



Taibah University

College of Engineering

Department of
Civil Engineering

Course CE 341:

Water resources

Chapter 3: Uniform flow

Introduction

- A flow is said to be uniform if its properties remain constant with respect to distance
- the term uniform flow in open channels is understood to mean steady uniform flow.
- The depth of flow remains constant at all sections in a uniform flow (Fig. 3.1).
- Considering two Sections 1 and 2, the depths

$$\begin{aligned}y_1 &= y_2 = y_0 \\ A_1 &= A_2 = A_0\end{aligned}$$

Since $Q = AV = \text{constant}$, it follows that in uniform flow $V_1 = V_2 = V$.

- in a uniform flow, the depth of flow, area of cross-section and velocity of flow remain constant along the channel.
- uniform flow is possible only in prismatic channels.
- The trace of the water surface and channel bottom slope are parallel in uniform flow (Fig. 3.1).
- Further, since $V = \text{constant}$, the energy line will be at a constant elevation above the water surface. As such, the slope of the energy line S_f , slope of the water surface S_w and bottom slope S_0 will all be equal to each other.

Uniform flow

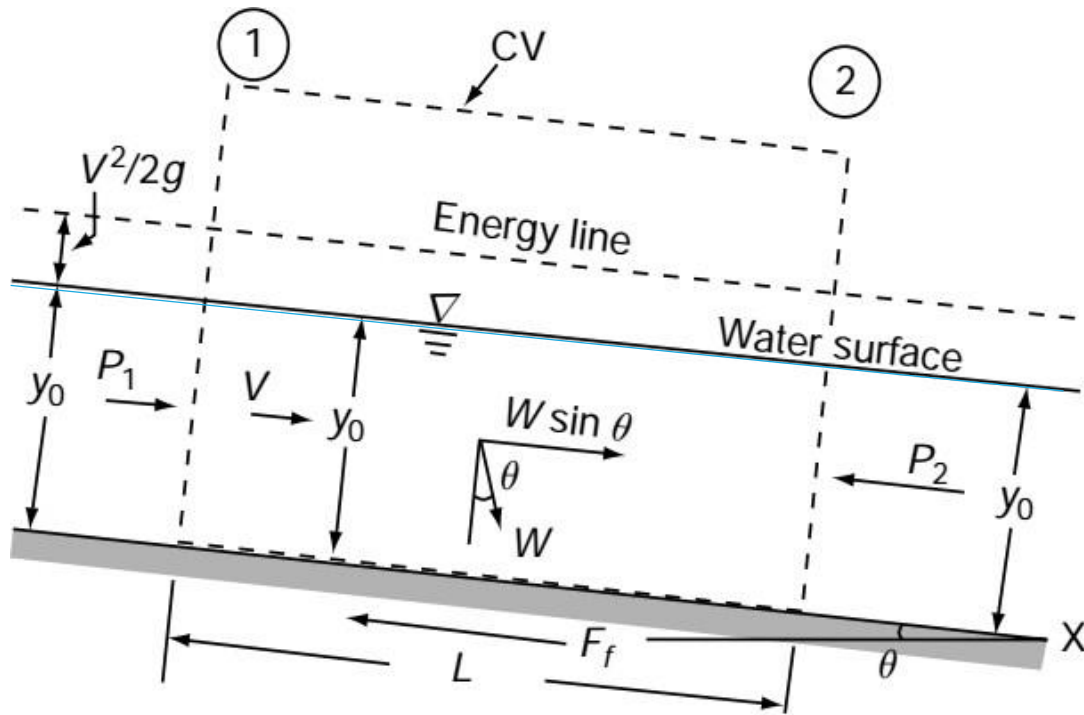
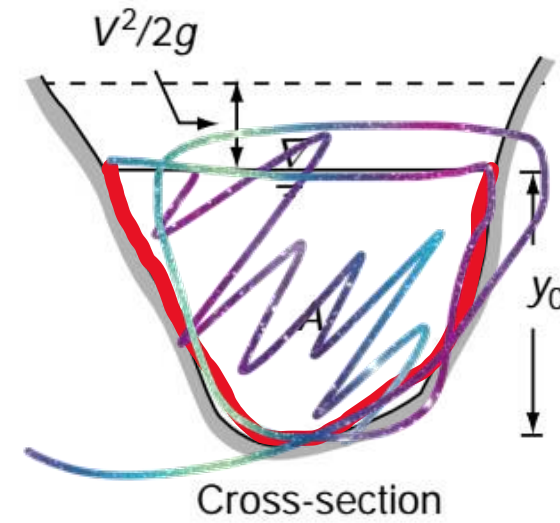


Fig. 3.1 Uniform flow



UNIFORM FLOW

MOMENTUM BALANCE:

$$\sum F_x = \rho Q (v_2 - v_1) = 0$$

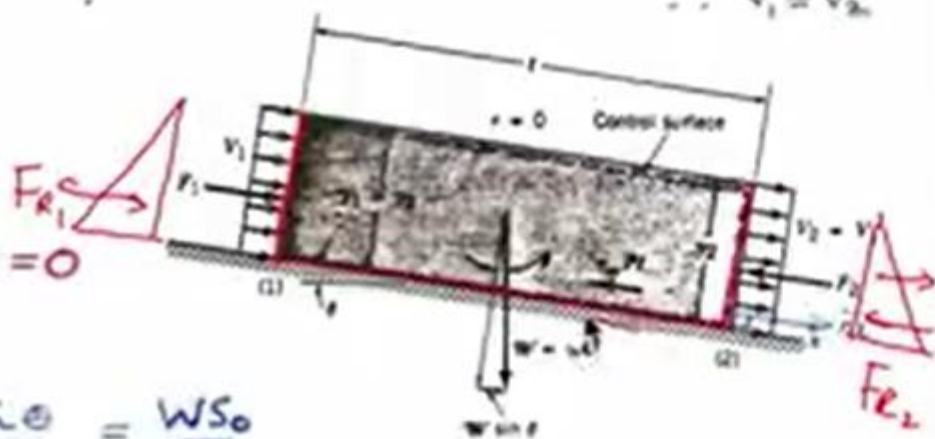
$\therefore \sum F_x = 0$

$$\frac{dy}{dx} = 0 \quad \text{ie. } y_1 = y_2$$

$$\therefore v_1 = v_2$$

FORCE BALANCE:

