



MECH-320 Kinematics of Mechanical Systems

Chapter 5 - Planar Linkages

Course lectures Slides prepared by

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➤ 5.1 Introduction

➤ 5.1.1 What are Linkage Mechanisms?

Have you ever wondered what kind of mechanism causes the windshield wiper on the front window of car to oscillate ([Figure 5-1a](#))? The mechanism, shown in [Figure 5-1b](#), transforms the rotary motion of the motor into an oscillating motion of the windshield wiper.

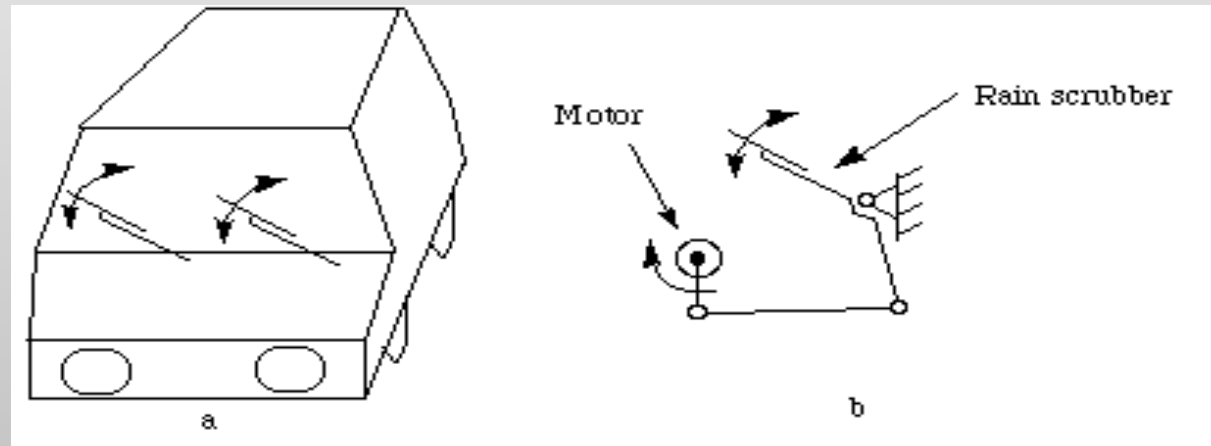


Figure 5-1 Windshield wiper

- Let's make a simple mechanism with similar behavior. Take some cardboard and make four strips as shown in [Figure 5-2a](#).
- Take 4 pins and assemble them as shown in [Figure 5-2b](#).
- Now, hold the 6 in. strip so it can't move and turn the 3 in. strip. You will see that the 4 in. strip oscillates.

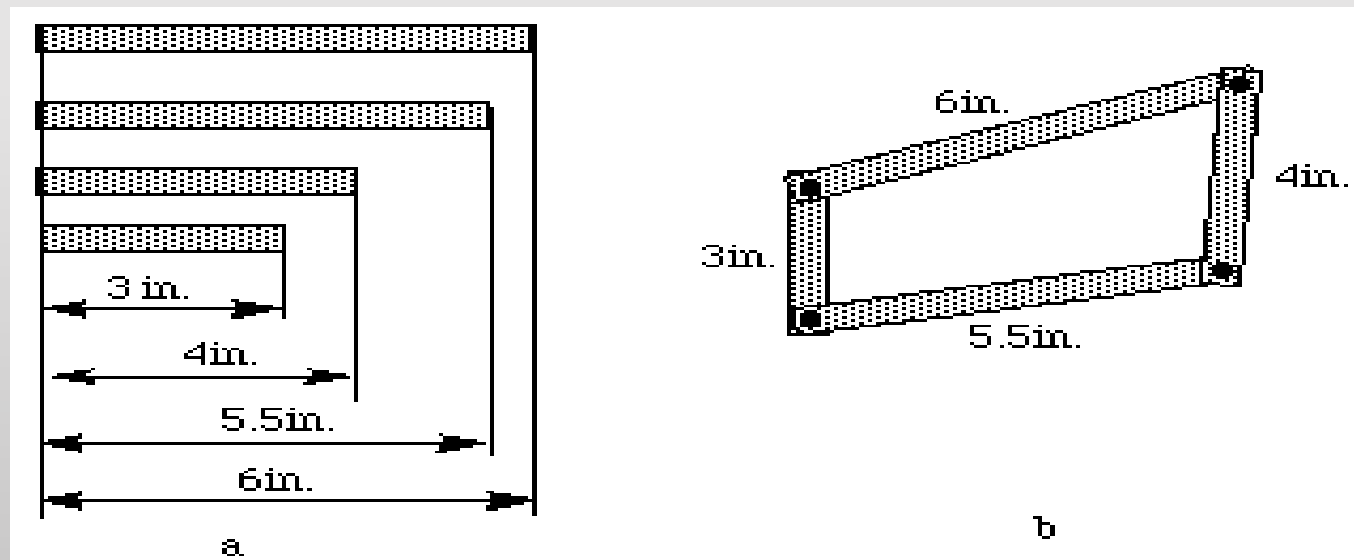
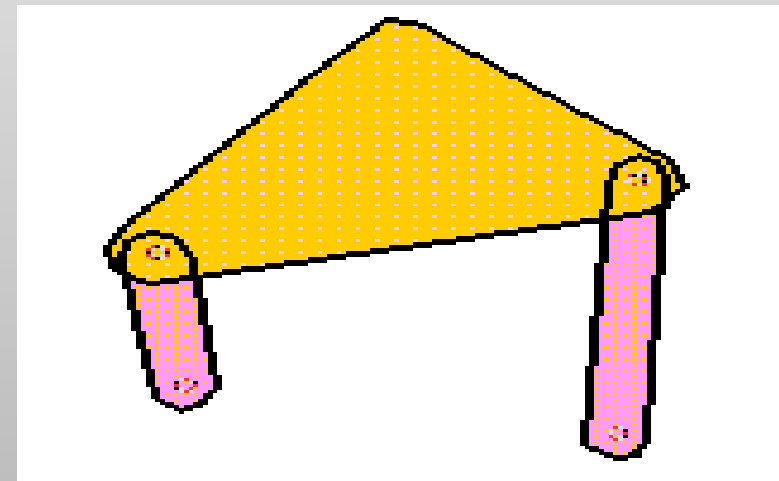


Figure 5-2 Do-it-yourself four bar linkage mechanism

- The *four bar linkage* is the simplest and often times, the most useful mechanism. As we mentioned before, a mechanism composed of rigid bodies and lower pairs is called a [linkage \(Hunt 78\)](#).
- In planar mechanisms, there are only two kinds of [lower pairs](#), [revolute pairs](#) and [prismatic pairs](#).
- The simplest closed-loop linkage is the *four bar linkage* which has four members, three moving links, **one fixed link and four pin joints**.
- *A linkage that has at least one fixed link is a mechanism.*
- The following example of a four bar linkage was created in [SimDesign](#) in *simdesign/fourbar.sim*

- Figure 5-3 Four bar linkage in SimDesign



- This mechanism has three moving links. Two of the links are pinned to the frame which is not shown in this picture. In SimDesign, links can be nailed to the background thereby making them into the [frame](#).
- How many [DOF](#) does this mechanism have? If we want it to have just one, we can impose one constraint on the linkage and it will have a definite motion. The four bar linkage is the simplest and the most useful mechanism.
- Reminder: A mechanism is composed of rigid bodies and lower pairs called linkages ([Hunt 78](#)). In planar mechanisms there are only two kinds of [lower pairs](#): turning pairs and prismatic pairs.

➤ 5.1.2 Functions of Linkages

- The function of a link mechanism is to produce rotating, oscillating, or reciprocating motion from the rotation of a crank or *vice versa* ([Ham et al. 58](#)).

- Stated more specifically linkages may be used to convert:
 - 1) Continuous rotation into continuous rotation, with a constant or variable angular velocity ratio.
 - 2) Continuous rotation into oscillation or reciprocation (or the reverse), with a constant or variable velocity ratio.
 - 3) Oscillation into oscillation, or reciprocation into reciprocation, with a constant or variable velocity ratio.

➤ Linkages have many different functions, which can be classified according on the primary goal of the mechanism:

1. **Function generation:** the relative motion between the links connected to the frame,
2. **Path generation:** the path of a tracer point, or
3. **Motion generation:** the motion of the coupler link.

