جامعة فهد بن سلطان
FAHAD BIN SULTAN UNIVERSITY

MECH-320 Kinematics of Mechanical Systems

4

Course lectures Slides prepared by

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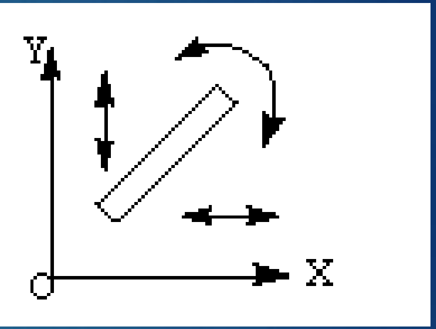
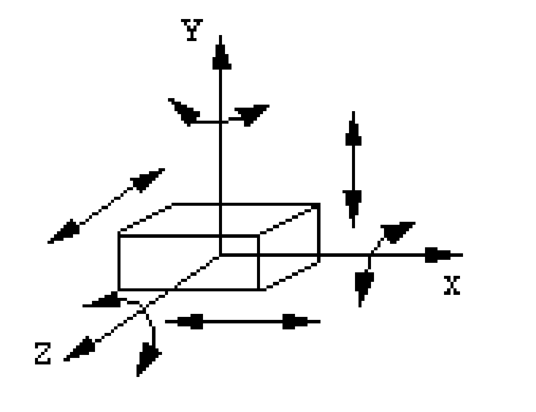
4 Basic Kinematics of Constrained Rigid Bodies

- **4.1 Degrees of Freedom of a Rigid Body**
- ***4.1.1 Degrees of Freedom of a Rigid Body in a Plane***
- The ***degrees of freedom*** (DOF) of a rigid body is defined as the number of independent movements it has.



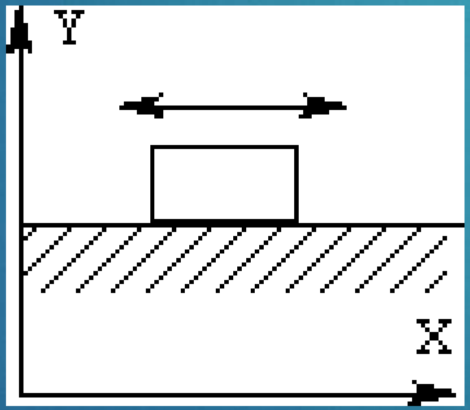
- Next figure shows a rigid body in a plane.
  - To determine the DOF of this body we must consider how many distinct ways the bar can be moved.
  - In a two dimensional plane such as this screen, there are 3 DOF.
  - The bar can be *translated* along the x axis, translated along the *y* axis, and *rotated* about its centroid.

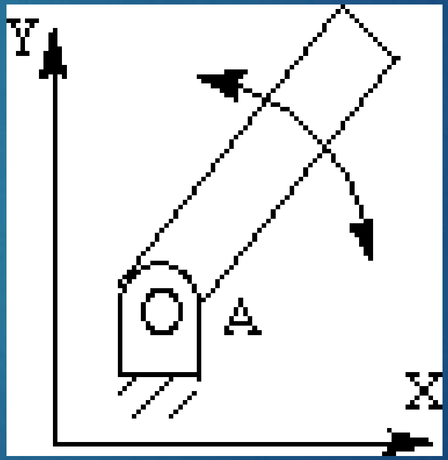
- ***4.1.2 Degrees of Freedom of a Rigid Body in Space***
- An unrestrained rigid body in space has six degrees of freedom: three translating motions along the x, *y* and z axes and three rotary motions around the x, *y* and z axes respectively.



- **4.2 Kinematic Constraints**
- Two or more rigid bodies in space are collectively called a *rigid body system*. We can hinder the motion of these independent rigid bodies with ***kinematic constraints***.
- ***Kinematic constraints*** are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system.
- The term **kinematic pairs** actually refers to *kinematic constraints* between rigid bodies. The kinematic pairs are divided into lower pairs and higher pairs, depending on how the two bodies are in contact.