$$F_1 - F_2 - \tau_w PL + W \sin \theta = 0$$



Rearranging: $\tau_w = \frac{W \sin \theta}{PL} = \frac{W S_0}{PL}$

Small θ : $\sin \theta \approx \tan \theta \approx S_0$

$$W = A L \gamma \Rightarrow \tau_w = \frac{A L}{P L} \gamma S_0 = R_h \gamma S_0$$

$$R_h = \frac{A}{P}$$

Turbulent flow $\tau_w = K_f \rho \frac{V^2}{2}$ (Darcy-Weisbach formula.)

Combining $V = \sqrt{\frac{2\gamma}{K_f} R_h S_0} = C \sqrt{R_h S_0} = V$

Uniform flow - CHEZY EQUATION

By definition there is no acceleration in uniform flow. By applying the momentum equation to a control volume encompassing Sections 1 and 2, distance L apart, as shown in Fig. 3.1,

$$P_1 - W \sin \theta - F_f - P_2 = M_2 - M_1$$

where P_1 and P_2 are the pressure forces and M_1 and M_2 are the momentum fluxes at Sections 1 and 2 respectively W = weight to fluid in the control volume and F_f = shear force at the boundary.

Since the flow is uniform, $P_1 = P_2$ and $M_1 = M_2$
 $W = \gamma AL$ and $F_f = \tau_0 PL$

where τ_0 = average shear stress on the wetted perimeter of length P and γ = unit weight of water. Replacing $\sin \theta$ by S_0 (= **bottom slope**), Eq. 3.1 can be written as

$$\begin{aligned} \gamma ALS_0 &= \tau_0 PL \\ \tau_0 &= \gamma \frac{A}{P} S_0 = \gamma RS_0 \end{aligned}$$

where **$R = A/P$ is defined as the hydraulic radius**. R is a length parameter accounting for the shape of the channel. It plays a very important role in developing flow equations which are common to all shapes of channels.

Uniform flow

$$\gamma ALS_0 = \tau_0 PL$$
$$\tau_0 = \gamma \frac{A}{P} S_0 = \gamma RS_0$$

Expressing the average **shear stress** τ_0 as $\tau_0 = k\rho V^2$, where k = a coefficient which depends on the nature of the surface and flow parameters, the above Eq. is written as

$$k\rho V^2 = \gamma RS_0 \quad \longrightarrow \quad V = C \sqrt{RS_0} \quad \text{where } C = \sqrt{\frac{\gamma}{\rho k}}$$

C = a coefficient which depends on the nature of the surface and the flow.

Above equation is known as the *Chezy formula* after the French engineer Antoine Chezy, who is credited with developing this basic simple relationship in 1769.

The dimensions of C are $[L^{1/2}T^{-1}]$ and it can be made dimensionless by dividing it by g .

$$C = \sqrt{8g/f}$$

The coefficient C is known as the **Chezy coefficient**.

Uniform flow - MANNING'S FORMULA

$$k\rho V^2 = \gamma RS_0$$

⇒

$$V = C \sqrt{RS_0}$$

$$\text{where } C = \sqrt{\frac{\gamma}{\rho} \frac{1}{k}}$$
$$C = \sqrt{8g/f}$$

A resistance formula proposed by Robert Manning, an Irish engineer, for uniform flow in open channels, is

$$V = \frac{1}{n} R^{2/3} S_0^{1/2}$$

where n = a roughness coefficient known as Manning's n . Dimensions of n are $[L^{-1/3} T]$.

This coefficient is essentially a function of the nature of boundary surface.

Owing to its simplicity and acceptable degree of accuracy in a variety of practical applications, the Manning's formula is probably the most widely used uniform flow formula in the world.

