Jazan University
Mechanical Engineering Department

CHAPTER 4

FLYWHEELS AND MOTOR SELECTION

EngM271 Theory of Machines

DR. Amr Assie

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FLYWHEELS

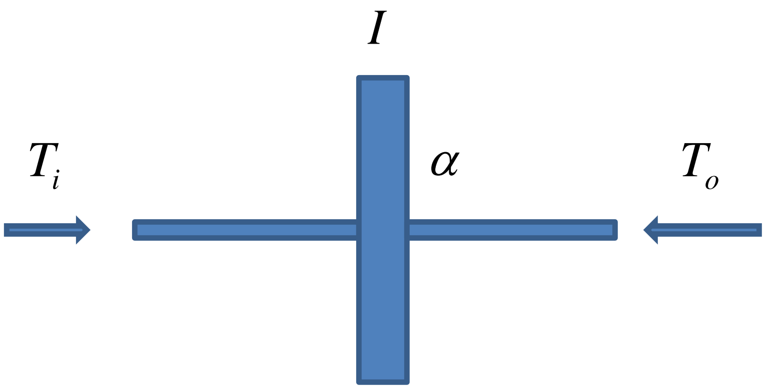
A flywheel is an energy storage device. It absorbs mechanical energy by increasing its angular velocity and delivers energy by decreasing its velocity.

Commonly, the flywheel is used to smooth out the flow of energy between a power source and its load.

If the power load happens to be a punch press, the actual punching operation requires energy only for a fraction of its motion cycle.

If the power source happens to be a two-cylinder four-cycle engine, the engine delivers energy during only about half of its motion cycle.

Flywheel can be used to absorb breaking energy and deliver accelerating energy for an automobile and to act as an energy smoothing devices for electric utilities as well as solar and wind power generating facilities.



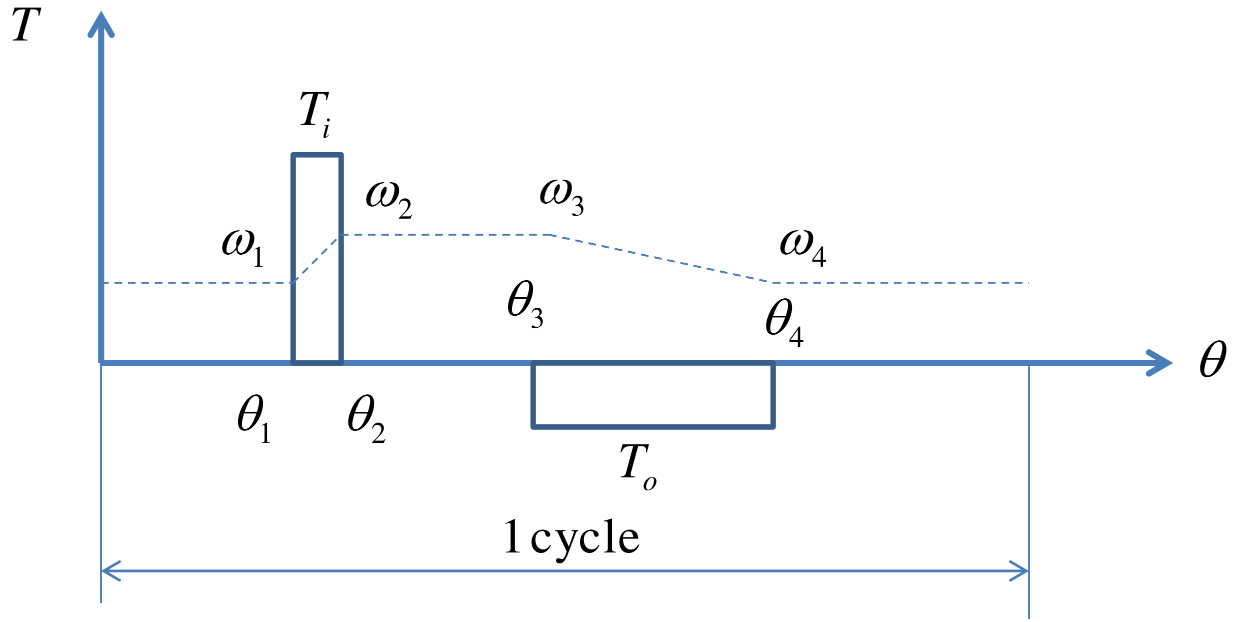
$$I\alpha = T_i(\theta,\omega) - T_o(\theta,\omega)$$

I : flywheel inertia, kgm^2

α : flywheel angular acceleration, $\mathrm{rad/s}^2$

T_i : input torque, Nm

T_o : output torque, Nm



T_i is applied from θ_1 to θ_2 ω_1 is increased to ω_2

No torque is applied from θ_2 to θ_3 ω_2 and ω_3 are same

T_o is applied from θ_3 to θ_4 ω_3 is decreased to ω_4

The work input to the flywheel is

$$U_i = T_i(\theta_2 - \theta_1)$$

The work output from the flywheel is

$$U_o = T_o(\theta_4 - \theta_3)$$

If $U_o > U_i$ flywheel energy is decreased $\omega_4 < \omega_1$

If $U_o = U_i$ flywheel energy is not changed $\omega_4 = \omega_1$

If $U_o < U_i$ flywheel energy is increased $\omega_4 > \omega_1$

flywheel energy at $\theta = \theta_1$ is

$$U_1 = \frac{1}{2} I \omega_1^2$$

flywheel energy at $\theta = \theta_2$ is

$$U_2 = \frac{1}{2} I \omega_2^2$$

energy stored to the flywheel is

$$U_2 - U_1 = \frac{1}{2} I (\omega_2^2 - \omega_1^2)$$

$$U_2 - U_1 = \frac{1}{2} I (\omega_2^2 - \omega_1^2) = \frac{1}{2} I (\omega_2 - \omega_1)(\omega_2 + \omega_1)$$

$$U_2 - U_1 = \frac{1}{2} I \frac{(\omega_2 - \omega_1)}{\omega} \omega \frac{(\omega_2 + \omega_1)}{2} 2 = I C_s \omega^2$$

$$C_s = \frac{(\omega_2 - \omega_1)}{\omega} \quad \text{coefficient of speed fluctuation}$$

$$\omega = \frac{(\omega_2 + \omega_1)}{2} \quad \text{nominal (average) angular velocity, rad/s}$$

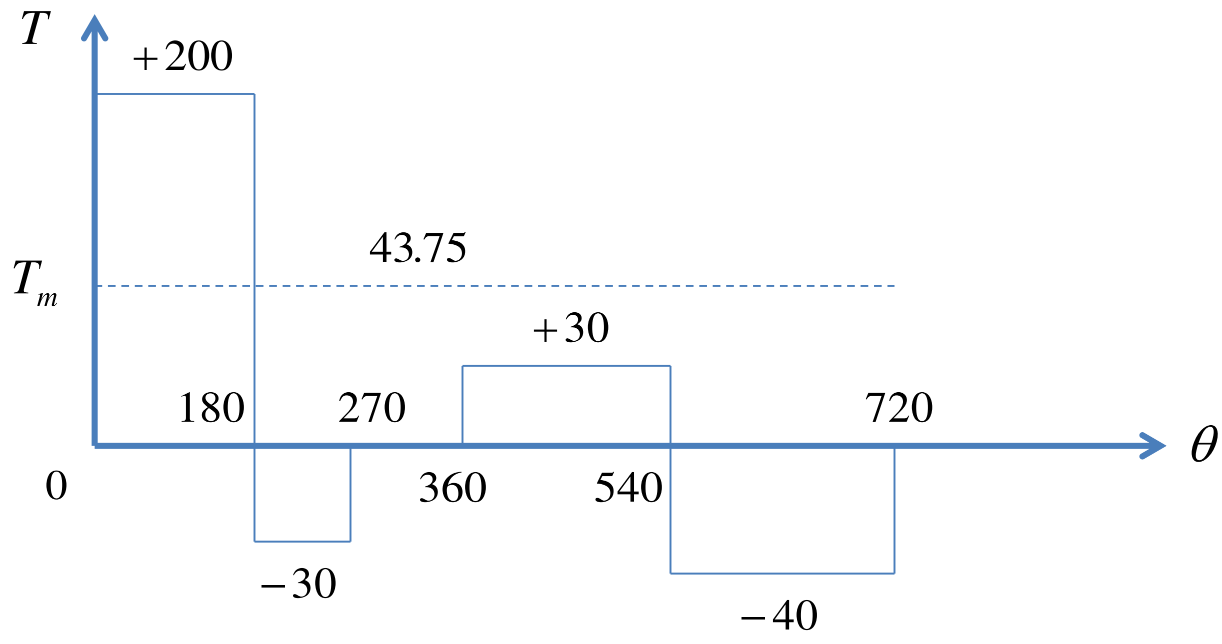
$$I = \frac{U_2 - U_1}{C_s \omega^2} \quad \text{flywheel inertia, kgm}^2$$

