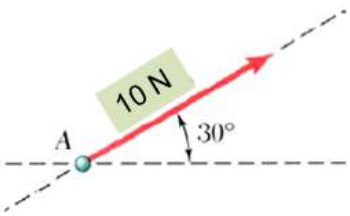
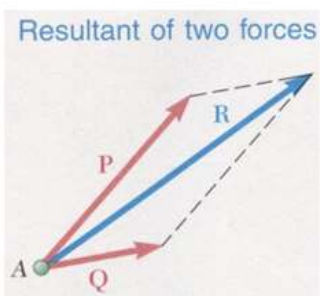
Static force analysis

In this chapter, some parts and examples are used from (Engineering Mechanics - Statics, 7th Edition, J. L. Meriam, L. G. Kraige),and (Machines & Mechanisms, Applied Kinematic Analysis, D.H. Myszka, Fourth Edition).

(FORCE & RESULTANT OF TWO FORCES)



- force: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense*.



- Experimental evidence shows that the combined effect of two forces may be represented by a single *resultant* force.
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.
- Force is a *vector* quantity.

(RECTANGULAR COMPONENTS)

Rectangular Components

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector **F** of Fig. 2/5 may be written as

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y \qquad (2/1)$$

where $\mathbf{F}_x$ and $\mathbf{F}_y$ are *vector components* of **F** in the *x*- and *y*-directions. $\mathbf{F}_x = F_x\mathbf{i}$ and $\mathbf{F}_y = F_y\mathbf{j}$, and thus we may write

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \qquad (2/2)$$

where the scalars F_x and F_y are the *x* and *y scalar components* of the vector **F**.

The scalar components can be positive or negative, depending on the quadrant into which **F** points. For the force vector of Fig. 2/5, the *x* and *y* scalar components are both positive and are related to the magnitude and direction of **F** by

$$F_x = F\cos\theta \qquad F = \sqrt{F_x{}^2 + F_y{}^2}$$
$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x} \qquad (2/3)$$

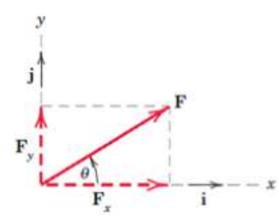
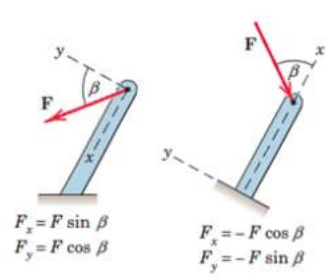
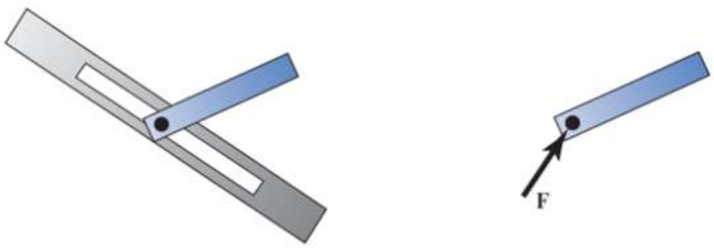


Figure 2/5



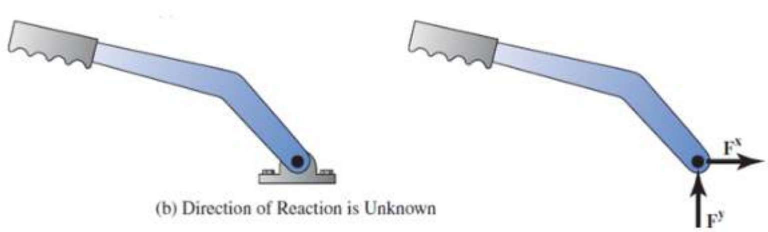
Characterizing Contact Forces

a. *Direction of Reaction is Known:* Components in this group include rollers, sliders, pins in slots, and cables. Each of these supports can prevent motion in only one direction. Reactions in this group involve only one unknown, namely the magnitude of the reaction force.



(a) Direction of Reaction is Known

b. *Direction of Reaction is Unknown:* Components in this group include frictionless pins, hinges, and sliders on rough surfaces. Each of these supports can prevent translation in both planar directions. Reactions in this group involve two unknowns, usually shown as the x- and y-components of the reaction force.



(b) Direction of Reaction is Unknown

c. *Reaction Prohibits Rotation:* Components in this group include fixed supports and pin joints at an actuator (motor or engine). Each of these supports can prevent translation in both planar directions and free rotation. Reactions in this group involve three unknowns, usually shown as the x- and y-components of the reaction force and a reaction moment.