Uniform flow

$$V = C \sqrt{RS_0}$$

By comparing

$$V = \frac{1}{n} R^{2/3} S_0^{1/2}$$

$$C = \frac{1}{n} R^{1/6}$$

Also

$$C = \sqrt{\frac{8g}{f}} = \frac{1}{n} R^{1/6}$$



$$f = \left(\frac{n^2}{R^{1/3}} \right) (8g)$$

Uniform flow

$$V = C \sqrt{RS_0}$$

$$V = \frac{1}{n} R^{2/3} S_0^{1/2}$$

Table 3.2 Values of Roughness Coefficient n

Sl. No.	Surface Characteristics	Range of n
(a)	Lined channels with straight alignment	
1	Concrete	
	(a) formed, no finish	0.013–0.017
	(b) Trowel finish	0.011–0.015
	(c) Float finish	0.013–0.015
	(d) Gunite, good section	0.016–0.019
	(e) Gunite, wavy section	0.018–0.022
2	Concrete bottom, float finish, sides as indicated	
	(a) Dressed stone in mortar	0.015–0.017
	(b) Random stone in mortar	0.017–0.020
	(c) Cement rubble masonry	0.020–0.025
	(d) Cement-rubble masonry, plastered	0.016–0.020
	(e) Dry rubble (rip-rap)	0.020–0.030
3	Tile	0.016–0.018
4	Brick	0.014–0.017
5	Sewers (concrete, A.C., vitrified-clay pipes)	0.012–0.015
6	Asphalt	
	(i) Smooth	0.013
	(ii) Rough	0.016
7	Concrete lined, excavated rock	
	(i) good section	0.017–0.020
	(ii) irregular section	0.022–0.027
8	Laboratory flumes-smooth metal bed and glass or perspex sides	0.009–0.010
(b)	Unlined, non-erodible channels	
1	Earth, straight and uniform	
	(i) clean, recently completed	0.016–0.020
	(ii) clean, after weathering	0.018–0.025
	(iii) gravel, uniform section, clean	0.022–0.030
	(iv) with short grass, few weeds	0.022–0.033
2	Channels with weeds and brush, uncut	
	(i) dense weeds, high as flow depth	0.05–0.12
	(ii) clean bottom, brush on sides	0.04–0.08
	(iii) dense weeds or aquatic plants in deep channels	0.03–0.035
	(iv) grass, some weeds	0.025–0.033
3	Rock	0.025–0.045
(c)	Natural channels	
1	Smooth natural earth channel, free from growth, little curvature	0.020
2	Earth channels, considerably covered with small growth	0.035
3	Mountain streams in clean loose cobbles, rivers with variable section with some vegetation on the banks	0.04–0.05
4	Rivers with fairly straight alignment, obstructed by small trees, very little under brush	0.06–0.075
5	Rivers with irregular alignment and cross-section, covered with growth of virgin timber and occasional patches of bushes and small trees	0.125

Uniform flow

Horton's Method of Equivalent Roughness Estimation

Consider a channel having its perimeter composed of N types of roughness, $P_1, P_2, \dots, P_i, \dots, P_N$ are the lengths of these N parts and $n_1, n_2, \dots, n_i, \dots, n_N$ are the respective roughness coefficients (Fig. 3.8). Let each part P_i be associated with a partial area A_i such that

$$\sum_{i=1}^N A_i = A_1 + A_2 + \dots + A_i + \dots + A_N = A = \text{total area}$$

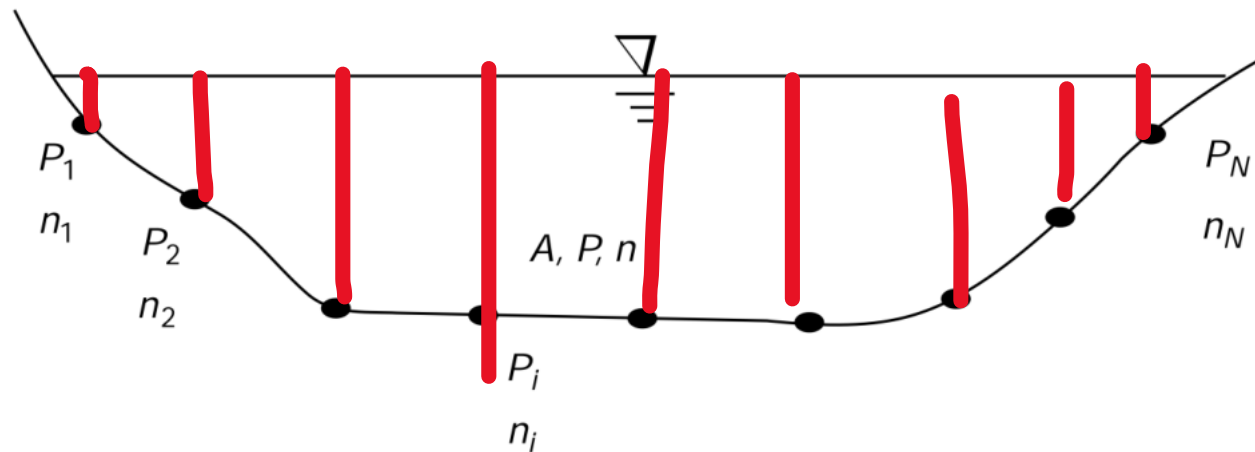


Fig. 3.8 Multi-roughness type perimeter

Uniform flow

Horton's Method of Equivalent Roughness Estimation:

It is assumed that the mean velocity in each partial area is the mean velocity V for the entire area of flow, i.e.,

$$V_1 = V_2 = \dots = V_i = \dots = V_N = V$$

By the Manning's formula $S_0^{1/2} = \frac{V_1 n_1}{R_1^{2/3}} = \frac{V_2 n_2}{R_2^{2/3}} = \dots = \frac{V_i n_i}{R_i^{2/3}} = \dots = \frac{V_N n_N}{R_N^{2/3}} = \frac{Vn}{R^{2/3}}$



$$\left(\frac{A_i}{A}\right)^{2/3} = \frac{n_i P_i^{2/3}}{nP^{2/3}}$$

$$A_i = A \frac{n_i^{3/2} P_i}{n^{3/2} P}$$

$$\sum A_i = A = A \frac{\sum (n_i^{3/2} P_i)}{n^{3/2} P}$$

$$n = \frac{(\sum n_i^{3/2} P_i)^{2/3}}{P^{2/3}}$$

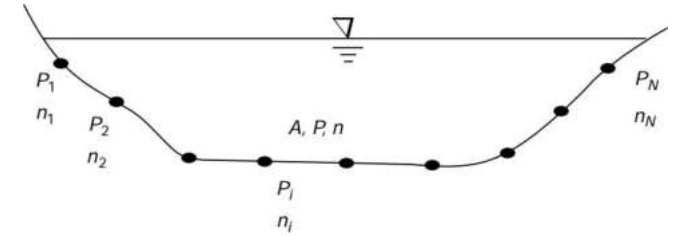


Fig. 3.8 Multi-roughness type perimeter

Where n = equivalent roughness

Uniform Flow – Computations

The Manning's formula and the continuity equation, $Q = AV$ form the basic equations for uniform-flow computations. The discharge Q is then given by

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$$
$$= K \sqrt{S_0}$$

where, $K = \frac{1}{n} AR^{2/3}$ is called the **conveyance** of the channel and expresses the discharge capacity of the channel per unit longitudinal slope. The term $nK = AR^{2/3}$ is sometimes called *the section factor for uniform-flow computations*.

