جامعة صحار
SOHAR UNIVERSITY

ELEC3100- Fundamentals of Electromagnetic Fields and Waves

Module 1: Learning Material
Week 3: Standing Waves and Power Flow

Course Learning Outcomes
Analyze the wave propagation and standing waves on a transmission line.

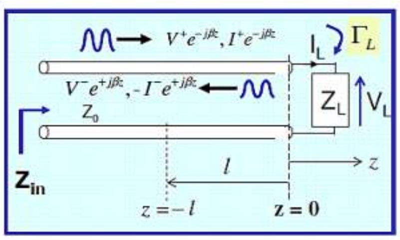
Lecturer Name: Dr. HILAL A. FADHIL
Academic Year: 2020-2021
Course Coordinator Name: Dr. HILAL A. FADHIL

Voltage Reflection Coefficient

We define the reflection coefficient as the ratio between the amplitude of the reverse (reflected) voltage to the amplitude of the forward (incident) voltage wave:

$$\Gamma = \frac{\bar{V}^-}{\bar{V}^+} = \left(\frac{Z_R - Z_O}{Z_R + Z_O}\right) e^{-2\gamma d}$$

$$\therefore \Gamma = |\Gamma| \angle \theta$$



Voltage reflection coefficient!

Note: Γ is complex, and it varies with d.

Reflection Coefficient (*cont.*)

At the receiving end (where $d = 0$):

$$\Gamma_R = |\Gamma_R| \angle \theta_R$$

$$= \frac{\overline{V}_R^-}{\overline{V}_R^+}$$

voltages at the receiving end of the line.

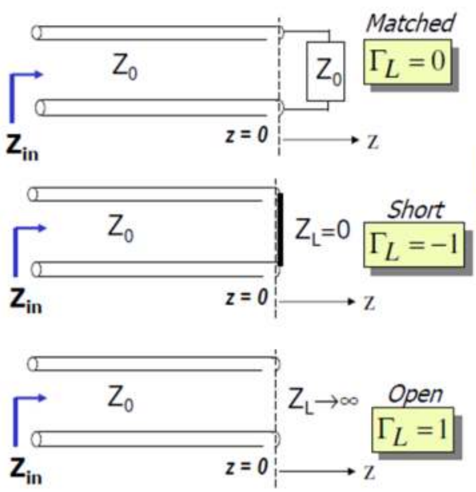
$$\therefore \Gamma_R = \frac{Z_R - Z_O}{Z_R + Z_O}$$

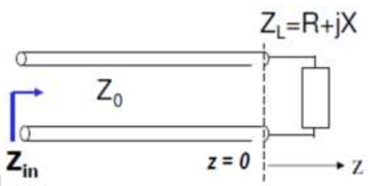
For lossy line, the reflection coefficient at distance *d* from the load is given by

$$\therefore \Gamma = |\Gamma_R| e^{-2\alpha d} \angle(\theta_R - 2\beta d)$$

Note that as *d* increases, the magnitude of the reflection coefficient reduces (due to the losses), and there is a negative phase shift of $2\beta d$ radians.

Reflection Coefficient for Various Loads

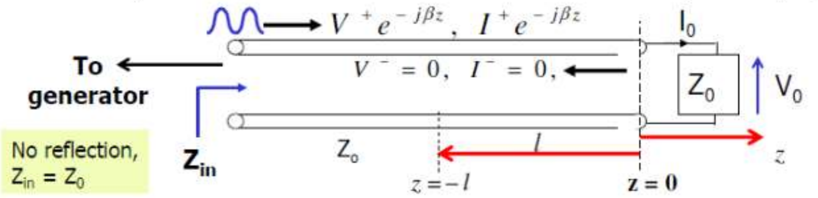




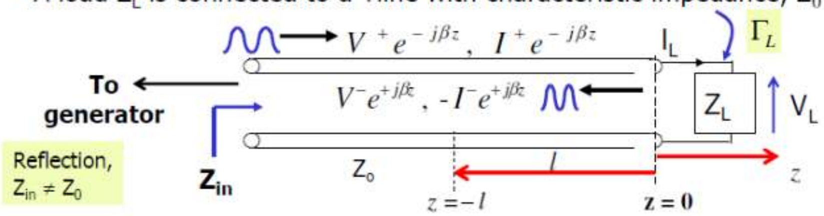
$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$