- **4.2.1 Lower Pairs in Planar Mechanisms**
- There are two kinds of lower pairs in planar mechanisms: revolute pairs and prismatic pairs.
- A rigid body in a plane has only three independent motions -- two translational and one rotary -- so introducing either a revolute pair or a prismatic pair between two rigid bodies removes two degrees of freedom.



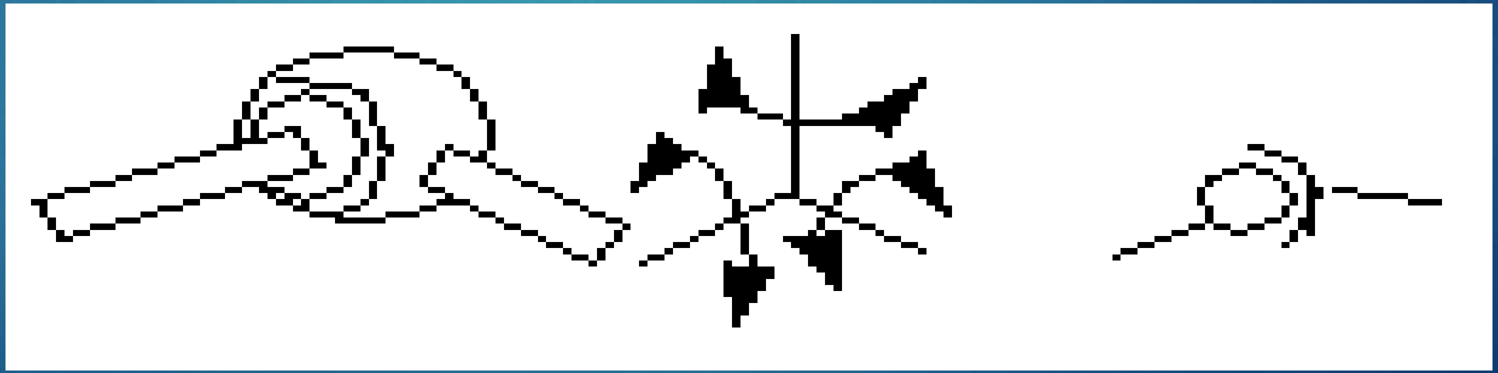


- **A planar prismatic pair (P-pair)** **A planar revolute pair (R-pair)**

4.2.2 Lower Pairs in Spatial Mechanisms

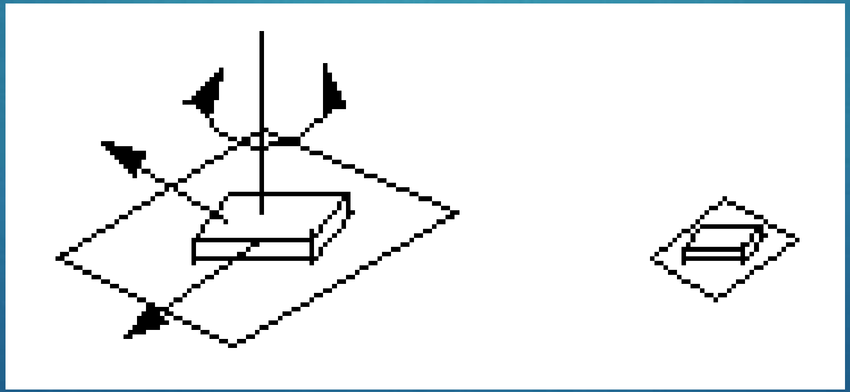
- There are six kinds of lower pairs under the category of spatial mechanisms.
- The types are: spherical pair, plane pair, cylindrical pair, revolute pair, prismatic pair, and screw pair.

- ***(1) A spherical pair*** keeps two spherical centers together. Two rigid bodies connected by this constraint will be able to *rotate* relatively around *x*, *y* and *z* axes, but there will be no relative translation along any of these axes. Therefore, a spherical pair removes three degrees of freedom in spatial mechanism. **DOF = 3**.