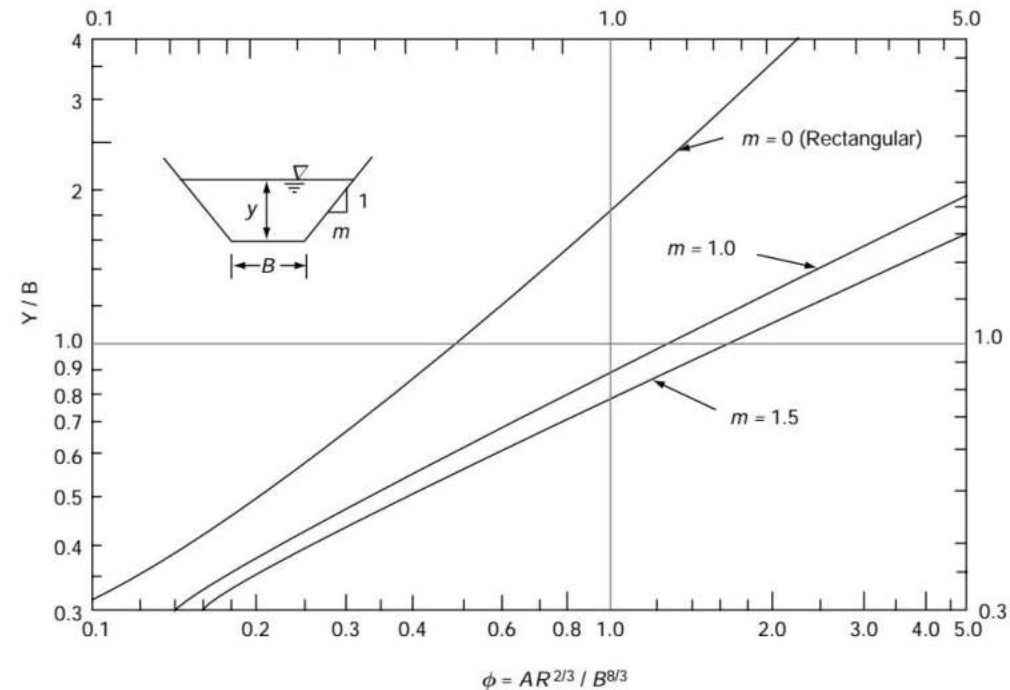
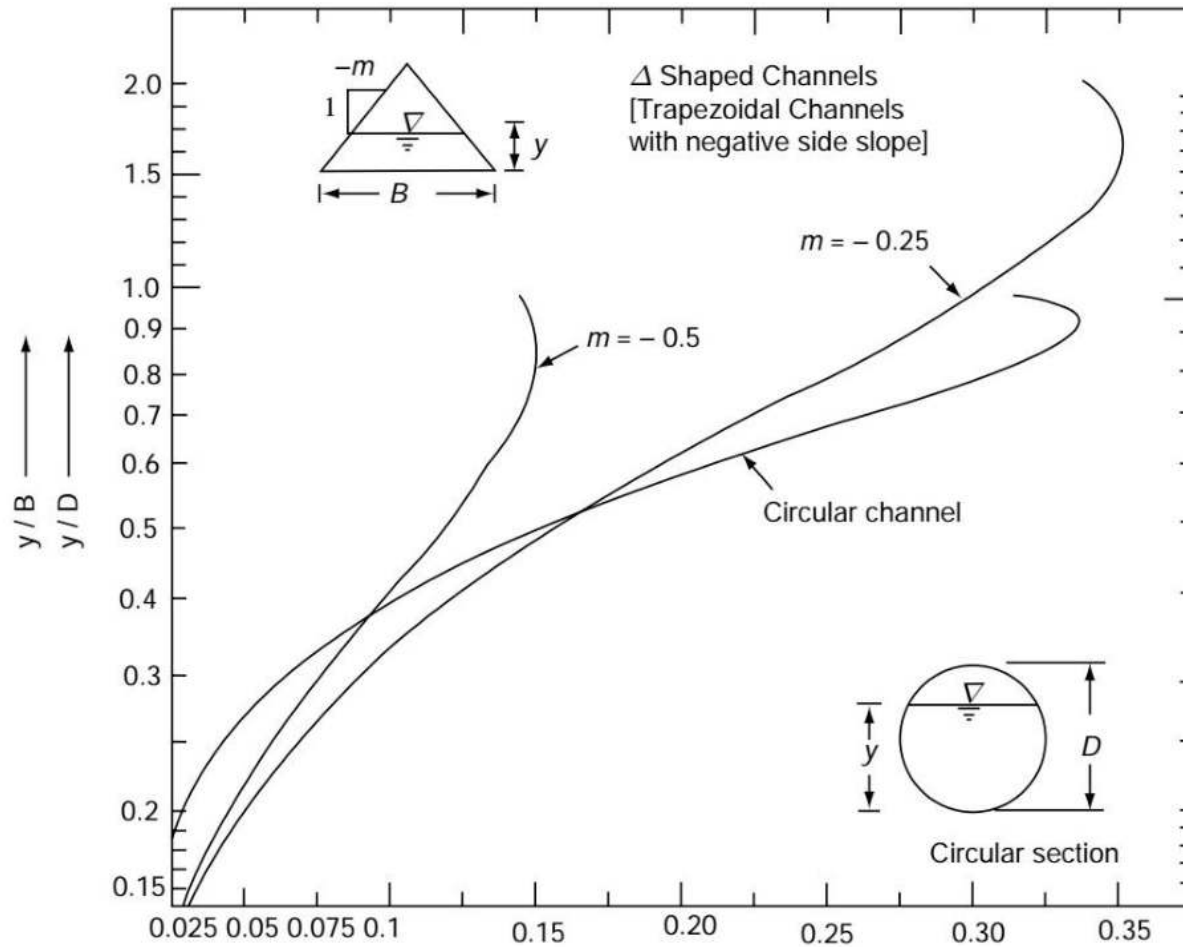