The Terminated Transmission Line

- A load Z_0 is connected to a Tline with characteristic impedance, Z_0



- A load Z_L is connected to a Tline with characteristic impedance, Z_0



** Note that at low frequency (Tline << λ), Zin = Z0, regardless of l*

Example 1

A 50 Ω lossless transmission line is terminated in a load impedance ZL=(30-j200) Ω. Calculate the voltage reflection coefficient at the load.

Answer:

$$\Gamma = \frac{30 - j200 - 50}{30 - j200 + 50}$$

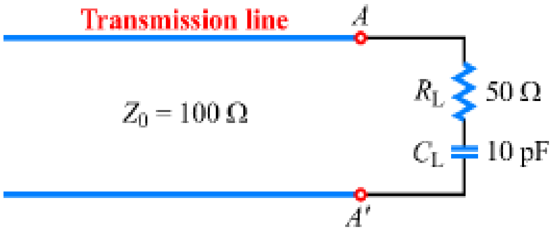
$$\Gamma = \frac{-20 - j200}{80 - j200} = 0.93 < -27.5^o$$

Exercise 1 (Test Your Understanding)

A 100-Ω transmission line is connected to a load consisting of 50-Ω resistor in series with a 10pF capacitor. Calculate the reflection coefficient at the load for a 100-MHz signal.

Answer:

$$\Gamma = 0.76 < -60.7^{o}$$

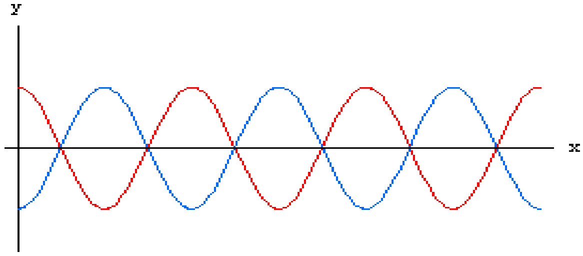


REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Standing waves on Transmission lines

Standing waves are **waves** of voltage and current generated as a result of interference between **incident** and **reflected** **waves** along a **transmission line**.

Standing wave



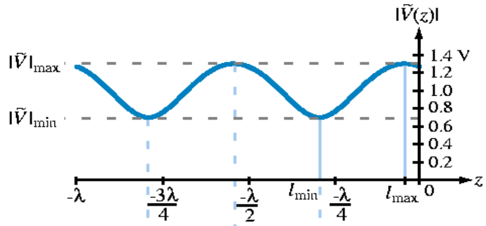
- **If the load Z_L is not equal to transmission line characteristic impedance Z_0, some of the incident wave is absorbed, and the rest is reflected.**
- **We have one set of waves, V and I, traveling toward the load, and the reflected set traveling back to the generator.**
- **These two sets of traveling waves, propagate in opposite directions (180° out of phase), set up an interference pattern known as Standing Waves in Transmission Lines**

REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Voltage Standing Wave Ratio

The voltage standing wave ratio (VSWR) is a figure of merit for a lossless transmission line system. It is defined as:

$$\mathrm{VSWR} = \mathrm{SWR} = \mathrm{S} = \frac{\mathrm{V_{max}}}{\mathrm{V_{min}}}$$



(a) $|\tilde{V}(z)|$ *versus* z

where

$$V_{max} = |\overline{V}^{+}| + |\overline{V}^{-}|$$

$$V_{min} = |\overline{V}^{+}| - |\overline{V}^{-}|$$

$Z_0 = 50\Omega$

$\Gamma = 0.3e^{j30^\circ}$

$|V_0^+| = 1$

$$VSWR = S = \frac{1+|\Gamma|}{1-|\Gamma|}, \quad \Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$1 \leq VSWR < \infty$ 　 *VSWR must be ≥ 1!* 　 $\Gamma \leq 1$

