Jazan University
Mechanical Engineering Department

CHAPTER 5

ROTARY BALANCING

EngM271 Theory of Machines

September 2018

BALANCING

Balancing is a technique of correcting or eliminating unwanted inertia forces and moments. Frame forces can vary significantly during a complete cycle of operation. Such forces can cause vibration which at times may reach dangerous amplitudes. Even if not dangerous vibration increases the component stresses and subject bearings to repeated loads which cause parts to fail prematurely by fatigue. Thus it is not sufficient in the design of machinery merely to avoid operation near the critical speed, we must also eliminate or at least reduce the inertia forces which produce these vibrations in the first place.

Unbalance is a condition where the center of mass is not coincident with the center of rotation.

Static unbalance: If unbalance condition occurs in a single plane it is called static unbalance.

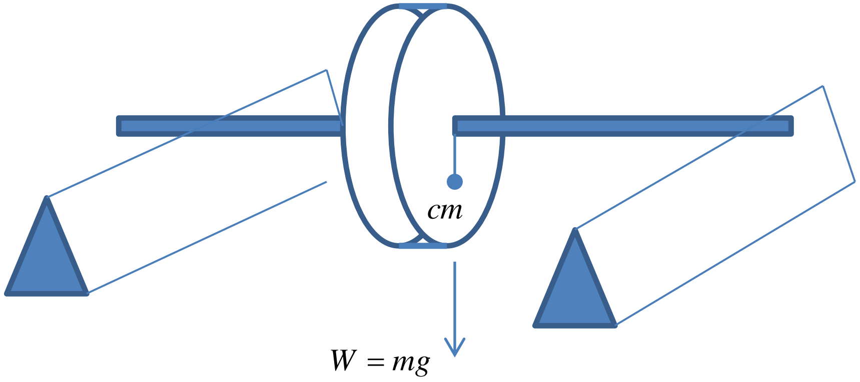
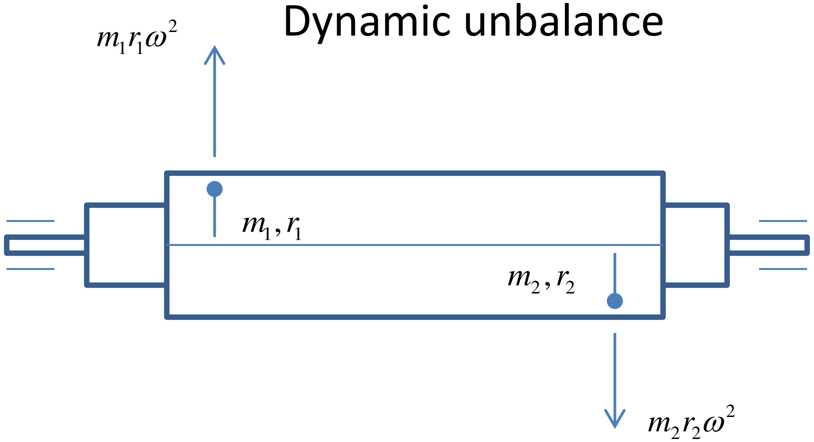


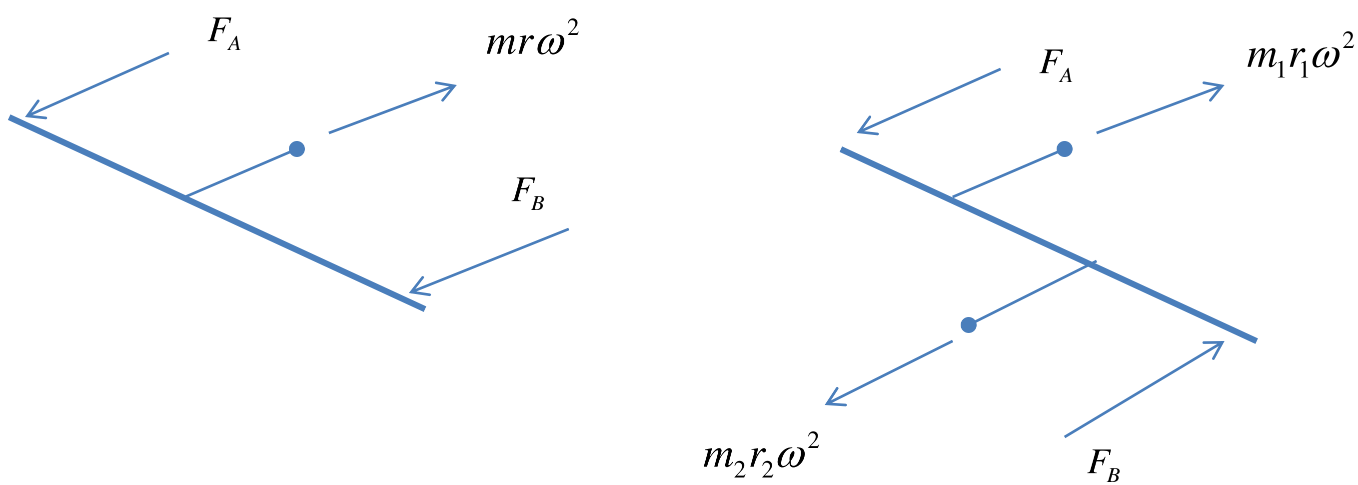
Figure shows a disk and shaft combination resting on hard rigid rail so that the shaft which is assumed to be perfectly straight can roll without friction. Simple experiment to determine whether the disk is statically unbalanced can be conducted as follows; roll the disk gently by hand and permit it to coast until it comes to rest. Then mark with chalk the lowest point of the periphery of the disk. Repeat four or five times. If the chalk marks are scattered at different places around the periphery, the disk on static balance. If the all chalk marks are coincident, the disk is statically unbalanced, which means that the axis of the shaft and the center of the mass of the disk are not coincident. The position of chalk marks indicates the angular rotation of the unbalance but not the amount.