Fig. 3.9 Variation of ϕ with y/B in trapezoidal channels

Uniform Flow – Computations



$AR^{2/3} / B^{8/3}$ for Δ shaped Channels

$AR^{2/3} / D^{8/3}$ for Circular Channels

Fig. 3.10 Variation of $AR^{2/3}$ in channels of the second kind

Uniform Flow – Computations

Types of Problems Uniform flow computation problems are relatively simple.

The available relations are

1. Manning's formula
2. Continuity equation
3. Geometry of the cross section

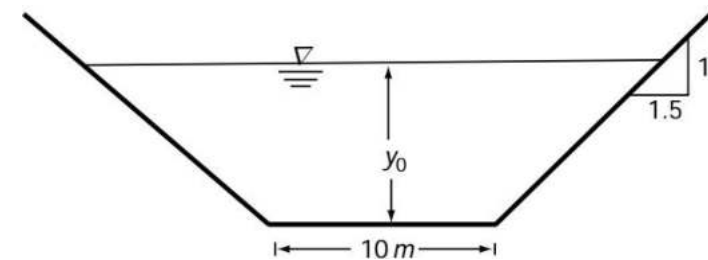
<i>Problem type</i>	<i>Given</i>	<i>Required</i>
<i>1</i>	<i>$y_0, n, S_0, \text{Geometric elements}$</i>	<i>$Q \text{ and } V$</i>
<i>2</i>	<i>$Q, y_0, n, \text{Geometric elements}$</i>	<i>S_0</i>
<i>3</i>	<i>$Q, y_0, S_0, \text{Geometric elements}$</i>	<i>n</i>
<i>4</i>	<i>$Q, n, S_0, \text{Geometric elements}$</i>	<i>y_0</i>
<i>5</i>	<i>$Q, y_0, n, S_0, \text{Geometry}$</i>	<i>$\text{Geometric elements}$</i>

- Problems of the types 1, 2 and 3 normally have **explicit** solutions. calculations.
- Problems of the types 4 and 5 usually do not have **explicit** solutions
 - ▶ involve trial-and-error solutions procedures.

Uniform flow

Example 3.4 A trapezoidal channel is 10.0 m wide and has a side slope of 1.5 horizontal: 1 vertical. The bed slope is 0.0003. The channel is lined with smooth concrete of $n = 0.012$. Compute the mean velocity and discharge for a depth of flow of 3.0 m.

Solution Let y_0 = uniform flow depth
Here $B = 10.0$ m and side slope $m = 1.5$
Area $A = (B + my) y$
 $= (10.0 + 1.5 \times 3.0) 3.0 = 43.50 \text{ m}^2$
Wetted perimeter $P = B + 2y\sqrt{m^2 + 1}$
 $= 10.0 + 2\sqrt{2.25 + 1} \times 3.0 = 20.817 \text{ m}$



Hydraulic radius

$$R = \frac{A}{P} = 2.090 \text{ m}$$

Mean velocity

$$V = \frac{1}{n} R^{2/3} S_0^{1/2}$$
$$= \frac{1}{0.012} \times (2.09)^{2.3} \times (0.0003)^{1/2}$$
$$= 2.36 \text{ m/s}$$

Discharge

$$Q = AV = 102.63 \text{ m}^3/\text{s}$$

Uniform flow

Example 3.5 In the channel of Example 3.4, find the bottom slope necessary to carry only $50 \text{ m}^3/\text{s}$ of the discharge at a depth of 3.0 m .

Solution $A = 43.50 \text{ m}^2$

$$P = 20.817 \text{ m}$$

$$R = 2.09 \text{ m}$$

$$S_0 = \frac{Q^2 n^2}{A^2 R^{4/3}} = \frac{(50.0)^2 \times (0.012)^2}{(43.5)^2 \times (2.09)^{4/3}}$$
$$= 0.0000712$$

Uniform flow

Example 3.6

A triangular channel with an apex angle of 75° carries a flow of $1.2 \text{ m}^3/\text{s}$ at a depth of 0.80 m . If the bed slope is 0.009 , find the roughness coefficient of the channel.

Solution $y_0 = \text{normal depth} = 0.80 \text{ m}$

Area

$$A = \frac{1}{2} \times 0.80 \times 2 \times 0.8 \tan\left(\frac{75}{2}\right)$$
$$= 0.491 \text{ m}^2$$