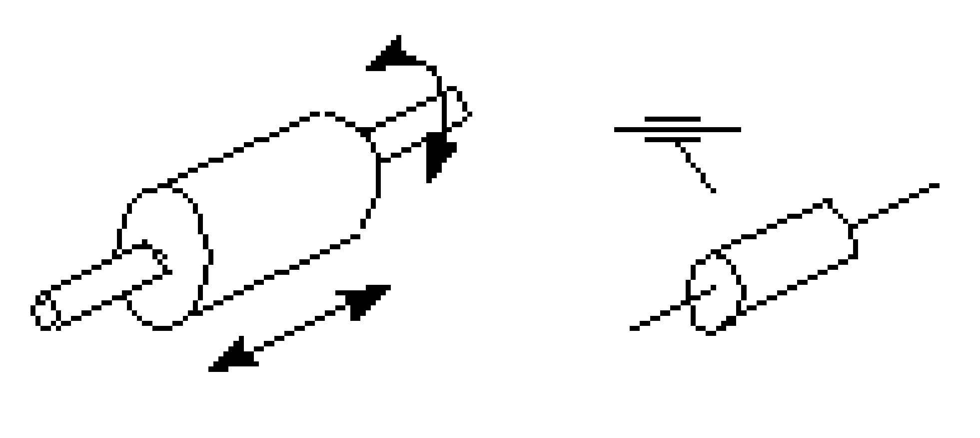
A spherical pair (S-pair)

- ***(2) A plane pair*** keeps the surfaces of two rigid bodies together. To visualize this, imagine a book lying on a table where is can move in any direction except off the table.
- Two rigid bodies connected by this kind of pair will have two independent translational motions in the plane, and a rotary motion around the axis that is perpendicular to the plane.
- Therefore, a plane pair removes three degrees of freedom in spatial mechanism. In our example, the book would not be able to raise off the table or to rotate into the table. **DOF = 3.**



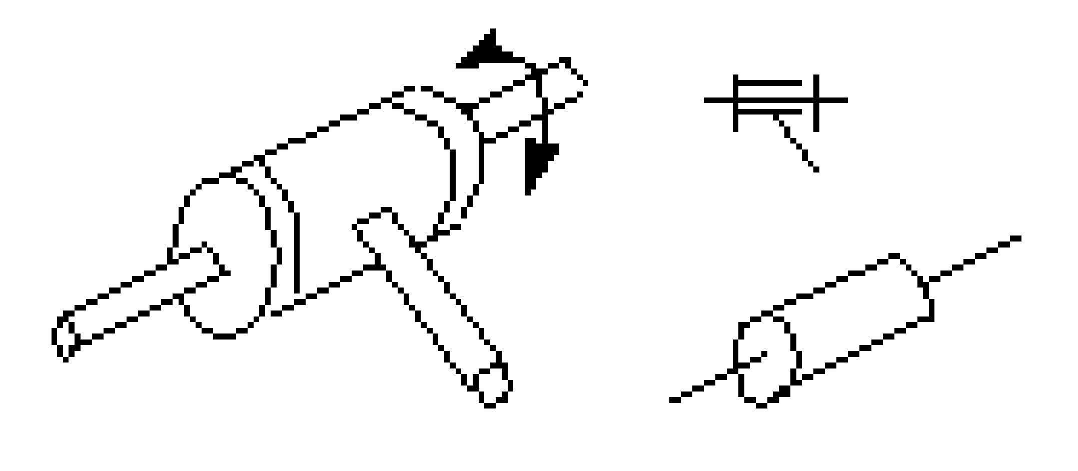
- **A planar pair (E-pair)**

- ***(3) A cylindrical pair*** keeps two axes of two rigid bodies aligned. Two rigid bodies that are part of this kind of system will have an independent translational motion along the axis and a relative rotary motion around the axis.
- Therefore, a cylindrical pair removes four degrees of freedom from spatial mechanism. **DOF = 2.**