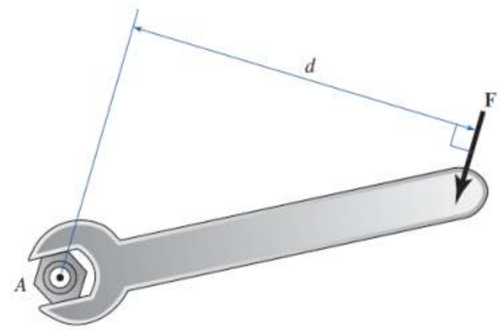
(c) Reaction Prohibits Translation and Rotation

Moments and torques

- ✓ *A **moment or torque** is the twisting action produced by a force.*
- ✓ *For example, in the following example, pushing on the handle of a wrench causes a twisting action around the center of a bolt. This resulting action is termed a moment or torque.*

$$M_A = F\,d$$

where F is the magnitude of the force, d is the perpendicular distance between the force and a reference point A. In the U.S. customary system the units for moments is (in.lb), In the International system (IS) the units are (N.mm).



Varignon’s Theorem in 2D

Varignon’s Theorem

One of the most useful principles of mechanics is *Varignon’s theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

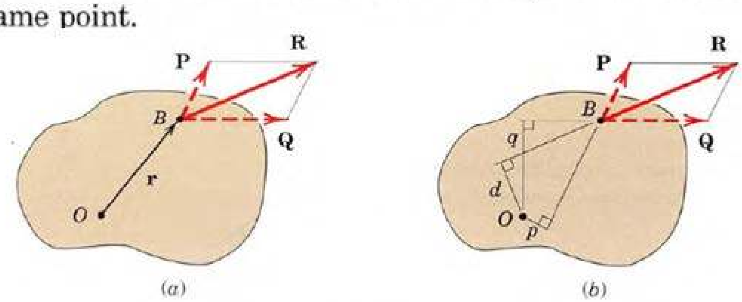


Figure 2/9

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R}$$

$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$$

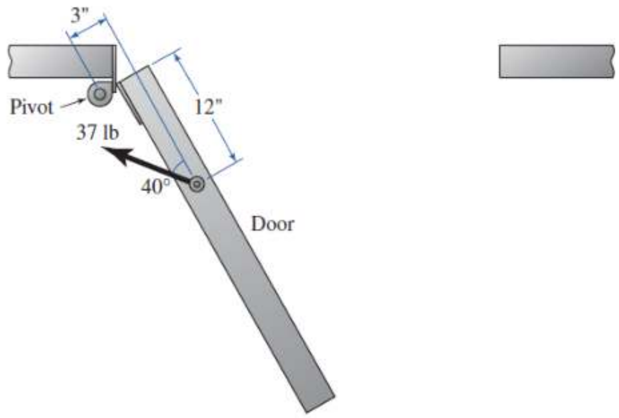
Figure 2/9*b* illustrates the usefulness of Varignon’s theorem. The moment of **R** about point O is Rd. However, if d is more difficult to determine than p and q, we can resolve **R** into the components **P** and **Q**, and compute the moment as

$$M_O = Rd = -pP + qQ$$

where we take the clockwise moment sense to be positive.

Example 1

A mechanism to automatically open a door exerts a 37-lb force on the door, applied in a direction as shown in Figure Determine the moment, relative to the pivot of the door, created by the force.



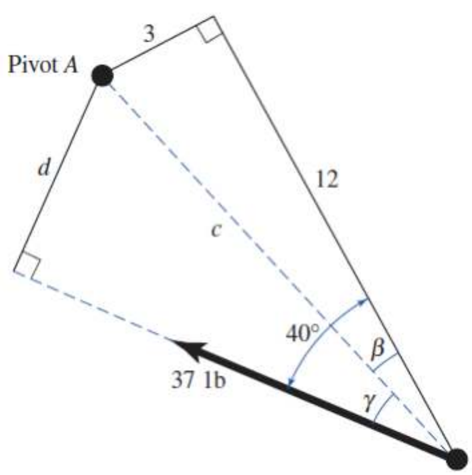
Solution 1

Calculate the perpendicular distance

Pythagorean theorem,

$$c = \sqrt{(12\,\text{in.})^2 + (3\,\text{in.})^2} = 12.37\text{ in.}$$

The included angle, β, can also be found from the trigonometric relations.



