Dynamic force analysis

In this chapter, some parts and examples are used from (Machines & Mechanisms, Applied Kinematic Analysis, D.H. Myszka, Fourth Edition).

INERTIAL FORCE

Section 13.4 listed Newton's three principal laws of mechanics. The second law is critical for all parts that experience acceleration. It is stated as

SECOND LAW: A body that has an unbalanced force has

a. An acceleration that is proportional to the force,
b. An acceleration that is in the direction of the force, and
c. An acceleration that is inversely proportional to the mass of the object.

For linear motion, this law can be stated in terms of the acceleration of the link's center of gravity, A_g; thus,

$$\Sigma F = mA_g \tag{14.7}$$

Equation (14.7) can be rewritten as

$$\Sigma F - mA_g = 0 \tag{14.8}$$

The second term in equation (14.8) is referred to as the inertia of a body. This term is defined as an *inertial force*, F_g^i

$$F_g^i = -mA_g \tag{14.9}$$

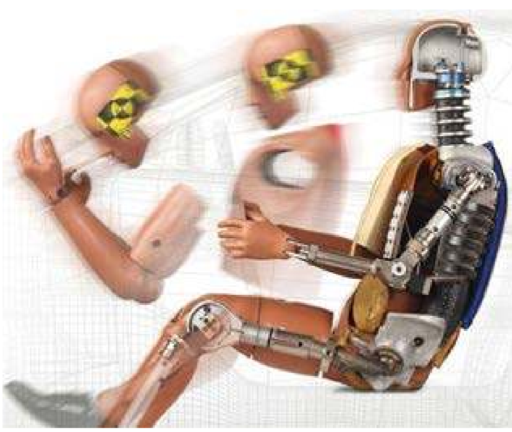
The negative sign indicates that the inertial force opposes acceleration (it acts in the opposite direction of the acceleration). Inertia is a passive property and does not enable a body to do anything except oppose acceleration.

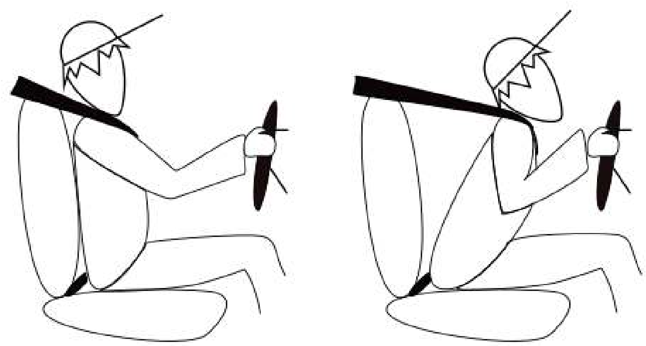
This notion is commonly observed. Imagine pounding on the gas pedal in an automobile, violently accelerating the vehicle. Envision the tendency for your head to lurch backward during the acceleration. This is the inertial force, acting in an opposite direction to the acceleration of the automobile. Further, the extent of the lurch is proportional to the magnitude of acceleration. Similarly, as the brakes in an automobile are slammed, decelerating the vehicle, your head lurches forward, again opposing the acceleration of the automobile. This is Newton's second law in practice.

Equation (14.8) can be rewritten as

$$\Sigma \mathbf{F} + \mathbf{F}_g^i = 0 \qquad (14.10)$$

This concept of rewriting equation (14.7) in the form of equation (14.8) is known as *d'Alembert's principle*. Using d'Alembert's principle in force analysis is referred to as *the inertia–force method of dynamic equilibrium*. It allows for analysis of accelerating links, using the same methods that are used in a static analysis.





NOTE THAT: for the given DYNAMICS question:

If the link has a MASS, then use $\Sigma \mathbf{F} + \mathbf{F}_g^i = 0$

If the link does not have a mass (negligible) then use $\Sigma \mathbf{F} = 0$

SEE THE FOLLOWING EXAMPLE

EXAMPLE PROBLEM 14.5

The compressor mechanism shown in Figure 14.7 is driven clockwise by a DC electric motor at a constant rate of 600 rpm. In the position shown, the cylinder pressure is 45 psi. The piston weighs 0.5 lb, and the coefficient of friction between the piston and the compressor cylinder is 0.1. The weight of all other links is negligible. At the instant shown, determine the torque required from the motor to operate the compressor.