Application	Coefficient of speed Fluctuation
Crushing and hammering machinery	0.2
Machine Tools	0.03
Pumps, shearing machinery	0.03-0.05
Textile machinery	0.025
Gearbox applications	0.02
Electrical generators	0.003

Example

The plot shows torque-crank angle diagram for a one-cylinder four-cycle engine. The nominal speed of the engine is 250 rad/s.

a) Integrate the torque-displacement function for one cycle to find the energy that can be delivered to the load during the cycle.
b) Determine the mean torque T_m
c) Using a coefficient of speed fluctuation of 0.1, find a suitable value for the flywheel inertia.
d) Find ω_1 and ω_2



$$a) \quad U = \int_0^{720} T(\theta)d\theta = \sum_i T_i \theta_i$$

$$U = 200\times180 - 30\times90 + 30\times180 - 40\times180 = 31500\,\text{Nm}^\circ = 31500\times\frac{\pi}{180} = 549.5\text{ Nm (J)}$$

$$b) \quad T_m = \frac{\sum_i T_i\theta_i}{\sum_i \theta_i} = \frac{31500}{720} = \frac{549.5}{4\pi} = 43.75\text{ Nm}$$

$c)$ Maximum speed change is between 0° and 180°

$$U_2 - U_1 = (200 - 43.75)(180^\circ) = 28125\,\text{Nm}^\circ = 28125\frac{\pi}{180} = 490.6\ \text{Nm}$$

$$I = \frac{U_2 - U_1}{C_s\omega^2} = \frac{490.6}{0.1\times 250^2} = 0.0785\ \ \text{kgm}^2$$

$d)$

$$\omega_2 - \omega_1 = C_s\omega$$

$$\omega_2 + \omega_1 = 2\omega$$

$$2\omega_2 = (2 + C_s)\omega$$

$$\omega_2 = \frac{(2 + C_s)}{2}\omega = \frac{(2 + 0.1)}{2}250 = 262.5\ \text{rad/s at}\ \theta = 180^\circ$$

$$2\omega_1 = (2 - C_s)\omega$$

$$\omega_1 = \frac{(2 - C_s)}{2}\omega = \frac{(2 - 0.1)}{2}250 = 237.5\ \text{rad/s}\ \ \text{at}\ \theta = 0^\circ$$

Punching press machines

Example

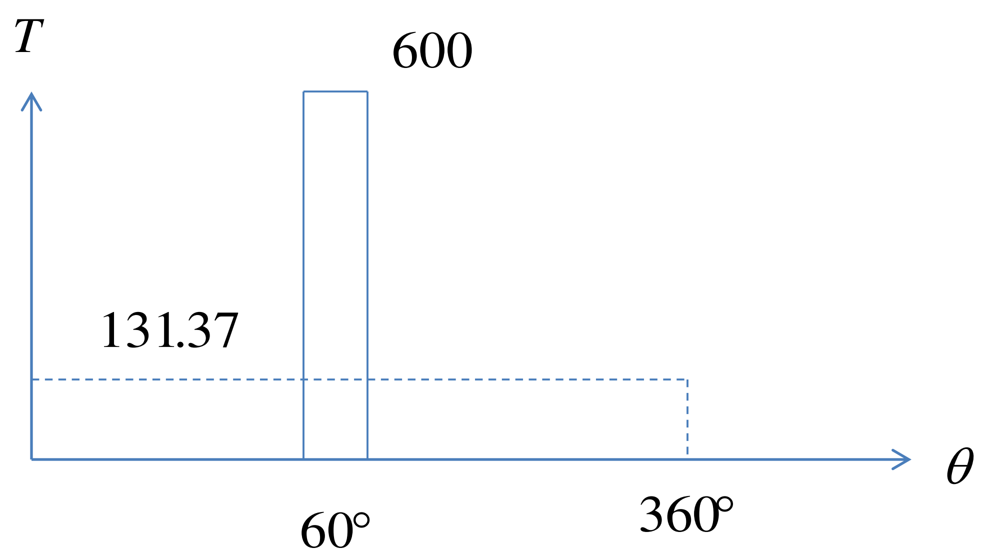
A 2.2 kW, 960 rpm motor powers the punching press through a gear box of gear ratio 6:1. The torque needed to do the job is 600 Nm for the period of 60 degrees. Assume coefficient of speed fluctuation of 0.02, determine suitable inertia for the flywheel.

Motor torque

$$T_m = \frac{2200 Nm/s}{960 \times \frac{2\pi}{60} rad/s} = 21.89 \text{ Nm}$$

Torque after gear box

$$T = 21.89 \times 6 = 131.37 \text{ Nm}$$



$$U_2 - U_1 = (600-131.37)(60°) = 28117.8 \text{ Nm°} = 28117.8 \frac{\pi}{180} = 490.5 \text{ Nm}$$

$$\omega = \frac{960 \times \frac{2\pi}{60}}{6} = 16.74 \text{ rad/s}$$

$$I = \frac{U_2 - U_1}{C_s \omega^2} = \frac{490.5}{0.02 \times 16.74^2} = 87.51 \text{ kgm}^2$$

Example

A punching press machine is run by an electrical motor delivering 2.24 kW continuously. At the beginning of an operation a flywheel of inertia 60 kgm2 on the machine is rotating at the speed of 250 rpm. The pressing operation requires 5000 J of energy and occupies 0.75s.

a) Find the maximum number of pressings that can be made in one hour.
b) Find reduction in speed of the flywheel after each pressing.

Energy supplied by motor in one hour

$a)\qquad U_m = 2.24kW \times 3600s = 8064kJ$

Work required per pressing is $U_p = 5kJ$

Number of pressing per hour is $\frac{8064\text{kJ}}{5\text{kJ}} = 1612$

$b)\qquad$ Motor supplies energy in 0.75 s $U_m = 2240W \times 0.75s = 1680J$

Energy taken from flywheel for each pressing is

$$U_p - U_m = 5000 - 1680 = 3320 = \frac{1}{2} I(\omega_1^2 - \omega_2^2) = \frac{1}{2} 60(250^2 - \omega_2^2)\left(\frac{2\pi}{60}\right)^2$$

$\omega_2 = 229\,\text{rpm}$

$\Delta\omega = 250 - 229 = 21\,\text{rpm}$

Example

The crankshaft torque of a multi-cylinder engine is given by

$$T_m = 60 + 8\sin 3\theta \text{ Nm}$$

Where θ is the crankshaft angle measured from a convenient datum. The engine is coupled directly to a machine which requires a torque given by

$$T_L = 60 + 32\sin\theta \text{ Nm}$$

All the components rotating with the crankshaft together have the mass of 50 kg and radius of gyration of 0.15 m.

