraulic jump in a horizontal apron aided by a two dimensional block on the apron. Obtain an expression for the drag force per unit length of the block.



MOMENTUM EQUATION



By momentum equation,

$$P_1 - F_D - P_2 = M_2 - M_1$$

$$\frac{1}{2}\gamma y_1^2 - F_D - \frac{1}{2}\gamma y_2^2 = \rho q(\beta_2 V_2 - \beta_1 V_1)$$

where q = discharge per unit width of apron $= y_1 V_1 = y_2 V_2$.

Assuming $\beta_1 = \beta_2 = 1.0$

$$F_D = \frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 = \rho q^2 \left(\frac{1}{y_2} - \frac{1}{y_1}\right)$$

$$= \frac{\gamma}{2}\left[y_1^2 - y_2^2 + \frac{q^2}{g}\left(\frac{y_2 - y_1}{y_1 y_2}\right)\right]$$

END OF LESSON

Thanks for listening….