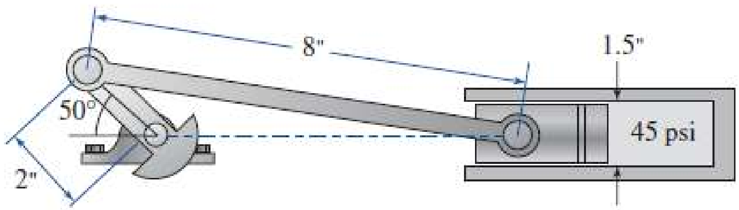


FIGURE 14.7 Mechanism for Example Problem 14.5.

SOLUTION:

1. *Draw a Kinematic Diagram*

   This is a common in-line, slider-crank mechanism, having a single degree of freedom. A scaled kinematic diagram is shown in Figure 14.8a.

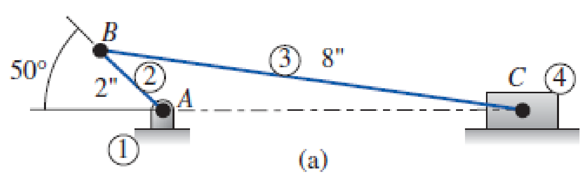


FIGURE 14.8 Diagrams for Example Problem 14.5.

2. ***Decide on a Method to Achieve the Required Motor Torque***

Because the piston is the only component without negligible weight, the inertial force, and the acceleration, of this component must be determined. The acceleration of the piston (link 4) is strictly translational and is identical to the motion of point *C*.

Once the acceleration of the piston has been obtained, the subsequent inertial forces can be calculated. Finally, free-body diagrams and the corresponding equations can be used to determine the required torque.

3. ***Determine the Velocity of Points* B *and* C**

The 2-in. crank is rotating at

600 rpm. The velocity of point *B* is

$$\omega_2 = \frac{\pi}{30}(600\text{ rev/min}) = 62.8\text{ rad/s, cw}$$

$$\mathbf{V}_B = \omega_2 r_{AB} = (62.8\text{ rad/s})(2\text{ in.}) = 125.6\text{ in./s} \quad \angle 40^\circ$$

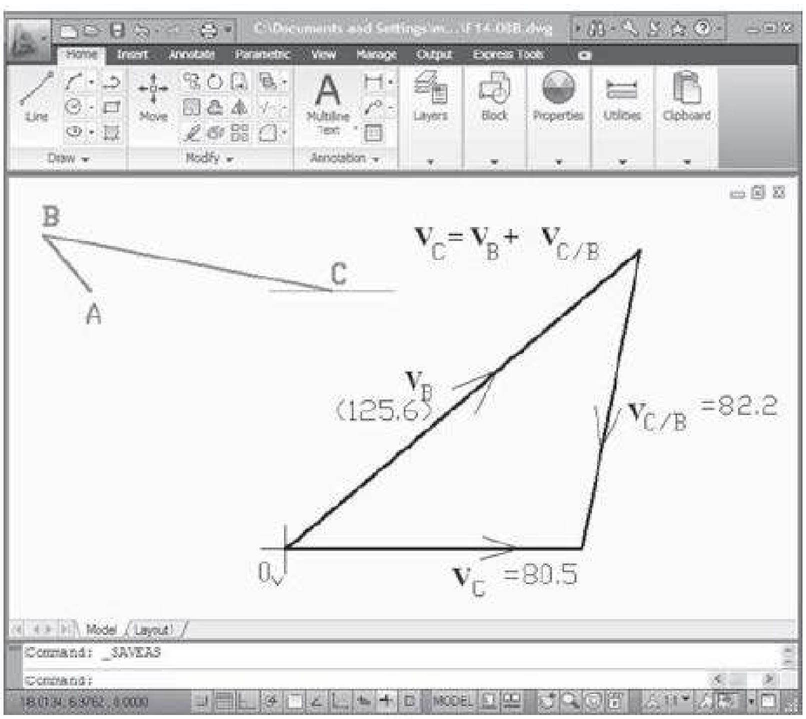
The direction of $\mathbf{V}_B$ is perpendicular to link 2 and consistent with the direction of ω_2, up and to the left.
The relative
velocity equation for points *B* and *C* can be written as