If the crankshaft speed is 300 rpm at $\theta = 0°$, find the speed when $\theta = 60°$.

$$\int_0^{60} (T_m - T_L)d\theta = \frac{1}{2} I(\omega_2^2 - \omega_1^2)$$

$$I = mk^2 = 50 \times 0.15^2 = 7.5 \text{ kgm}^2$$

$$\int_0^{60} [60 + 8\sin 3\theta - (60 + 32\sin\theta)]d\theta = \frac{1}{2} 7.5(\omega_2^2 - 300^2)\left(\frac{2\pi}{60}\right)^2$$

$$\int_0^{60} [8\sin 3\theta - 32\sin\theta] d\theta = \frac{1}{2} 7.5(\omega_2^2 - 300^2)\left(\frac{2\pi}{60}\right)^2$$

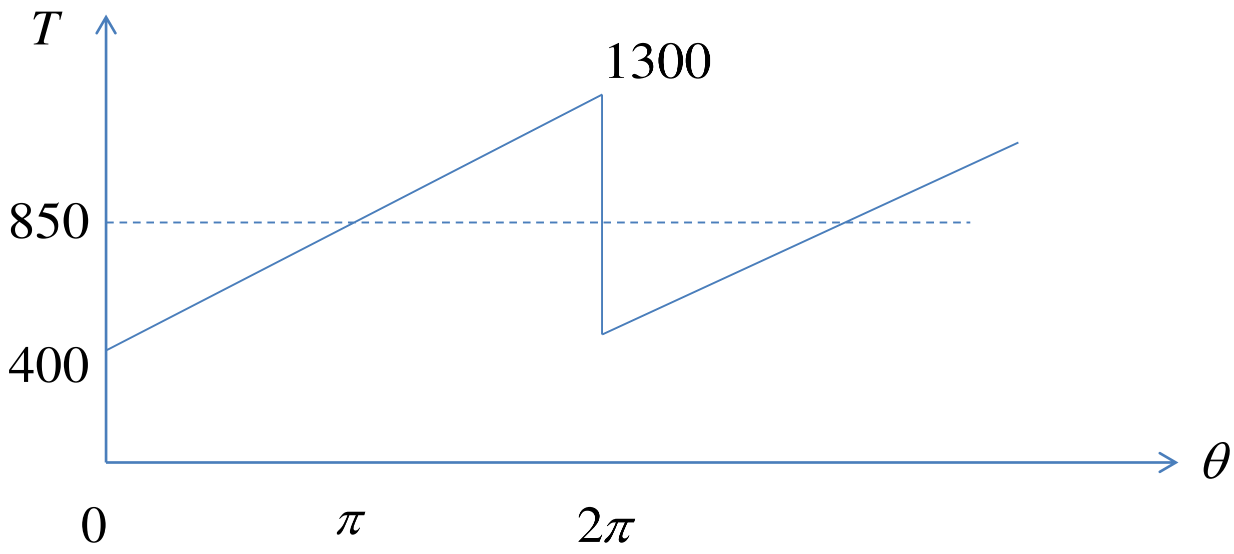
$$\left| 8(-\frac{1}{3})\cos 3\theta + 32\cos\theta \right|_0^{60} = \frac{1}{2} 7.5(\omega_2^2 - 300^2)\left(\frac{2\pi}{60}\right)^2$$

$$18.66 - 29.33 = -10.67 = 6.16 \times 10^{-3}(\omega_2^2 - 300^2)$$

$$\omega_2 = 297\,\text{rpm}$$

Example

A machine is driven by a motor which exerts a constant torque of 850 Nm. The resisting torque (load torque) within the machine increases uniformly from 400 Nm to 1300 Nm throughout 360 deg Rotation of the driving shaft then drops suddenly to 400 Nm again at the beginning of the next revolution. The mean speed of the machine is 25 rad/s. The machine is fitted with a cast iron flywheel of 4.7 kgm2 inertia. Calculate the maximum and the minimum angular velocities of the machine and the maximum angular acceleration of the machine.



between 0 and π load energy is less than motor energy, flywheel speed is increased

$$U_m - U_L = 850 \times \pi - \frac{1}{2}(400 + 850) \times \pi = 225\pi \text{ J}$$

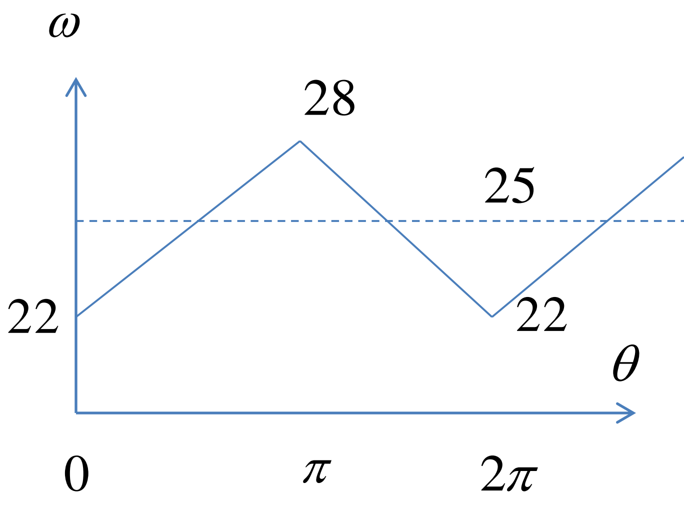
between π and 2π load energy is more than motor energy, flywheel speed is decreased

$$U_L - U_m = \frac{1}{2}(850 + 1300) \times \pi - 850 \times \pi = 225\pi \text{ J}$$

$$C_s = \frac{U_L - U_m}{I\omega^2} = \frac{225\pi}{4.7 \times 25^2} = 0.24$$

$$\omega_2 = \frac{(2 + C_s)}{2}\omega = \frac{(2 + 0.24)}{2} 25 = 28 \text{ rad/s at } \theta = \pi$$

$$\omega_1 = \frac{(2 - C_s)}{2}\omega = \frac{(2 - 0.24)}{2} 25 = 22 \text{ rad/s at } \theta = 2\pi$$



Maximum accelerating torque is at $\theta = 0°$

$$T_{\max} = 850 - 400 = 450Nm = I\alpha_{\max}$$

$$\alpha_{\max} = \frac{450Nm}{4.7km^2} = 95.74 rad / s^2$$

MOTION PROFILE

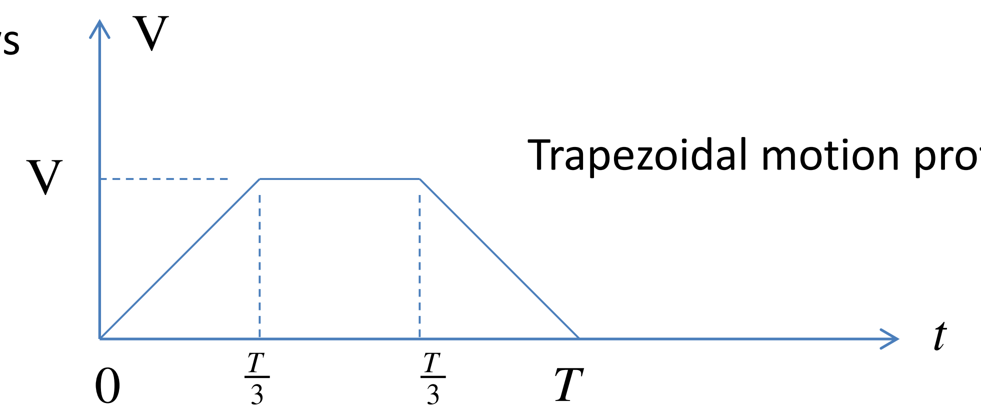
Before calculating torque requirements of an application you need to know the velocities and accelerations needed. For those positioning applications where only a distance X and time T to move that distance is known, the trapezoidal motion profile is a good starting point for determining the requirements.

