# Balancing of Reciprocating masses

# Balancing of Primary Forces of Multi-cylinder In-line Engines

- The multi-cylinder engines with the cylinder centre lines in the same plane and on the same side of the centre line of the crankshaft, are known as *In-line engines.*
- *The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts of a multi-cylinder engine :*
  1) The algebraic sum of the primary forces must be equal to zero. In other words, the primary force polygon must close ; and
  2) The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero. In other words, the primary couple polygon must close.

We have already discussed, that the primary unbalanced force due to the reciprocating masses is equal to the component parallel to the line of stroke, of the centrifugal force produced by the equal mass placed at the crankpin and revolving with it.

Therefore, in order to give the *primary balance of the reciprocating parts of a multi-cylinder engine, it is convenient to imagine the reciprocating masses to be transferred to their respective crankpins and to treat the problem as one of revolving masses.*

## Notes :

1) For a two cylinder engine with cranks at 180°, condition (1) may be satisfied, but this will result in an unbalanced couple. Thus the above method of primary balancing cannot be applied in this case.

2) For a three cylinder engine with cranks at 120° and if the reciprocating masses per cylinder are same, then condition (1) will be satisfied because the forces may be represented by the sides of an equilateral triangle. However, by taking a reference plane through one of the cylinder centre lines, two couples with non-parallel axes will remain and these cannot vanish vectorially. Hence the above method of balancing fails in this case also.

3) For a four cylinder engine, similar reasoning will show that complete primary balance is possible and it follows that

   **For a multi-cylinder engine, the primary forces may be completely balanced by suitably arranging the crank angles, provided that the number of cranks are not less than four'.**

- The closing side of the primary force polygon gives the maximum unbalanced primary force and the closing side of the primary couple polygon gives the maximum un-balanced primary couple.

# Balancing of Secondary Forces of Multi-cylinder In-line Engines

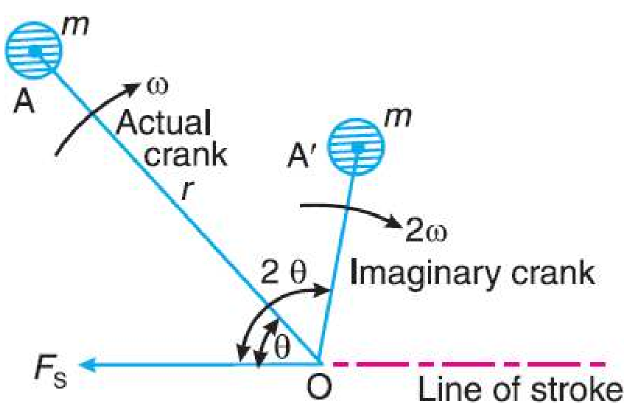
- When the connecting rod is not too long *(i.e. when the obliquity of the connecting rod is considered), then the secondary disturbing force due to the reciprocating mass arises.*
- We have discussed that the secondary force,

$$F_S = m.\omega^2.r \times \frac{\cos 2\theta}{n}$$

$$n = \frac{L}{2}$$

This expression may be written as

$$F_S = m.(2\omega)^2 \times \frac{r}{4n} \times \cos 2\theta$$



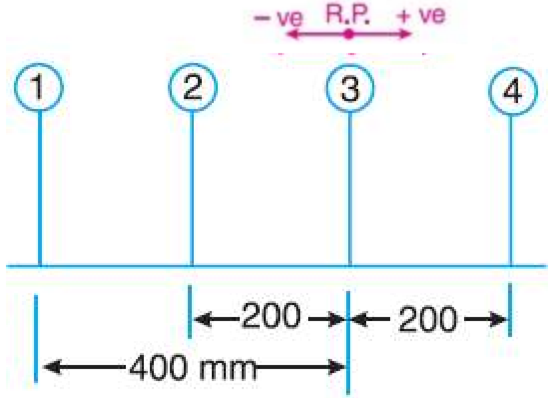

- As in case of primary forces, the secondary forces may be considered to be equivalent to the component, parallel to the line of stroke, of the centrifugal force produced by an equal mass placed at the imaginary crank of length *r/4n and revolving at twice the speed of the actual crank (i.e. 2 ω) as shown in Fig.*

- Thus, in multi-cylinder in-line engines, each imaginary secondary crank with a mass attached to the crankpin is inclined to the line of stroke at twice the angle of the actual crank.
- The values of the secondary forces and couples may be obtained by considering the revolving mass. This is done in the similar way as discussed for primary forces.
- The following two conditions must be satisfied in order to give a complete secondary balance of an engine :
  1. The algebraic sum of the secondary forces must be equal to zero. In other words, the secondary force polygon must close, and
  2. The algebraic sum of the couples about any point in the plane of the secondary forces must be equal to zero. In other words, the secondary couple polygon must close.

Note : The closing side of the secondary force polygon gives the maximum unbalanced secondary force and the closing side of the secondary couple polygon gives the maximum unbalanced secondary couple.