• 5.2 Four Link Mechanisms

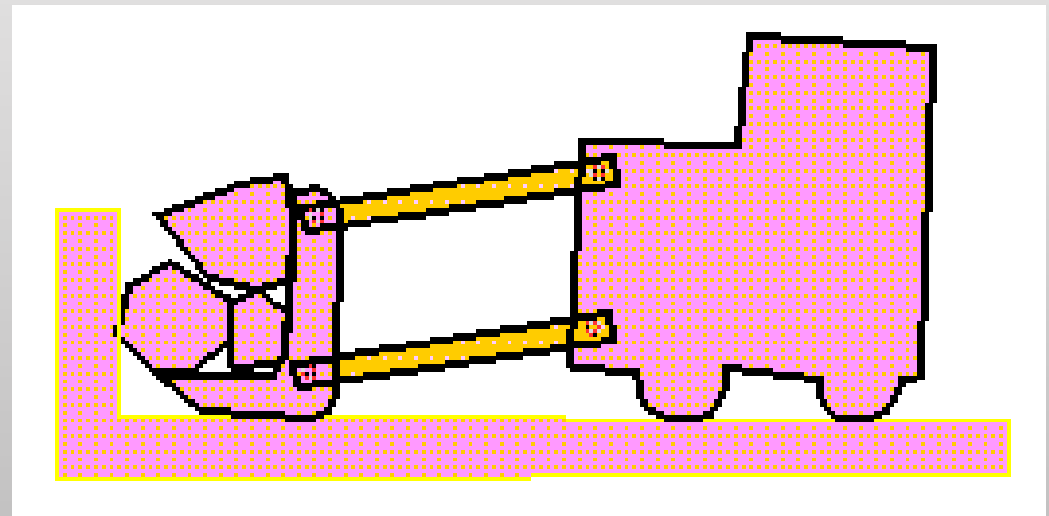
- One of the simplest examples of a constrained linkage is the ***four-link mechanism***. A variety of useful mechanisms can be formed from a four-link mechanism through slight variations, such as changing the character of the pairs, proportions of links, *etc.* Furthermore, many complex link mechanisms are combinations of two or more such mechanisms. The majority of four-link mechanisms fall into one of the following two classes:
 - the four-bar linkage mechanism, and
 - the [slider-crank](#) mechanism.

➤ 5.2.1 Examples

1) Parallelogram Mechanism

- In a parallelogram four-bar linkage, the orientation of the coupler does not change during the motion.
- The figure illustrates a loader. Obviously the behavior of maintaining parallelism is important in a loader.
- The bucket should not rotate as it is raised and lowered. The corresponding SimDesign file is `simdesign/loader.sim`.

➤ Figure 5-4 Front loader mechanism



2) Slider-Crank Mechanism

- The four-bar mechanism has some special configurations created by making one or more links infinite in length.
- The slider-crank (or crank and slider) mechanism shown below is a four-bar linkage with the slider replacing an infinitely long output link. The corresponding SimDesign file is *simdesign/slider.crank.sim*.
- This configuration translates a rotational motion into a translational one.
- Most mechanisms are driven by motors, and slider-cranks are often used to transform rotary motion into linear motion.

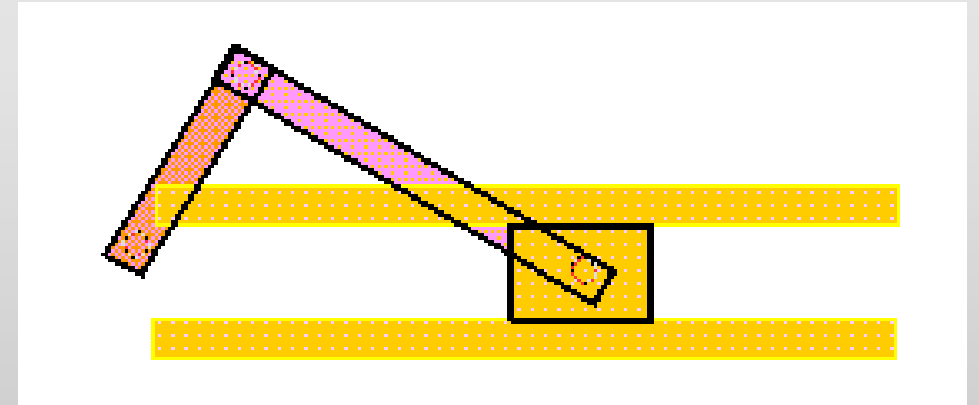
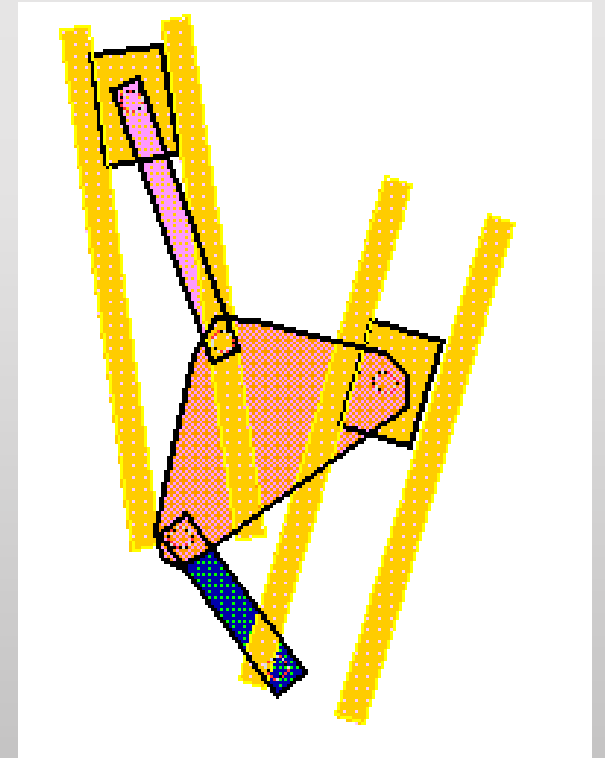


Figure 5-5 Crank and Slider Mechanism

3) Crank and Piston

- You can also use the slider as the input link and the crank as the output link. In this case, the mechanism transfers translational motion into rotary motion. The pistons and crank in an internal combustion engine are an example of this type of mechanism. The corresponding *SimDesign* file is *simdesign/combustion.sim*.
- You might wonder why there is another slider and a link on the left. This mechanism has two dead points.
- The slider and link on the left help the mechanism to overcome these dead points.

Figure 5-6 Crank and Piston



4) Block Feeder

- One interesting application of slider-crank is the block feeder. The SimDesign file can be found in *simdesign/block-feeder.sim*

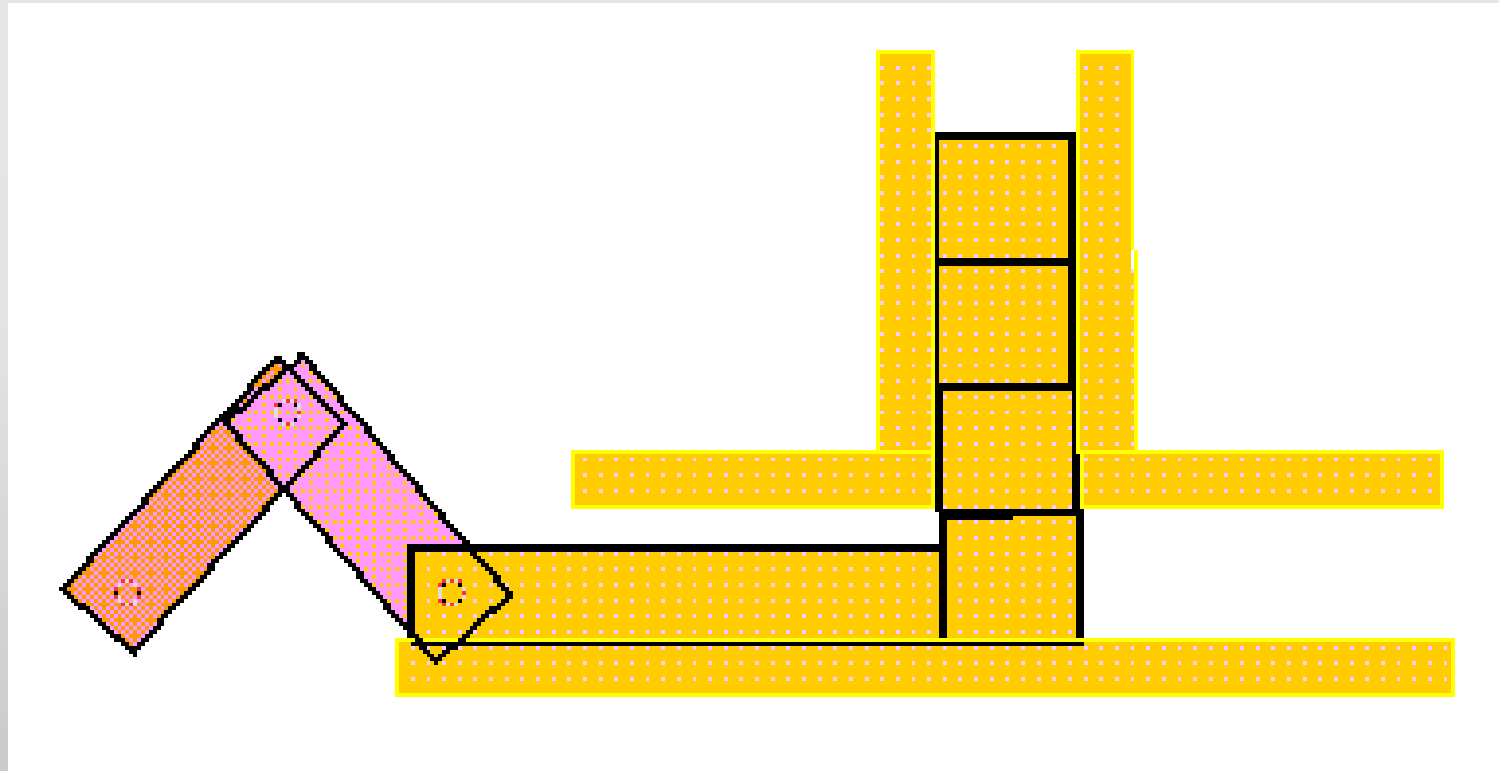


Figure 5-7 Block Feeder

5.2.2 Definitions

➤ In the range of planar mechanisms, the simplest group of lower pair mechanisms are four bar linkages. A **four bar linkage** comprises four bar-shaped links and four turning pairs as shown in [Figure 5-8](#).