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{C/B}$$

A completed velocity diagram is shown in Figure 14.8b. Scaling the vector magnitudes from the diagram,

$$\mathbf{V}_C = 80.5\text{ in./s} \rightarrow$$

$$\mathbf{V}_{C/B} = 82.2\text{ in./s} \quad 79^\circ$$



(b)

4. ***Calculate Acceleration Components***

The next step is to construct an acceleration diagram, which includes points B and C. Calculating the magnitudes of the known accelerations,

$$A_B^n = \frac{(V_B)^2}{r_{AB}} = \frac{(125.6\text{ in./s})^2}{2.0\text{ in.}} = 7888\text{ in./s}^2 \ \angle 50^\circ \quad \text{(directed toward the center of rotation, point } A\text{)}$$

$$a_B^t = 0 \quad \text{(no angular acceleration of the 2-in. crank)}$$

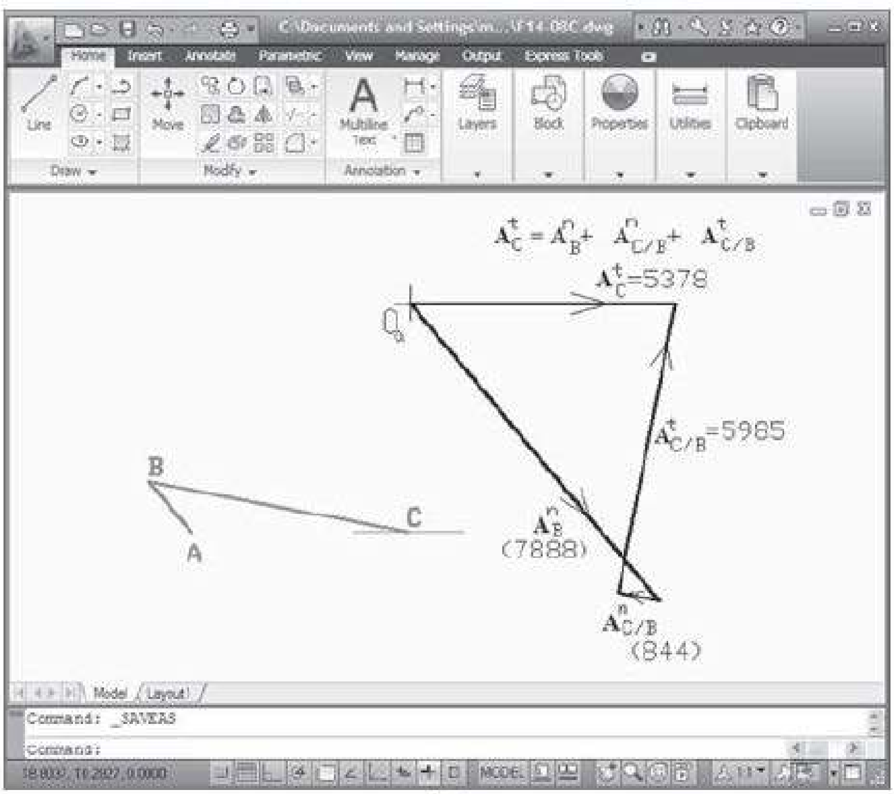
$$A_{C/B}^n = \frac{(V_{C/B})^2}{r_{BC}} = \frac{(82.2\text{ in./s})^2}{8.0\text{ in.}} = 844\text{ in./s}^2 \ \angle 11^\circ \quad \text{directed from } C \text{ toward } B,$$

5. ***Construct an Acceleration Diagram***

The relative acceleration equation for points B and C can be written as

$$A_C^n + A_C^t = A_B^n + A_B^t + A_{C/B}^n + A_{C/B}^t$$

The completed acceleration diagram is shown in Figure 14.8c.



(c)

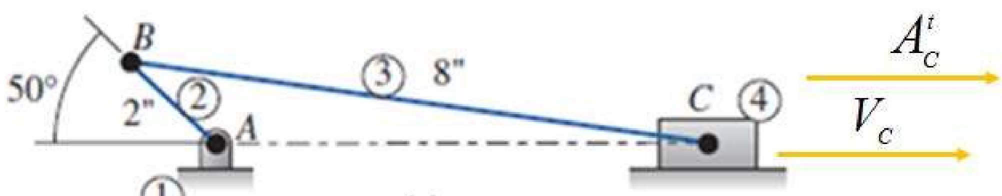
6. ***Measure the Piston Acceleration***

Scaling the vector magnitudes from the diagram,

$$A^{t}_{C/B} = 5985 \text{ in./s}^2 \angle 79^\circ$$

$$A^{t}_{C} = 5378 \text{ in./s}^2 \rightarrow$$

Because the tangential acceleration of point C is in the same direction as the velocity, the piston is accelerating (speeding up), not decelerating.