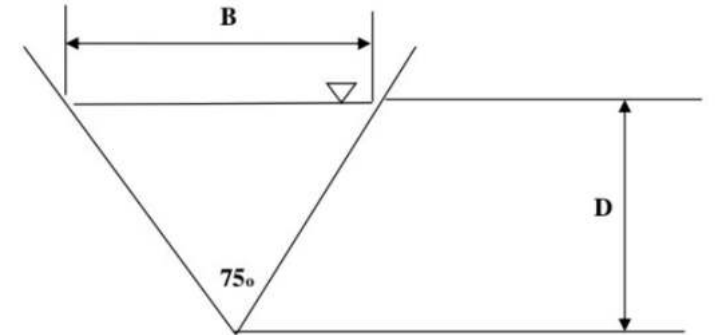
Wetted perimeter

$$P = 2 \times 0.8 \times \sec 37.5^\circ = 2.0168 \text{ m}$$

$$R = A/P = 0.243 \text{ m}$$

$$n = \frac{AR^{2/3}S_0^{1/2}}{Q} = \frac{(0.491) \times (0.243)^{2/3} \times (0.009)^{1/2}}{1.20}$$

$$n = 0.0151$$



Uniform flow

Example 3.7 A 5.0-m wide trapezoidal channel having a side slope of 1.5 horizontal: 1 vertical is laid on a slope of 0.00035. The roughness coefficient $n = 0.015$. Find the normal depth for a discharge of $20 \text{ m}^3/\text{s}$ through this channel.

Solution Let

$y_0 =$ normal depth

Area $A = (5.0 + 1.5 y_0) y_0$

Wetted perimeter $P = 5.0 + 2 \sqrt{3.25} y_0$
 $= 5.0 + 3.606 y_0$

$R = A/P = \frac{(5.0 + 1.5 y_0) y_0}{(5.0 + 3.606 y_0)}$

The section factor $AR^{2/3} = \frac{Qn}{\sqrt{S_0}}$

$$\frac{(5.0 + 1.5 y_0)^{5/3} y_0^{5/3}}{(5.0 + 3.606 y_0)^{2/3}} = \frac{20 \times 0.015}{(0.00035)^{1/2}} = 16.036$$

Algebraically, y_0 can be found from the above equation by the trial-and-error method. The normal depth is found to be 1.820 m

Uniform flow

Example 3.8 | A concrete-lined trapezoidal channel ($n = 0.015$) is to have a side slope of 1.0 horizontal: 1 vertical. The bottom slope is to be 0.0004. Find the bottom width of the channel necessary to carry $100 \text{ m}^3/\text{s}$ of discharge at a normal depth of 2.50 m.

Solution Let B = bottom width. Here, y_0 = normal depth = 2.50 m, $m = 1$.

Area $A = (B + 2.5) \times 2.5$

Wetted perimeter $P = (B + 2\sqrt{2} \times 2.5) = B + 7.071$

$$\frac{Qn}{\sqrt{S_0}} = \frac{100 \times 0.015}{\sqrt{0.0004}} = 75 = AR^{2/3}$$

$$\frac{[(B + 2.5) \times 2.5]^{5/3}}{(B + 7.071)^{2/3}} = 75.0$$

By trial-and-error $B = 16.33 \text{ m}$.

Uniform Flow – LINED CANAL SECTIONS

STANDARD LINED CANAL SECTIONS

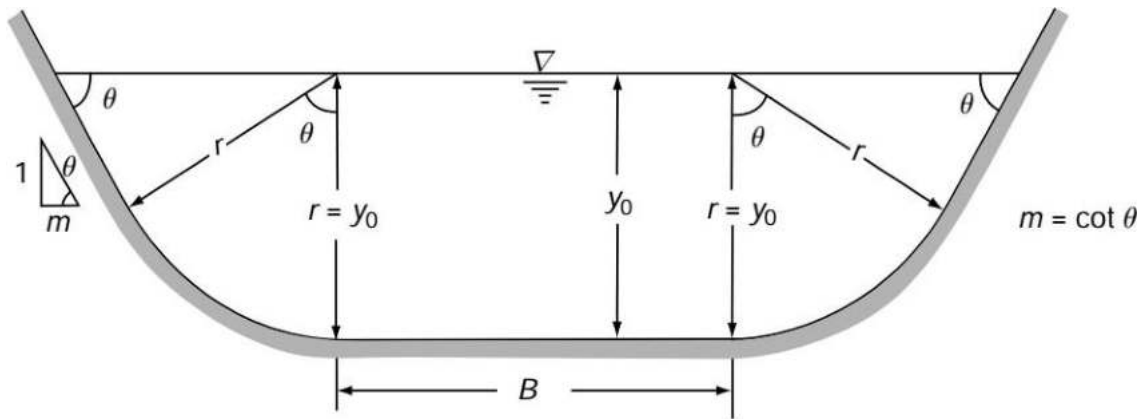


Fig. 3.16 Standard lined trapezoidal channel section for $Q > 55 \text{ m}^3/\text{s}$

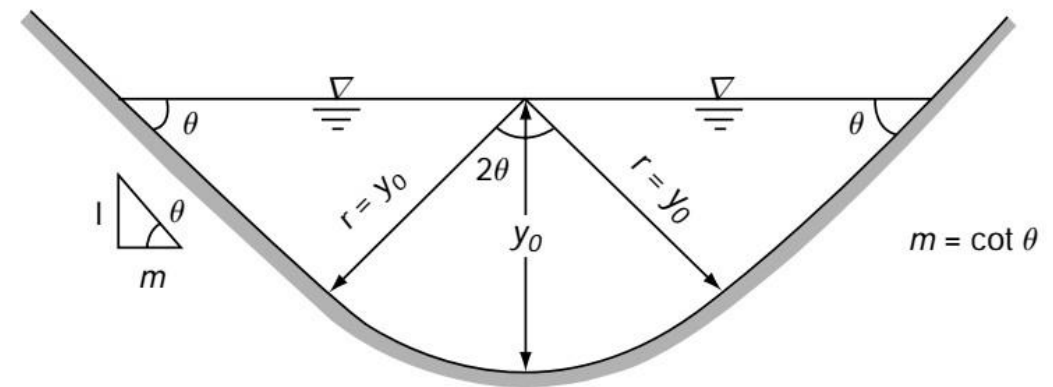


Fig. 3.17 Standard lined triangular channel section for $Q \leq 55 \text{ m}^3/\text{s}$