**Example 22.7.** *A four cylinder vertical engine has cranks 150 mm long. The planes of rotation of the first, second and fourth cranks are 400 mm, 200 mm and 200 mm respectively from the third crank and their reciprocating masses are 50 kg, 60 kg and 50 kg respectively. Find the mass of the reciprocating parts for the third cylinder and the relative angular positions of the cranks in order that the engine may be in complete primary balance.*

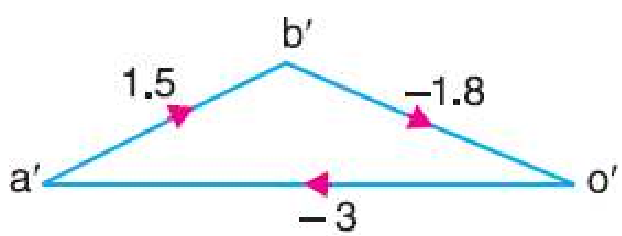
**Solution.** Given $r_1 = r_2 = r_3 = r_4 = 150$ mm $= 0.15$ m ; $m_1 = 50$ kg ; $m_2 = 60$ kg ; $m_4 = 50$ kg



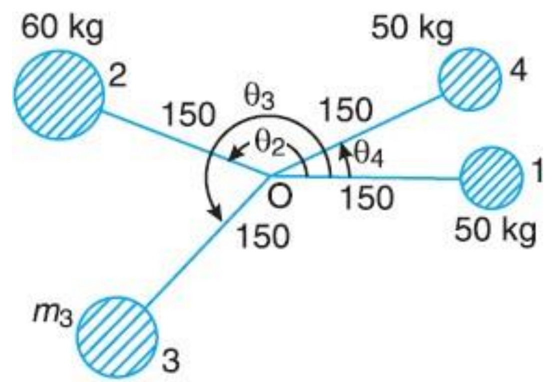

(*a*) Position of planes.

- In order to give the primary balance of the reciprocating parts of a multi-cylinder engine, the problem may be treated as that of revolving masses with the reciprocating masses transferred to their respective crank pins.
- The position of planes is shown in Fig. a

| Plane (1) | Mass (m) kg (2) | Radius (r) m (3) | Cent. force $\div \omega^2$ (m.r) kg-m (4) | Distance from plane 3(l) m (5) | Couple $\div \omega^2$ (m.r.l) kg-m$^2$ (6) |
|---|---|---|---|---|---|
| 1 | 50 | 0.15 | 7.5 | – 0.4 | – 3 |
| 2 | 60 | 0.15 | 9 | – 0.2 | – 1.8 |
| 3(R.P.) | $m_3$ | 0.15 | $0.15m_3$ | 0 | 0 |
| 4 | 50 | 0.15 | 7.5 | 0.2 | 1.5 |




(c) Couple polygon.



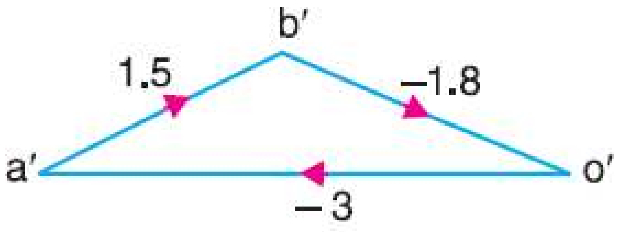

(b) Angular position of cranks.

By measurement, we find that the angular position of crank 2 from crank 1 in the anticlockwise direction is

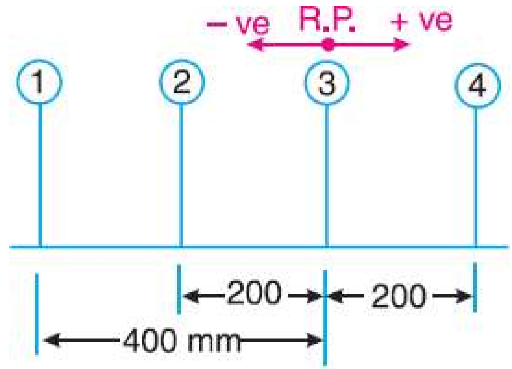
$$\theta_2 = 160° \text{ **Ans.**}$$

and the angular position of crank 4 from crank 1 in the anticlockwise direction is

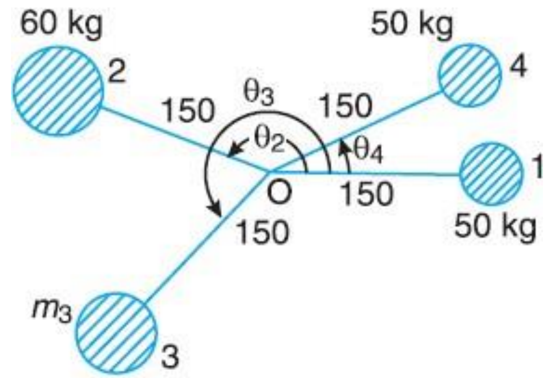
$$\theta_4 = 26° \text{ **Ans.**}$$



(c) Couple polygon.