If static unbalance is found to exist, it can be corrected by drilling out material at the chalk mark or by adding mass to the periphery 180 degree from the mark. Since the amount of unbalance is unknown these corrections must be made by trial and error.

Dynamic unbalance

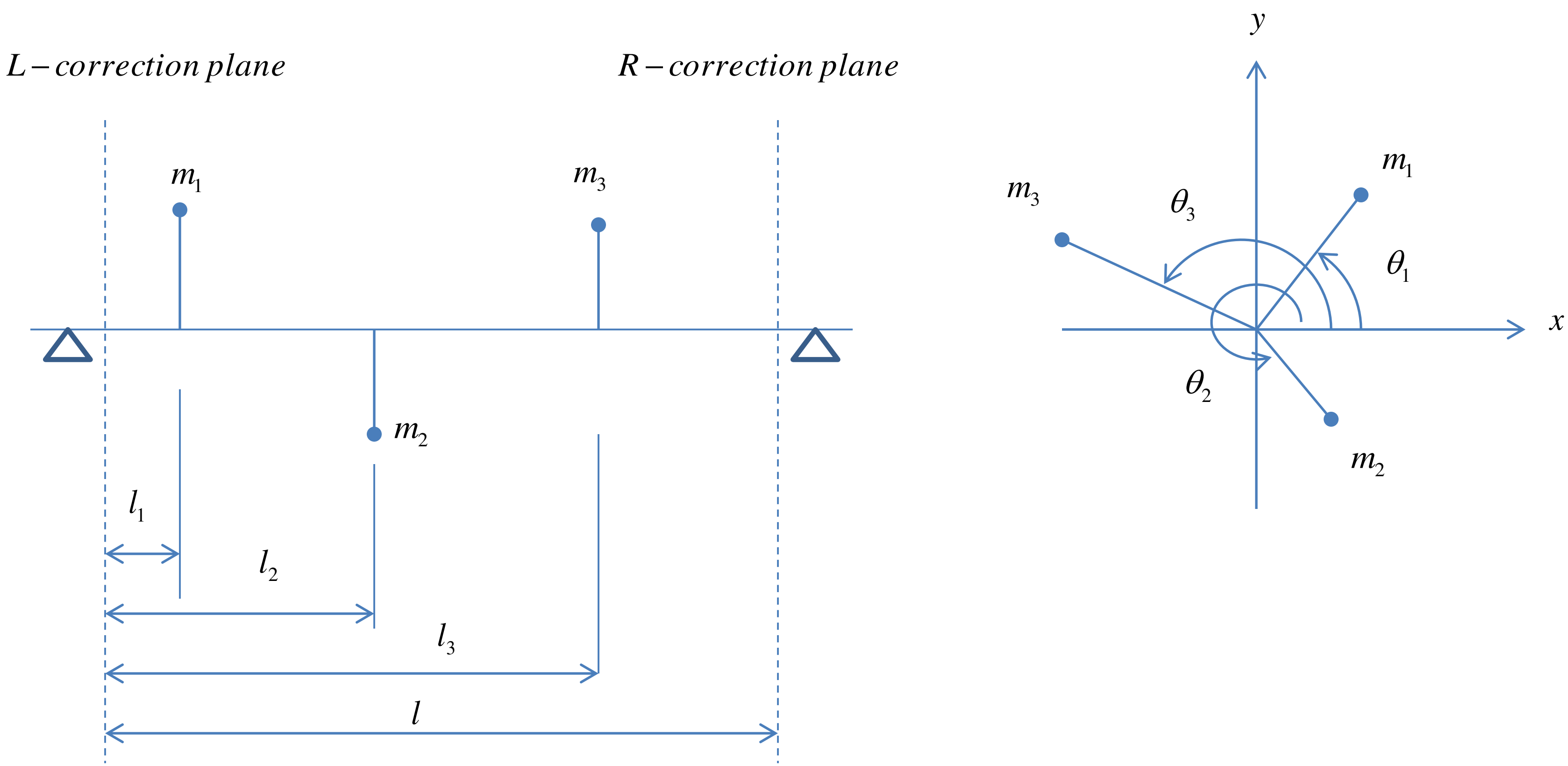


Suppose two equal masses m_1 and m_2 are placed at opposite ends of the rotor and at equal distances r_1 and r_2 from the axis of rotation. Since the masses are equal and on opposite sides of the rotational axis the rotor can be placed on rails as described earlier to show that it is statically balanced in all angular positions. If the rotor is placed in bearings and caused to rotate at an angular velocity ω rad/s, the centrifugal forces $m_1 r_1 \omega^2$ and $m_2 r_2 \omega^2$ act respectively at m_1 and m_2 on the rotor ends. These centrifugal forces produce the unequal bearing reactions F_A and F_B the entire system of forces rotates with the rotor at the angular velocity ω . Thus a rotor may be statically balanced and at the same time dynamically unbalanced.



In general case, distribution of the mass along the axis of the part depends upon the configuration of the part but errors occur in machining and also in casting and forging. Other errors and unbalance may be caused by improper boring, by keys and by assembly.