➤ The link opposite the [frame](#) is called the **coupler link**, and the links which are hinged to the frame are called **side links**.

A link which is free to rotate through 360 degree with respect to a second link will be said to **revolve** relative to the second link (not necessarily a frame). If it is possible for all four bars to become simultaneously aligned, such a state is called a **change point**.

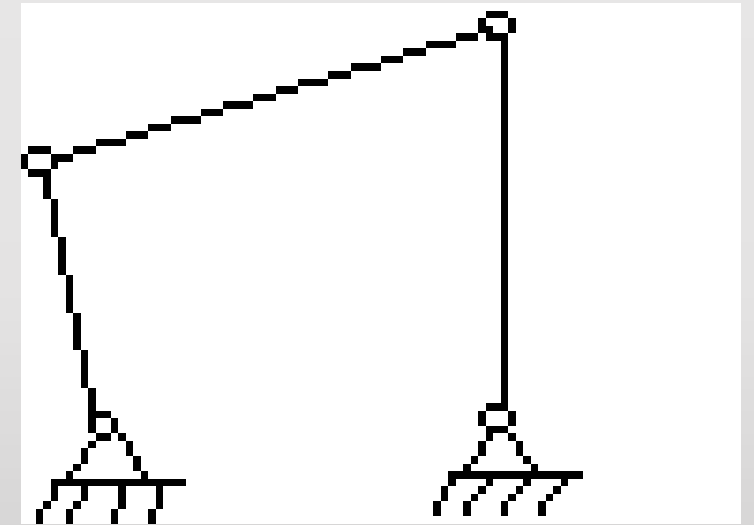
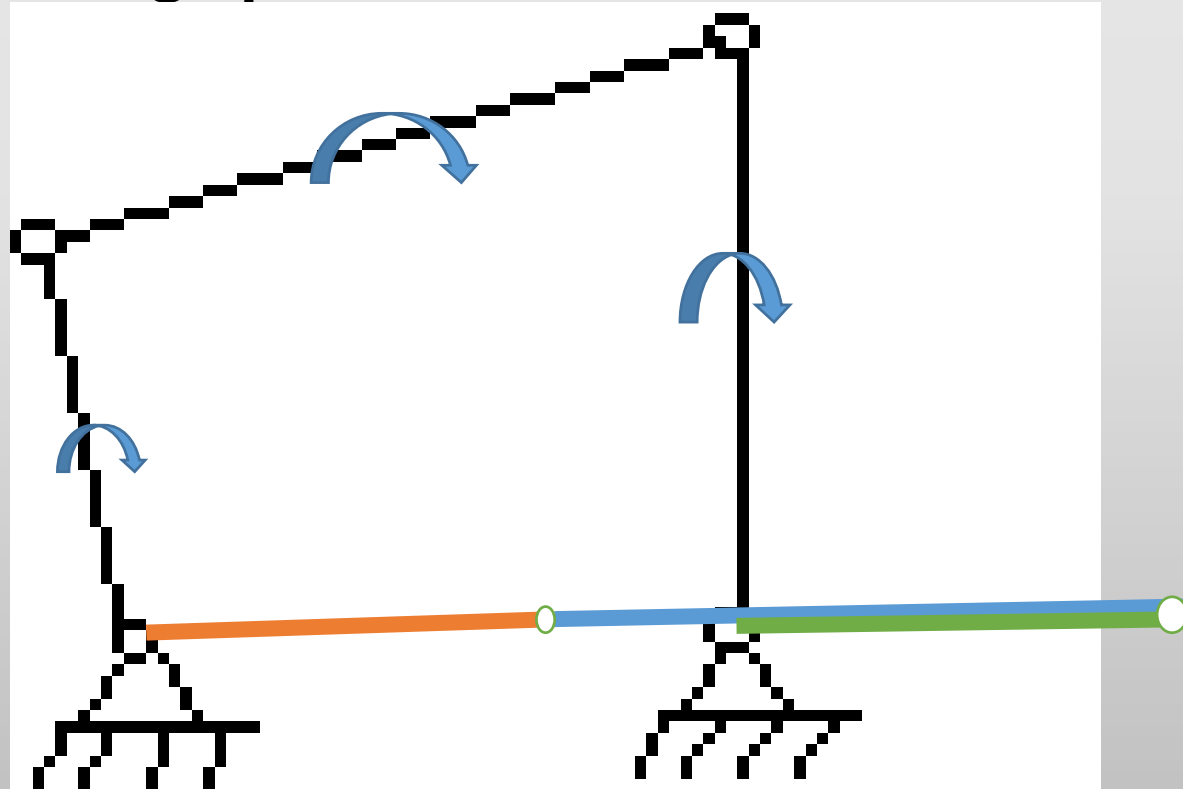
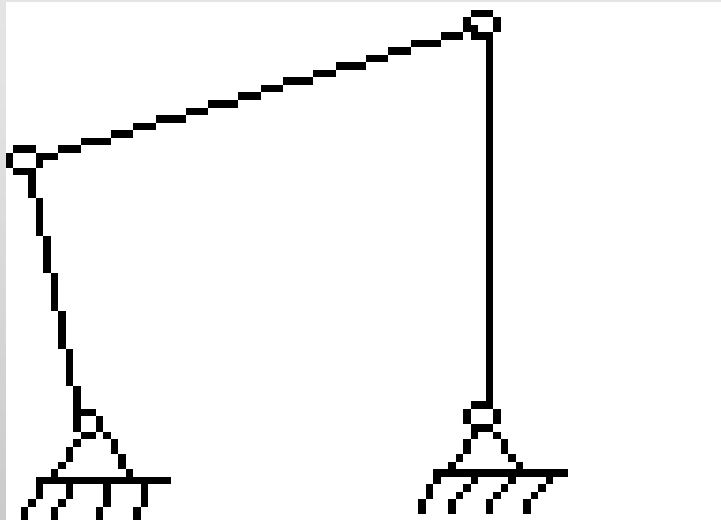


Figure 5-8 Four bar linkage

change point

If it is possible for all four bars to become simultaneously aligned, such a state is called a **change point**.



➤ **Some important concepts in link mechanisms are:**

- **Crank:** A side link which revolves relative to the frame is called a *crank*.
- **Rocker:** Any link which does not revolve is called a *rocker*.
- **Crank-rocker mechanism:** In a four bar linkage, if the shorter side link revolves and the other one rocks (*i.e.*, oscillates), it is called a *crank-rocker mechanism*.
- **Double-crank mechanism:** In a four bar linkage, if both of the side links revolve, it is called a *double-crank mechanism*.
- **Double-rocker mechanism:** In a four bar linkage, if both of the side links rock, it is called a *double-rocker mechanism*.

5.2.3 Classification

- Before classifying four-bar linkages, we need to introduce some basic nomenclature.
- In a four-bar linkage, we refer to the *line segment between hinges* on a given link as a **bar** where:
 - s = length of shortest bar
 - l = length of longest bar
 - p, q = lengths of intermediate bar

- **Grashof's theorem**

- states that a **four-bar** mechanism has *at least one revolving link* **if**

$$s + l \leq p + q$$

and all three mobile links will **rock** if

$$s + l > p + q$$

- The inequality 5-1 is **Grashof's criterion**.
- All four-bar mechanisms fall into one of the four categories listed in Table 5-1:

- All four-bar mechanisms fall into one of the four categories listed in Table 5-1:

Case	<u>l + s</u> vers. <u>p + q</u>	Shortest Bar	Type
1	<	Frame	Double-crank
2	<	Side	Rocker-crank
3	<	Coupler	Double rocker
4	=	Any	Change point
5	>	Any	Double-rocker

Table 5-1 Classification of Four-Bar Mechanisms

- From Table 5-1 we can see that for a mechanism to have a crank, the sum of the length of its shortest and longest links must be less than or equal to the sum of the length of the other two links.