$$\beta = \tan^{-1}\left(\frac{3\,\text{in.}}{12\,\text{in.}}\right) = 14.0°$$

$$\gamma = 40° - \beta = 26°$$

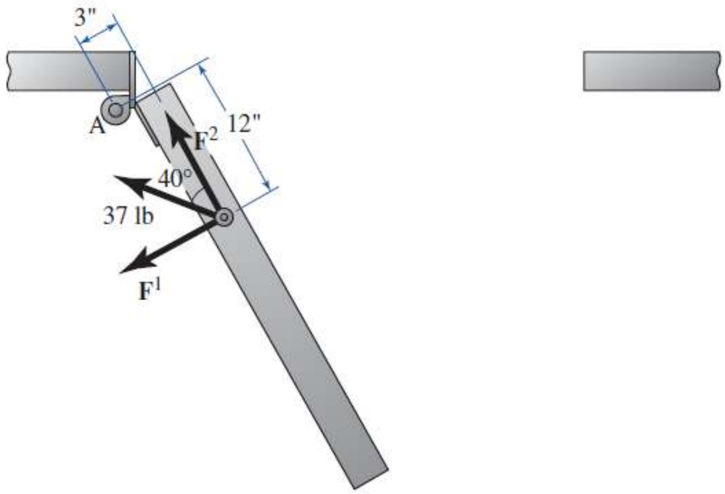
$$d = c\sin(\gamma) = (12.37\,\text{in.})\sin(26°) = 5.42\text{ in.}$$

$$M_A = F(d) = 37\,\text{lb}(5.42\text{ in.}) = 200.5\text{ in.lb, cw}$$

SOLUTION 2

Resolve force into rectangular components

The magnitude of F^1 and F^2 can be computed as



$$F^1 = (37\,\text{lb})\sin 40° = 23.8\ \text{lb}$$

$$F^2 = (37\,\text{lb})\cos 40° = 28.3\ \text{lb}$$

Calculate Moment

Calculating the moment relative to point *A*,

$$\begin{aligned} M_A &= -\ F^1(12\,\text{in.}) + F^2(3\ \text{in.}) \\ &= -[(23.8\,\text{lb})(12\,\text{in.})] + [(28.3\,\text{lb})(3\ \text{in.})] \\ &= -200.5\ \text{in. lb} \\ &= 200.5\ \text{in. lb, cw} \end{aligned}$$

LAWS OF MOTION

Sir Isaac Newton developed three laws of motion that serve as the basis of all analysis of forces acting on machines and components. These laws are stated as follows:

FIRST LAW: Every object remains at rest, or moves with constant velocity, unless an unbalanced force acts upon it.

SECOND LAW: A body that has an unbalanced force has

a. Acceleration that is proportional to the force,

b. Acceleration that is in the direction of the force, and

c. Acceleration that is inversely proportional to the mass of the object.

THIRD LAW: For every action, there is an equal and opposite reaction.

All of these laws are utilized in the study of mechanisms. However, in this chapter dealing with static force analysis, only the first and third laws are applicable. The following chapter incorporates the second law into the analysis.

STATIC EQUILIBRIUM

Newton's first law applies to all links that are at rest or moving at constant velocity; thus, the condition is referred to as *static equilibrium*. For an object to be in static equilibrium, the following two necessary and sufficient conditions must be met:

Condition I:

The combination, or resultant, of all external forces acting on the object is equivalent to zero and does not cause it to translate. Mathematically, the first condition of equilibrium can be summarized as

$$\Sigma \mathbf{F} = 0$$

$$\Sigma F^{x} = 0$$

$$\Sigma F^{y} = 0$$

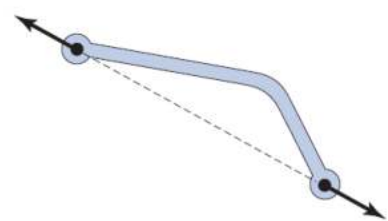
Condition II:
The moment due to any external force is canceled by the moments of the other forces acting on the object and do not cause it to rotate about any point. The second condition of equilibrium can be mathematically summarized as

$$\Sigma M_A = 0$$

This condition indicates that all the moments acting on the component are balanced.

ANALYSIS OF A TWO-FORCE MEMBER

A special case of equilibrium, which is of considerable interest, is that of a member that is subjected to only two forces. This type of machine component is termed a *two-force member*. Many mechanism links, particularly couplers and connecting rods, are two-force members. A two-force member is shown in Figure


Two-force member.