Analysis of unbalance



$$\vec{F}_1 = m_1 r_1 \omega^2 e^{j\theta_1}$$

$$\vec{F}_2 = m_2 r_2 \omega^2 e^{j\theta_2}$$

$$\vec{F}_3 = m_3 r_3 \omega^2 e^{j\theta_3}$$

$$\sum M_L = 0$$

$$l_1 \vec{F}_1 + l_2 \vec{F}_2 + l_3 \vec{F}_3 + l \vec{F}_R = 0$$

$$\vec{F}_R = -\frac{l_1}{l}\vec{F}_1 - \frac{l_2}{l}\vec{F}_2 - \frac{l_3}{l}\vec{F}_3$$

$$\vec{F}_R = F_R e^{j\theta_R} = -\frac{l_1}{l} m_1 r_1 \omega^2 e^{j\theta_1} - \frac{l_2}{l} m_2 r_2 \omega^2 e^{j\theta_2} - \frac{l_3}{l} m_3 r_3 \omega^2 e^{j\theta_3}$$

$$F_R(\cos\theta_R + j\sin\theta_R) =$$

$$-\frac{l_1}{l} m_1 r_1 \omega^2 (\cos\theta_1 + j\sin\theta_1) - \frac{l_2}{l} m_2 r_2 \omega^2 (\cos\theta_2 + j\sin\theta_2) - \frac{l_3}{l} m_3 r_3 \omega^2 (\cos\theta_3 + j\sin\theta_3)$$

$$F_R \cos\theta_R = -\frac{l_1}{l} m_1 r_1 \omega^2 \cos\theta_1 - \frac{l_2}{l} m_2 r_2 \omega^2 \cos\theta_2 - \frac{l_3}{l} m_3 r_3 \omega^2 \cos\theta_3 = -\frac{\omega^2}{l} \sum_1^3 l_i m_i r_i \cos\theta_i$$

$$F_R \sin\theta_R = -\frac{l_1}{l} m_1 r_1 \omega^2 \sin\theta_1 - \frac{l_2}{l} m_2 r_2 \omega^2 \sin\theta_2 - \frac{l_3}{l} m_3 r_3 \omega^2 \sin\theta_3 = -\frac{\omega^2}{l} \sum_1^3 l_i m_i r_i \sin\theta_i$$

$$F_R = m_R r_R \omega^2 = \frac{\omega^2}{l} \sqrt{\left(\sum_1^3 l_i m_i r_i \cos\theta_i\right)^2 + \left(\sum_1^3 l_i m_i r_i \sin\theta_i\right)^2}$$

$$m_R r_R = \frac{1}{l} \sqrt{\left(\sum_1^3 l_i m_i r_i \cos\theta_i\right)^2 + \left(\sum_1^3 l_i m_i r_i \sin\theta_i\right)^2}$$

$$\theta_R = \tan^{-1} \frac{\left(\sum_1^3 l_i m_i r_i \sin\theta_i\right)}{\left(\sum_1^3 l_i m_i r_i \cos\theta_i\right)}$$

$$\sum \vec{F} = 0$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_R + \vec{F}_L = 0$$

$$\vec{F}_L = -\sum_1^3 \vec{F}_i - \vec{F}_R$$

$$F_L e^{j\theta_L} = -\sum_1^3 F_i e^{j\theta_i} - F_R e^{j\theta_R}$$

$$m_L r_L (\cos\theta_L + j\sin\theta_L) = -\sum_1^3 m_i r_i (\cos\theta_i + j\sin\theta_i) - m_R r_R (\cos\theta_R + j\sin\theta_R)$$

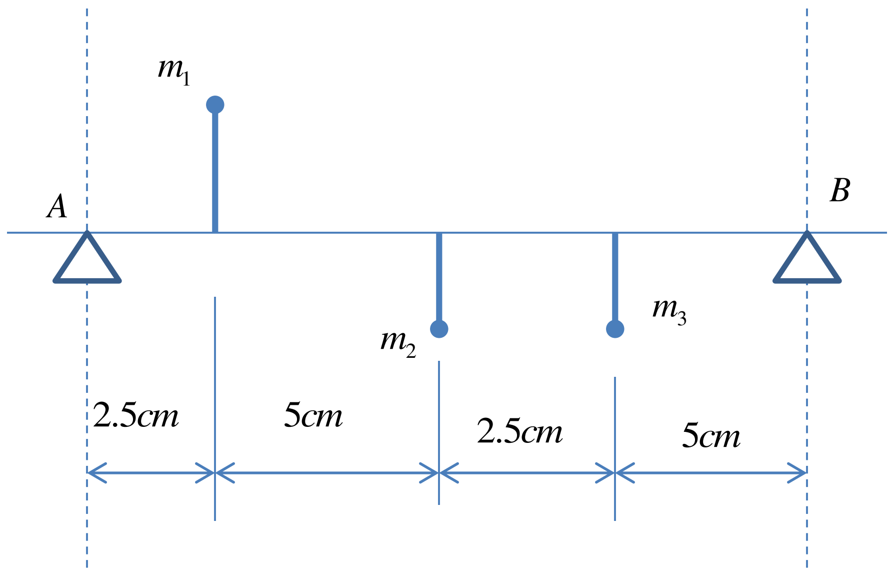
$$m_L r_L \cos\theta_R = -\sum_1^3 m_i r_i \cos\theta_i - m_R r_R \cos\theta_R$$