➤ However, this condition is **necessary** but **not sufficient**. Mechanisms satisfying this condition fall into the following three categories:

- 1) When the *shortest link is a side link*, the mechanism is a **crank-rocker** mechanism. *The shortest link is the crank in the mechanism.*
- 2) When the *shortest link is the frame* of the mechanism, the mechanism is a **double-crank mechanism**.
- 3) When the *shortest link is the coupler link*, the mechanism is a **double-rocker mechanism**.

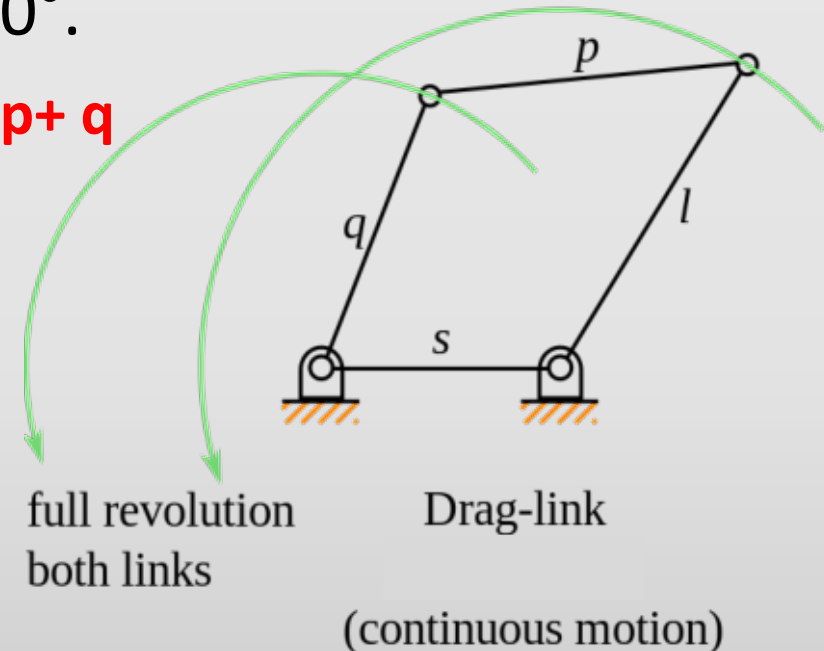
Examples

- **1. Double crank mechanism**

In double crank mechanism, the shortest link 'S' is a ground link. Both input crank and output crank rotate at 360°.

- **Grashof's condition for double crank mechanism: $s+l < p+q$**

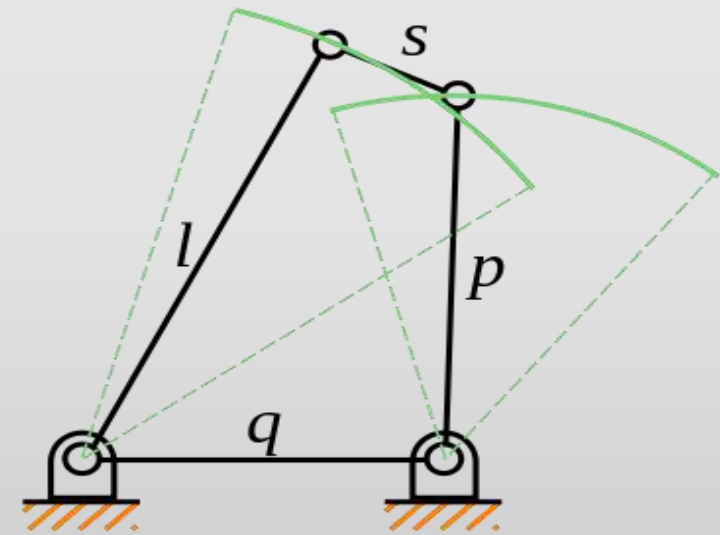
- Let: 's' = length of shortest link,
'l' = length of longest link,
'p' = length of one remaining link and
'q' = length of other remaining link.



- **2. Double-rocker mechanism**

In double-rocker mechanism, the shortest link 'S' is coupler link. The coupler link can rotate 360°.

- **Grashof's condition for double crank mechanism: $s+l < p+q$**

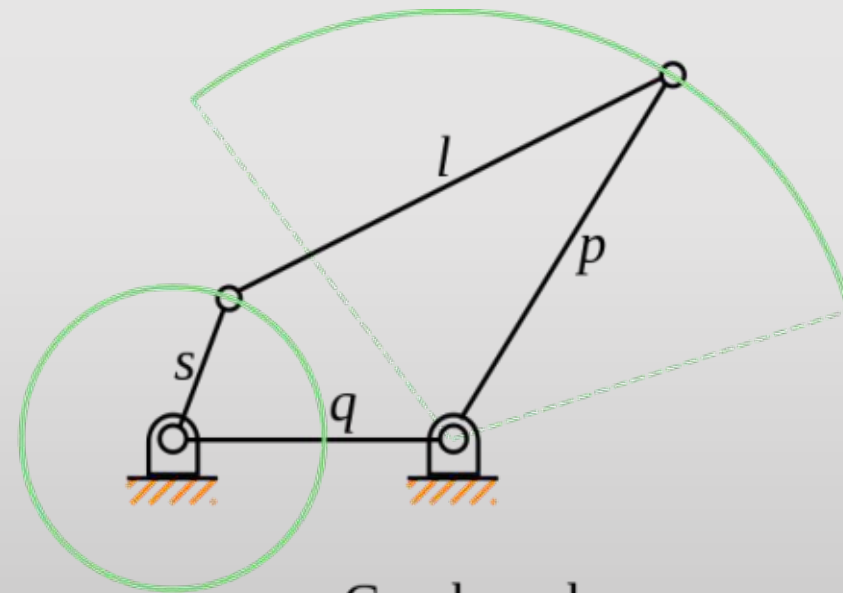


Double-rocker
 $s+l > p+q$
(no continuous motion)

- **3. Crank and rocker mechanism**

In crank and rocker mechanism, the shortest link “S” is input crank or output crank. Input crank or output crank rotates 360°.

- **Grashof’s condition for double crank mechanism: $s+l < p+q$**

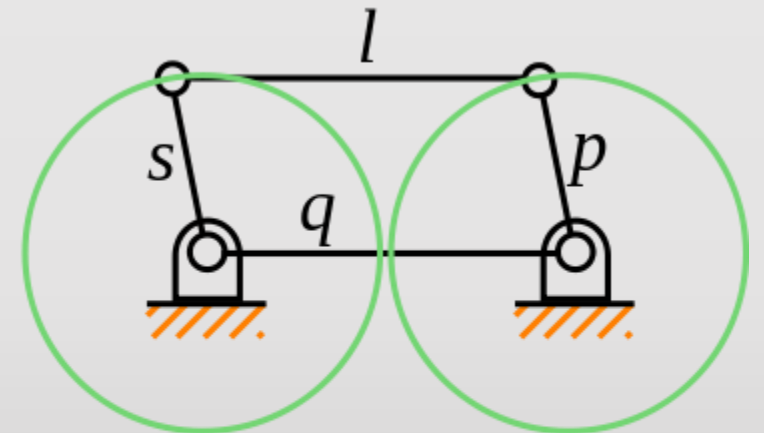


Crank-rocker
 $s+l < p+q$
(continuous motion)

- **4. Parallel linkage mechanism**

the parallel linkage mechanism is a special case of Grashof's criteria, where the sum of the shortest link 'S' and longest link 'L' of a planar quadrilateral linkage is less than or equal to the sum of the remaining two links 'P+Q'.