(a)

7. ***Calculate the Inertial Force***

Because the piston is the only link of considerable weight, its inertial force is computed by combining equations (14.9) and (14.1).

$$F^{i}_{g4} = - m_4 A_{g4} = \frac{W_4}{g}(- A_{g4})$$

$$= \frac{(0.5 \text{ lb})}{386 \text{ in./s}^2}(5378 \text{ in./s}^2) = 6.96 \text{ lb} \leftarrow$$

NOTE THAT *the acceleration of gravity is as follows*

The magnitude of weight and mass can be related through Newton's gravitational law.

$$W = mg$$

In most analyses on earth, the acceleration of gravity is assumed to be

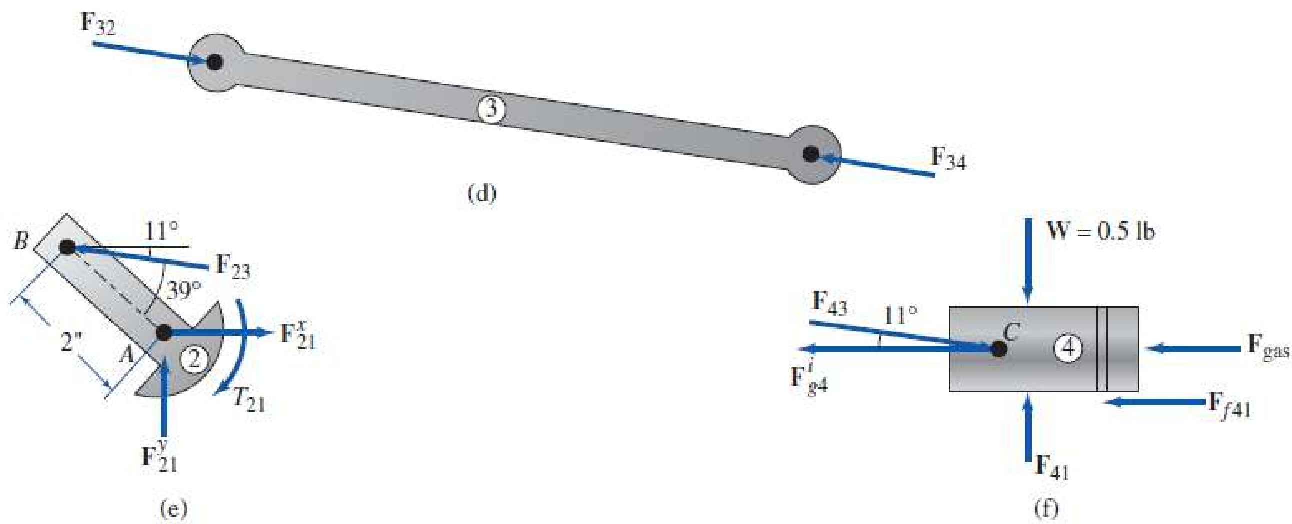
$$g = 32.2 \text{ ft/s}^2 = 386.4 \text{ in./s}^2 = 9.81 \text{ m/s}^2 = 9810 \text{ mm/s}^2$$

8. *Sketch Free-Body Diagrams of the Mechanism Links*

Notice that link 3 *(BC)* is a simple link, containing only two pin joints. In addition, no other force is acting on this link. Thus, it is a two-force member, and the forces acting on the link must be equal and along the line that connects the two pins. The free-body diagram for link 3 is shown as Figure 14.8d. As before, the notation used is that $\mathbf{F}_{32}$ is a force that is applied to link 3 as a result of contact from link 2.