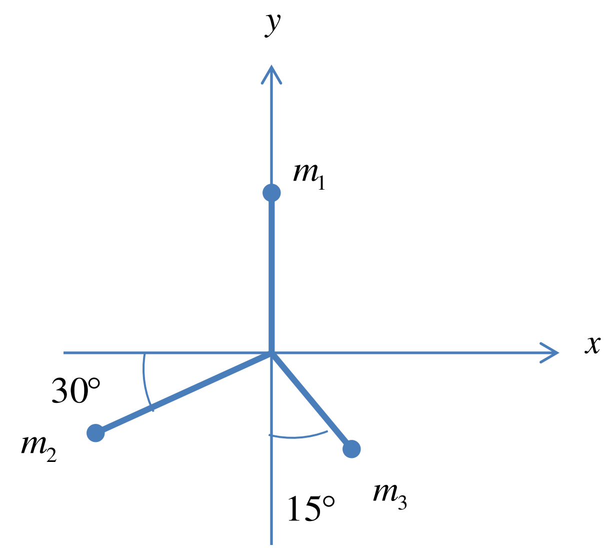
$$m_L r_L \sin\theta_R = -\sum_1^3 m_i r_i \sin\theta_i - m_R r_R \sin\theta_R$$

$$m_L r_L = \sqrt{\left(\sum_1^3 m_i r_i \cos\theta_i - m_R r_R \cos\theta_R\right)^2 + \left(\sum_1^3 m_i r_i \sin\theta_i - m_R r_R \sin\theta_R\right)^2}$$

$$\theta_L = \tan^{-1} \frac{\sum_1^3 m_i r_i \sin\theta_i - m_R r_R \sin\theta_R}{\sum_1^3 m_i r_i \cos\theta_i - m_R r_R \cos\theta_R}$$

Example





$m_1 = 60g \quad r_1 = 8cm$

$m_2 = 30g \quad r_2 = 5cm$

$m_3 = 40g \quad r_3 = 6cm$

$\omega = 100 rad/s$

Determine the bearing reactions at A and B

$$\vec{F}_1 = m_1 r_1 \omega^2 e^{j\theta_1} = (0.06kg)(0.08m)(100rad/s)^2 e^{j90} = 48e^{j90} N$$

$$\vec{F}_2 = m_2 r_2 \omega^2 e^{j\theta_2} = (0.03kg)(0.05m)(100rad/s)^2 e^{j210} = 15e^{j210} N$$

$$\vec{F}_3 = m_3 r_3 \omega^2 e^{j\theta_3} = (0.04kg)(0.06m)(100rad/s)^2 e^{j285} = 24e^{j285} N$$

$$\sum M_A = 0$$

$$l_1 \vec{F}_1 + l_2 \vec{F}_2 + l_3 \vec{F}_3 + l \vec{F}_B = 0$$

$$\vec{F}_B = -\frac{l_1}{l}\vec{F}_1 - \frac{l_2}{l}\vec{F}_2 - \frac{l_3}{l}\vec{F}_3 = -\frac{2.5}{15}48e^{j90} - \frac{7.5}{15}15e^{j210} - \frac{10}{15}24e^{j285}$$

$$F_B e^{j\theta_B} = -8e^{j90} - 7.5e^{j210} - 16e^{j285}$$

$$F_B \cos\theta_B = -8\cos 90 - 7.5\cos 210 - 16\cos 285 = 2.35$$

$$F_B \sin\theta_B = -8\sin 90 - 7.5\sin 210 - 16\sin 285 = 11.20$$

$$F_B = \sqrt{(2.35)^2 + (11.2)^2} = 11.44N$$

$$\theta_B = \tan^{-1}\frac{11.2}{2.35} = 78°$$

$$\vec{F}_B = 11.44e^{j78}N$$

$$\sum \vec{F} = 0 \qquad \vec{F}_A + \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_B = 0$$

$$\vec{F}_A = -\vec{F}_1 - \vec{F}_2 - \vec{F}_3 - \vec{F}_B = -48e^{j90} - 15e^{j210} - 24e^{j285} - 11.44e^{j78}$$

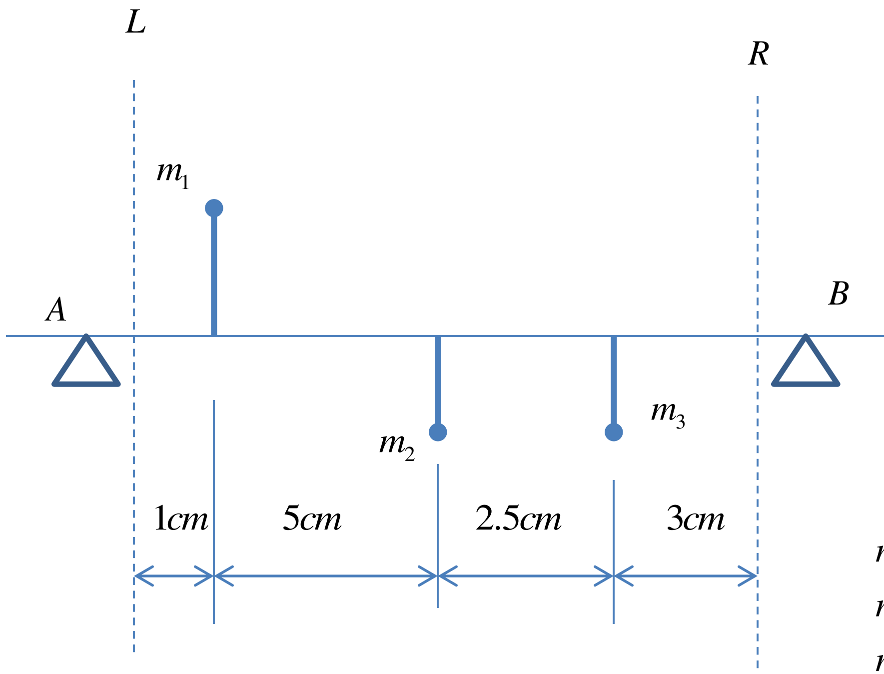
$$F_A \cos\theta_A = -48\cos 90 - 15\cos 210 - 24\cos 285 - 11.44\cos 78 = 4.4$$