Move distance X in time T

Assume that

1. Distance X/4 is moved in time T/3 (acceleration)
2. Distance X/2 is moved in time T/3 (run)
3. Distance X/4 is moved in time T/3 (deceleration)

The graph would appear as follows



Trapezoidal motion profile

For constant acceleration

$$V = at \quad S = \tfrac{1}{2} at^2$$

$$a = -d = \frac{2S}{t^2} = \frac{2\left(\frac{X}{4}\right)}{\left(\frac{T}{3}\right)^2} = \frac{18}{4}\frac{X}{T^2} = 4.5\frac{X}{T^2}$$

$$V = at = \frac{18}{4}\frac{X}{T^2}\left(\frac{T}{3}\right) = \frac{18}{12}\frac{X}{T} = 1.5\frac{X}{T}$$

Example

You need to move 15 cm in 2 seconds, what is a and V.

$$a = -d = 4.5\frac{X}{T^2} = 4.5\frac{15cm}{(2s)^2} = 16.87cm/s^2$$

$$V = at = 1.5\frac{X}{T} = 1.5\frac{15cm}{2s} = 11.25cm/s$$

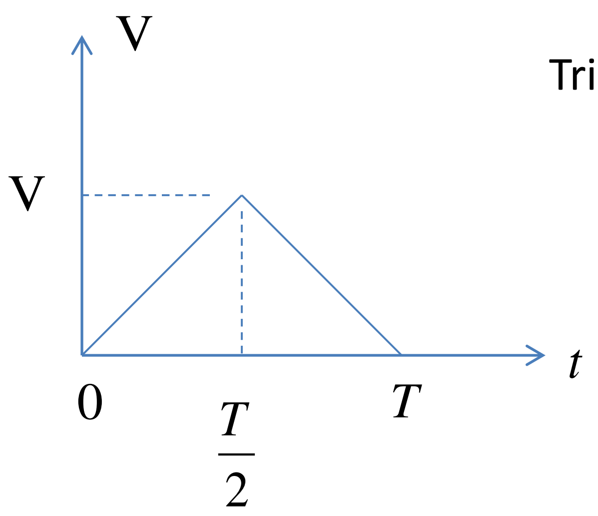
Triangular motion profile

Move distance X in time T

Assume that

1. Distance X/2 is moved in time T/2 (acceleration)
2. Distance X/2 is moved in time T/2 (deceleration)

The graph would appear as follows



Triangular motion profile

For constant acceleration

$$V = at \quad S = \tfrac{1}{2} a t^2$$

$$a = -d = \frac{2S}{t^2} = \frac{2\left(\frac{X}{2}\right)}{\left(\frac{T}{2}\right)^2} = 4\frac{X}{T^2}$$

$$V = at = 4\frac{X}{T^2}\left(\frac{T}{2}\right) = 2\frac{X}{T}$$

Example

You need to move 15 cm in 2 seconds, what is a and V.

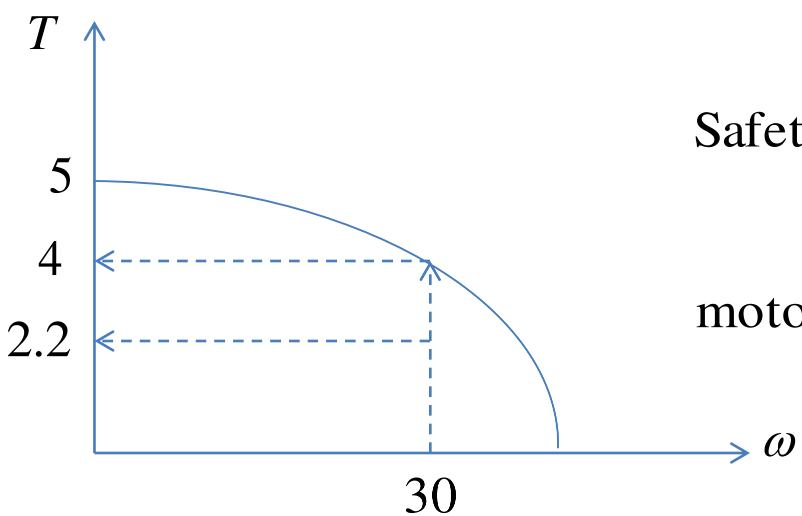
$$a = -d = 4\frac{X}{T^2} = 4\frac{15cm}{(2s)^2} = 15cm/s^2$$

$$V = at = 2\frac{X}{T} = 2\frac{15cm}{2s} = 15cm/s$$

Motor selection based on continuous torque requirements:
Having calculated the torque requirements for an application, you can select the motor suited to your needs. Motors have speed-torque curves based on continuous duty cycle. To chose a motor simply plot total torque vs. velocity on the speed-torque curve. This point should fall under the curve and allow a 50% margin for safety.

Example

T_F = 0.2 Nm Friction torque
T_A = 2 Nm Acceleration torque
T_T = 2.2 Nm total torque
ω = 30 rad/s maximum velocity



$$\text{Safety margin} = \frac{4 - 2.2}{2.2} = 0.82$$

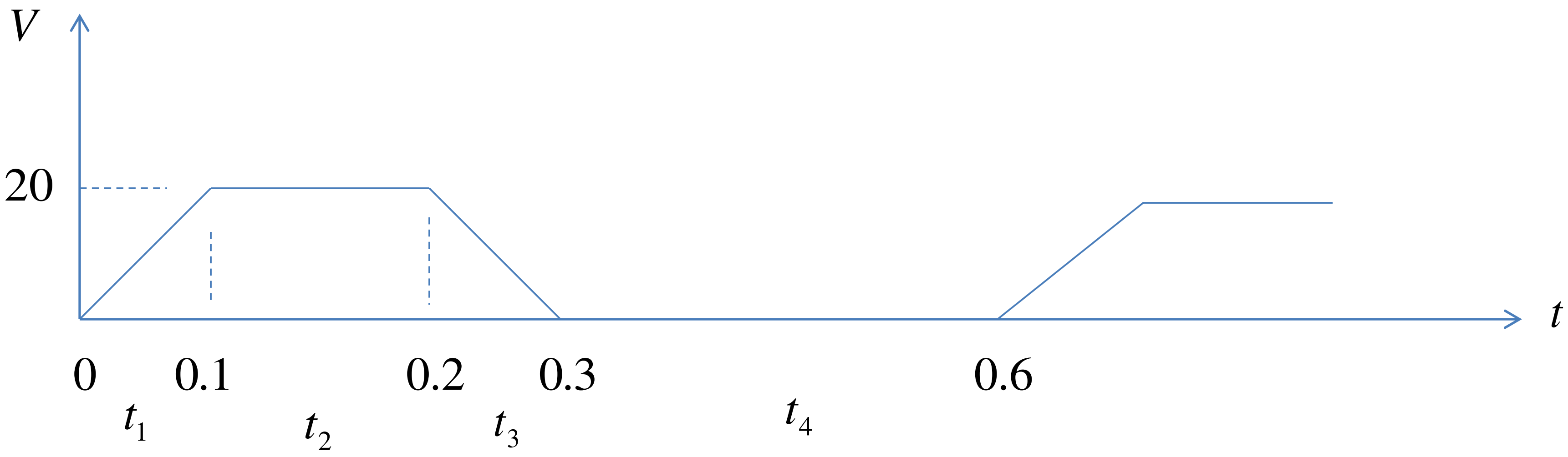
motor torque is 82% more than the required torque

Motor selection based on peak torque requirements:
Servomotors have two speed-torque curves. One for continuous duty cycle (continuous torque curve)and another for intermittent duty cycle (peak torque curve). A servomotor can be selected according to the total torque and maximum velocity indicated by the continuous torque curve. However, by calculating the root mean square (rms) torque based on the duty cycle, you may be able to take advantage of the higher peak torque available in the peak torque range.

$$T_{rms} = \sqrt{\frac{\sum T_i^2 t_i}{\sum t_i}}$$

Example
Assume the following results from load calculations
T_F=0.2 Nm Friction torque
T_A=5.5 Nm Acceleration torque
T_T=5.7 Nm Total torque
V_M=20 cps (Hz) Maximum velocity

Motion profile

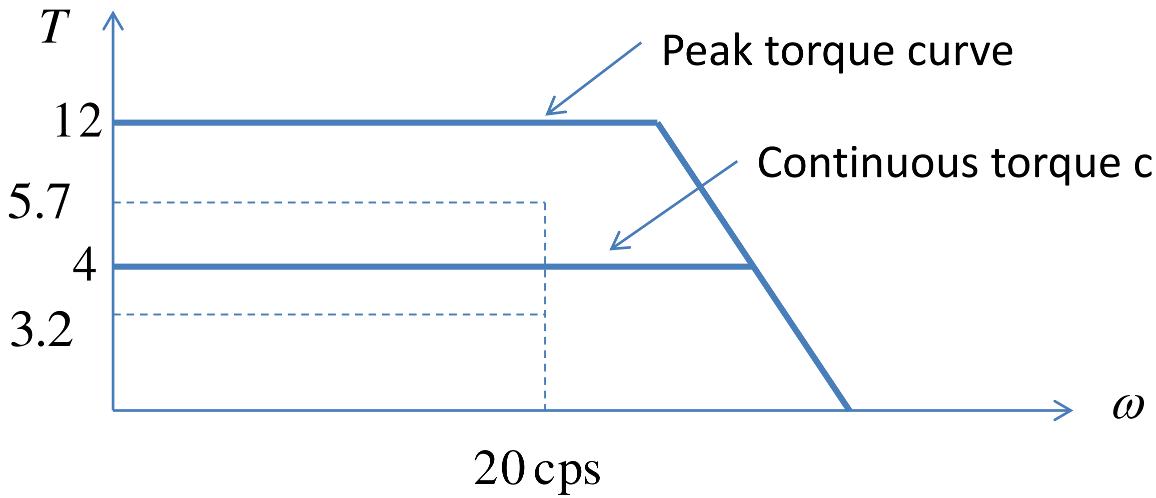


Duty cycle is rotate 4 revolutions in 0.3 seconds, dwell 0.3 seconds then repeat

t_1= Torque required to accelerate the load from zero speed to maximum speed ($T_1=T_F+T_A$)
t_2= Torque required to keep the motor moving ($T_2=T_F$)
t_3= Torque required to decelerate the load from maximum speed to zero speed ($T_3=T_F-T_A$)
t_4= Torque required while motor is sitting still at zero speed ($T_4=0$)