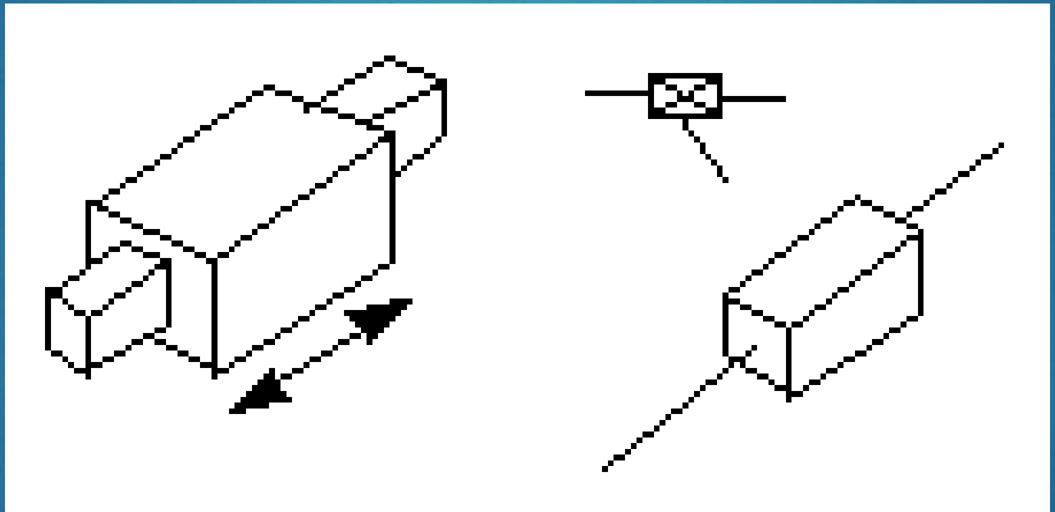
A cylindrical pair (C-pair)

- ***(4) A revolute pair*** keeps the axes of two rigid bodies together.
- Two rigid bodies constrained by a revolute pair have an independent rotary motion around their common axis.
- Therefore, a revolute pair removes five degrees of freedom in spatial mechanism. **DOF = 1.**



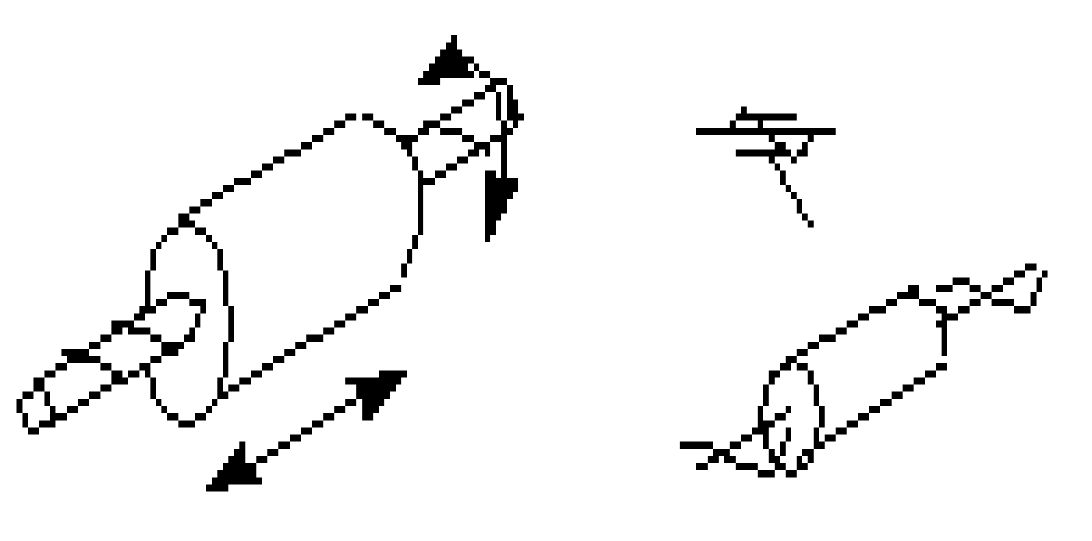
A revolute pair (R-pair)

- **(5) A *prismatic pair*** keeps two axes of two rigid bodies align and allow no relative rotation. Two rigid bodies constrained by this kind of constraint will be able to have an independent translational motion along the axis.
- Therefore, a prismatic pair removes five degrees of freedom in spatial mechanism. **DOF = 1.**



A prismatic pair (P-pair)