Maxima and minima are spaced $\lambda/4$ apart

Example 2

Calculate the reflection coefficient and VSWR for different load (a) $Z_R = \infty\ \Omega$, (b) For $Z_R = Z_O/3\ \Omega$

(a) For an open circuit termination ($Z_R = \infty\ \Omega$):

$$\left|\Gamma_R\right| = \lim_{Z_R \to \infty}\left|\frac{\boldsymbol{Z_R} - \boldsymbol{Z_O}}{\boldsymbol{Z_R} + \boldsymbol{Z_O}}\right| = 1$$

$$\therefore \mathrm{VSWR} = \infty$$

(b) For $Z_R = Z_O/3\ \Omega$):

$$|\Gamma_R| = \frac{Z_O/3 - Z_O}{Z_O/3 + Z_O} = \frac{-2/3}{4/3} = 0.5$$

$$\therefore VSWR = 3$$

Distance of voltage Maximum (l_{max} and voltage minimum (l_{min})

- **Maximum voltage occurs when the incident and reflected waves are in phase. The distance (l_{max}) from the load to the point on the line where the voltage is at maximum is given by :**

$$l_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} \qquad \begin{cases} n = 1,2,3,\ldots. & if\theta_r < 0 \\ n = 0,1,2,3,\ldots. & if\theta_r \geq 0 \end{cases}$$

θ_r is the angle of the reflection coefficient.

- **Minimum voltage occurs when the incident and reflected waves are in phase opposition. The distance (l_{min}) from the load to the point on the line where the voltage is at minimum is given by :**

$$l_{min} = \frac{\theta_r \lambda}{4\pi} + \frac{(2n+1)\lambda}{4} \qquad n = 0,1,2,3,\ldots$$

The period of repetition of the standing wave is $\lambda/2$.
But no reflection….no standing wave!

Example 3

A 72 cm lossless transmission line with characteristic impedance=140 Ω is terminated in a load impedance ZL = (280+ j182) Ω. Analyze the wave across the line by considering the following:

(a) Calculate the reflection coefficient.

(b) Calculate the voltage standing-wave ratio, S.

(c) Determine the locations of voltage maximum.

REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Answer: given $Z_0 = 140\ \Omega, \qquad Z_L = (280 + j182)\ \Omega$

(a) The reflection coefficient.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$
$$= \frac{280 + j182 - 140}{280 + j182 + 140} = \frac{140 + j182}{420 + j182} = 0.5\angle 29^\circ .$$

(b) The voltage standing-wave ratio, S.

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.5}{1 - 0.5} = \frac{1.5}{0.5} = 3.$$

(c) The locations of voltage maxima l_{max}

$$l_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}, \qquad n = 0, 1, 2, \ldots$$
$$= \frac{(29\pi/180) \times 0.72}{4\pi} + \frac{n \times 0.72}{2}$$
$$= (2.9 + 36n) \quad (\text{cm}), \qquad n = 0, 1, 2, \ldots$$

REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Power Flow on a Lossless Transmission Line

- We have seen that a **transmission line** can **support forward** and **reverse travelling voltage** and **current waves**.
- In **application**, we are concerned with **conveying energy** (usually a signal carrying information) **from one point to another** (e.g. to a satellite and back down to a receiver located in another location).
- We will examine the **flow of power** carried by the forward (**incident**) and reverse (**reflected**) waves, and see how we can **maximise the power delivered to the load termination.**
- We will limit our discussion to **lossless lines**.