- **Grashof's condition for parallel linkage mechanism: $s+l = p+q$**



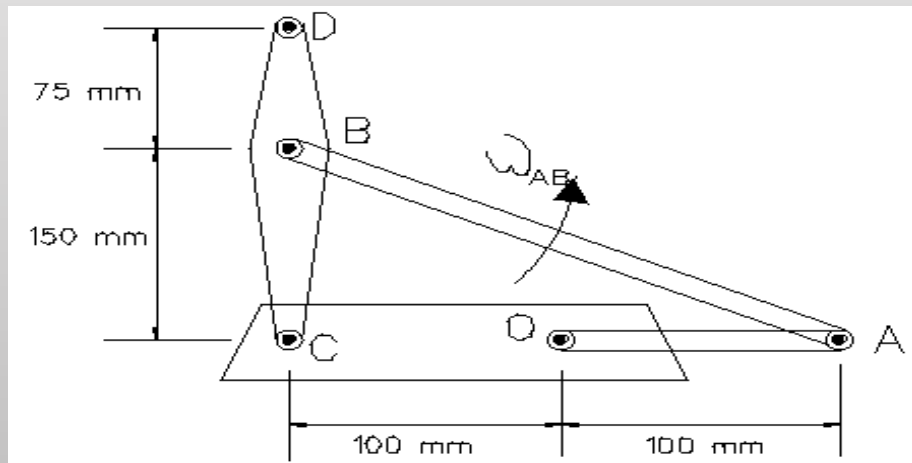
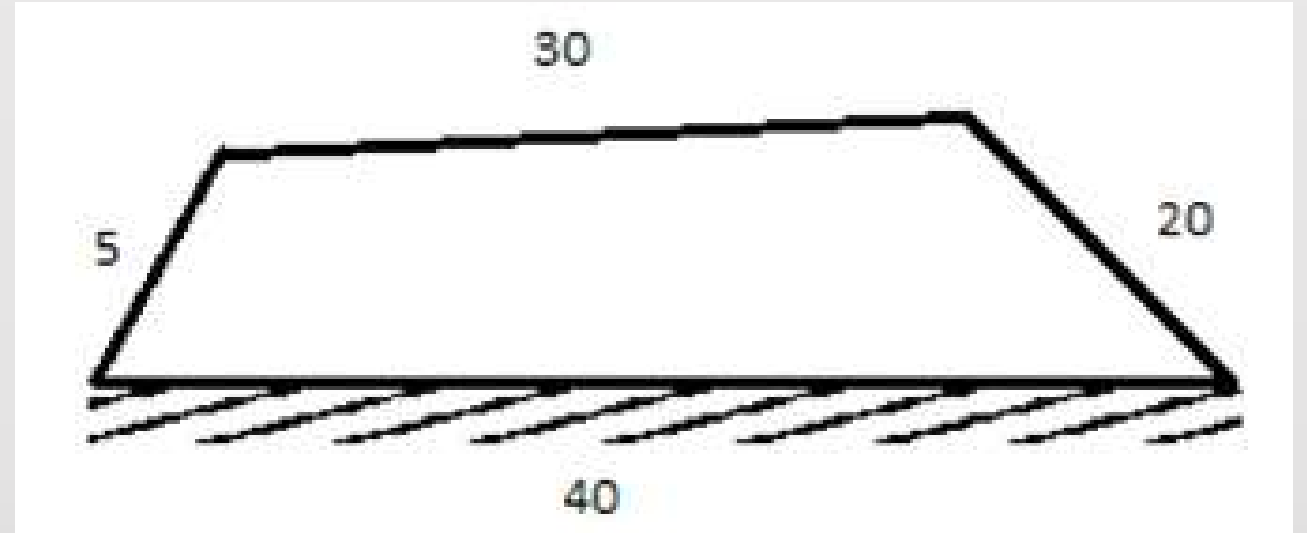
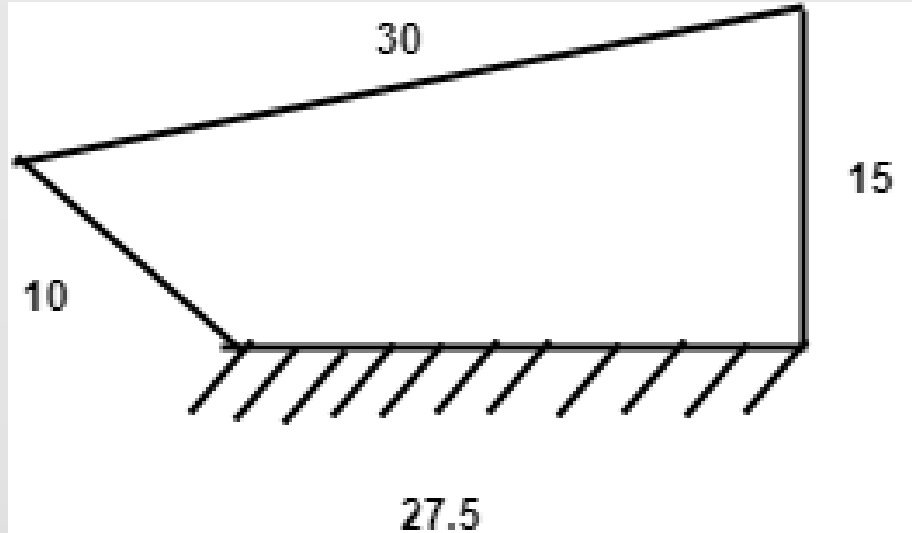
Parallelogram linkage

$$s+l = p+q$$

(continuous motion)

Problems:

- What type of mechanism is shown in the given diagram?



www.mekanizmalar.com

1 Rotation Speed

300 Length of the AC and BD Linkages
500 Length of the AB and CD linkages
50 100° CE/CB
0 EF Distance

Characters Visible
 CE and EF Visible

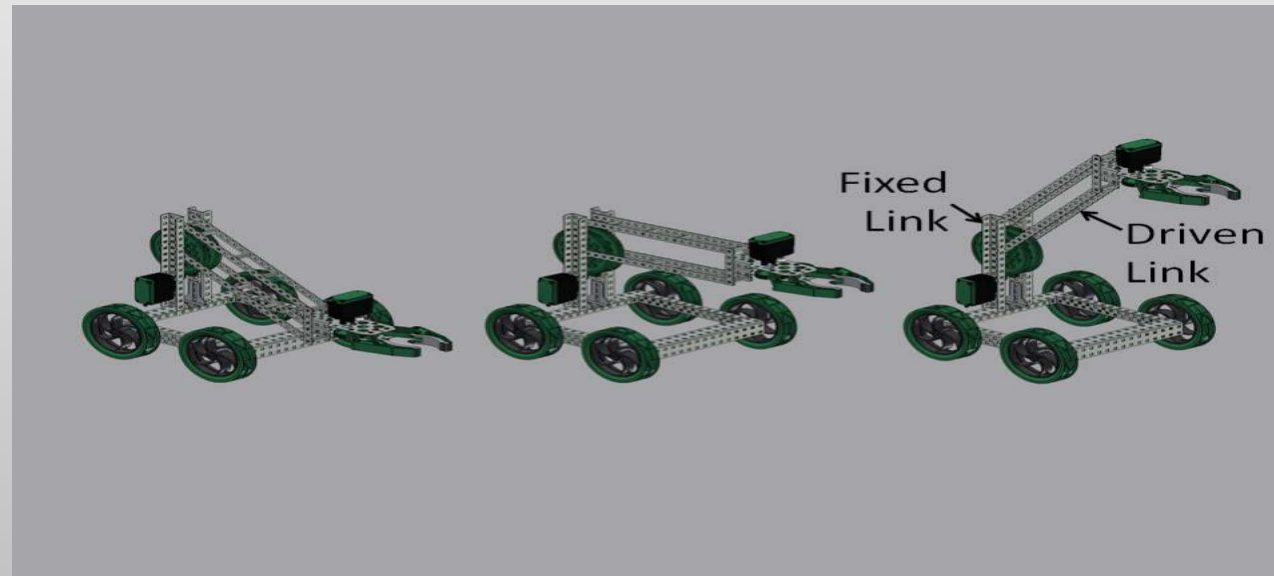
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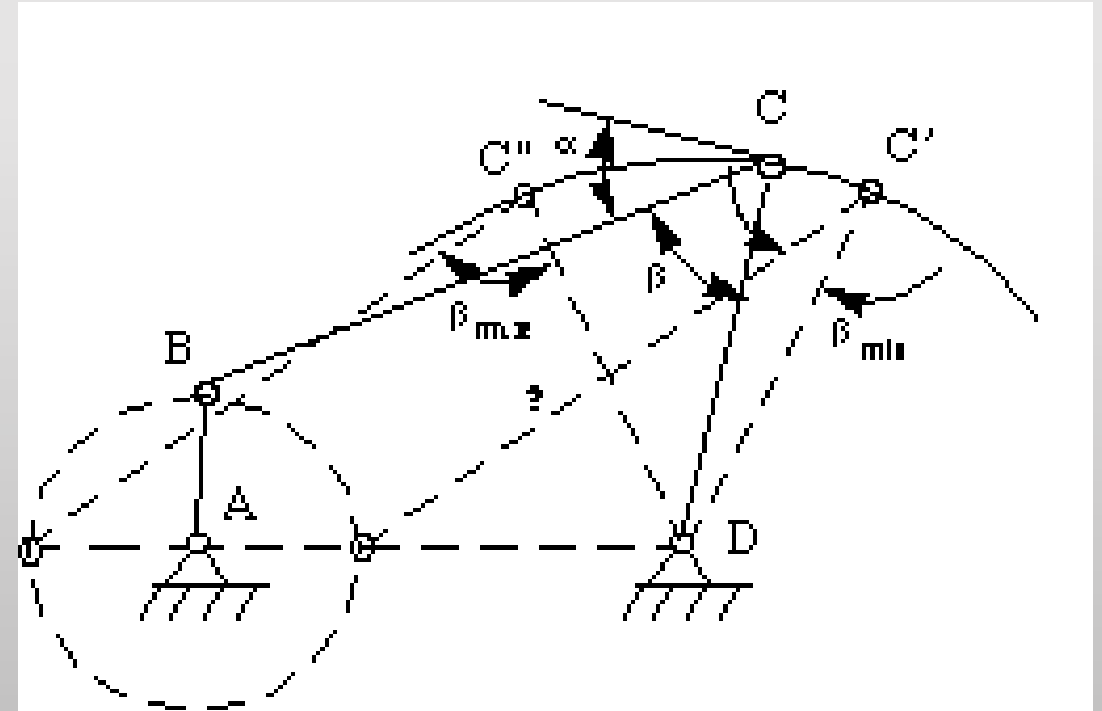
- https://www.youtube.com/watch?v=h8bz4ni6mdY&feature=emb_rel_end
- <https://www.mekanizmalar.com/four-bar-infinity-curve.html>
- *The change point*
- <https://grabcad.com/library/four-bar-mechanism-by-catia-v5-dmu-kinametics>



5.2.4 Transmission Angle β

- In [Figure 5-9](#), if
 - AB is the input link, the force applied to the output link, CD , is transmitted through the coupler link BC .
 - (That is, pushing on the link CD imposes a force on the link AB , which is transmitted through the link BC).