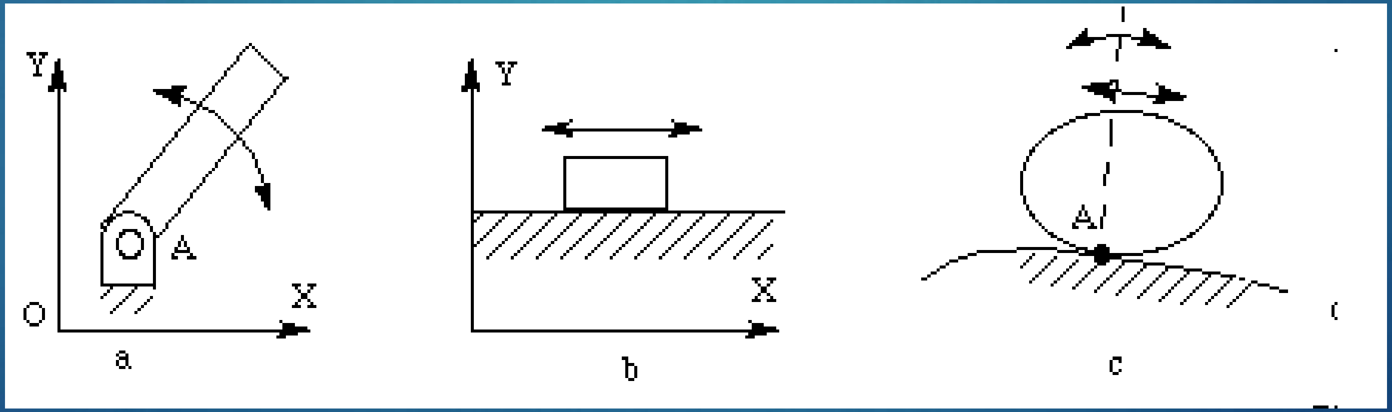
- ***(6) The screw pair*** keeps two axes of two rigid bodies aligned and allows a relative screw motion. Two rigid bodies constrained by a screw pair a motion which is a composition of a translational motion along the axis and a corresponding rotary motion around the axis.
- Therefore, a screw pair removes five degrees of freedom in spatial mechanism. **DOF = 5**



A screw pair (H-pair)

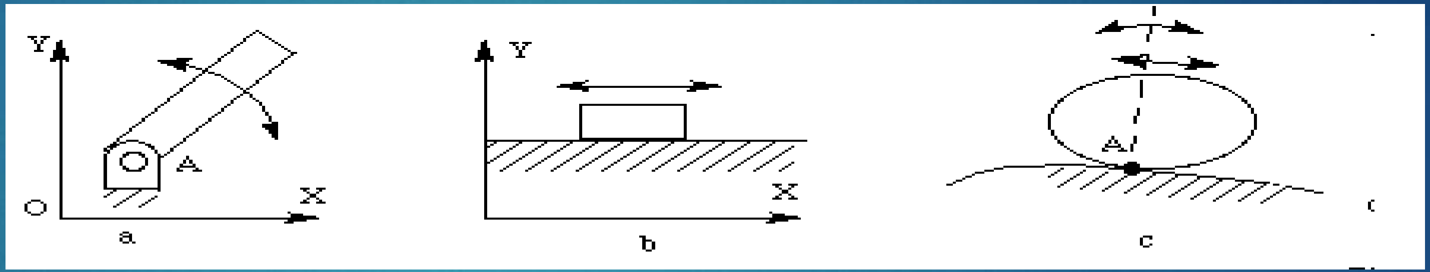
4.4 Degrees of Freedom of Planar Mechanisms

- **4.4.1 Gruebler's Equation**
- The definition of the ***degrees of freedom*** of a mechanism is ***the number of independent relative motions among the rigid bodies.***
- Examples: rigid body constrained by different kinds of pairs:
- Ex. (1): a rigid body is constrained by a revolute pair which allows only rotational movement around an axis. It has one degree of freedom, turning around point A. The two lost degrees of freedom are translational movements along the x and y axes. The only way the rigid body can move is to rotate about the fixed point A.