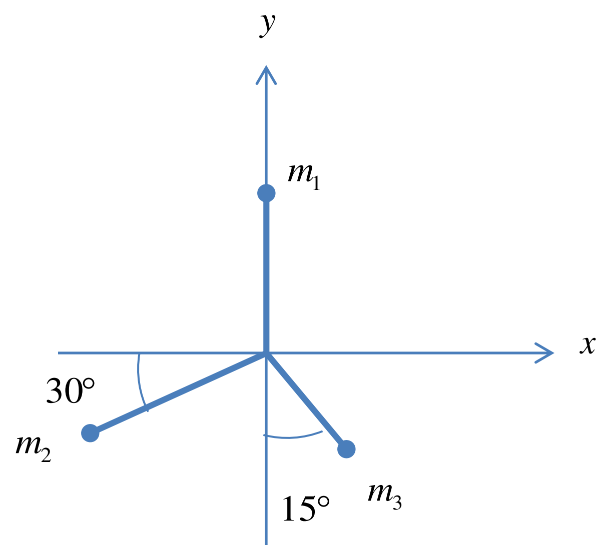
$$F_A \sin\theta_A = -48\sin 90 - 15\sin 210 - 24\sin 285 - 11.44\sin 78 = -28.5$$

$$F_A = \sqrt{(4.4)^2 + (28.5)^2} = 28.84N$$

$$\theta_A = \tan^{-1}\frac{-28.5}{4.4} = -81.22° \qquad \vec{F}_A = 28.84e^{-j81.22}N$$

Example





$m_1 = 60g \quad r_1 = 8cm$

$m_2 = 30g \quad r_2 = 5cm$

$m_3 = 40g \quad r_3 = 6cm$

$\omega = 100 rad/s$

Determine amount of balancing masses at right and left planes if balancing masses are placed at the radius of 4cm.

$$\vec{F}_1 = m_1 r_1 \omega^2 e^{j\theta_1} = (0.06kg)(0.08m)(100rad/s)^2 e^{j90} = 48e^{j90} N$$

$$\vec{F}_2 = m_2 r_2 \omega^2 e^{j\theta_2} = (0.03kg)(0.05m)(100rad/s)^2 e^{j210} = 15e^{j210} N$$

$$\vec{F}_3 = m_3 r_3 \omega^2 e^{j\theta_3} = (0.04kg)(0.06m)(100rad/s)^2 e^{j285} = 24e^{j285} N$$

$$\sum M_L = 0$$

$$l_1 \vec{F}_1 + l_2 \vec{F}_2 + l_3 \vec{F}_3 + l \vec{F}_R = 0$$

$$\vec{F}_R = -\frac{l_1}{l} \vec{F}_1 - \frac{l_2}{l} \vec{F}_2 - \frac{l_3}{l} \vec{F}_3 = -\frac{1}{11.5} 48e^{j90} - \frac{6}{11.5} 15e^{j210} - \frac{8.5}{11.5} 24e^{j285}$$

$$F_R e^{j\theta_R} = -4.17e^{j90} - 7.83e^{j210} - 17.74e^{j285}$$

$$F_R \cos\theta_R = -4.17\cos 90 - 7.83\cos 210 - 17.74\cos 285 = 2.19$$

$$F_R \sin\theta_R = -4.17\sin 90 - 7.83\sin 210 - 17.74\sin 285 = 16.88$$

$$F_R = \sqrt{(2.19)^2 + (16.88)^2} = 17.02N$$

$$\theta_R = \tan^{-1}\frac{16.88}{2.19} = 82.6°$$

$$\vec{F}_R = 17.02e^{j82.6}N$$

$$\sum \vec{F} = 0 \qquad \vec{F}_L + \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_R = 0$$

$$\vec{F}_L = -\vec{F}_1 - \vec{F}_2 - \vec{F}_3 - \vec{F}_R = -48e^{j90} - 15e^{j210} - 24e^{j285} - 17.02e^{j82.6}$$

$$F_L \cos\theta_L = -48\cos 90 - 15\cos 210 - 24\cos 285 - 17.02\cos 82.6 = 4.58$$

$$F_L \sin\theta_L = -48\sin 90 - 15\sin 210 - 24\sin 285 - 17.02\sin 82.6 = -34.19$$