Figure 5-9 Transmission angle

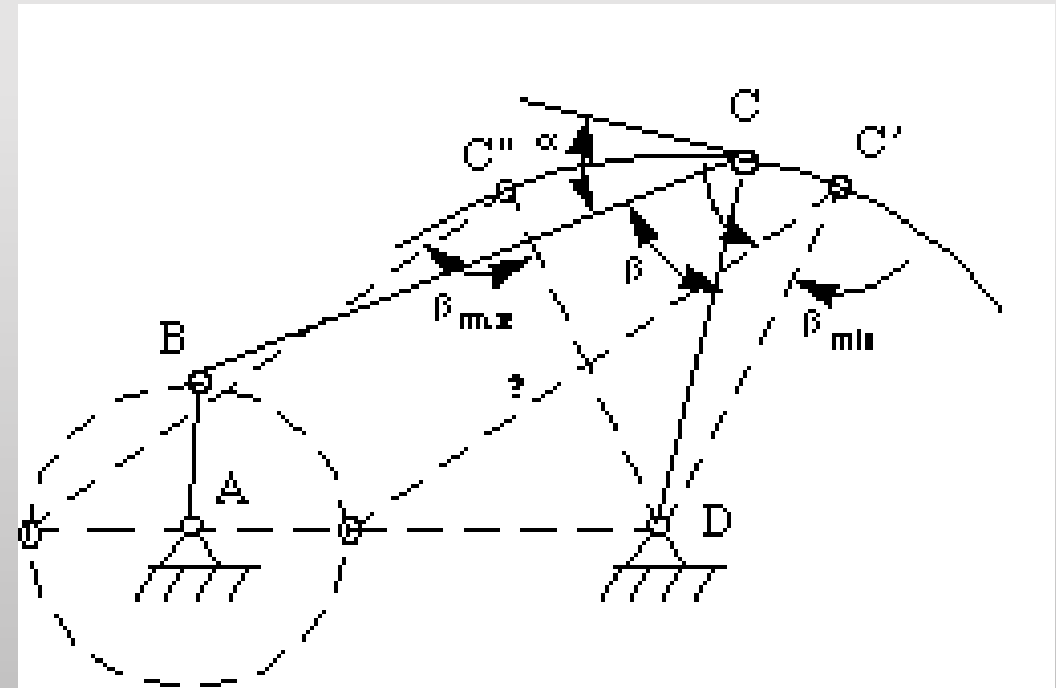


- For sufficiently slow motions (negligible inertia forces), the force in the coupler link is pure tension or compression (negligible bending action) and is directed along BC .
- For a given force in the coupler link, the torque transmitted to the output bar (about point D) is maximum when the angle β between coupler bar BC and output bar CD is $\pi/2$. Therefore, angle BCD is called **transmission angle β**

$$\alpha_{\max} = |90^\circ - \beta|_{\min} < 50^\circ$$

- When the *transmission angle* deviates significantly from $\pi/2$, the torque on the output bar decreases and may not be sufficient to overcome the friction in the system.
- For this reason, the **deviation angle** $\alpha = |\pi/2 - \beta|$ should not be too great. In practice, there is no definite upper limit for α , because the existence of the inertia forces may eliminate the undesirable force relationships that is present under static conditions.
- Nevertheless, the following criterion can be followed.

Figure 5-9 Transmission angle

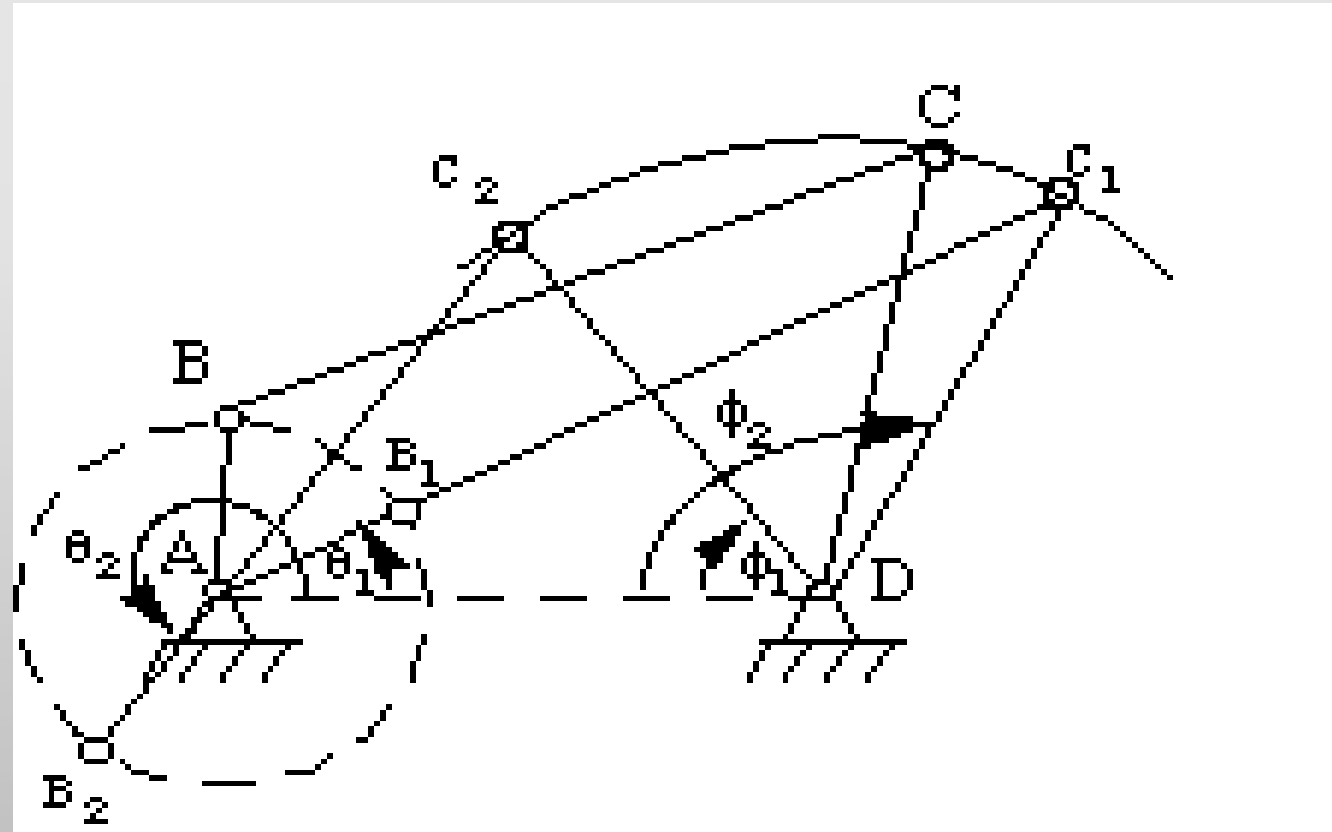


5.2.5 Dead Point

- When a side link such as AB in [Figure 5-10](#), becomes aligned with the coupler link BC , it can only be compressed or extended by the coupler.
- In this configuration, a torque applied to the link on the other side, CD , cannot induce rotation in link AB .

This link is therefore said to be at a **dead point** (sometimes called a **toggle point**).

Figure 5-10 Dead point



- In **Figure 5-10**, if AB is a crank, it can become aligned with BC in full extension along the line AB_1C_1 or in flexion with AB_2 folded over B_2C_2 . We denote the angle ADC by Φ and the angle DAB by θ . We use the subscript 1 to denote the extended state and 2 to denote the flexed state of links AB and BC . In the extended state, link CD cannot rotate clockwise without stretching or compressing the theoretically rigid line AC_1 . Therefore, link CD cannot move into the *forbidden zone* below C_1D , and Φ must be at one of its two extreme positions;

in other words, link CD is at an extremum.

A second extremum of link CD occurs with

$$\Phi = \Phi_1.$$

Note that the extreme positions of a side link occur simultaneously with the dead points of the opposite link.