Incident Instantaneous Power

- **The instantaneous power carried by the incident wave as it arrives at the termination, is calculated as:**

$$P_R^+(t) = v_R^+(t) \times i_R^+(t)$$

$$= \mathrm{Re}\left\{\overline{V}_R^+ e^{j\omega t}\right\} \times \mathrm{Re}\left\{\overline{I}_R^+ e^{j\omega t}\right\}$$

$$= \mathrm{Re}\left\{\left|\overline{V}_R^+\right| \angle \phi_R^+ e^{j\omega t}\right\} \times \mathrm{Re}\left\{\frac{\left|\overline{V}_R^+\right|}{Z_O} \angle \phi_R^+ e^{j\omega t}\right\}$$

$$= \left|\overline{V}_R^+\right| \cos\left(\omega t + \phi_R^+\right) \times \frac{\left|\overline{V}_R^+\right|}{Z_O} \cos\left(\omega t + \phi_R^+\right)$$

$$so \quad \boxed{P_R^+(t) = \frac{\left|\mathbf{V}_R^+\right|^2}{\mathbf{Z}_O} \cos^2\left(\omega t + \phi_R^+\right) \; Watts}$$

REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Reflected Instantaneous Power

- **The instantaneous power carried by the reflected wave as it leaves the termination, is calculated as:**

$$\boxed{\begin{aligned}\Gamma_R &= \left|\Gamma_R\right|\angle\theta_R \\ &= \left|\Gamma\right|\angle\theta_R\end{aligned}}$$

$$P_R^-(t) = v_R^-(t) \times i_R^-(t)$$

$$= \mathrm{Re}\left\{\overline{V}_R^- e^{j\omega t}\right\} \times \mathrm{Re}\left\{\overline{I}_R^- e^{j\omega t}\right\}$$

$$= \mathrm{Re}\left\{\left|\Gamma\right|\left|\overline{V}_R^+\right| \angle\left(\phi_R^+ + \theta_R\right) e^{j\omega t}\right\} \times \mathrm{Re}\left\{-\left|\Gamma\right| \frac{\left|\overline{V}_R^+\right|}{Z_O} \angle\left(\phi_R^+ + \theta_R\right) e^{j\omega t}\right\}$$

Negative sign means power flows away from the termination

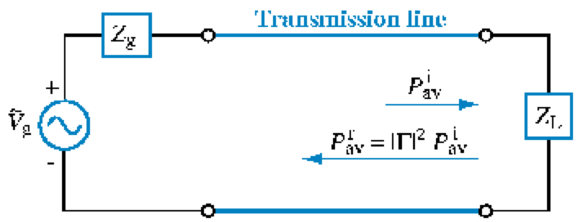
$$= \left|\Gamma\right|\left|\overline{V}_R^+\right| \cos\left(\omega t + \phi_R^+ + \theta_R\right) \times \left[-\left|\Gamma\right| \frac{\left|\overline{V}_R^+\right|}{Z_O} \cos\left(\omega t + \phi_R^+ + \theta_R\right)\right]$$

$$\boxed{P_R^-(t) = -\left|\Gamma\right|^2 \frac{\left|\overline{V}_R^+\right|^2}{Z_O} \cos^2\left(\omega t + \phi_R^+ + \theta_R\right) \quad \text{Watts}}$$

REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Average Power Flow Calculation

- In practice, we are more interested in the average power flowing along the line than the instantaneous power.
- We can calculate the average power using either a time-domain approach or the simpler phasor-domain approach.



REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Time-Domain Calculation of the Average Incident Power

- **We need to average the incident instantaneous power over one time period $T = 1/f = 2\pi/\omega$.**

$$P^+_{av} = \frac{1}{T}\int_0^T P^+_R(t)\,dt$$

$$= \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left[\frac{|\bar{V}^+_R|^2}{Z_o}\cos^2(\omega t + \phi^+_R)\right]dt$$

so

$$\boxed{P^+_{av} = \frac{|\bar{V}^+_R|^2}{2Z_o}\ Watts}$$

Factor of 1/2 comes from the integration of the $\cos^2$ term.