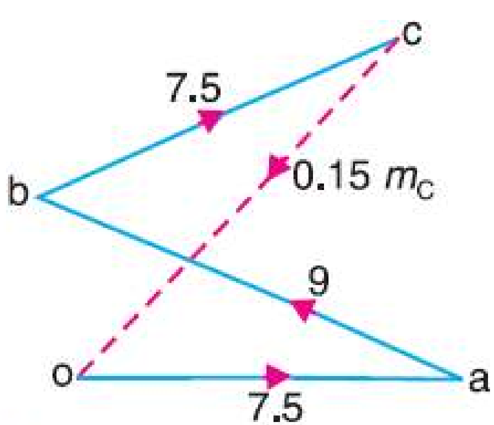


(a) Position of planes.



(b) Angular position of cranks.

In order to find the mass of the third cylinder ($m_3$) and its angular position, draw the force polygon, to some suitable scale, as shown in Fig. d, from the data given in Table (column 4).



(d) Force polygon.

$0.15m_3 = 9$ kg-m or $m_3 = 60$ kg **Ans.**

$\theta_3 = 227°$ **Ans.**

| *Cent. force ÷ $\omega^2$ (m.r) kg-m (4)* |
|---|
| 7.5 |
| 9 |
| $0.15m_3$ |
| 7.5 |

**Example 22.8.** *A four crank engine has the two outer cranks set at 120° to each other, and their reciprocating masses are each 400 kg. The distance between the planes of rotation of adjacent cranks are 450 mm, 750 mm and 600 mm. If the engine is to be in complete primary balance, find the reciprocating mass and the relative angular position for each of the inner cranks.*

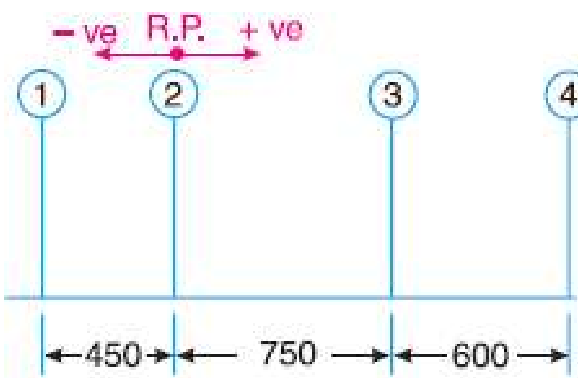
*If the length of each crank is 300 mm, the length of each connecting rod is 1.2 m and the speed of rotation is 240 r.p.m., what is the maximum secondary unbalanced force ?*

**Solution.** Given : $m_1 = m_4 = 400$ kg ; $r = 300$ mm $= 0.3$ m ; $l = 1.2$ m ; $N = 240$ r.p.m. or $\omega = 2\pi \times 240/60 = 25.14$ rad/s

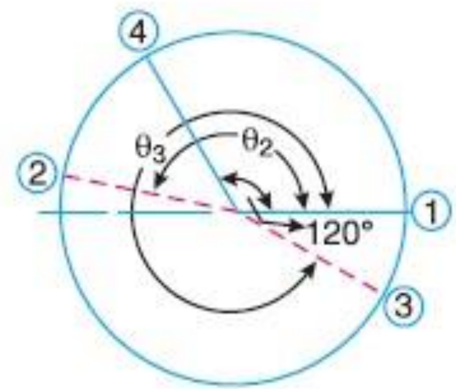
***Reciprocating mass and the relative angular position for each of the inner cranks***

Let $m_2$ and $m_3$ = Reciprocating mass for the inner cranks 2 and 3 respectively, and

$\theta_2$ and $\theta_3$ = Angular positions of the cranks 2 and 3 with respect to crank 1 respectively.

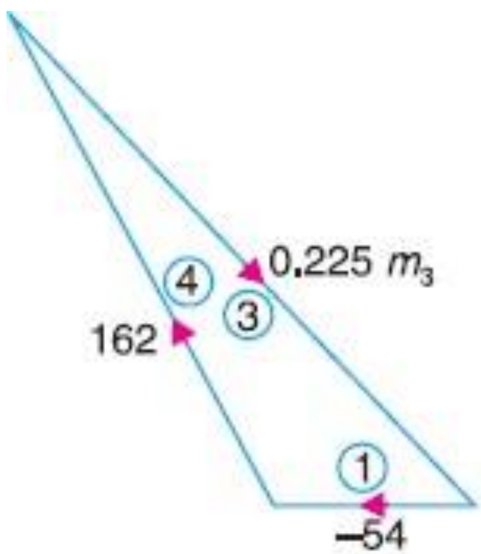



(*a*) Positions of planes.