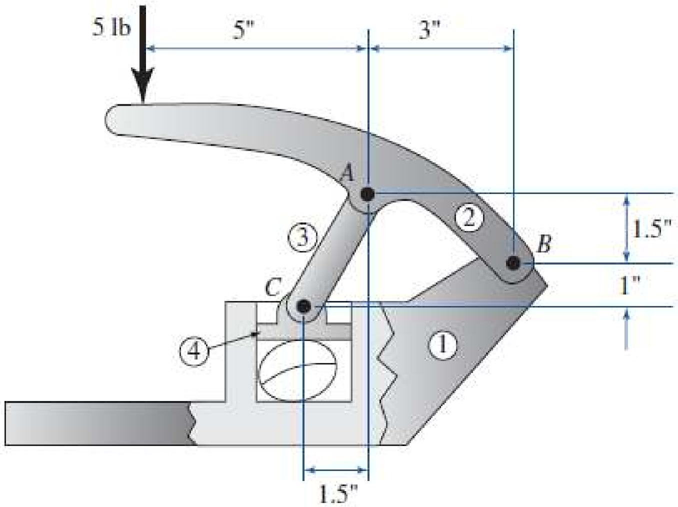
In order for a two-force member to be in equilibrium the two forces must:

1. Have the same magnitude,
2. Act along the same line, and
3. Be opposite in sense.

Because the two forces must act along the same line, the only line that can satisfy this constraint is the line that extends between the points where the forces are applied. Thus, a link with only two forces simply exhibits either tension or compression.

Example 2

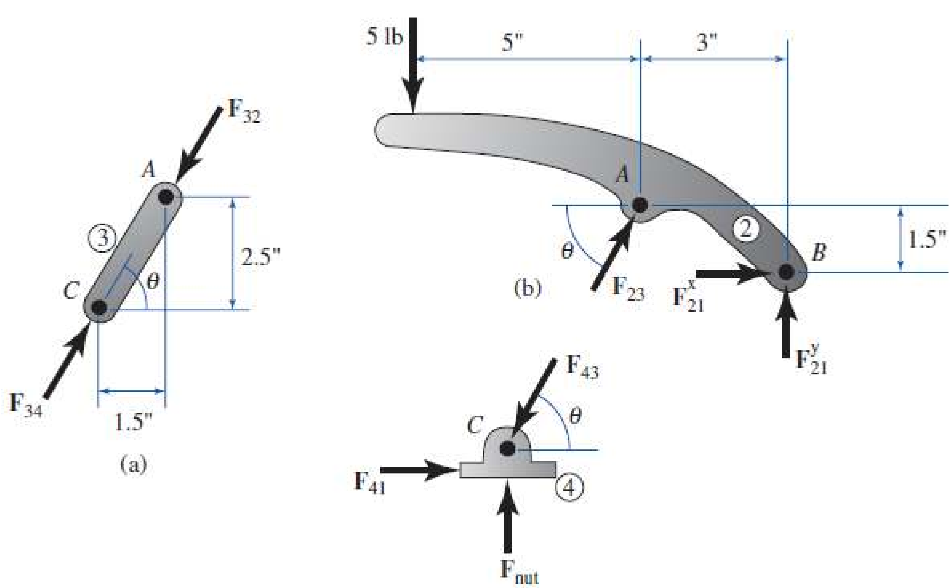
A novelty nutcracker is shown in Figure . A force of 5 lb is applied to the top handle, as shown, and the mechanism does not move (static). Draw a free-body diagram and determine the forces on each link. For this analysis, the weight of each link can be considered negligible.



Nutcracker

1. *Sketch the Free-Body Diagrams for the Mechanism Links*

   Notice that link 3 *(AC)* is a simple link, containing only two pin joints. In addition, no other force is acting on this link. Thus, it is a two-force member and the forces acting on the link must be equal and along the line that connects the two pins.



The free-body diagram for link 3 is shown As stated previously, the notation used is that F_{32} is a force applied to link 3 as a result of contact from link 2.

Being a two-force member, the direction of the two forces, F_{34} and F_{32}, is along the line that connects the two pins. The angle of inclination, θ, of this line can be determined.

$$\theta = \tan^{-1}\left(\frac{2.5}{1.5}\right) = 59.0^\circ$$

Link 2 is also a simple link that contains only two pin joints; however, an additional force is applied to the handle. Thus, this link is not a two-force member. Newton's third law stipulates that a force that is acting at *A* will be equal and opposite to F_{32}. Thus, the direction of F_{23} is known The general pin joint at point *B* dictates that two reaction forces will be used. The free-body diagram for link 2 is shown .

