The previous chapters explored methods to determine the displacement of a mechanism whose link lengths are given.

For example in the following example we have the dimensions of the wheel assembly for a small aircraft. We need to classify the displacement of this four-bar mechanism.

(Machines & Mechanisms, David H. Myszka, Fourth Edition)

The lengths of the links are:
$S = 12 \text{ in.}, \ L = 32.4 \text{ in.}, \ P = 30 \text{ in.}, \ Q = 26 \text{ in.}$

**Check the case I:** $S + L < P + Q$

$12 + 32.4 < 30 + 26 \Rightarrow 44.4 < 56 \ (\text{YES})$

The shortest link $AB$ is adjacent to the ground (frame). **This is the class I.2** (GCRR)

**Class I.2**

*Ground either link adjacent to the shortest and you get a crank-rocker:* the shortest link will fully rotate and the other link pivoted to ground will oscillate.

* Prepared by Dr. Eyyup Aras, KSU, 2015 Spring
Compared to the previous chapters the **graphical linkage synthesis** presents the **opposite task**: That is, **GIVEN A DESIRED MOTION, A MECHANISM FORM AND DIMENSIONS MUST BE DETERMINED.**

- For example in the following example two desired positions of a body are given. We need to choose a mechanism with appropriate link lengths for obtaining these positions. The four-bar mechanism can be used.

![Figure 1: Constructing a four-bar linkage for two desired positions.](image)

- Use the simplest mechanism capable of achieving the desired motion.
- The four-bar and slider crank mechanisms are the most widely used.

* Prepared by Dr. Eyyup Aras, KSU, 2015 Spring
Importance of the synthesis is design a mechanism that produces a desired output motion for a given input motion.

**Type synthesis:**
Proper type of mechanism best suited to a given problem.

For example linkages, gears, cam and follower, belt and pulley etc. each of these solutions, while possible may not be optimal or practical.

**Number synthesis:**
How many links should the mechanism have? How many degrees of freedom are desired?

**Dimensional synthesis**
Dimensional synthesis of a linkage is determination of the lengths of the links necessary to accomplish the desired motions.

* Prepared by Dr. Eyyup Aras, KSU, 2015 Spring
Tasks of mechanisms can be classified in three groups
(1) Function generation, (2) Path generation, (3) Motion generation

Function generation

- A function generator is conceptually a “black box” which delivers some predictable output in response to a known input.

- Historically, before the advent of electronic computers, mechanical function generators found wide applications.

Figure 2: Log function generator

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Path generation

- Control of a point in a plane to follow a certain path.
- Typically a point on the coupler traces the desired path

![Path generator](image)

**Figure 3:** Path generator

Motion generation

- Control of a line in the plane such that it follows some prescribed set of sequential positions.
- A simple case: coupler output

![Motion generator](image)

**Figure 4:** Motion generator

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EXAMPLE 1

Design a **four-bar Grashof crank-rocker mechanism** to give $45^\circ$ of rocker rotation with equal time forward and back, from a constant speed motor input (see the Figure 5).

**Key pieces of statement**

- **Rocker output** – two positions with angular displacement $45^\circ$ (Function generation)
- **Four-bar linkage**, three in motion, one grounded
- **Grashof** – at least one link can rotate fully
- **Crank-rocker**: $(S + L < P + Q)$ and link adjacent to shortest link is ground

**Note that**: the sentence “with equal time forward and back” will be discussed later in the quick return mechanisms section.

*Prepared by Dr. Eyyup Aras, KSU, 2015 Spring*
Step 1
Draw the rocker $O_4B$ in both extreme positions $B_1$ and $B_2$ in any convenient location with angle $45^0$.

Step 2
Select a convenient point $O_2$ on line $B_1B_2$ extended.

Step 3
- Bisect line $B_1B_2$ and draw a circle with that radius about $O_2$.
- Label the two intersection of circle with $B_1B_2$ extended, $A_1$ and $A_2$.
- Measure $O_2A_1$ (Link 2: crank) and $A_1B_1$ (Link 3: coupler).

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Step 4

- Measure the ground length (Link 1), crank length (Link 2) and rocker length (Link 4).
- Check the Grashof condition \(S + L \leq P + Q\). If non-Grashof then redo steps 2 to 4 with \(O_2\) further from \(O_4\).

Figure 7: Limiting positions
**Example 2**

Rocker output - Design a Grashof four-bar linkage to move link CD from position $C_1D_1$ to $C_2D_2$ (see the Figure 8).

**Key pieces of statement**

- **Rocker output** – two positions with complex displacement (motion generation)
- **Four-bar linkage**, three in motion, one grounded
- Only the locations of the lines $C_1D_1$ and $C_2D_2$ are given.
- **Grashof** – at least one link can rotate fully ($S + L < P + Q$)

![Figure 8: finished linkage](image)

Figure 8: finished linkage

![Figure 9: kinematic diagram of the linkage](image)

Figure 9: kinematic diagram of the linkage

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Step 1

*Draw the link CD in its two desired positions $C_1D_1$ and $C_2D_2$, in the plane as shown.*

Step 2

*Draw construction lines from point $C_1$ to $C_2$, and from point $D_1$ to $D_2.*

Step 3

*Bisect line $C_1C_2$ and line $D_1D_2$ and extend their perpendicular bisectors to intersect at $O_4$. Their intersection is the rotopole.*

Step 4

*Since this example is the rocker output, draw the lines from $O_4$ to the lines $C_1D_1$ and $C_2D_2.*

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**Step 5**
Select a convenient radius and draw an arc about the rotopole to intersect both lines $O_4C_1$ and $O_4C_2$. Label the intersection $B_1$ and $B_2$.