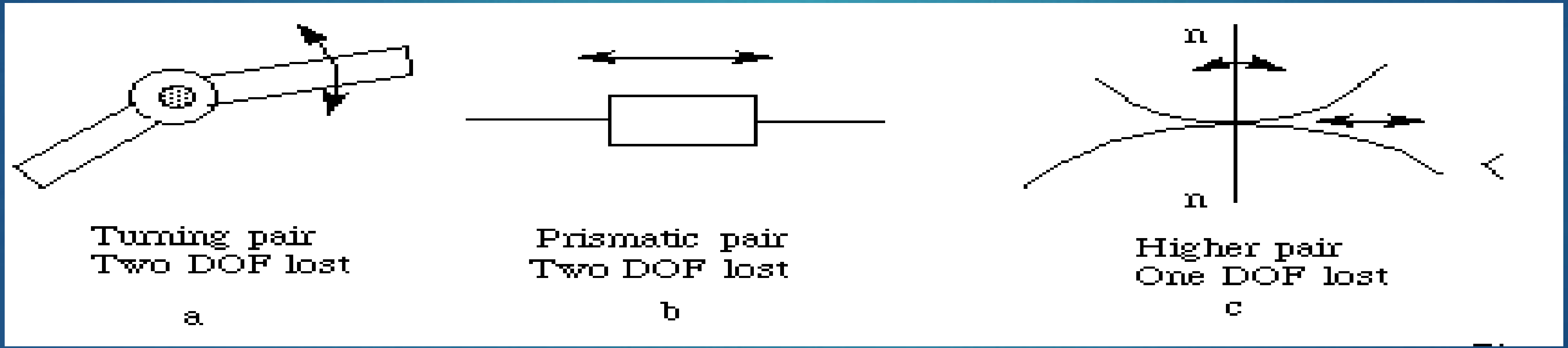
revolute pair

- In Figure 4-11a, a rigid body is constrained by a revolute pair which allows only rotational movement around an axis. It has one degree of freedom, turning around point A.



- The two lost degrees of freedom are translational movements along the x and *y* axes. The only way the rigid body can move is to rotate about the fixed point A.
- In Figure 4-11b, a rigid body is constrained by a prismatic pair which allows only translational motion. In two dimensions, it has one degree of freedom, translating along the x axis. In this example, the body has lost the ability to rotate about any axis, and it cannot move along the *y* axis.
- In Figure 4-11c, a rigid body is constrained by a higher pair. It has two degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

- In general, a rigid body in a plane has three degrees of freedom. Kinematic pairs are constraints on rigid bodies that reduce the degrees of freedom of a mechanism. Figure 4-11 shows the three kinds of pairs in planar mechanisms. These pairs reduce the number of the degrees of freedom. If we create a lower pair (Figure 4-11a,b), the degrees of freedom are reduced to 2. Similarly, if we create a higher pair (Figure 4-11c), the degrees of freedom are reduced to 1.



- Figure 4-12 Kinematic Pairs in Planar Mechanisms

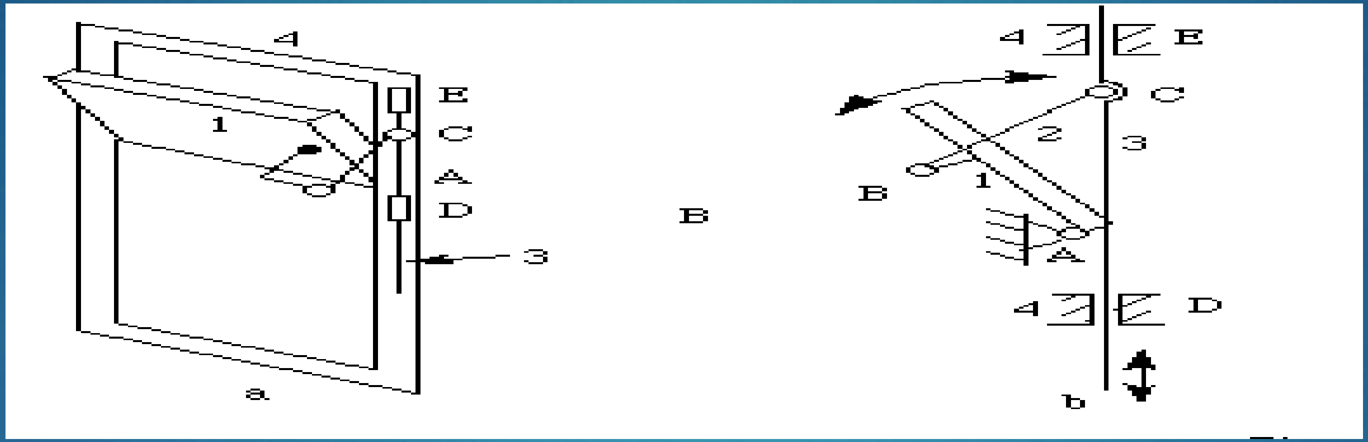
- **Therefore, we can write the following equation:**