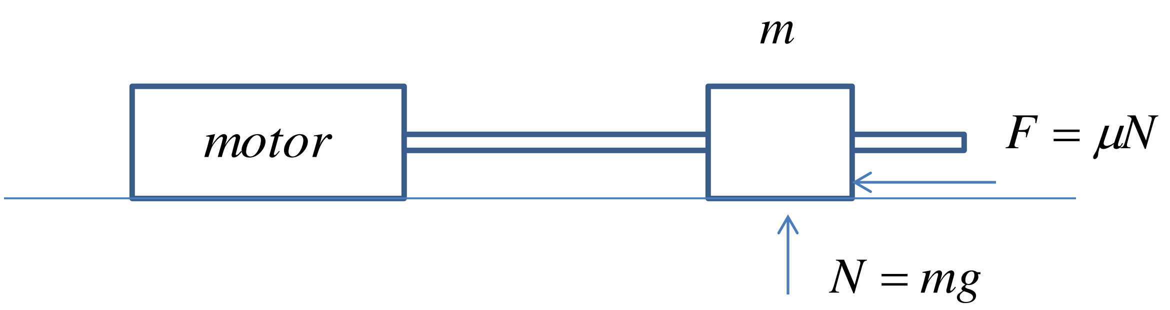
$$T_{rms} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3 + T_4^2 t_4}{t_1 + t_2 + t_3 + t_4}}$$

$$T_{rms} = \sqrt{\frac{5.7^2 \times 0.1 + 0.2^2 \times 0.1 + 5.3^2 \times 0.1 + 0^2 \times 0.3}{0.1 + 0.1 + 0.1 + 0.3}} = 3.2 Nm$$



This motor meets the requirements. rms torque value falls within the continuous torque curve, maximum torque value falls within the peak torque curve.

Lead-screw drives:



$$T_{motor} = T_{Friction} + T_{Acceleration}$$

$$T_F = \frac{F}{2\pi p \eta} = \frac{\mu m g}{2\pi p \eta}$$

F : Frictional force, N

p : pitch, rev/m

η : leadscrew efficiency

$$\frac{1}{2}J_{eq}\omega_m^2 = \frac{1}{2}J_m\omega_m^2 + \frac{1}{2}J_{LS}\omega_m^2 + \frac{1}{2}mv^2$$

$$J_{eq} = J_m + J_{LS} + m\frac{v^2}{\omega_m^2} \qquad \omega_m = 2\pi pv$$

$$J_{eq} = J_m + J_{LS} + \frac{m}{(2\pi p)^2}$$

$$T_A = J_{eq}\alpha$$

Example

Find the torque required to accelerate a 90 kg load sliding on a steel table to a speed of 5 cm/s in 0.2 seconds using steel leadscrew having pitch of 2 rev/cm. Assume that leadscrew has an Acme thread and uses a plastic nut (efficiency is 65%). Friction coefficient is 0.15 (steel on steel lubricated). Leadscrew inertia is $150\times10^{-5}\ \mathrm{kgm}^2$ motor inertia is $12\times10^{-5}\ \mathrm{kgm}^2$

$$F = \mu mg = 0.15\times90\times9.8 = 132.3\ \mathrm{N}$$

$$T_F = \frac{F}{2\pi p\eta} = \frac{132.3}{2\pi(200)(0.65)} = 0.16Nm$$

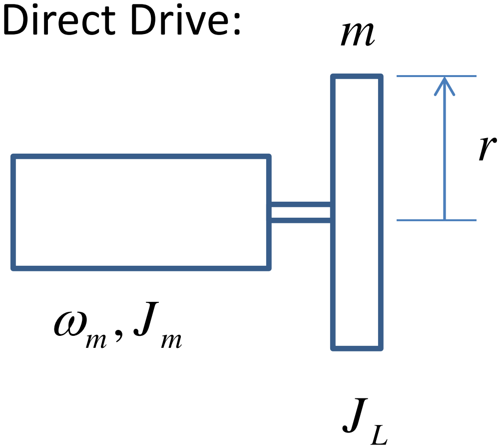
$$J_{eq} = J_m + J_{LS} + \frac{m}{(2\pi p)^2} = 12\times10^{-5} + 150\times10^{-5} + \frac{90}{(2\pi 200)^2} = 167.7\times10^{-5}\, kgm^2$$

$$\omega_m = 2\pi p v = 2\pi(200 rev/m)(0.05 m/s) = 62.8 rad/s$$

$$T_A = J_{eq}\alpha = J_{eq}\frac{\omega_m}{t} = 167.6\times10^{-5}\,\frac{62.8 rad/s}{0.2s} = 0.53Nm$$

$$T_m = T_F + T_A = 0.16 + 0.53 = 0.69Nm$$

Direct Drive:



$$\frac{1}{2}J_{eq}\omega_m^2 = \frac{1}{2}J_m\omega_m^2 + \frac{1}{2}J_L\omega_m^2 \qquad J_{eq} = J_m + J_L$$

$$T_A = J_{eq}\alpha = J_{eq}\frac{\omega_m}{t}$$

$$J_L = \frac{1}{2}mr^2$$

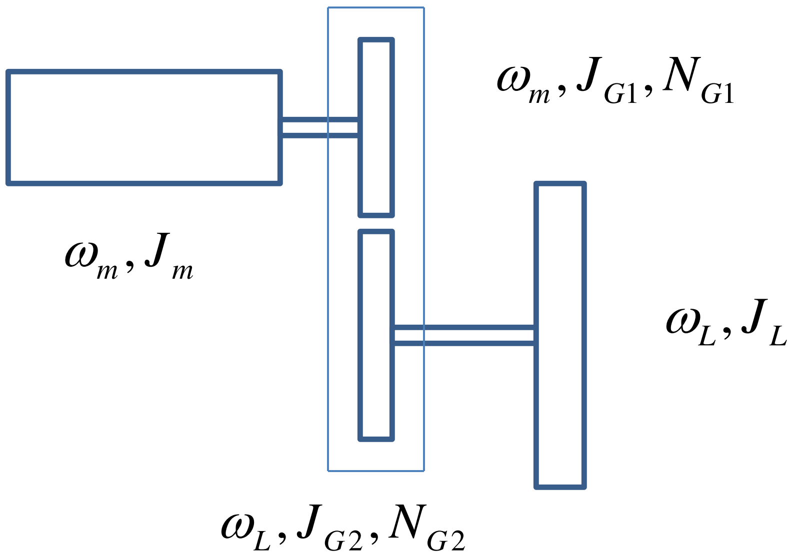
Example

Calculate the motor torque required to accelerate a solid cylinder of aluminum 13 cm in radius and 0.8 kg in mass from rest to 2 rad/s in 0.25 seconds. Motor inertia is $70\times10^{-5}\ \mathrm{kgm}^2$

$$J_L = \frac{1}{2}mr^2 = \frac{1}{2}(0.8kg)(0.13m)^2 = 676\times10^{-5}\,kgm^2$$

$$T_A = (70\times10^{-5} + 676\times10^{-5})\frac{2rad/s}{0.25s} = 59.68\times10^{-3}\,Nm$$

Gear drive formulas:



