جامعة صحار
SOHAR UNIVERSITY

ELEC3100- Fundamentals of Electromagnetic Fields and Waves

Module 1: Learning Material

Week 2: Wave Propagation on Transmission Line

Course Learning Outcomes

1. Calculate the transmission lines characteristic parameters.
2. Analyze the wave propagation and standing waves on a transmission line.

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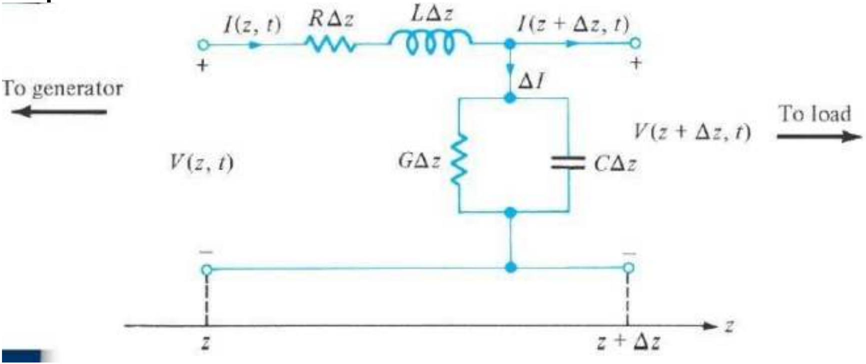
Wave propagation on a transmission line

- **The generalized lumped-element model of a transmission line can be used to derive the wave propagation along the line. In addition, it can be used to calculate its characteristic parameters.**
- **These parameters are: characteristic impedance, propagation constant (phase and attenuation), phase velocity, and input impedance .**



REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Wave propagation on a transmission line

General Solutions of Transmission-Line Equations

The voltage v(z,t) and current i(z,t) along the transmission line are functions of position and time.

For uniform transmission line with time-harmonic variation $e^{j\omega t}$, the transmission-line equations (Voltage and Current) are given by:

$$\frac{d^2V(z)}{dz^2} - \gamma^2 \ V(z) = 0 \longrightarrow \text{Voltage}$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 \ I(z) = 0 \longrightarrow \text{Current}$$

Where γ is the propagation constant and can be determined by the following equation

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

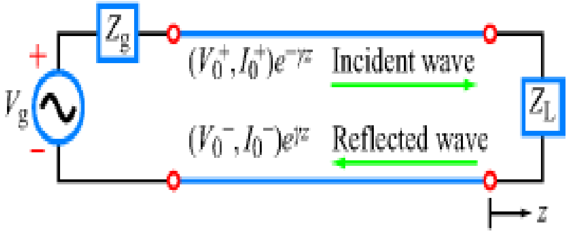
$$\gamma = \alpha + j\beta$$

$$\alpha = \text{attenuation constant } [\text{neper/m}]$$

$$\beta = \text{phase constant } [\text{radians/m}]$$

Solutions of Transmission-Line Equations

Steady state wave propagation along a transmission line (TL), voltage *V(z)* and current *I(z)* is usually determined by employing phasor analysis (phasor domain). The figure below shows the wave propagation along a TL.



The general solution of the TL wave equations is given as below:

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$$I(z) = I_o^+ e^{-\gamma z} - I_o^- e^{\gamma z}$$

$$I_o^+ = \frac{V_o^+}{Zo}$$

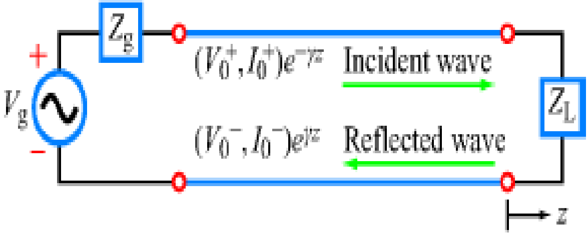
$$I_o^- = \frac{V_o^-}{Zo}$$

Forward Travelling Wave

Backward Travelling Wave

REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Solutions of Transmission-Line Equations



Where V_o^+, V_o^-, I_o^+, I_o^- are the initial amplitude of the voltages and currents. Zo is the characteristic impedance of the line and γ is the propagation constant .

In general, transmission line support two waves (forward wave and backward wave) traveling in opposite directions with their relative phase changes along the line.

REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Solutions of Transmission- Instantaneous Expression

The instantaneous expression of the voltage v(z,t) across the transmission line can be determined from the phasor of voltage as follow:

$$v(z,t) = Rel\left[V(z)e^{j\omega t}\right]$$

$$v(z,t) = Rel[V_o^+e^{-\gamma z}e^{j\omega t}+V_o^-e^{\gamma z}e^{j\omega t}]$$

$$v(z,t) = Rel[V_o^+e^{-\alpha z}e^{-j\beta z}e^{j\omega t}+V_o^-e^{\alpha z}e^{j\beta z}e^{j\omega t}]$$

$$v(z,t) = Rel[V_o^+e^{-\alpha z}e^{j(\omega t-\beta z)}+V_o^-e^{\alpha z}e^{j(\omega t+\beta z)}]$$

Using the trigonometric identity $e^{j\theta} = cos\theta + jsin\theta$ **, the instantaneous voltage is given by**

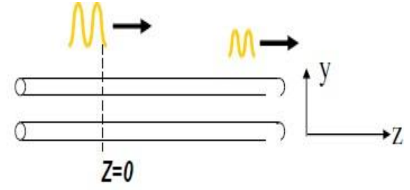
$$v(z,t) = V_o^+e^{-\alpha z}\cos(wt - \beta z)+ V_o^-e^{\alpha z}\cos(wt + \beta z)$$

Attenuation Constant α

The attenuation factor α decreases the amplitude of the voltage and current wave along the TL.