Link 2 is also a simple link; it contains only two pin joints. However, a moment (torque) is also applied to this crank. Thus, this link is not a simple, two-force member. Newton's third law stipulates that a force that is acting at *B* will be equal and opposite to $\mathbf{F}_{32}$. Thus, the direction of $\mathbf{F}_{23}$ is known as a result of Figure 14.8d. The angle between links 2 and 3 was measured from the CAD model. A general pin joint at point *A* dictates that two reaction forces will be present. The free-body diagram for link 2 is shown as Figure 14.8e.

Link 4 has sliding contact with link 1. This contact force will act perpendicular to the contact surface. The force from the compressed gas will, similarly, act perpendicular to the piston surface. A friction force will oppose the motion (velocity) of link 4. Also, Newton's third law stipulates that a force that is acting at *C* will be equal and opposite to $\mathbf{F}_{34}$. Thus, the direction of $\mathbf{F}_{43}$ is known as a result of Figure 14.8d. The free-body diagram for link 4 is shown as Figure 14.8f.



NOTE THAT for the link 4, the **direction of the inertial force** is opposite to the direction of the acceleration

YOU can calculate the angles 39^0 **and** 11^0 **using he cosine and sine laws**

9. ***Solve the Dynamic Equilibrium Equations for Link 4***

Link 4 is examined first because it contains the applied force. The gas force is calculated as

$$F_{gas} = p_{gas} A_{piston} = p_{gas}\left[\frac{\pi(d_{piston})^2}{4}\right] = 45\ \text{lb/in.}^2\left[\frac{\pi(1.5\ \text{in.})^2}{4}\right] = 79.5\ \text{lb} \leftarrow$$

The friction force is

$$F_f = \mu F_{41} = 0.1\, F_{41}$$

The two unknown forces on this link (Figure 14.8f) are solved by using the following equilibrium equations:

$\overset{+}{\rightarrow}$ $\Sigma F^x + \quad F^i = 0$:

$$F_{43}\cos 11.0^\circ - F_{gas} - F^i_{g4} - F_f = 0$$

$+\uparrow$ $\Sigma F^y = 0$:

$$-F_{43}\cos 11.0^\circ + F_{41} - 0.5\ \text{lb} = 0$$

Solving these equations yields

$$F_{43} = +89.8\,\text{lb} = 89.8\ \text{lb}\ \overline{\searrow 11^\circ}$$

$$F_{41} = +16.6\,\text{lb} = 16.6\ \text{lb}\uparrow$$

NOTE THAT since the Link 4 has only translational motion we don't use the moment equation.

10. ***Solve for Equilibrium of Link 3***

Because link 3 is a two-force member (Figure 14.8d.), the equilibrium equations dictate that the forces have the same magnitude, act along the same line, and are opposite in sense. Of course, Newton's third law dictates that $F_{32} = F_{23}$. Thus, the forces acting on link 3 are

$$F_{34} = 89.8\,\text{lb}\ \underline{11^\circ \nwarrow}$$

$$F_{32} = 89.8\,\text{lb}\ \overline{\searrow 11^\circ}$$

NOTE THAT for the link 3, since there is no mass (negligible) the inertial force is zero.

11. ***Solve for Equilibrium of Link 2***

The free-body diagram of link 2 (Figure 14.8e) will reveal the required motor torque. Of course, Newton's third law dictates that $F_{32} = F_{23}$. The unknown forces and moment on this link are solved using the following equilibrium equations:

$\overset{+}{\rightarrow}$ $\Sigma F^x = 0$:

$$F_{21}^x - F_{23}\cos 11° = 0$$

$+\uparrow$ $\Sigma F^y = 0$:

$$F_{21}^y + F_{23}\sin 11° = 0$$

$+\circlearrowright$ $\Sigma M_A = 0$:

$$-T_{21} + (F_{23}\sin 39°)(2\text{ in.}) = 0$$

NOTE THAT for the link 2, since there is no mass (negligible) the inertial force is zero use $\Sigma F = 0$

ALSO since this is the dynamic case the above moment equation is $\sum M_A = I\alpha$. But for the constant rotational speed of the link 2, $\alpha = 0$.

Solving the three equations yields

$$F_{21}^x = +88.1\text{ lb} = 88.1\text{ lb} \rightarrow$$

$$F_{21}^y = -17.1\text{ lb} = 17.1\text{ lb} \downarrow$$

$$T_{21} = +113.0\text{ lb in.} = 113.0\text{ lb in., cw}$$

Because the torque is the desired value, solving only the moment equation was necessary.