Link 4 has sliding contact with link 1. Neglecting any friction force, this contact force will act perpendicular to the contact surface. The contact force from the nut itself will similarly act perpendicular to the mating surface. Also, Newton's third law stipulates that a force acting at C will be equal and opposite to F_{34}. Thus, the direction of F_{43} is known . The free-body diagram for link 4 is shown

2. ***Solve the Equilibrium Equations for Link 2***

Link 2 is examined first because it contains the applied force. The three unknown forces on this link are solved by using the three equilibrium equations.

$\xrightarrow{+}\ \Sigma F^x = 0$:

$$F_{23}\cos 59.0^\circ + F_{21}^x = 0$$

$+\uparrow\ \Sigma F^y = 0$:

$$F_{23}\sin 59.0^\circ + F_{21}^y - 5\,\text{lb} = 0$$

$+\circlearrowleft\ \Sigma M_B = 0$:

$$(5\,\text{lb})(8\,\text{in.}) - (F_{23}\cos 59.0^\circ)(1.5\,\text{in.}) - (F_{23}\sin 59.0^\circ)(3\,\text{in.}) = 0$$

Solving the three equations yields

$$F_{23} = +11.96\,\text{lb} = 11.96\ \text{lb}\ \angle 59^\circ$$

$$F_{21}^x = -6.16\,\text{lb} = 6.16\ \text{lb} \leftarrow$$

$$F_{21}^y = -5.25\,\text{lb} = 5.25\ \text{lb} \downarrow$$

3. ***Solve the Equilibrium Equations for Link 3***

Because link 3 is a two-force member , the equilibrium equations dictate that the forces have the same magnitude, act along the same line, and are opposite in sense. Of course, Newton's third law dictates that $F_{32} = F_{23}$. Thus, the forces acting on link 3 are

$$F_{32} = 11.96 \text{ lb } \swarrow 59°$$

$$F_{34} = 11.96 \text{ lb } \nearrow 59°$$

4. ***Solve the Equilibrium Equations for Link 4***

The free-body diagram of link 4 will reveal the force exerted on the nut. Of course, Newton's third law dictates that $F_{34} = F_{43}$. Because the forces on link 4 all converge at a point, the moment equation of equilibrium does not apply. The two unknown forces on this link are solved by using two component equilibrium equations.

$\overset{+}{\rightarrow} \ \Sigma F^x = 0$:

$$F_{41} - F_{43} \cos 59.0° = 0$$

$+\uparrow \ \Sigma F^y = 0$:

$$F_{nut} - F_{43} \sin 59.0° = 0$$

Solving the two equations yields

$$F_{41} = +6.16 \text{ lb} = 6.16 \text{ lb} \rightarrow$$

$$F_{nut} = +10.25 \text{ lb} = 10.25 \text{ lb} \uparrow$$

EXAMPLE PROBLEM 13.4

Figure 13.12 shows a mechanism used to crush rocks. The 60-mm mechanism crank is moving slowly, and inertial forces can be neglected. In the position shown, determine the torque required to drive the 60-mm crank and crush the rocks.

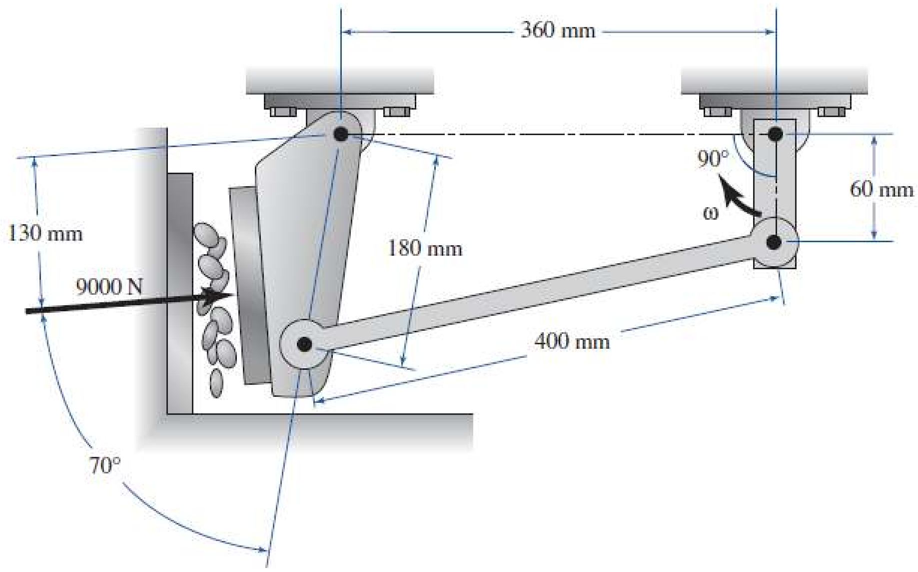


FIGURE 13.12 Rock crusher for Example Problem 13.4.