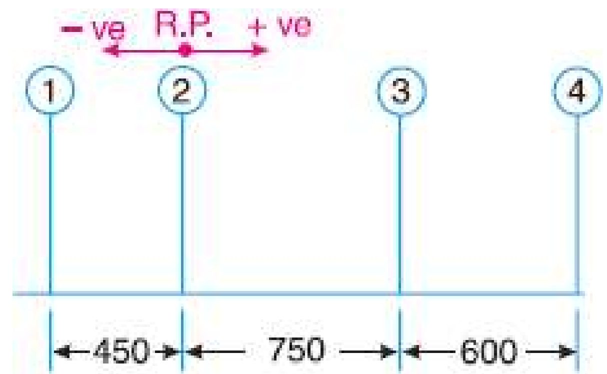



(*b*) Primary crank positions.

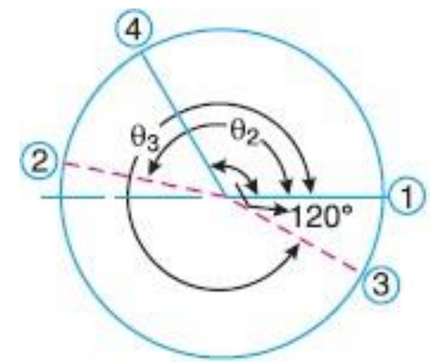
| *Plane* (1) | *Mass* (m) kg (2) | *Radius* (r) m (3) | *Cent. force* $\div \omega^2$ (m.r) kg-m (4) | *Distance from plane* (2) (l) m (5) | *Couple* $\div \omega^2$ (m.r.l.) kg-m$^2$ (6) |
|---|---|---|---|---|---|
| 1 | 400 | 0.3 | 120 | – 0.45 | – 54 |
| 2(R.P.) | $m_2$ | 0.3 | 0.3 $m_2$ | 0 | 0 |
| 3 | $m_3$ | 0.3 | 0.3 $m_3$ | 0.75 | 0.225 $m_3$ |
| 4 | 400 | 0.3 | 120 | 1.35 | 162 |




(c) Primary couple polygon.




(*a*) Positions of planes.



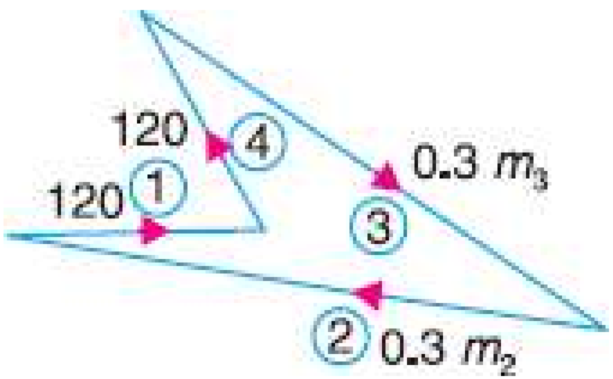

(*b*) Primary crank positions.

$m_3$ = 871 kg **Ans.**

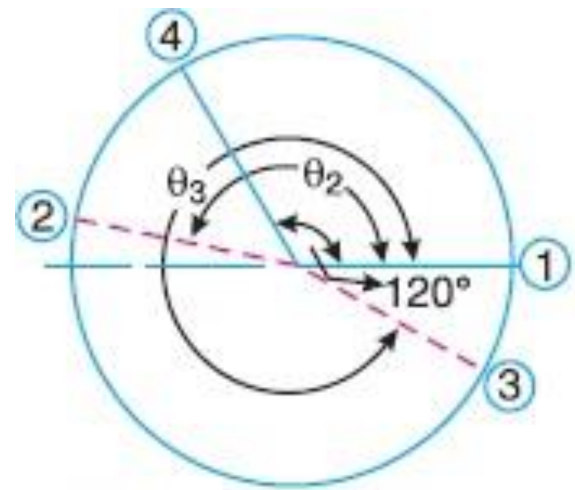
angular position with respect to crank 1 in the anticlockwise direction,

$\theta_3$ = 326° **Ans.**

| Plane (1) | Mass (m) kg (2) | Radius (r) m (3) | Cent. force $\div \omega^2$ (m.r) kg-m (4) | Distance from plane (2) (l) m (5) | Couple $\div \omega^2$ (m.r.l.) kg-m$^2$ (6) |
|---|---|---|---|---|---|
| 1 | 400 | 0.3 | 120 | – 0.45 | – 54 |
| 2(R.P.) | $m_2$ | 0.3 | 0.3 $m_2$ | 0 | 0 |
| 3 | $m_3$ | 0.3 | 0.3 $m_3$ | 0.75 | 0.225 $m_3$ |
| 4 | 400 | 0.3 | 120 | 1.35 | 162 |




(*d*) Primary force polygon.




(*b*) Primary crank positions.