Equivalent inertia with respect to the motor shaft

$$\frac{1}{2} J_{eq} \omega_m^2 = \frac{1}{2} J_m \omega_m^2 + \frac{1}{2} J_{G1} \omega_m^2 + \frac{1}{2} J_{G2} \omega_L^2 + \frac{1}{2} J_L \omega_L^2$$

$$J_{eq} = J_m + J_{G1} + (J_{G2} + J_L) \frac{\omega_L^2}{\omega_m^2}$$

$$T_A = J_{eq} \alpha$$

$$\text{Speed ratio} = \frac{\omega_L}{\omega_m}$$

$$\omega_m N_{G1} = \omega_L N_{G2}$$

$$\text{Gear ratio} = \frac{N_{G2}}{N_{G1}} = \frac{1}{\text{speed ratio}}$$

Example

Motor is coupled to a gearbox and flywheel. The inertias are given. If the motor has starting torque of 5 Nm, what is the angular acceleration of the flywheel? How long it will take to reach the speed of 1200 rpm.

$$J_m = 9.8 \times 10^{-3}\, kgm^2$$

$$J_{G1} = 0.4 \times 10^{-3}\, kgm^2$$

$$J_{G2} = 19.4 \times 10^{-3}\, kgm^2$$

$$J_f = 308 \times 10^{-3}\, kgm^2$$

$$N_{G1} = 50T$$

$$N_{G2} = 230T$$

$$\frac{\omega_L}{\omega_m} = \frac{N_{G1}}{N_{G2}} = \frac{50}{230}$$

$$J_{eq} = J_m + J_{G1} + (J_{G2} + J_L)\frac{\omega_L^2}{\omega_m^2}$$

$$= 9.8\times10^{-3} + 0.4\times10^{-3} + (19.4\times10^{-3} + 308\times10^{-3})\left(\frac{50}{230}\right)^2 = 0.02567 kgm^2$$

$$T_A = J_{eq}\alpha = 0.02567\alpha = 2Nm$$

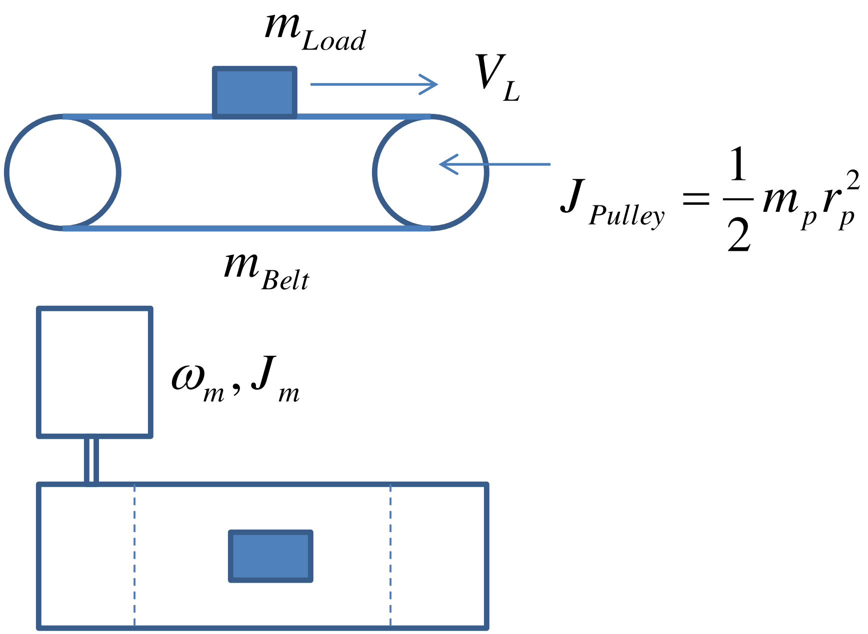
$$\alpha = \frac{5Nm}{0.02567 kgm^2} = 194.78 rad/s^2$$

$$\omega_m = \alpha t$$

$$1200\left(\frac{2\pi}{60}\right) = 194.78t$$

$$t = 0.64s$$

Belt and pulley derives:



Equivalent inertia with respect to the motor shaft

$$\frac{1}{2} J_{eq} \omega_m^2 = \frac{1}{2} J_m \omega_m^2 + 2\left(\frac{1}{2} J_P \omega_m^2\right) + \frac{1}{2} m_B V_B^2 + \frac{1}{2} m_L V_L^2$$

$$V_B = R\omega_m$$

$$V_L = R\omega_m$$

$$J_{eq} = J_m + 2J_P + (m_B + m_L) R^2$$

$$T_A = J_{eq} \alpha = J_{eq} \frac{\omega_m}{t}$$

Example

What torque is required to accelerate a 2 kg mass load to a velocity of 50 cm/s in 10 milliseconds using a flat timing belt? The motor drives a 10 cm diameter steel pulley which has mass of 0.1 kg. Friction torque is 0.2 Nm, belt mass is 0.8 kg, motor inertia is $19\times10^{-5}\ \mathrm{kgm}^2$

$$J_{Pulley} = \frac{1}{2} m_p r_p^2 = \frac{1}{2}(0.8kg)(0.05m)^2 = 12.5\times10^{-5} kgm^2$$

$$J_{eq} = J_m + 2J_P + (m_B + m_L)R^2$$

$$= 19\times10^{-5} + 2\times12.5\times10^{-5} + (0.8+2)(0.05)^2 = 744\times10^{-5} kgm^2$$

$$V_L = R\omega_m$$

$$50cm/s = (0.05m)\omega_m$$

$$\omega_m = 10 rad/s$$

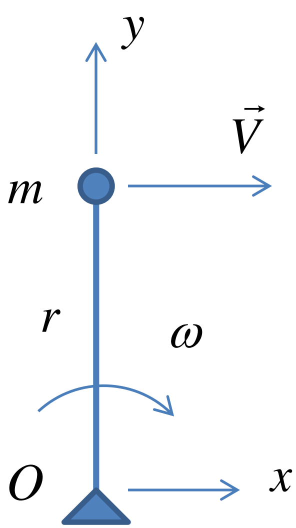
$$T_A = J_{eq}\frac{\omega_m}{t} = 744\times10^{-5}\frac{10 rad/s}{10\times10^{-3} s} = 7.44Nm$$

$$T_m = T_A + T_F = 7.44 + 0.2 = 7.64Nm$$

GYROSCOPIC EFFECTS

A body rotating about an axis of symmetry offers a resistance to a change in the direction of these axis. This is known as the "gyroscopic effect". Important applications of the gyroscopic effect are the gyrocompass used on airplanes and ships and in inertial guidance control systems for missiles and space travel, where gyroscopes are used to sense the angular motion of the body.

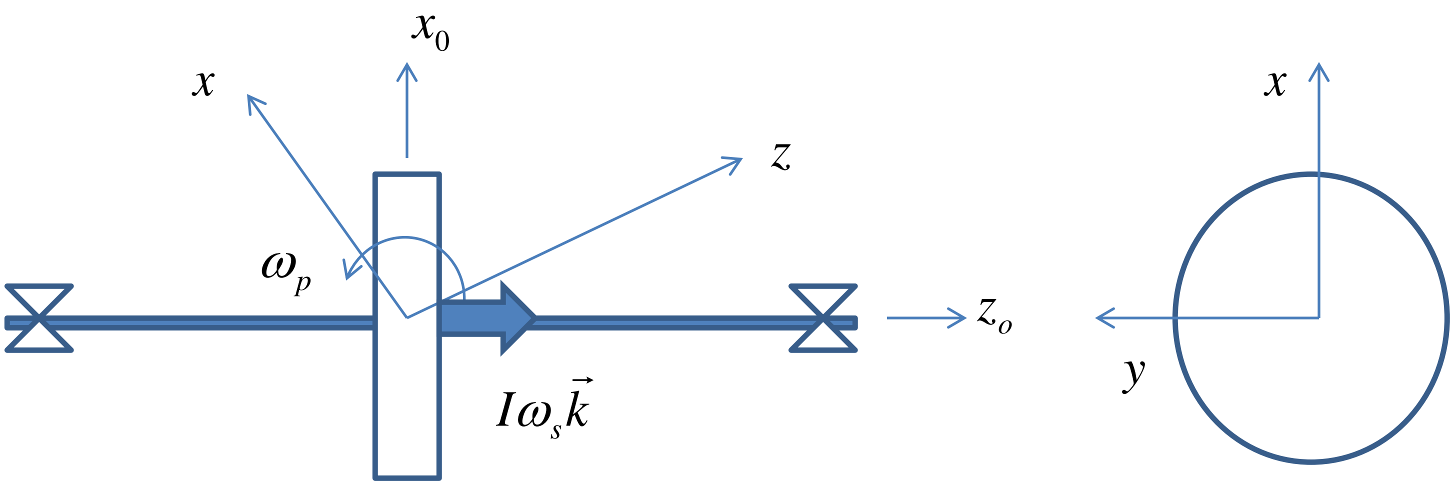
Forces due to gyroscopic effects must often be taken into account in the design of machines. These forces are encountered in the bearings of an automobile engine as the automobile engine makes a turn, in marine turbines as the ship pitches in a heavy sea, and in a jet airplane engine shaft as the airplane changes direction.