SOLUTION: 1. ***Sketch Free-Body Diagrams for the Mechanism Links***

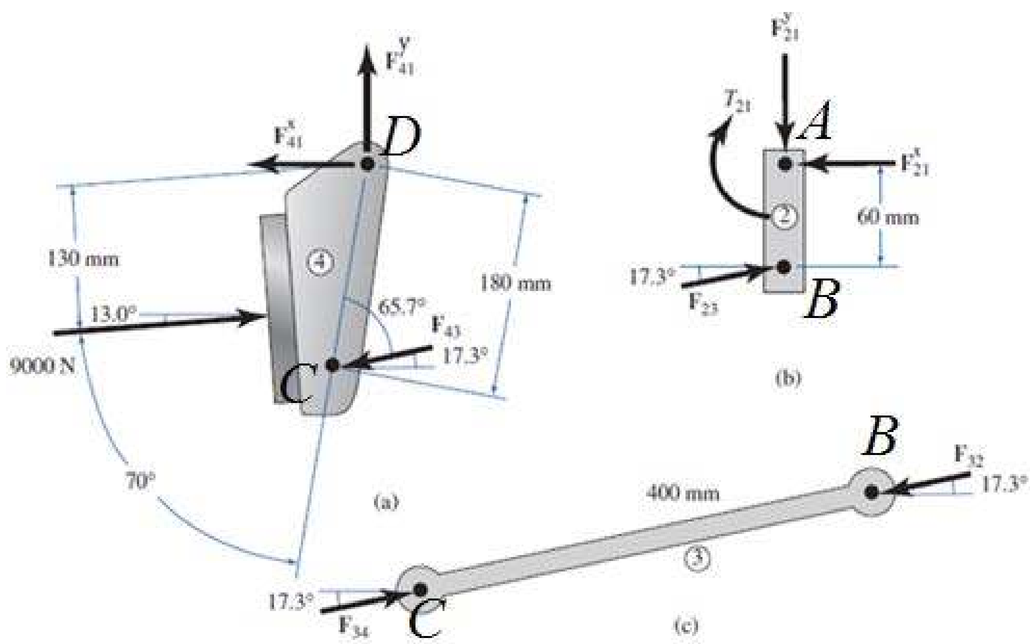


FIGURE 13.14 Free-body diagrams for Example Problem 13.4.

The free-body diagram for link 3 is shown . As stated previously, the notation used is that F_{32} is a force applied to link 3 as a result of contact from link 2.

Link 2 is also a simple link that contains only two pin joints; however, a drive torque is applied to the link at the shaft (point *A*). Thus, this link is not a two-force member. Newton's third law stipulates that a force that is acting on link 2 at point *B* will be equal and opposite to F_{32}. Thus, the direction of F_{23} is known as a result of Figure 13.14c. The general pin joint at point *A* dictates that two reaction forces will be used. The free-body diagram for link 2 is shown as Figure 13.14b.

Also, Newton's third law stipulates that a force that is acting onto link 4 at point *C* will be equal and opposite to F_{34}. The general pin joint at point *D* dictates that two reaction forces must be used. The free-body diagram for link 4 is shown as Figure 13.14a.

2. ***Solve the Equilibrium Equations for Link 4***

Link 4 is examined first because it contains the applied force. The three unknown forces on this link (Figure 13.11a) are solved by using the three equilibrium equations.