$$F_L = \sqrt{(4.58)^2 + (34.19)^2} = 34.49N$$

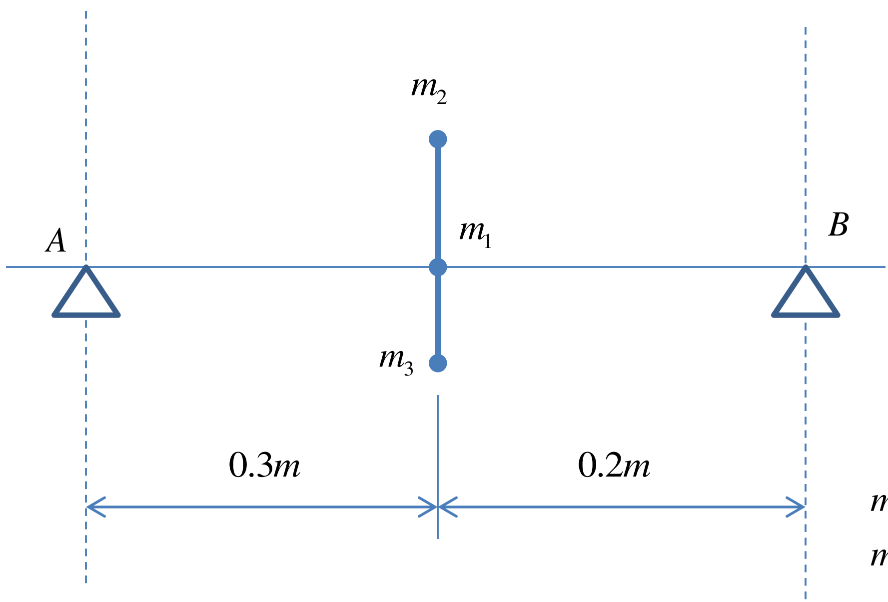
$$\theta_L = \tan^{-1}\frac{-34.19}{4.58} = -82.37° \qquad \vec{F}_L = 34.49e^{-j82.37}N$$

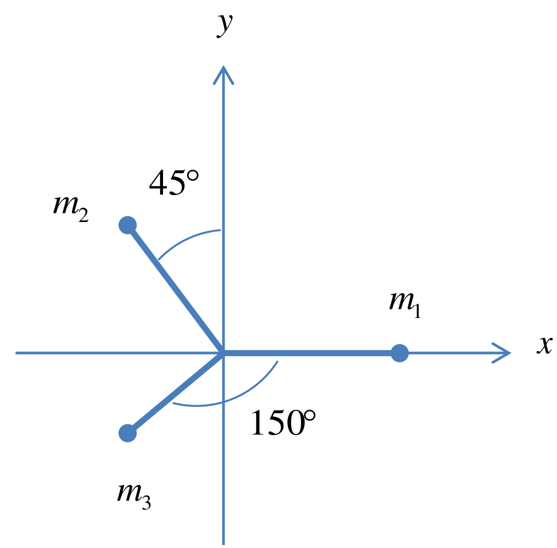
$$F = mr\omega^2$$

$$m_R = \frac{F_R}{r_R\omega^2} = \frac{17.02N}{(0.04m)(100rad/s)^2} = 0.04255kg = 42.55g$$

$$m_L = \frac{F_L}{r_L\omega^2} = \frac{34.49N}{(0.04m)(100rad/s)^2} = 0.08622kg = 86.22g$$

Example





$$m_1 = 12kg \quad r_1 = 0.2m$$
$$m_2 = 3kg \quad r_2 = 0.3m$$
$$m_3 = 10kg \quad r_3 = 0.15m$$
$$\omega = 750rpm$$

a)Determine bearing reactions at bearings A and B

b)Determine the location and magnitude of the balancing mass if it is to be placed at a radius of 0.25m

$$a)\quad \omega = 750rpm\left(\frac{2\pi}{60}\right) = 78.5rad/s$$

$$\vec{F}_1 = m_1 r_1 \omega^2 e^{j\theta_1} = (12kg)(0.2m)(78.5rad/s)^2 e^{j0} = 14789e^{j0} N$$

$$\vec{F}_2 = m_2 r_2 \omega^2 e^{j\theta_2} = (3kg)(0.3m)(78.5rad/s)^2 e^{j135} = 5546e^{j135} N$$

$$\vec{F}_3 = m_3 r_3 \omega^2 e^{j\theta_3} = (10kg)(0.15m)(78.5rad/s)^2 e^{j210} = 9243e^{j210} N$$

$$\sum M_A = 0$$

$$l_1\vec{F}_1 + l_2\vec{F}_2 + l_3\vec{F}_3 + l\vec{F}_B = 0$$

$$\vec{F}_B = -\frac{l_1}{l}\vec{F}_1 - \frac{l_2}{l}\vec{F}_2 - \frac{l_3}{l}\vec{F}_3 = -\frac{0.3}{0.5}14789e^{j0} - \frac{0.3}{0.5}5546e^{j135} - \frac{0.3}{0.5}9243e^{j210}$$

$$F_B e^{j\theta_B} = -8873.4e^{j0} - 3327.6e^{j135} - 5545.8e^{j210}$$

$$F_B \cos\theta_B = -8873.4\cos 0 - 3327.6\cos 135 - 5545.8\cos 210 = -1717.62$$

$$F_B \sin\theta_B = -8873.4\sin 0 - 3327.6\sin 135 - 5545.8\sin 210 = 419.93$$

$$F_B = \sqrt{(1717.62)^2 + (419.93)^2} = 1768.21N$$

$$\theta_B = \tan^{-1}\frac{419.93}{-1717.93} = (180-13.74) = 166.26°$$

$$\vec{F}_B = 1768.21e^{j166.26}N$$

$$\sum M_B = 0$$

$$l_1\vec{F}_1 + l_2\vec{F}_2 + l_3\vec{F}_3 + l\vec{F}_A = 0$$

$$\vec{F}_A = -\frac{l_1}{l}\vec{F}_1 - \frac{l_2}{l}\vec{F}_2 - \frac{l_3}{l}\vec{F}_3 = -\frac{0.2}{0.5}14789e^{j0} - \frac{0.2}{0.5}5546e^{j135} - \frac{0.2}{0.5}9243e^{j210}$$