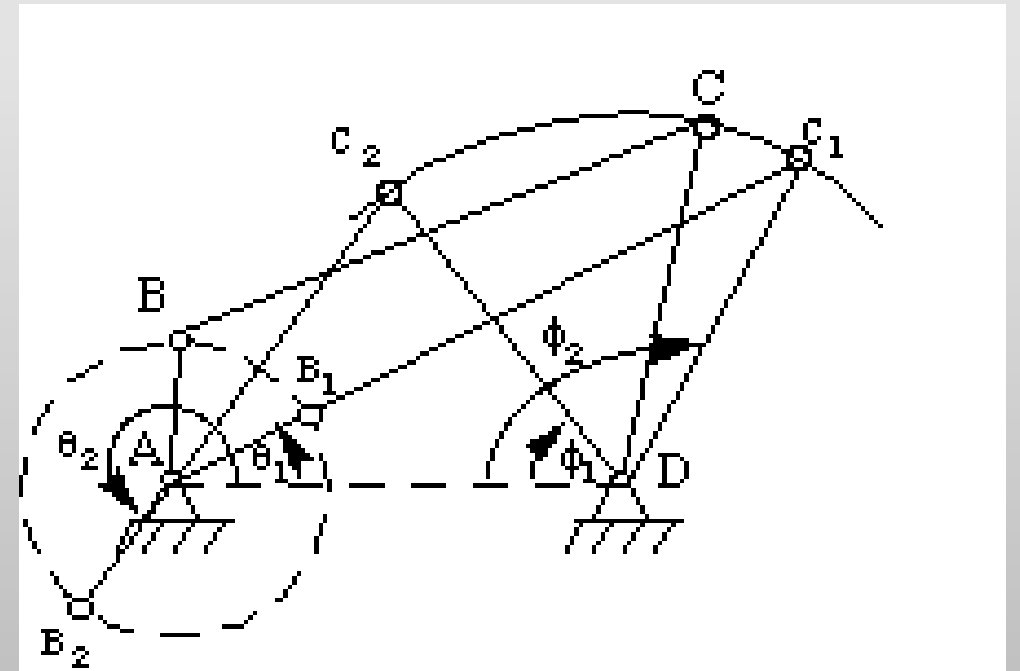
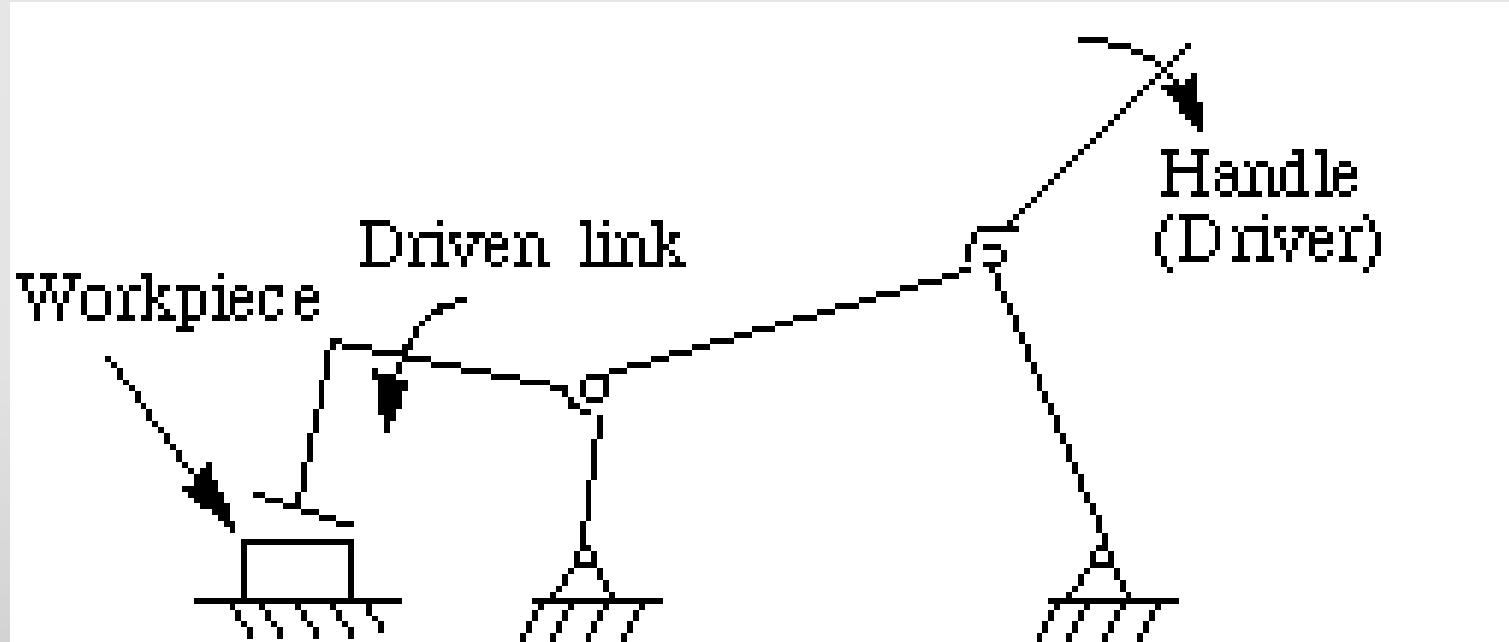


Figure 5-10 Dead point

▪ In some cases:

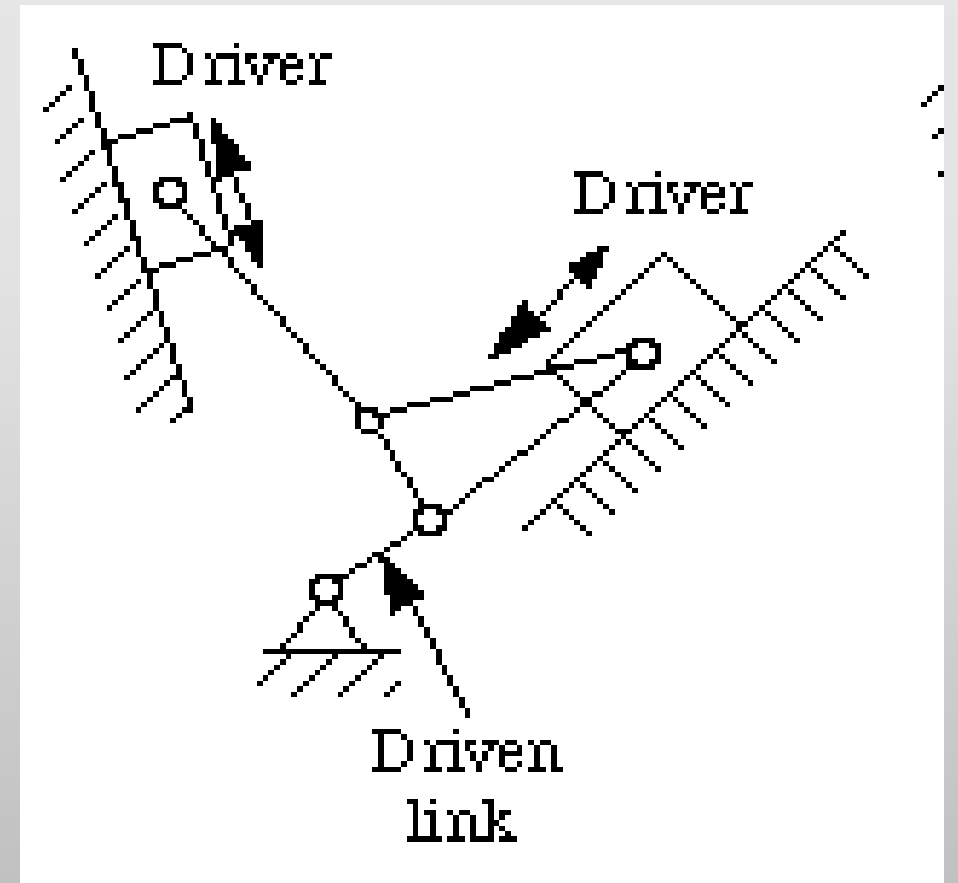
- 1) The dead point can be useful for tasks such as work fix-Turing ([Figure 5-11](#)).



- In some cases:

2) Dead point should be and can be overcome with the moment of inertia of links or with the asymmetrical deployment of the mechanism ([Figure 5-12](#)).

Figure 5-12 Overcoming the dead point by asymmetrical deployment (V engine)



5.2.6 Slider-Crank Mechanism

- The slider-crank mechanism, which has a well-known application in engines, is a special case of the [crank-rocker](#) mechanism. Notice that if rocker 3 in [Figure 5-13a](#) is very long, it can be replaced by a block sliding in a curved slot or guide as shown.