$\overset{+}{\rightarrow}\ \Sigma F^x = 0$:

$$(9000\text{ N})\cos 13.0^\circ - F_{43}\cos 17.3^\circ - F_{41}^x = 0$$

$+\uparrow\ \Sigma F^y = 0$:

$$(9000\text{ N})\sin 13.0^\circ - F_{43}\sin 17.3^\circ + F_{41}^y = 0$$

$+\circlearrowleft\ \Sigma M_D = 0$:

$$(9000\text{ N})(130\text{ mm}) - (F_{43}\sin 65.7^\circ)(180\text{ mm}) = 0$$

Solving the three equations yields

$$F_{43} = +7132\text{ N} = 7132\text{ N}\ \ \overline{17.3^\circ\swarrow}$$

$$F_{41}^x = +1960\text{ N} = 1960\text{ N}\leftarrow$$

$$F_{41}^y = +96.3\text{ N} = 96.3\text{ N}\uparrow$$

3. ***Solve the Equilibrium Equations for Link 3***

Because link 3 is a two-force member (Figure 13.14c), the equilibrium equations dictate that the forces have the same magnitude, act along the same line, and are opposite in sense. Of course, Newton's third law dictates that $F_{34} = F_{43}$. Thus, the forces acting on link 3 are

$$F_{34} = 7132\text{ N}\ \ \nearrow\underline{17.3^\circ}$$

$$F_{32} = 7132\text{ N}\ \ \overline{17.3^\circ\swarrow}$$

4. ***Solve the Equilibrium Equations for Link 2***

The free-body diagram of link 2 (Figure 13.14b) will reveal the instantaneous torque required to operate the device. Of course, Newton's third law dictates that $F_{23} = F_{32}$.

$\overset{+}{\rightarrow}$ $\Sigma F^x = 0$:

$$- F^x_{21} + F_{23} \cos 17.3° = 0$$

$+\uparrow$ $\Sigma F^y = 0$:

$$- F^y_{21} + F_{23} \sin 17.3° = 0$$

$+\circlearrowright$ $\Sigma M^A = 0$:

$$- T_{21} + (F_{23} \cos 17.3°)(60 \text{ mm}) = 0$$

Solving the three equations yields

$$F^x_{21} = +6809 \text{ N} = 6809 \text{ N} \leftarrow$$

$$F^y_{21} = +2121 \text{ N} = 212 \text{ N}\downarrow$$

$$T_{21} = +408{,}561 \text{ Nmm} = 409 \text{ Nm, cw}$$

Because the torque is the desired value, solving only the moment equation was necessary.

SLIDING FRICTION FORCE

As stated a contact force, as a result of a sliding joint, always acts perpendicular to the surface in contact. This contact force is commonly referred to as a *normal force* because it acts perpendicular to the surfaces in contact.

When friction cannot be neglected in machine analysis, an additional force, *friction force*, F_f, is observed. Friction always acts to impede motion. Therefore, a friction force acts on a sliding link, perpendicular to the normal force, and in a direction opposite to the motion (velocity).

For a stationary object, friction works to prevent motion until the maximum attainable friction has been reached. This maximum value is a function of a *coefficient of friction*, μ. The coefficient of friction is a property that is determined experimentally and is dependent on the materials and surface conditions of the contacting links.

$$F_f = \mu N$$

EXAMPLE PROBLEM 13.5

The scotch-yoke mechanism shown in Figure 13.15 is used in a valve actuator. As fluid is pumped into the cylinder, the increased pressure drives the mechanism and applies a torque to the output shaft. This torque can be used to actuate (open and close) valves. At the instant shown, the pressure load on the piston is300 *lb*Determine the torque generated on the output shaft. The coefficient of friction between the follower pin and crosshead slot is 0.15.

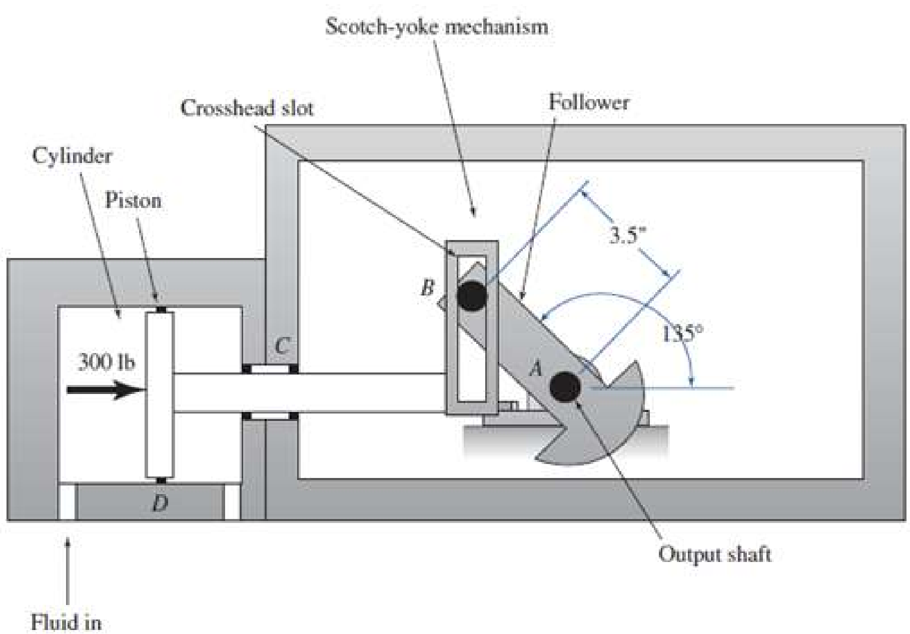


FIGURE 13.15 Valve actuator for Example Problem 13.5.