$$F = 3(n-1) - 2l - h$$

- Where
  - F = total degrees of freedom in the mechanism
  - n = number of links (including the frame)
  - l = number of lower pairs (one degree of freedom)
  - h = number of higher pairs (two degrees of freedom)

- **This equation is also known as *Gruebler's equation*.**

- **Example 1**
- **Look at the transom above the door in Figure 4-13a. The opening and closing mechanism is shown in Figure 4-13b. Let's calculate its degree of freedom.**



- **n = 4 (link 1,3,3 and frame 4), L = 4 (at A, B, C, D), h = 0**

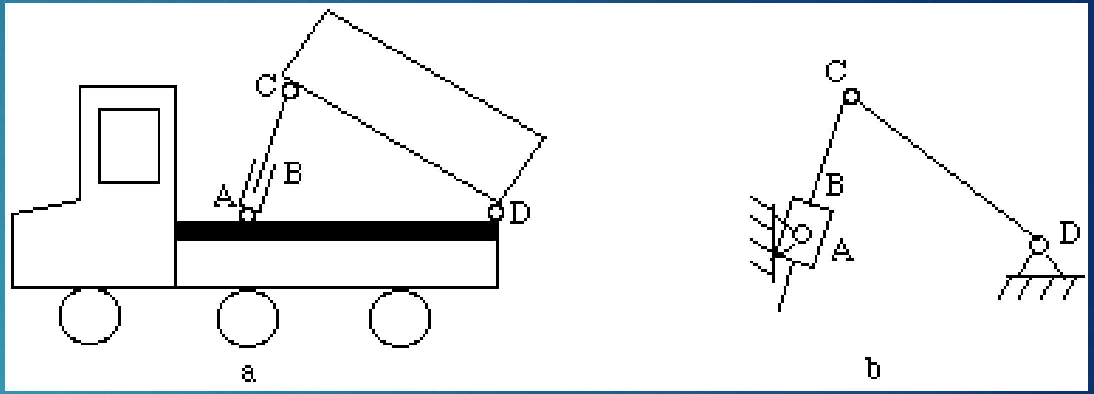
$$F = 3(n-1) - 2l - h$$

$$F = 3(4-1) - 2\times 4 - 1\times 0 = 1$$

- **Note: D and E function as a same prismatic pair, so they only count as one lower pair.**

- **Example 2**
- Calculate the degrees of freedom of the mechanisms shown in Figure 4-14b. Figure 4-14a is an application of the mechanism.
- n = 4, l = 4 (at A, B, C, D), h = 0

$$F = 3(4-1) - 2\times 4 - 1\times 0 = 1$$



- **Figure 4-14 Dump truck**
- **Example 3**
- Calculate the degrees of freedom of the mechanisms shown in Figure 4-15.
- For the mechanism in Figure 4-15a
- n = 6, l = 7, h = 0 , $F = 3(6-1) - 2\times 7 - 1\times 0 = 1$
- For the mechanism in Figure 4-15b
- n = 4, l = 3, h = 2 $F = 3(4-1) - 2\times 3 - 1\times 2 = 1$