$$\vec{L} = m\vec{V} = \text{ linear momentum}$$

$$\vec{H}_o = \vec{r} \times m\vec{V} = r\vec{j} \times mV\vec{i} = -rmV\vec{k} = -rm\omega r\vec{k} = -mr^2\omega\vec{k} = -I\omega\vec{k}$$

angular momentum(moment of linear momentum)



$\vec{H} = I\omega_s\vec{k}$ $\quad I$ is mass moment of inertia, ω_s is spin of the disk

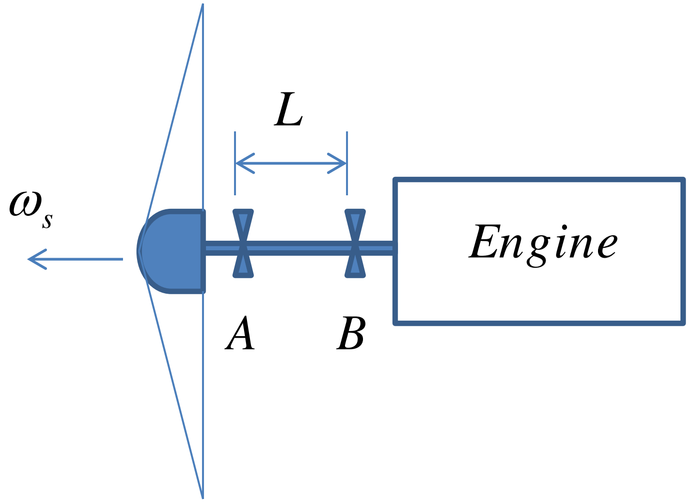
$$\vec{M} = \dot{\vec{H}} = \omega_p\vec{j} \times I\omega_s\vec{k} = I\omega_s\omega_p\vec{i}$$

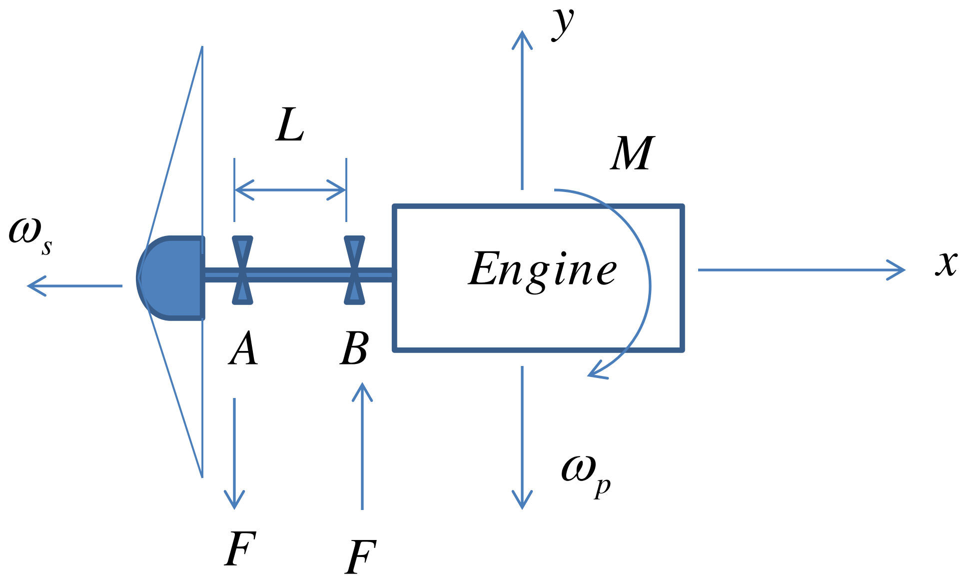
ω_p is precession of the disk, M is the gyroscopic moment

If there is a precession in the direction of y, it will create a moment M in the direction of x or if a moment is applied in the direction of x it will cause a precession in the direction of y.

Example

The propeller shaft of an airplane is shown. The propeller rotates at 2000 rpm cw when viewed from the rear and is driven by the engine through reduction gears. Suppose the airplane is flying horizontally and is making a turn to the right at a 0.2 rad/s when viewed from above. The propeller mass is 30 kg and radius of gyration is 1 m. It is desired to find the gyroscopic forces which the propeller shaft exerts against bearings A and B which are 15cm apart.





$$\omega_s = 2000\frac{2\pi}{60} = 209 rad/s$$

$$\omega_p = 0.2 rad/s \quad \text{precessionspeed}$$

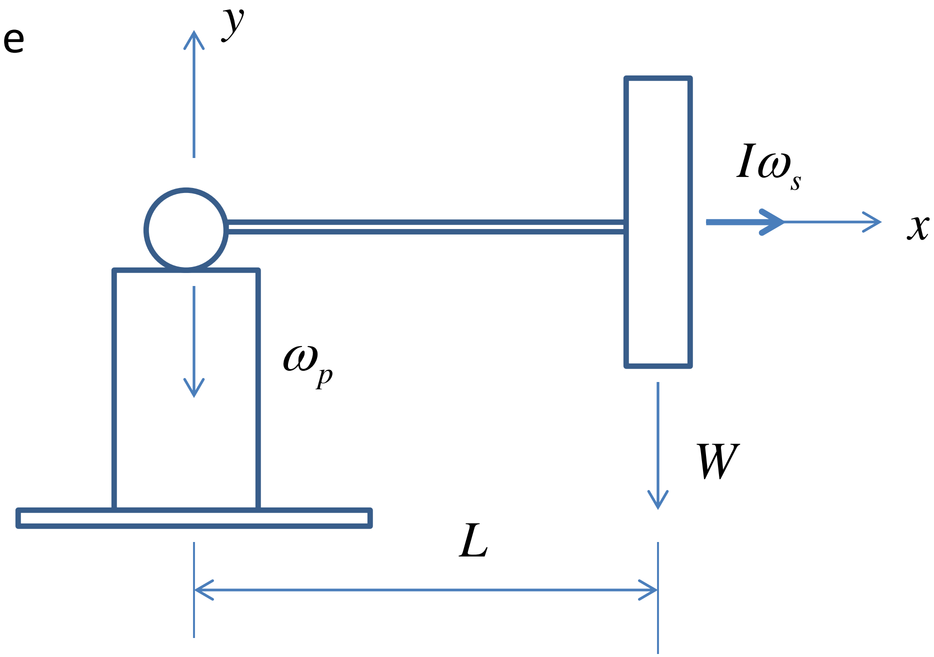
$$\vec{M} = \dot{\vec{H}} = -\omega_p \vec{j} \times -I\omega_s \vec{i} = -I\omega_s\omega_p \vec{k} = -(30kg)(1m)^2(209\text{rad/s})(0.2\text{rad/s})\vec{k} = -1254\vec{k}\ \text{Nm}$$

$$I = mk^2$$

$$M = FL$$

$$F = \frac{M}{L} = \frac{1254Nm}{0.15m} = 8360N$$

Example



A toy gyroscopic top is shown. If the disk of weight W is given a high angular velocity of ω_s about its axis and one end of the shaft placed on a pedestal, the shaft and disk will not fall but will precess around the axis y because of the torque WL acting on the top.

$$\vec{M} = -\omega_p \vec{j} \times I\omega_s \vec{i} = I\omega_s\omega_p \vec{k}$$