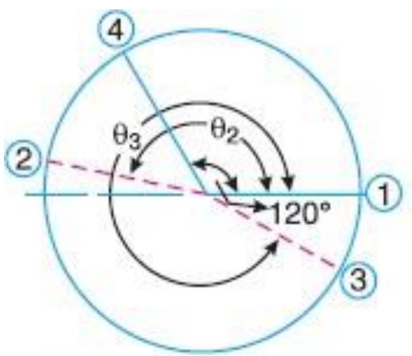
angular position with respect to crank 1 in the anticlockwise direction,

$$\theta_2 = 168° \text{ Ans.}$$

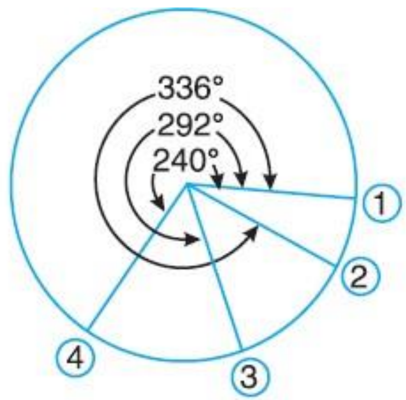
$$m_2 = 947 \text{ kg Ans.}$$

- The secondary crank positions obtained by rotating the primary cranks at twice the angle, is shown in Fig (*e).*
- *Now draw the secondary force polygon, as shown in Fig. ( f ), to* some suitable scale, from the data given in Table (column 4).
- The closing side of the polygon shown dotted in Fig. ( *f ) represents the maximum secondary unbalanced force.*
- *By measurement,* we find that the maximum secondary unbalanced force is proportional to 582 kg-m.

| Cent. force $\div \omega^2$ (m.r) kg-m (4) |
|---|
| 120 |
| $0.3\ m_2$ |
| $0.3\ m_3$ |
| 120 |



(*b*) Primary crank positions.



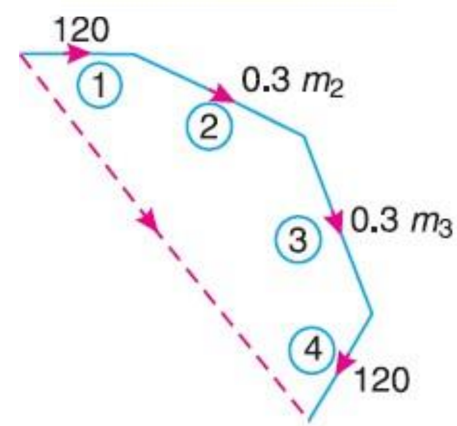
(*e*) Secondary crank positions.

$\theta_2 = 168°$ **Ans.**

$\theta_3 = 326°$ **Ans.**

$m_2 = 947$ kg **Ans.**

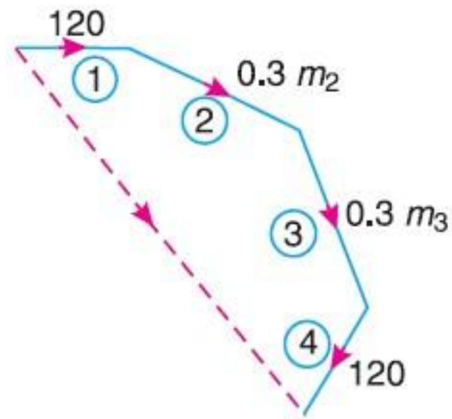
$m_3 = 871$ kg **Ans.**



( *f* ) Secondary force polygon.

- *By measurement,* the maximum secondary unbalanced force
  m.r. Cos 2θ= 582 kg-m.

$$F_S = m.\omega^2.r \times \frac{\cos 2\theta}{n}$$




( *f* ) Secondary force polygon.

∴ Maximum secondary unbalanced force

$$= 582 \times \frac{\omega^2}{n} = \frac{582(25.14)^2}{1.2/0.3} = 91\,960\text{N} = 91.96 \text{ kN } \textbf{Ans.} \quad \ldots (\because\ n = l/r)$$

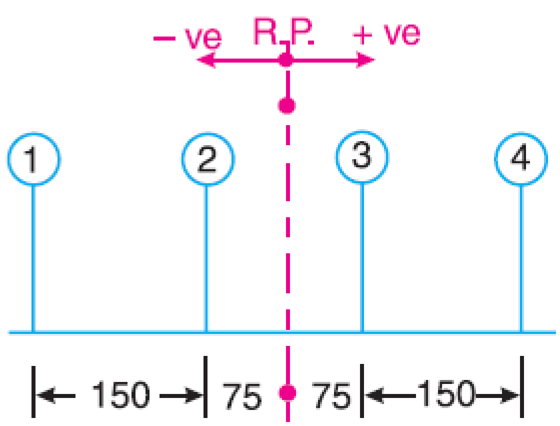
**Solution.** Given : $m_1 = m_4 = 400$ kg ; $r = 300$ mm $= 0.3$ m ; $l = 1.2$ m ; $N = 240$ r.p.m. or $\omega = 2\pi \times 240/60 = 25.14$ rad/s