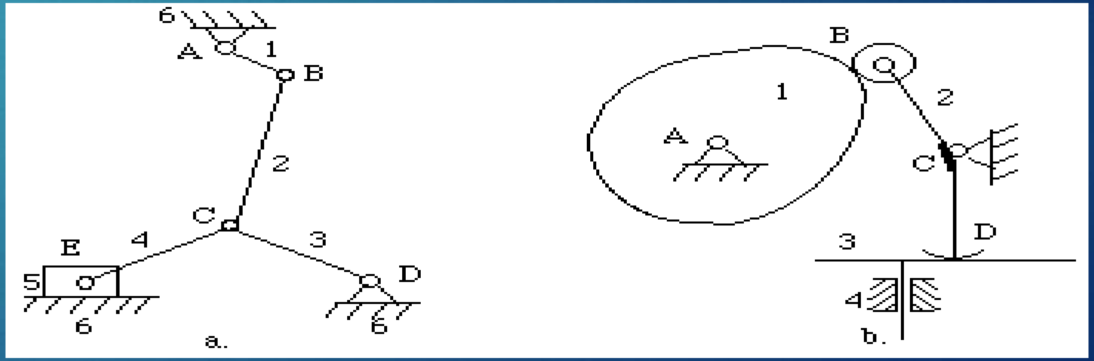


Figure 4-15 Degrees of freedom calculation

- **Note: The rotation of the roller does not influence the relationship of the input and output motion of the mechanism. Hence, the freedom of the roller will not be considered; It is called a *passive* or *redundant* degree of freedom. Imagine that the roller is welded to link 2 when counting the degrees of freedom for the mechanism.**

- **4.4.2 Kutzbach Criterion**
- The number of **degrees of freedom** of a mechanism is also called the *mobility* of the device. The ***mobility*** is the number of input parameters (usually pair variables) that must be independently controlled to bring the device into a particular position. The ***Kutzbach criterion***, which is similar to Gruebler's equation, calculates the *mobility*.
- In order to control a mechanism, the number of independent input motions must equal the number of degrees of freedom of the mechanism. For example, the transom in Figure 4-13a has a single degree of freedom, so it needs one independent input motion to open or close the window. That is, you just push or pull rod 3 to operate the window.
- To see another example, the mechanism in Figure 4-15a also has 1 degree of freedom. If an independent input is applied to link 1 (*e.g.*, a motor is mounted on joint A to drive link 1), the mechanism will have the a prescribed motion.

- **4.5 Finite Transformation**
- Finite transformation is used to describe the motion of a point on rigid body and the motion of the rigid body itself.
- **4.5.1 Finite Planar Rotational Transformation**
- Suppose that a point *P* on a rigid body goes through

a rotation describing a circular path from P_1 to P_2 around the origin of a coordinate system. We can describe this motion with a ***rotation operator*** R_{12}:

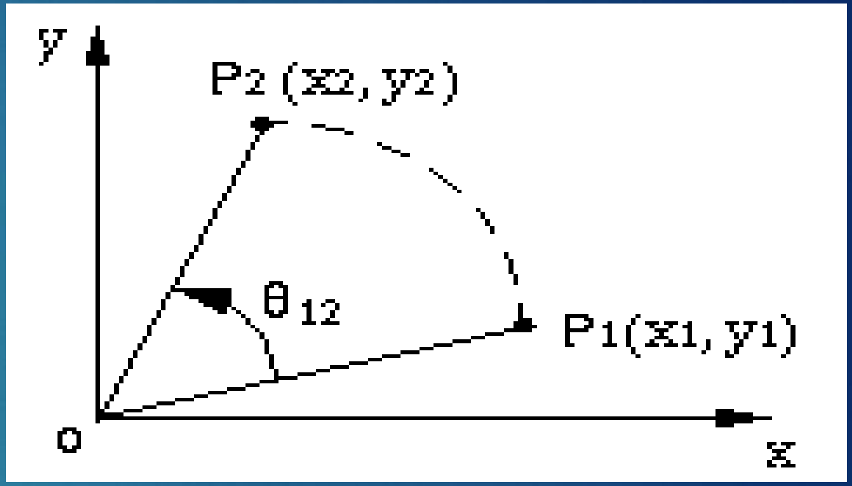


Figure 4-16 Point on a planar rigid body rotated through an angle

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = R_{12} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

$$R_{12} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} & 0 \\ \sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$