Uniform Flow – HYDRAULICALLY EFFICIENT CHANNEL SECTION

HYDRAULICALLY EFFICIENT CHANNEL SECTION

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$$
$$= K \sqrt{S_0}$$

where, $K = \frac{1}{n} AR^{2/3}$ is called the *conveyance* of the channel

- The conveyance of a channel section of a given area **increases** with a **decrease** in its perimeter. Hence a channel section having the minimum perimeter for a given area of flow provides the maximum value of the conveyance.
- With the slope, roughness coefficient and area of flow fixed, a **minimum perimeter section** will represent the hydraulically efficient section as it conveys the maximum discharge. This channel section is also called the **best section**.
- Of all the various possible open channel sections, the semicircular shape has the least amount of perimeter for a given area.
- High conveyance = Save money

Uniform Flow – HYDRAULICALLY EFFICIENT CHANNEL SECTION

(a) Rectangular Section

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$$

$$= K \sqrt{S_0} \quad \text{where, } K = \frac{1}{n} AR^{2/3} \text{ is called the conveyance of the channel}$$

Rectangular Section

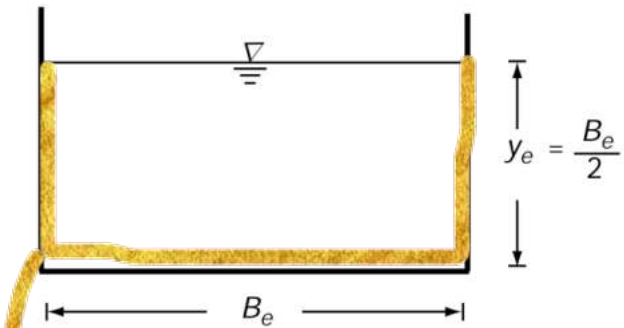


Fig. 3.18 Hydraulically efficient rectangular channel

P lining cost money

$$\begin{aligned} \text{Area of flow } A &= By = \text{constant} \\ \text{Wetted perimeter } P &= B + 2y \\ &= \frac{A}{y} + 2y \end{aligned}$$

If P is to be minimum with $A = \text{constant}$,

$$\frac{dP}{dy} = -\frac{A}{y^2} + 2 = 0$$

Which gives

$$A = 2y_e^2$$

i.e.

$$y_e = B_e/2, B_e = 2y_e \text{ and } R_e = \frac{y_e}{2}$$

$$\text{hydraulic radius} = \frac{\text{cross-sectional area}}{\text{wetted perimeter}}$$

$$A = By$$

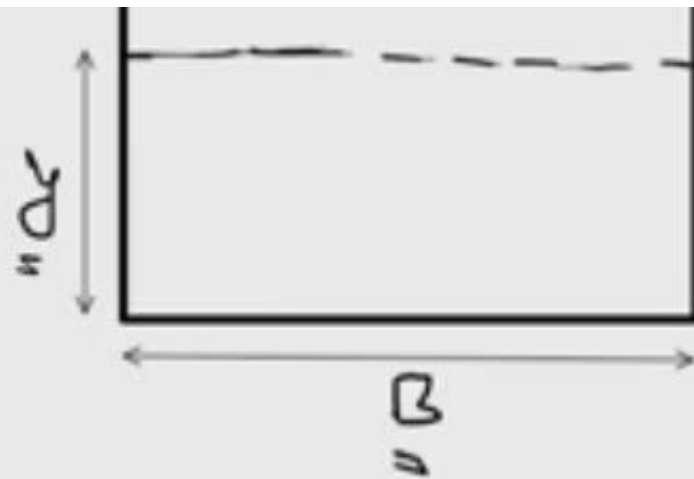
$A \rightarrow$ constant

$$P = B + 2y$$

$Q \rightarrow$ max.

$$P = \frac{A}{y} + 2y$$

$P \rightarrow$ min



$$\frac{dP}{dy} = 0 = -\frac{A}{y^2} + 2 = 0$$

$$2y^2 = A$$

$$2y^2 = By$$

$$y = \frac{B}{2}$$

① The depth of flow should be half of the bed width

$$R = \frac{A}{P} = \frac{By}{B+2y} = \frac{2y^2}{4y}$$

$$R = \frac{y}{2}$$

② Hydraulic radius should be half of flow depth.

Uniform Flow – HYDRAULICALLY EFFICIENT CHANNEL SECTION

(b) Trapezoidal Section

$$m_{em} = \frac{1}{\sqrt{3}} = \cot \theta$$

$$\theta_{em} = 60^\circ$$

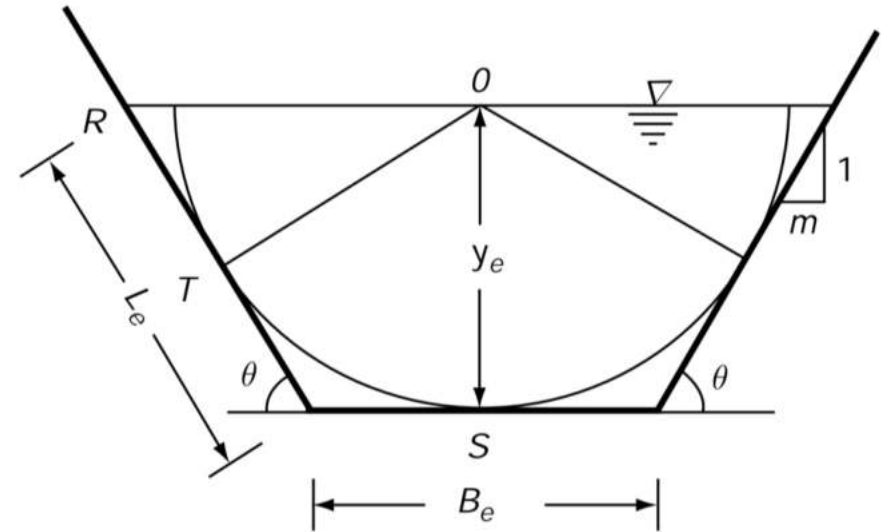


Fig. 3.19 Hydraulically efficient trapezoidal channel

Where the suffix '*em*' denotes the most efficient section, Further.