**Step 6** Do steps 2 to 4 of example 1 to complete the linkage.

![Diagram](image1)

**Figure 10**: Final four-bar Grashof crank-rocker mechanism with complex motion

![Diagram](image2)

**Figure 11**: Limiting positions

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**Example 3**

**Coupler output** - Design a Grashof four bar linkage to move the link $CD$ from position $C_1D_1$ to $C_2D_2$ (with moving pivots at $C$ and $D$) (see the Figure 12).

**Key pieces of statement**

- **Coupler output** – two positions with complex displacement (motion generation)
- **Four-bar linkage, three in motion, one grounded**
- **Only the locations of the lines $C_1D_1$ and $C_2D_2$ are given.**
- **Grashof** – at least one link can rotate fully ($S + L < P + Q$)

---

**Figure 12:** (a) finished linkage, (b) kinematic diagram of the linkage

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Step 1

*Draw the link CD in its two desired positions $C_1D_1$ and $C_2D_2$, in the plane as shown.*

Step 2

*Draw construction lines from point $C_1$ to $C_2$, and from point $D_1$ to $D_2$*

Step 3

*Bisect line $C_1C_2$ and line $D_1D_2$ and extend their perpendicular bisectors in convenient directions. The rotopole will not be used in this question.*

Step 4

*Select any convenient point on each bisector as the fixed pivots $O_2$ and $O_4$ respectively. Connect $O_2$ with $C_1$ and call it Link 2. Connect $O_4$ with $D_1$ and call it Link 4.*

Step 5

*Line $C_1D_1$ is Link 3. Line $O_2O_4$ is Link 1.*

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Step 6
Check the Grashof condition \((S + L \leq P + Q)\), if unsatisfied, and then repeat steps 4 and 5. Note that any Grashof condition is potentially acceptable in this case.

Step 7 Construct the model for the positions \(C_1D_1\) and \(C_2D_2\).
THREE MOTION SYNTHESIS

Example 4

Coupler output – design a Grashof four bar linkage to move the link CD shown from position \( C_1D_1 \) to \( C_2D_2 \) and then to position \( C_3D_3 \). Moving pivots are at \( C \) and \( D \). Find the fixed pivot locations (see the Figure 13).

Key pieces of statement

- Coupler output – three positions with complex displacement (motion generation)
- Four-bar linkage, three in motion, one grounded
- Only the locations of the lines \( C_1D_1, C_2D_2 \) and \( C_3D_3 \) are given.
- Need to find fixed pivot locations \( O_2 \) and \( O_4 \).

Figure 13: finished linkage

* Prepared by Dr. Eyyup Aras, KSU, 2015 Spring
Step 1
*Draw link CD in three positions 
$C_1D_1, C_2D_2$ and $C_3D_3$.

Step 2
*Draw construction lines from point $C_1$ to $C_2$, and from $C_2$ to $C_3$.

Step 3
*Bisect line $C_1C_2$ and line $C_2C_3$ and extend their perpendicular bisector until they intersect. Label their intersection $O_2$.

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Step 4

Repeat steps 2 and 3 for lines $D_1D_2$ and $D_2D_3$. Label the intersection $O_4$.

Step 5 Connect $O_2$ with $C_1$ and call Link 2. Connect $O_4$ with $D_1$ and call Link 4.

Step 6 Check the Grashof condition $(S + L \leq P + Q)$. Note that any Grashof condition is potentially acceptable in this case.

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**Example 5**

Design a **dyad** to control and limit the extremes of motion of the linkage in example 3 to its two design positions (see the Figure 14).

*Key pieces of statement*

- Adding a **Dyad (two bar chain)** to control motion in example 3.

**Step 1**

Select a convenient point on Link 2 of the linkage designed in example 3. *Note that it need not be on the line \( O_2C_1 \).* Label this point \( B_1 \).

**Step 2**

Draw an arc about center \( O_2 \) through \( B_1 \) to intersect the corresponding line \( O_2B_2 \) in the second position of Link 2. Label this point \( B_2 \). The chord \( B_1B_2 \) provides us with the same problem as given by Example 1 (see page 6).

**Step 3**

Do steps 2 and 3 of Example 1 to complete the linkage. Name the new links as Link 5 and Link 6 and the center is \( O_6 \). Link 6 will be the driver crank. The four-bar sub chain of links \( O_6, A_1, B_1, O_2 \) must be a Grashof crank-rocker.

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Figure 14: Driver dyad with motor at $O_6$

Figure 15: Alternative location of the driver dyad with motor at $O_6$

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