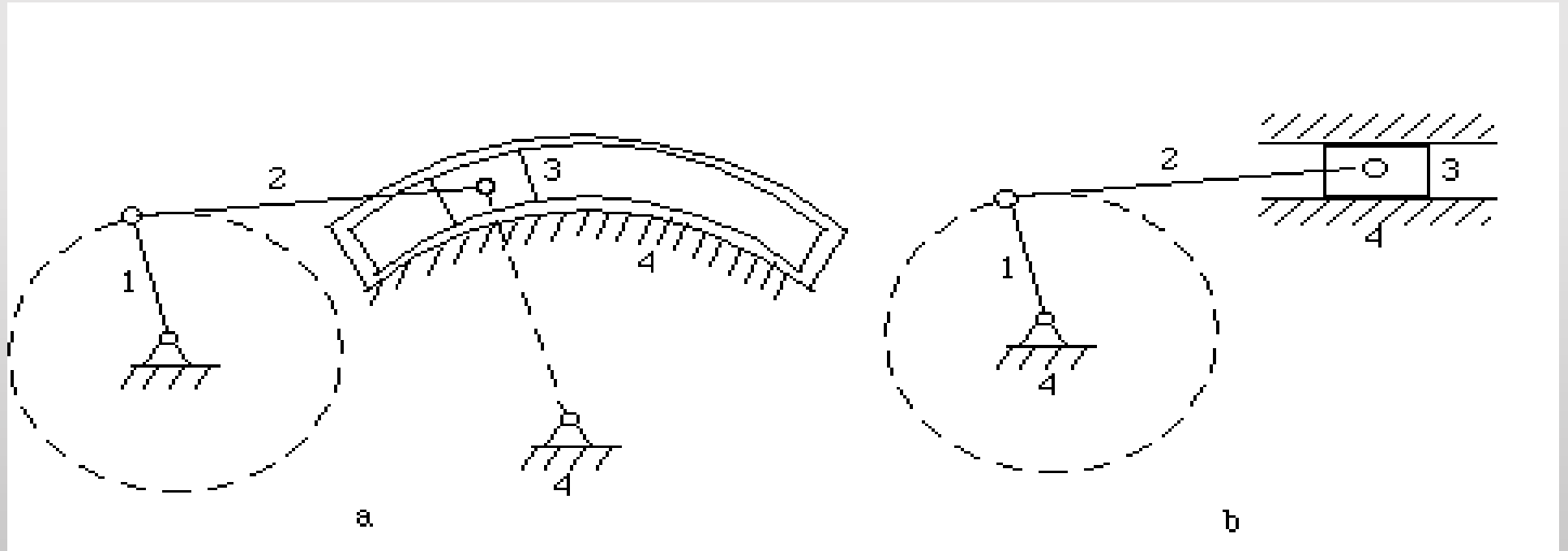


Figure 5-13 Slider-Crank mechanism

- If the length of the rocker is infinite, the guide and block are no longer curved. Rather, they are apparently straight, as shown in [Figure 5-13b](#), and the linkage takes the form of the ordinary **slider-crank mechanism**.

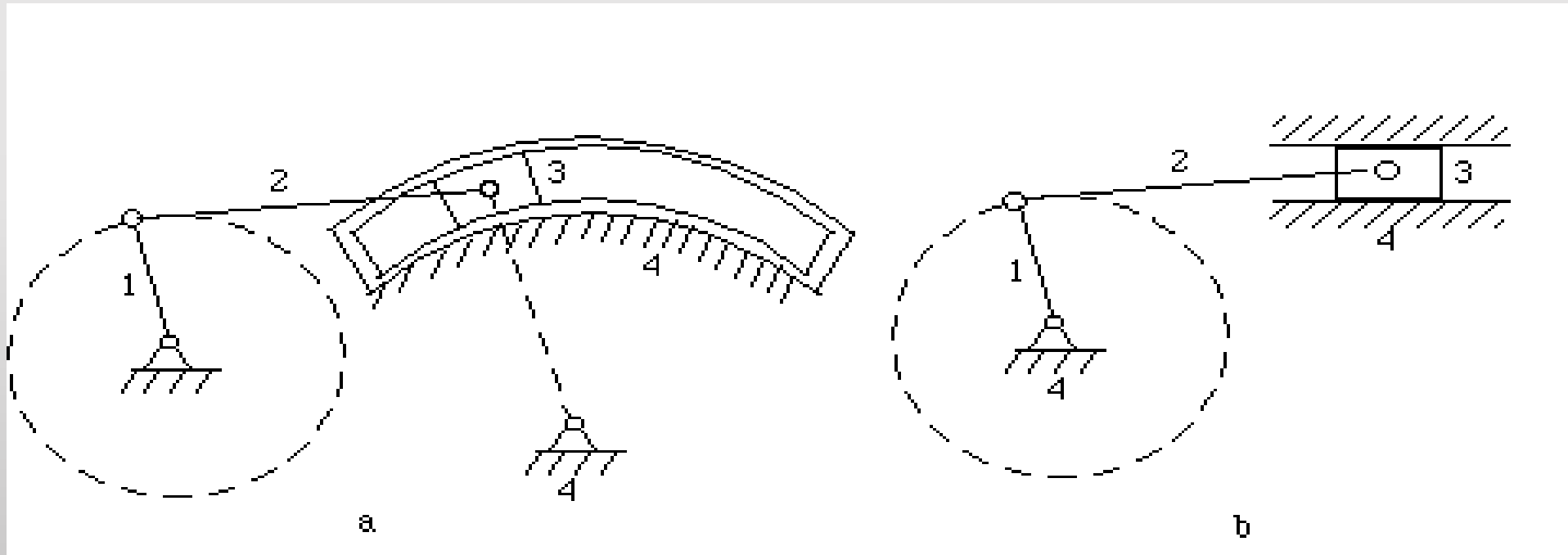


Figure 5-13 Slider-Crank mechanism

5.2.7 Inversion of the Slider-Crank Mechanism

- **Inversion** is a term used in kinematics for a reversal or interchange of form or function as applied to [kinematic chains](#) and mechanisms. For example, taking a different link as the fixed link, the slider-crank mechanism shown in [Figure 5-14a](#) can be inverted into the mechanisms shown in Figure 5-14b, c, and d.

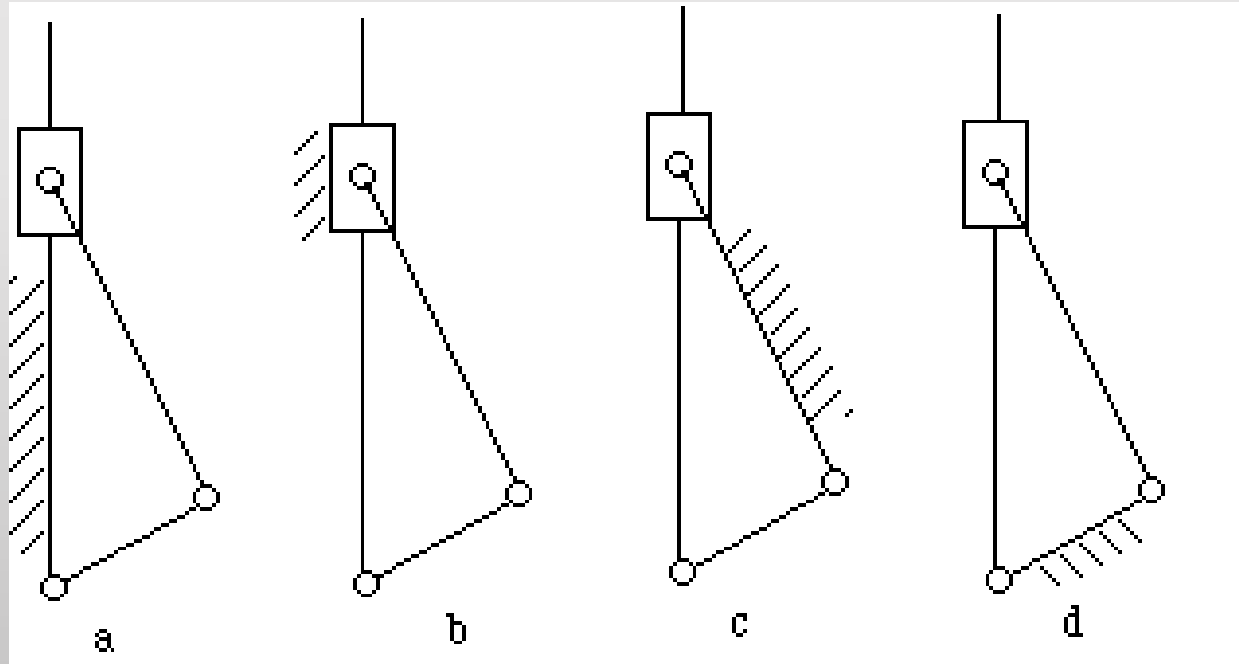


Figure 5-14 Inversions of the crank-slide mechanism

Different examples can be found in the application of these mechanisms. For example, the mechanism of the pump device in [Figure 5-15](#) is the same as that in Figure 5-14b.

Keep in mind that the inversion of a mechanism does not change the motions of its links relative to each other but does change their absolute motions.

Figure 5-15 A pump device