$$P_{em} = 2y_{em} \left(2\sqrt{1+1/3} - \frac{1}{\sqrt{3}} \right) = 2\sqrt{3} y_{em}$$

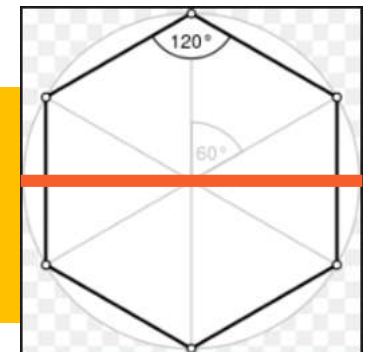
$$B_{em} = 2y_{em} \left(2\sqrt{1+1/3} - \frac{1}{\sqrt{3}} \right) = \frac{2}{\sqrt{3}} y_{em}$$

$$A = \left(2\sqrt{1+1/3} - \frac{1}{\sqrt{3}} \right) y_{em}^2 = \sqrt{3} y_{em}^2$$

If L = length of the inclined side of the canal, it is easily seen that

$$L_{em} = \frac{2}{\sqrt{3}} y_{em} = B_{em}$$

Thus the hydraulically most efficient trapezoidal section is one-half of a regular hexagon.



Uniform Flow – HYDRAULICALLY EFFICIENT CHANNEL SECTION

Table 3.4 Proportions of Some Most Efficient Sections

Sl. No	Channel Shape	Area (A_{em})	Wetted Perimeter (P_{em})	Width (B_{em})	Hydraulic Radius (R_{em})	Top width (T_{em})	$\frac{Qn}{y_{em}^{8/3} S_0^{1/2}} = K_{em}$
1	Rectangle (Half square)	$2 y_{em}^2$	$4 y_{em}$	$2 y_{em}$	$\frac{y_{em}}{2}$	$2 y_{em}$	1.260
2	Trapezoidal (Half regular hexagon, $m = \frac{1}{\sqrt{3}}$)	$\sqrt{3} y_{em}^2$	$2\sqrt{3} y_{em}$	$\frac{2}{\sqrt{3}} y_{em}$	$\frac{y_{em}}{2}$	$\frac{4 y_{em}}{\sqrt{3}}$	1.091
3	Circular (semi-circular)	$\frac{\pi}{2} y_{em}^2$	πy_{em}	$D = 2y_{em}$	$\frac{y_{em}}{2}$	$2 y_{em}$	0.9895
4	Triangle (Vertex angle = 90°)	y_{em}^2	$2\sqrt{3} y_{em}$	–	$\frac{y_{em}}{2\sqrt{2}}$	$2 y_{em}$	0.500

Uniform Flow – Computations

Example 3.14 | A slightly rough brick-lined ($n = 0.017$) trapezoidal channel carrying a discharge of $25.0 \text{ m}^3/\text{s}$ is to have a longitudinal slope of 0.0004 . Analyse the proportions of an efficient trapezoidal channel section having a side slope of 1.5 horizontal: 1 vertical.

Solution For an efficient trapezoidal section having a side slope of m , by Eq. 3.53

$$A_e = \left(2 \times \sqrt{1 + m^2} - m\right) y_e^2 = \left(2 \times \sqrt{1 + (1.5)^2} - 1.5\right) y_e^2$$

$$R_e = y_e/2 \text{ and } Q = 25.0 \text{ m}^3/\text{s}$$

Remember m was given so you are not free to do me (most efficient)

Substituting a Manning's formula,

$$25.0 = \frac{1}{0.017} \times (2.1056 y_e^2) \times (y_e / 2)^{2/3} \times (0.0004)^{1/2}$$

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$$

$$y_e = 2.830 \text{ m}$$

$$\text{By Eq. (3.54), } B_e = 2 y_e (\sqrt{1 + m^2} - m) = 2 \times 2.830 \times (\sqrt{1 + (1.5)^2} - (1.5)) = 1.714 \text{ m}$$

Uniform Flow – Computations

Example 3.16 Determine the normal depth, bed width and sides slopes of a most efficient trapezoidal channel section to carry a discharge of 25 m³/s. The longitudinal slope of the channel is to be 0.0009 and Manning's n can be taken as 0.015.

Solution For the most efficient trapezoidal section

Remember m was not given so you are free to do me (most efficient)

From Table 3.4, $\frac{Qn}{y_{em}^{8/3} S_0^{1/2}} = 1.091$. In the present case $Q = 25.0$ m³/s and $S_0 = 0.0009$.

$$\text{Hence } y_{em}^{-8/3} = \frac{1.091 \times (0.0009)^{1/2}}{25.0 \times 0.015} = 0.08728$$

$$y_{em} = 2.50 \text{ m}$$

$$\text{Also, from Table 3.4, } B_{em} = \frac{2}{\sqrt{3}} y_{em} = \frac{2}{\sqrt{3}} \times 2.50 = 2.887 \text{ m}$$

$$\text{Sides slope of most efficient trapezoidal channel section, } m = \frac{1}{\sqrt{3}} = 0.57735$$

Continue...

Thanks for listening....