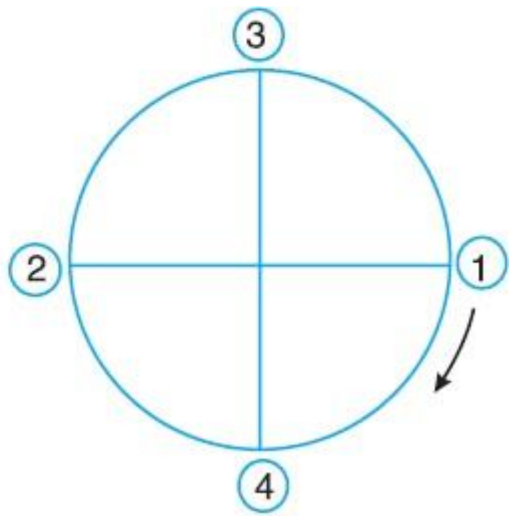
**Example 22.9.** *The cranks and connecting rods of a 4-cylinder in-line engine running at 1800 r.p.m. are 60 mm and 240 mm each respectively and the cylinders are spaced 150 mm apart. If the cylinders are numbered 1 to 4 in sequence from one end, the cranks appear at intervals of 90° in an end view in the order 1-4-2-3. The reciprocating mass corresponding to each cylinder is 1.5 kg.*

*Determine :* **1.** *Unbalanced primary and secondary forces, if any, and* **2.** *Unbalanced primary and secondary couples with reference to central plane of the engine.*

**Solution.** Given : $N$ = 1800 r.p.m. or $\omega = 2\pi \times 1800/60 = 188.52$ rad/s ; $r$ = 60 mm = 0.6 m ; $l$ = 240 mm = 0.24 m ; $m$ = 1.5 kg

**1.** ***Unbalanced primary and secondary forces***



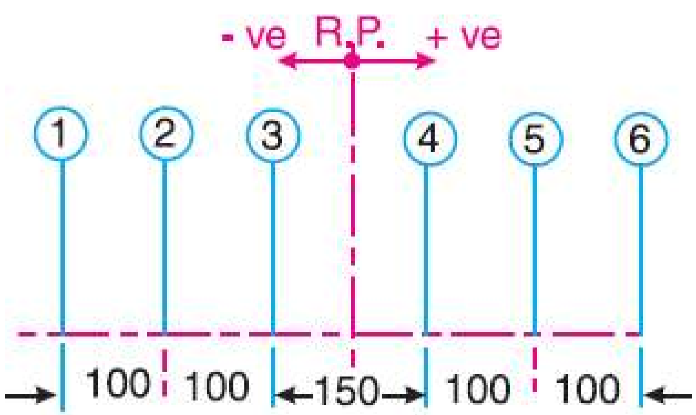

(*b*) Primary crank positions.

**Example 22.13.** *The firing order in a 6 cylinder vertical four stroke in-line engine is 1-4-2-6-3-5. The piston stroke is 100 mm and the length of each connecting rod is 200 mm. The pitch distances between the cylinder centre lines are 100 mm, 100 mm, 150 mm, 100 mm, and 100 mm respectively. The reciprocating mass per cylinder is 1 kg and the engine runs at 3000 r.p.m.*

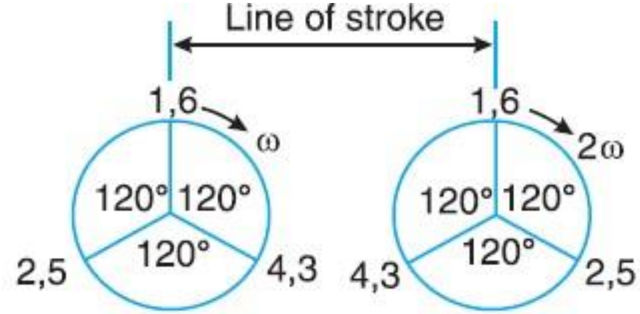
*Determine the out-of-balance primary and secondary forces and couples on this engine, taking a plane midway between the cylinder 3 and 4 as the reference plane.*

**Solution.** Given : $L = 100$ mm or $r = L / 2 = 50$ mm $= 0.05$ m ; $l = 200$ mm ; $m = 1$ kg ; $N = 3000$ r.p.m.

- The position of the cylinders and the cranks are shown in Fig. *(a), (b) and (c).*



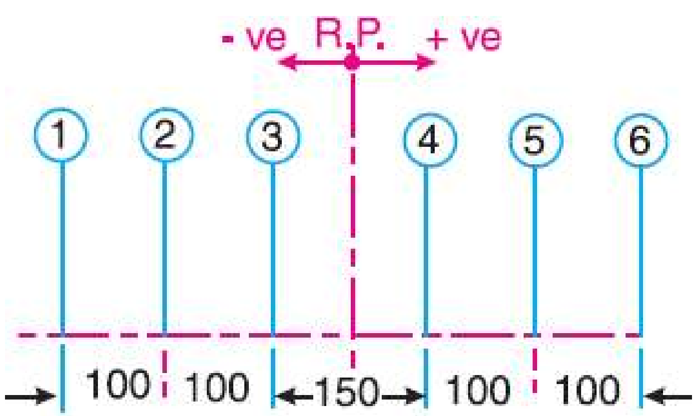

(*a*) Positions of planes.