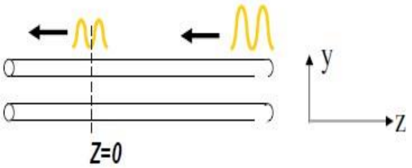
- For +ve traveling wave:

$$\left|V_0^+\right|\cos(\omega t-\beta z+\phi_+)e^{-\alpha z}$$



- For -ve traveling wave:

$$\left|V_0^-\right|\cos(\omega t+\beta z+\phi_-)e^{\alpha z}$$

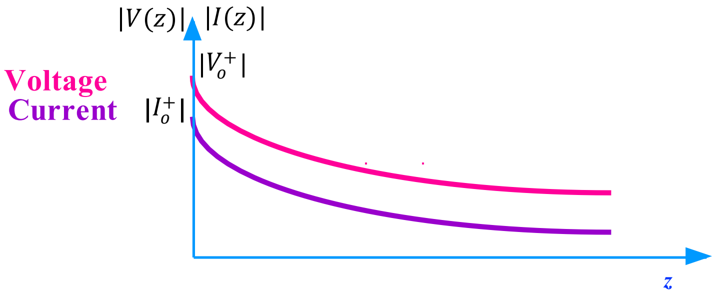


Attenuation Constant α

- Losses are due to:
  - Conductor loss (which usually increases with frequency)
  - Insulation (dielectric) loss.

Let V_o^+ is the input to the TL

Then, $|V(z)| = |V_o^+|e^{-\alpha z}$ and $|I(z)| = |I_o^+|e^{-\alpha z}$



Phase Constant β

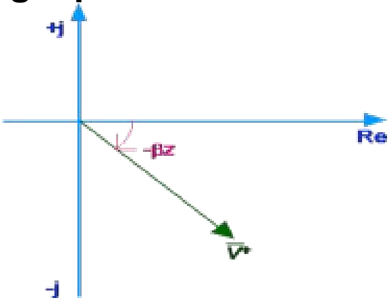
- The **phase shift** along the transmission line is due to the **time delay** caused by the distance *z*.

- $e^{-j\beta z}$ represents this phase shift.

  - The phase constant (phase velocity) β can be determined by the following equation

$$\boxed{\beta = \frac{2\pi}{\lambda} = \frac{\omega}{u_p} \ \mathrm{rad/m}}$$

λ: *wavelength (m)*
ω: $2\pi f$, *f is frequency*
u_p: *is the velocity of the wave*



Characteristic Impedance Z_o

- The characteristic impedance of the line is the ratio of positively traveling voltage wave to current wave at any point on the line.

- Characteristic impedance (Z_o) is the most important parameter for any transmission line.

$$Z_o = \frac{V_o^+}{I_o^+}$$

$$Z_o = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

Z_o is frequency dependent and complex

The characteristic admittance $Y_0 = 1/Z_0$

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Variation of Z_o vs. Frequency

At $\omega = 0$:

$$\boxed{Z_o = \sqrt{\frac{R'}{G'}} \rightarrow \infty \quad \text{since } G' \text{ is very small.}}$$

At high frequencies:

$$\omega L' >> R' \quad \text{and}$$

$$\omega C' >> G'$$

$$\therefore \boxed{\lim_{\omega \to \infty} Z_o = \sqrt{\frac{L'}{C'}} \quad \text{which is also real.}}$$

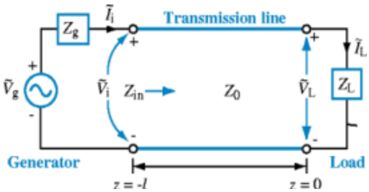
Characteristics of RG58-C/U

- Parameters for RG58-C/U coaxial cable
  - R' = 1 Ω / m
  - L' = 252 nH / m
  - G' = 0.0004 S / m
  - C' = 101 pF / m

Line Impedance

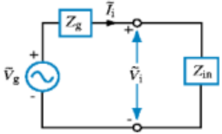
We can define a line impedance as the ratio of the line voltage and line current at a distance *d* from the load:

$$Zin = \frac{V_{in}}{I_{in}}$$



For lossy line, the line impedance (Z) can be determined by the following equation:

$$Z = Z_O \left[\frac{Z_R + Z_O \tanh \gamma d}{Z_O + Z_R \tanh \gamma d} \right]$$



For lossless line, the line impedance (Z) can be determined by the following equation:

$$Z = Z_O \left[\frac{Z_R + jZ_O \tan \beta d}{Z_O + jZ_R \tan \beta d} \right]$$

*** Z varies along the line with distance d and frequency**

REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Line Impedance vs. Z_R

Consider $Z_R = 0\ \Omega$ (i.e. a short circuit termination):

$$\boxed{Z = jZ \quad \mathrm{t}}$$

For $Z_R = \infty\ \Omega$ (i.e. an open circuit termination):

$$\boxed{Z = -jZ_O \cot \beta d}$$

Impedance is purely reactive

For $Z_R = Z_O$ (i.e. a matched termination):

$$\boxed{Z = Z_O}$$

for the matched line, Z is constant and equal to Z_O.

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