Time-Domain Calculation of the Average Reflected Power

- We need to average the reflected instantaneous power over one time period $T = 1/f = 2\pi/\omega$.

$$P_{av}^{-} = \frac{1}{T}\int_0^T P_R^{-}(t)\,dt$$

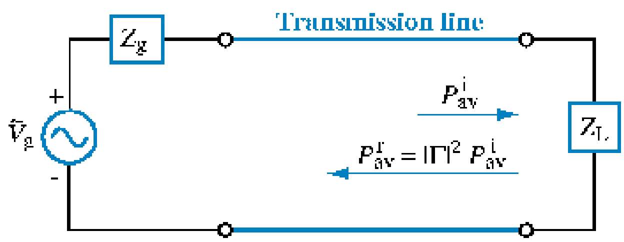
$$= -|\Gamma|^2 \frac{\left|\overline{V}_R^{+}\right|^2}{2Z_O}$$

so $\boxed{P_{av}^{-} = -|\Gamma|^2 P_{av}^{+} \qquad \text{Watts}}$

Net Average Power Delivered to the Termination

- The net average power delivered to the termination is therefore:

$$\boxed{P_{av,term} = P_{av\,i}^{+} - P_{av\,r}^{-} = \frac{\left|\overline{V}_R^{+}\right|^2}{2Z_O}\left[1-|\Gamma|^2\right] \text{Watts}}$$



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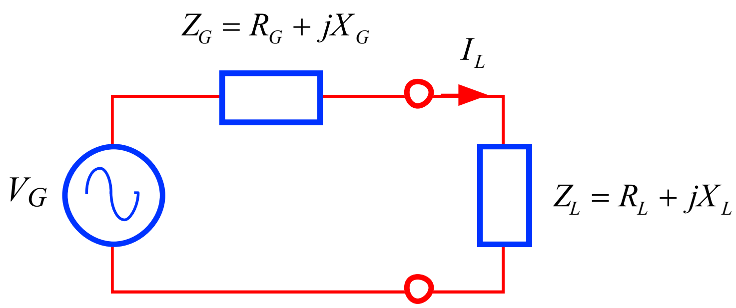
Maximum Power Transfer

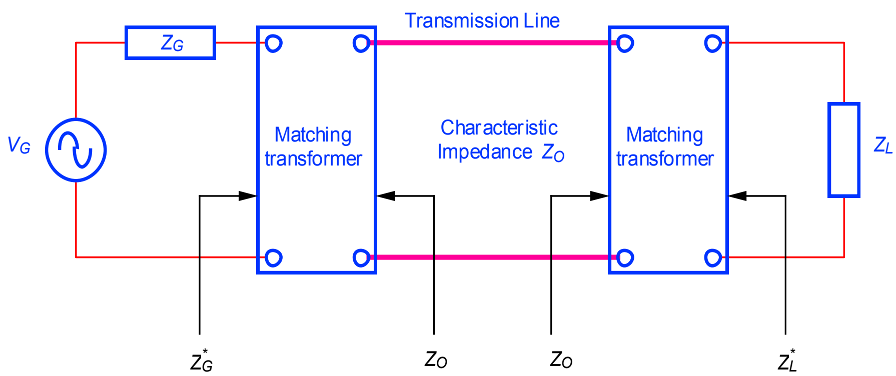
From circuit theory, maximum power transfer occurs when the load impedance is the complex conjugate of the generator impedance.



where $R_L = R_G$ and $X_L = -X_G$

Maximum Power Transfer

In a transmission line system, matching must be done both from the generator to the line and from the line to the load, as shown below:



REF. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London.

Maximum Power Transfer

- **In the above case, Z_o is real, and the impedances at every point are conjugates of the impedances seen when looking in opposite directions.**
- **In practice, commercially available RF signal generators and power amplifiers have a standard output impedance which corresponds closely to the Z_o of manufactured lines.**

For example:

General RF & microwave:	50 Ω
TV:	75 Ω
Telephony:	600 Ω

∴ only the load needs to be matched.

Matching Methods

- At radio frequency (RF):
  - Iron-cored transformers
  - Lumped L's and C's
- At ultra high frequency (UHF) and microwave frequency, sections of transmission line are used:
  - Quarter-wave transformer
  - Single and double stub tuners

This technology gives the best performance and it is the cheapest.

References

1. Fawwaz T. Ulaby (2005). Electromagnetics for Engineers, Prentice Hall International, London. Library code (QC760 .U488 2005).
2. Fawwaz T. Ulaby (2007). Fundamental of Applied Electromagnetics, Library code (QC760 .U49 2007).