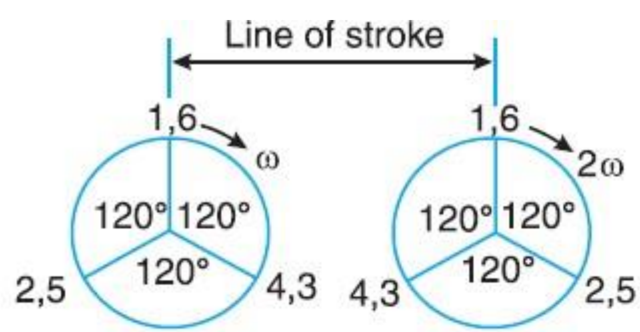

(*b*) Primary crank positions. (*c*) Secondary crank positions

- *With* the reference plane midway between the cylinders 3 and 4, the data may be tabulated as given in the following table :

| *Plane* (1) | *Mass* (*m*) *kg* (2) | *Radius* (*r*) *m* (3) | *Cent. force* $\div \omega^2$ (*m.r*) *kg-m* (4) | *Distance from plane 3* (*l*)*m* (5) | *Couple* $\div \omega^2$ (*m.r.l*) *kg-m*$^2$ (6) |
|---|---|---|---|---|---|
| 1 | 1 | 0.05 | 0.05 | – 0.275 | – 0.01375 |
| 2 | 1 | 0.05 | 0.05 | – 0.175 | – 0.00875 |
| 3 | 1 | 0.05 | 0.05 | – 0.075 | – 0.00375 |
| 4 | 1 | 0.05 | 0.05 | + 0.075 | + 0.00375 |
| 5 | 1 | 0.05 | 0.05 | + 0.175 | + 0.00875 |
| 6 | 1 | 0.05 | 0.05 | + 0.275 | + 0.01375 |

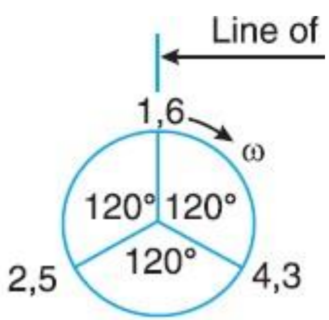


(*a*) Positions of planes.

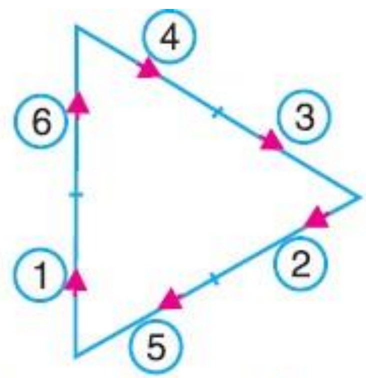


(*b*) Primary crank positions. (*c*) Secondary crank positions

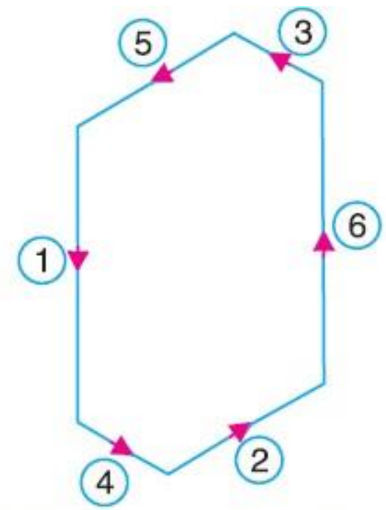
| Plane (1) | Mass (m) kg (2) | Radius (r) m (3) | Cent. force $\div \omega^2$ (m.r) kg-m (4) | Distance from plane 3 (l)m (5) | Couple $\div \omega^2$ (m.r.l) kg-m$^2$ (6) |
|---|---|---|---|---|---|
| 1 | 1 | 0.05 | 0.05 | – 0.275 | – 0.01375 |
| 2 | 1 | 0.05 | 0.05 | – 0.175 | – 0.00875 |
| 3 | 1 | 0.05 | 0.05 | – 0.075 | – 0.00375 |
| 4 | 1 | 0.05 | 0.05 | + 0.075 | + 0.00375 |
| 5 | 1 | 0.05 | 0.05 | + 0.175 | + 0.00875 |
| 6 | 1 | 0.05 | 0.05 | + 0.275 | + 0.01375 |



(*b*) Primary crank positions.



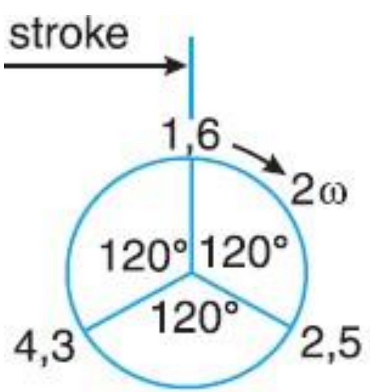
(*d*) Primary force polygon.