Sketch Free-Body Diagrams of the Mechanism Links

Link 2 is the piston/rod assembly/crosshead slot. Link 4 is the follower.

The free-body diagrams for links 2 and 4 are shown in Figure 13.17. Link 3 is not required for force analysis. Notice that a friction force is shown opposing relative motion. The directions may seem confusing and warrant further explanation.

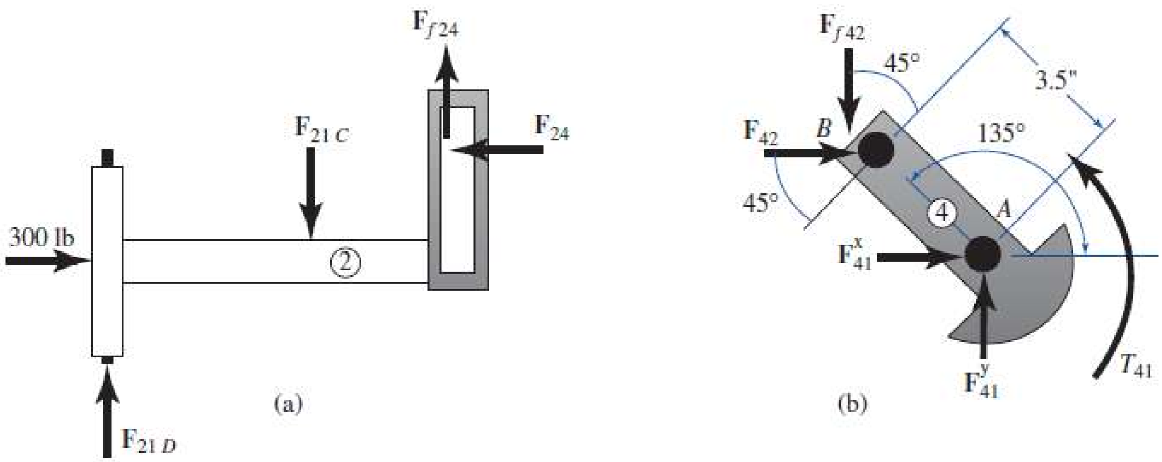


FIGURE 13.17 Free-body diagrams for Example Problem 13.5.

Consider link 4 (Figure 13.17b). The pin moves upward relative to the crosshead slot. Therefore, friction will act to prevent this motion of the pin by acting downward. Similarly, consider link 2 (Figure 13.17a). The slot moves downward relative to the pin (recall the definition of relative motion). Therefore, friction will act to prevent this motion of the slot by acting upward.

3. ***Solve the Equilibrium Equations for Link 2***

Link 2 (Figure 13.17a) is examined first because it contains the applied force. For this analysis, only the *x*-equilibrium equation is required.

$\overset{+}{\rightarrow} \; \Sigma F^x = 0$:

$$F_{24} = 300 \text{ lb} \leftarrow$$

4. ***Solve the Equilibrium Equations for Link 4***

The free-body diagram of link 4 (Figure 13.17b) will reveal the torque on the output shaft. Of course, Newton's first law dictates that $F_{42} = F_{24}$.

$$F_{f42} = \mu F_{42} = (0.15)(300 \text{ lb}) = 45 \text{ lb}$$

The torque can be determined by using the moment equilibrium equation.

$+\circlearrowleft \; \Sigma M_A = 0$:

$$-(F_{42}\cos 45°)(3.5 \text{ in.}) + (\mu F_{42}\cos 45°)(3.5 \text{ in.}) + T_{41} = 0$$

$$-[(300 \text{ lb})\cos 45°](3.5 \text{ in.}) + [(45 \text{ lb})\cos 45°(3.5 \text{ in.})] + T_{41} = 0$$

Finally, the torque exerted on the output shaft is

$$T_{41} = +631 \text{ lb-in.}$$