$$F_A e^{j\theta_A} = -5915.6e^{j0} - 2218.4e^{j135} - 3697.2e^{j210}$$

$$F_A\cos\theta_A = -5915.6\cos0 - 2218.4\cos135 - 3697.2\cos210 = -1145.08$$

$$F_A\sin\theta_A = -5915.6\sin0 - 2218.4\sin135 - 3697.2\sin210 = 279.95$$

$$F_A = \sqrt{(1145.08)^2 + (279.95)^2} = 1178.8N$$

$$\theta_A = \tan^{-1} \frac{279.95}{-1145.08} = (180-13.74) = 166.26°$$

$$\vec{F}_A = 1178.8e^{j166.26} N$$

$b)$ Let $\vec{F}_C$ be correctingforce

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_C = 0$$

$$\vec{F}_C = -\vec{F}_1 - \vec{F}_2 - \vec{F}_3$$

$$F_C e^{j\theta_C} = -14789e^{j0} - 5546e^{j135} - 9243e^{j210}$$

$$F_C \cos\theta_C = -14789\cos 0 - 5546\cos 135 - 9243\cos 210 = -2862.7$$

$$F_C \sin\theta_C = -14789\sin 0 - 5546\sin 135 - 9243\sin 210 = 699.88$$

$$F_C = \sqrt{(2862.7)^2 + (699.88)^2} = 2947N$$

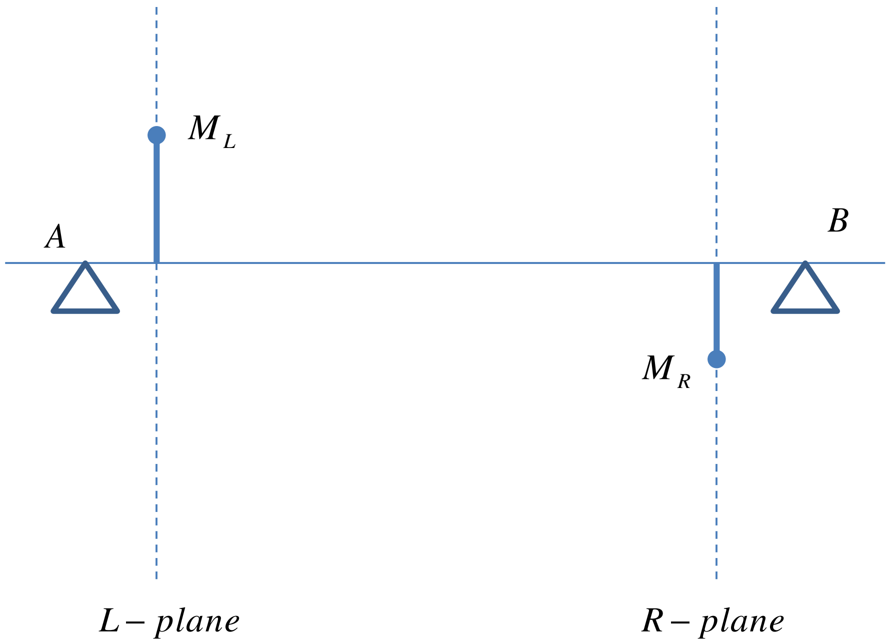
$$\theta_C = \tan^{-1} \frac{699.88}{-2862.7} = (180-13.74) = 166.26°$$

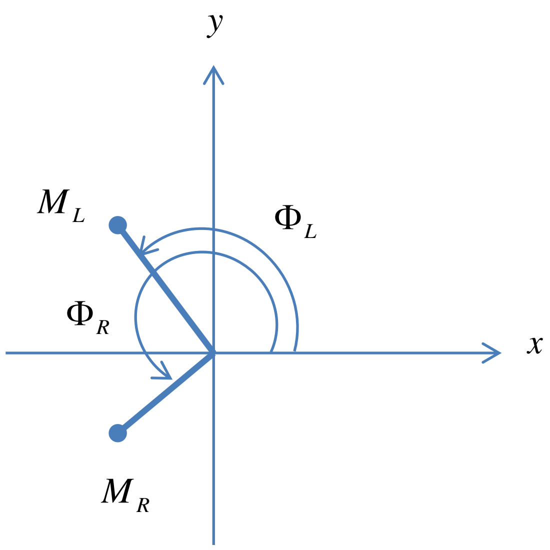
$$m_C = \frac{F_C}{r_C \omega^2} = \frac{2947N}{(0.25m)(78.5rad/s)^2} = 1.91kg$$

Field Balancing

It is possible to balance a machine in the field by balancing a single plane at a time. But cross effects and correction plane interference often require balancing each end of a rotor two or three times to obtain satisfactory results. Field balancing is necessary for very large rotors for which balancing machines are impractical and even though high-speed rotors are balanced in the shop during manufacture, it is frequently necessary to rebalance them in the field because of slight deformations brought on by shipping, by creep, or by high operating temperatures.

Two-plane field balancing:





Unknown unbalances U_L and U_R are assumed to exist in the left and right-hand correction planes, respectively. When these unbalances have been found, their negatives are located in the left and right planes to achieve balance. In the field balancing three runs or tests are made as follows;

First run : measure amplitudes $\vec{X}_A = X_A e^{j\Phi_A}$ and $\vec{X}_B = X_B e^{j\Phi_B}$ at bearings A and B due only to the original unbalances.