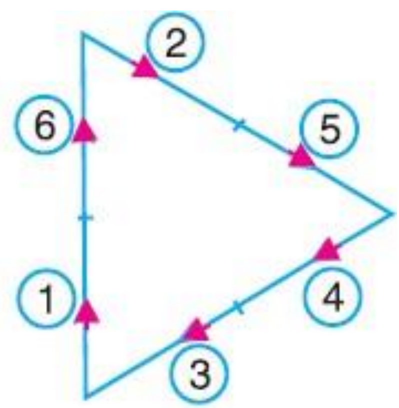
(*f*) Primary couple polygon.

- From Fig. (*d) and (f), we see that the primary force polygons and* couple polygons *are* closed figures, therefore there are no out-of-balance primary forces.
- Thus the engine is balanced for primary forces and couples.

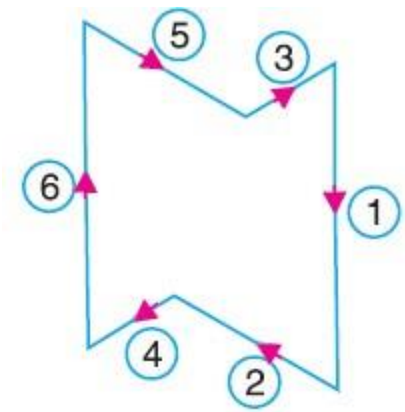
| *Plane* (*1*) | *Mass* (*m*) *kg* (*2*) | *Radius* (*r*) *m* (*3*) | *Cent. force* $\div \omega^2$ (*m.r*) *kg-m* (*4*) | *Distance from plane 3* (*l*)*m* (*5*) | *Couple* $\div \omega^2$ (*m.r.l*) *kg-m*$^2$ (*6*) |
|---|---|---|---|---|---|
| 1 | 1 | 0.05 | 0.05 | – 0.275 | – 0.01375 |
| 2 | 1 | 0.05 | 0.05 | – 0.175 | – 0.00875 |
| 3 | 1 | 0.05 | 0.05 | – 0.075 | – 0.00375 |
| 4 | 1 | 0.05 | 0.05 | + 0.075 | + 0.00375 |
| 5 | 1 | 0.05 | 0.05 | + 0.175 | + 0.00875 |
| 6 | 1 | 0.05 | 0.05 | + 0.275 | + 0.01375 |



(*c*) Secondary crank positions



(*e*) Secondary force polygon.



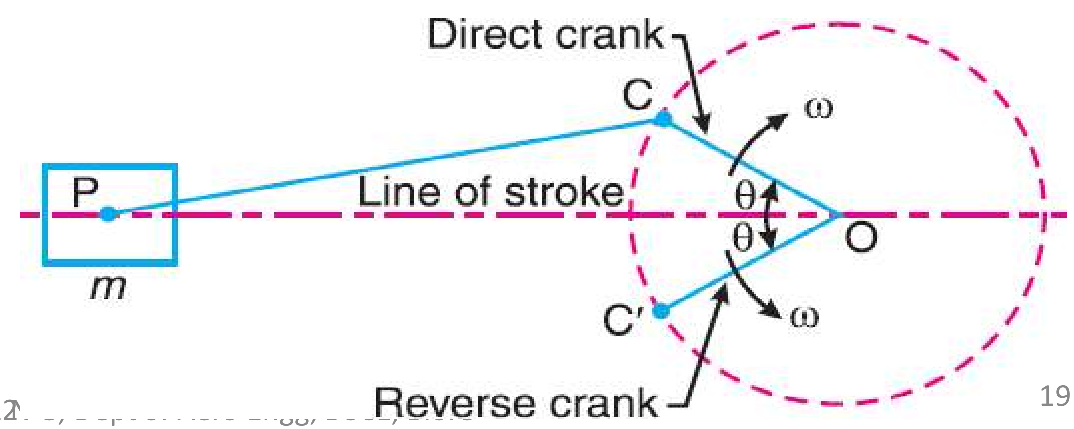
(*g*) Secondary couple polygon.

- From Fig. (*e) and (g), we see that the secondary force polygons and* couple polygons *are* closed figures, therefore there are no out-of-balance secondary forces and couples.
- Thus the engine is balanced for secondary forces and couples.

# Balancing of Radial Engines

# (Direct and Reverse Cranks Method )

- The method of direct and reverse cranks is used in balancing of radial or V-engines, in which the connecting rods are Connected to a common crank. Since the plane of rotation of the various cranks (in radial or V-engines) is same, therefore there is no unbalanced primary or secondary couple.
- Consider a reciprocating engine mechanism as shown in Fig.
- Let the crank *OC (known as the direct crank) rotates uniformly at* $\omega$ *radians per second in a clockwise direction. Let at any instant the crank makes an angle* $\theta$ *with the line of stroke OP.*



- *The indirect or reverse crank OC' is the image of the direct crank OC, when seen through the mirror placed at the line of stroke.*
- *A little consideration will show that when the direct crank revolves in a clockwise direction, the reverse crank will revolve in the anticlockwise direction.*
- *We shall now discuss the primary and secondary forces due to the mass (m) of the reciprocating parts at P.*