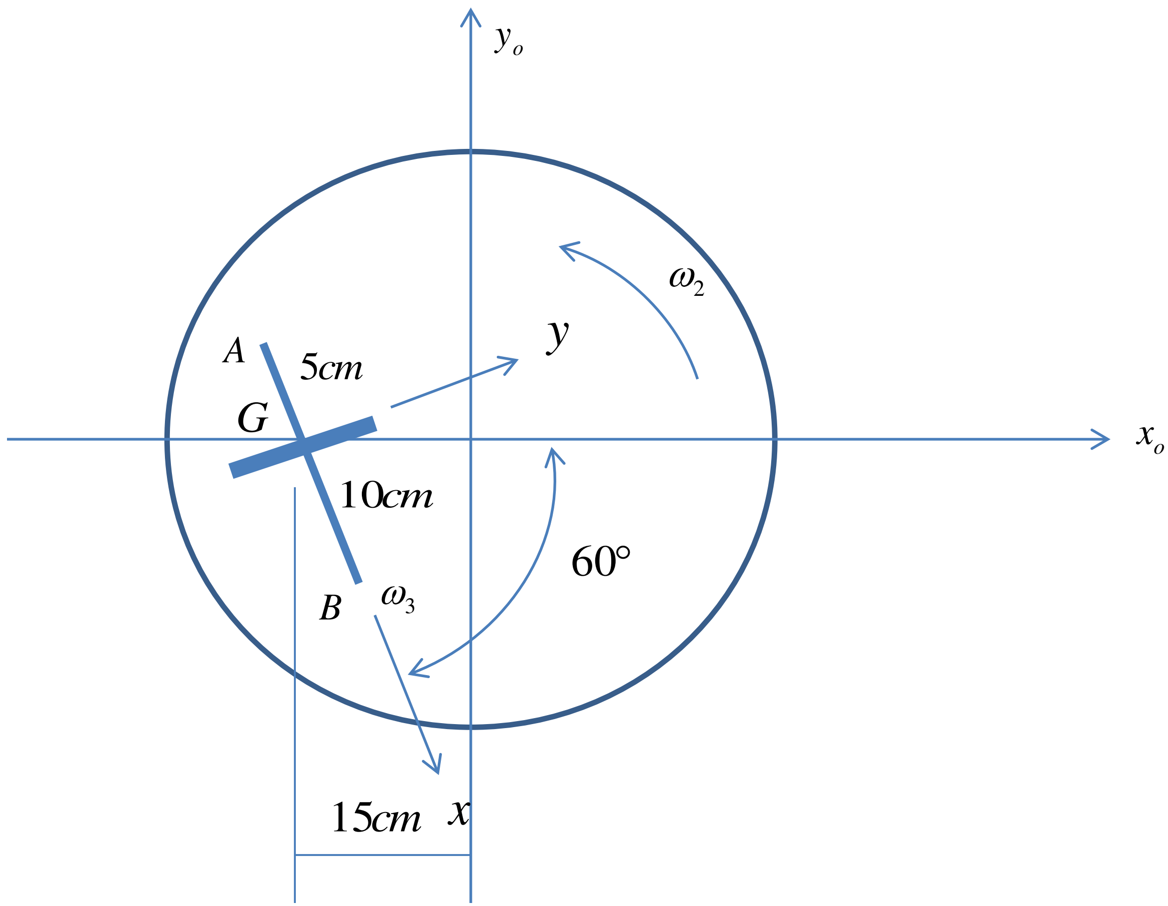
$$\vec{M} = -WL\vec{k}$$

$$I\omega_s\omega_p = WL$$

$$\omega_p = \frac{WL}{I\omega_s} \quad \text{precession of the top}$$

If the shaft were prevented from precessing by some means such as holding one's finger in its path it would immediately fall. Friction on the pedestal dissipates some of the energy and the top gradually drops lower. Also friction between the disk and its shaft causes a gradual decrease in ω_s and because of this the angular velocity of precession increases.

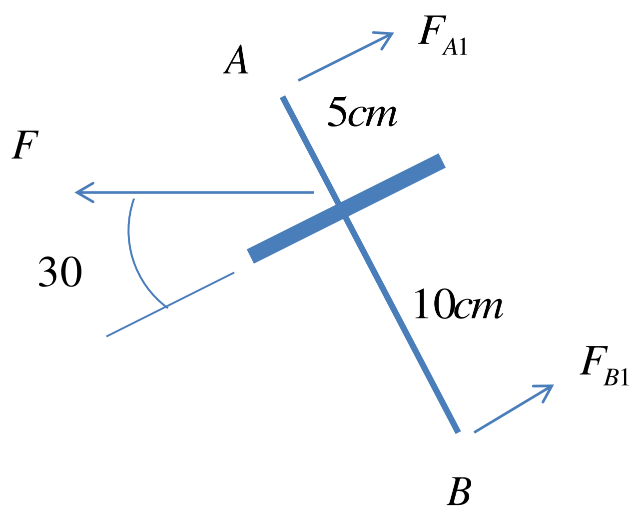
Example



A round plate rotates about the z_o axis with angular velocity ω_2. Mounted on thin revolving plate are two bearings A and B, which retain a shaft and a disk which is rotating with the angular velocity of ω_3. An xyz system is fixed to the shaft-disk system and rotating with it. The mass center G of the disk defines the origin of the system and the x axis is coincident with the axis of rotation. The mass of the disk is 5kg and its radius of gyration is 5cm and angular velocity of the disk is 350 rad/s. Assuming plate rotation as 5 rad/s find the bearing reactions.

Centrifugal force at G because of the plate rotation is

$$F = mr\omega^2 = (5kg)(0.15m)(5rad/s)^2 = 18.75N$$



$$\sum M_A = 0$$

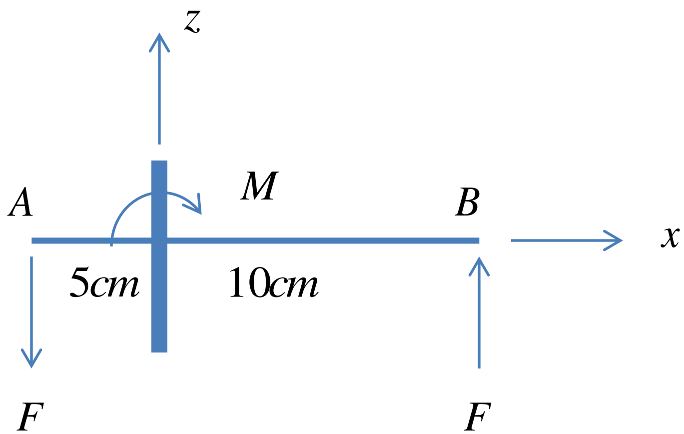
$$F_{B1}(0.15m) - F\cos 30(0.05m) = 0$$

$$F_{B1} = \frac{0.05}{0.15} 18.75\cos 30 = 7.93N$$

$$F_{A1} = 18.75\cos 30 - 7.93 = 10.82N$$

$$I = mk^2 = (5kg)(0.05m)^2 = 12.5\times10^{-3}\,kgm^2$$

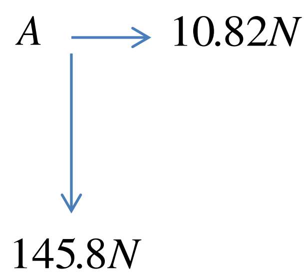
$$\vec{M} = \omega_2\vec{k} \times I\omega_3\vec{i} = I\omega_3\omega_2\vec{j} = (12.5\times10^{-3}\,kgm^2)(350rad/s)(5rad/s)\vec{j} = 21.87\vec{j}N$$

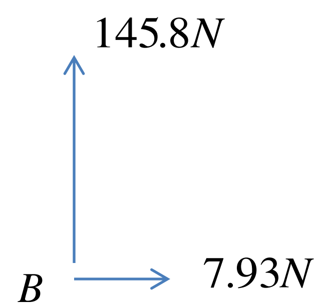


$$M = FL$$

$$-F(0.15m) + 21.87 = 0$$

$$F = \frac{21.87}{0.15} = 145.8N$$





$$F_A = \sqrt{10.82^2 + 145.8^2} = 146.2N$$

$$F_B = \sqrt{7.93^2 + 145.8^2} = 146.0N$$