Example : Compute **f(0.3)** for the data

x	0	1	3	4	7
f	1	3	49	129	813

using Lagrange's interpolation formula (Analytic value is **1.831**)

$$f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)(x - x_4)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4)} f_0 + \ldots + \frac{(x - x_0)(x - x_1)(x - x_2)(x - x_3)}{(x_4 - x_0)(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)} f_4$$

$$= \frac{(0.3 - 1)(0.3 - 3)(0.3 - 4)(0.3 - 7)}{(-1)(-3)(-4)(-7)} 1 + \frac{(0.3 - 0)(0.3 - 3)(0.3 - 4)(0.3 - 7)}{1 \times (-2)(-3)(-6)} 3 +$$

$$\frac{(0.3 - 0)(0.3 - 1)(0.3 - 4)(0.3 - 7)}{3 \times 2 \times (-1)(-4)} 49 + \frac{(0.3 - 0)(0.3 - 1)(0.3 - 3)(0.3 - 7)}{4 \times 3 \times 1 (-3)} 129 +$$

$$\frac{(0.3 - 0)(0.3 - 1)(0.3 - 3)(0.3 - 4)}{7 \times 6 \times 4 \times 3} 813$$

$$= 1.831$$

Example 5.22

Using Lagrange's interpolation formula find y(10) from the following table:

x	5	6	9	11
y	12	13	14	16

Solution:

Here the intervals are unequal. By Lagrange's interpolation formula we ha

$$x_0 = 5, x_1 = 6, x_2 = 9, x_3 = 11$$

$$y_0 = 12, y_1 = 13, y_2 = 14, y_3 = 16$$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$= \frac{(x-6)(x-9)(x-11)}{(5-6)(5-6)(5-11)}(12) + \frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-9)}(13)$$

$$+ \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)}(14) + \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)}(16)$$

Put $x = 10$

$$y(10) = f(10) = \frac{4(1)(-1)}{(-1)(-4)(-6)}(12) + \frac{(5)(1)(-1)}{(1)(-3)(-5)}(13) + \frac{5(4)(-1)}{4(3)(-2)}(14) + \frac{(5)(4)(1)}{6(5)(2)}(16)$$

$$= \frac{1}{6}(12) - \frac{13}{3} + \frac{5(14)}{3\times 2} + \frac{4\times 16}{12}$$

$$= 14.6663$$

10. Using Lagrange's interpolation formula find a polynomial which passes through the points (0, –12), (1, 0), (3, 6) and (4,12).

Sol.

Given

x	0	1	3	4
y	–12	0	6	12

Here the intervals are unequal.

$\therefore$ By Lagranges interpolation formula, we have

$x_0 = 0, \quad x_1 = 1, \quad x_2 = 3, \quad x_3 = 4$

$y_0 = -12, \quad y_1 = 0, \quad y_2 = 6, \quad y_3 = 12$ and $x = x$.

$$\therefore y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$$

$$= \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)}(-12)$$

$$+ \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)}(0)$$

$$+ \frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)}(6)$$

$$+ \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)}(12)$$

$$= \frac{(x-1)(x-3)(x-4)}{(-1)(-3)(-4)}(-12) + 0$$

$$+ \frac{x(x-1)(x-4)}{(3)(2)(-1)}(6)$$

$$+ \frac{x(x-1)(x-3)}{(4)(3)(1)}(12)$$

$$= +[(x-1)(x-3)(x-4)] - x(x-1)(x-4) + x(x-1)(x-3)$$

$$= +[(x^2-4x+3)(x-4)] - x(x^2-5x+4) + x(x^2-4x+3)$$

$$= -(x^3-8x^2+19x-12) - 4x^2 + 3x$$

$$= (x-4)(x^2-4x+3) - x(x^2-5x+4) + x(x^2-4x+3)$$

$$= x^3 - 4x^2 + 3x - 4x^2 + 16x - 12 - x^3 + 5x^2 - 4x + x^3 - 4x^2 + 4x$$

$$= x^3 - 7x^2 + 19x - 12.$$

10. Use Lagrange's formula and estimate from the following data the number of workers getting income not exceeding Rs. 26 per month.

Income not exceeding (₹)	15	25	30	35
No. of workers	36	40	45	48