Second run : add trial mass $\vec{m}_L = m_L e^{j\Phi_L}$ to the left correction plane measure the amplitudes $\vec{X}_{AL} = X_{AL} e^{j\Phi_{AL}}$ and $\vec{X}_{BL} = X_{BL} e^{j\Phi_{BL}}$

Third run : remove trial mass m_L and add trial mass $\vec{m}_R = m_R e^{j\Phi_R}$ to the right correction plane measure the amplitudes $\vec{X}_{AR} = X_{AR} e^{j\Phi_{AR}}$ and $\vec{X}_{BR} = X_{BR} e^{j\Phi_{BR}}$

To develop the equations for the unbalance to be found we first define the complex stiffness, by which we mean the amplitude that would result at either bearing due to a unit unbalance located at the intersection of the rotating reference mark and one of the correction planes. Thus we need to find the complex stiffnesses A_L and B_L due to a unit unbalance located at the intersection of the rotating reference mark and plane L. And we require the complex stiffness A_R and B_R due to a unit unbalance located at the intersection of the rotating reference mark and plane R. If these stiffnesses were known we could write the following sets of complex equations

$$\vec{X}_{AL} = \vec{X}_A + \vec{A}_L \cdot \vec{m}_L \qquad \vec{X}_{BL} = \vec{X}_B + \vec{B}_L \cdot \vec{m}_L$$

$$\vec{X}_{AR} = \vec{X}_A + \vec{A}_R \cdot \vec{m}_R \qquad \vec{X}_{BR} = \vec{X}_B + \vec{B}_R \cdot \vec{m}_R$$

$$\vec{A}_L = \frac{\vec{X}_{AL} - \vec{X}_A}{\vec{m}_L} \qquad \vec{B}_L = \frac{\vec{X}_{BL} - \vec{X}_B}{\vec{m}_L}$$

$$\vec{A}_R = \frac{\vec{X}_{AR} - \vec{X}_A}{\vec{m}_R} \qquad \vec{B}_R = \frac{\vec{X}_{BR} - \vec{X}_B}{\vec{m}_R}$$

From the definition of stiffness we can write

$$\vec{X}_A = \vec{A}_L \cdot \vec{U}_L + \vec{A}_R \cdot \vec{U}_R$$

$$\vec{X}_B = \vec{B}_L \cdot \vec{U}_L + \vec{B}_R \cdot \vec{U}_R$$

$$\vec{U}_L = \frac{\vec{X}_A \cdot \vec{B}_R - \vec{X}_B \cdot \vec{A}_R}{\vec{A}_L \cdot \vec{B}_R - \vec{A}_R \cdot \vec{B}_L} \qquad \vec{U}_R = \frac{\vec{X}_B \cdot \vec{A}_L - \vec{X}_A \cdot \vec{B}_L}{\vec{A}_L \cdot \vec{B}_R - \vec{A}_R \cdot \vec{B}_L}$$

Example

$$\vec{X}_A = 8.6e^{j63}\mu m \quad \vec{X}_B = 6.5e^{j206}\mu m \text{ first run results}$$

$$\vec{m}_L = 10e^{j270}g \quad \vec{X}_{AL} = 5.9e^{j123}\mu m \quad \vec{X}_{BL} = 4.5e^{j228}\mu m \quad \text{second run results}$$

$$\vec{m}_R = 12e^{j180}g \quad \vec{X}_{AR} = 6.2e^{j36}\mu m \quad \vec{X}_{BR} = 10.4e^{j162}\mu m \quad \text{third run results}$$

Find amount of balances at left and right planes.

remember

$$\vec{X} = Xe^{j\theta_X} \quad \vec{Y} = Ye^{j\theta_Y}$$

$$\vec{X} \cdot \vec{Y} = Xe^{j\theta_X} \cdot Ye^{j\theta_Y} = XYe^{j(\theta_X + \theta_Y)}$$

$$\frac{\vec{X}}{\vec{Y}} = \frac{X}{Y} e^{j(\theta_X - \theta_Y)}$$

$$\vec{X} \pm \vec{Y} = (X\cos\theta_X \pm Y\cos\theta_Y) + j(X\sin\theta_X \pm Y\sin\theta_Y)$$

$$\vec{A}_L = \frac{\vec{X}_{AL} - \vec{X}_A}{\vec{m}_L} = \frac{5.9e^{j123} - 8.6e^{j63}}{10e^{j270}} = 0.76e^{-j69}$$

$$\vec{B}_L = \frac{\vec{X}_{BL} - \vec{X}_B}{\vec{m}_L} = \frac{4.5e^{j228} - 6.5e^{j206}}{10e^{j270}} = 0.29e^{j80}$$

$$\vec{A}_R = \frac{\vec{X}_{AR} - \vec{X}_A}{\vec{m}_R} = \frac{6.2e^{j36} - 8.6e^{j63}}{12e^{j180}} = 0.35e^{j105.5}$$

$$\vec{B}_R = \frac{\vec{X}_{BR} - \vec{X}_B}{\vec{m}_R} = \frac{10.4e^{j162} - 6.5e^{j206}}{12e^{j180}} = 0.61e^{-j56.2}$$

Amount of unbalances

$$\vec{U}_L = \frac{\vec{X}_A \cdot \vec{B}_R - \vec{X}_B \cdot \vec{A}_R}{\vec{A}_L \cdot \vec{B}_R - \vec{A}_R \cdot \vec{B}_L} = \frac{8.6e^{j63} \cdot 0.61e^{-j56.2} - 6.5e^{j206} \cdot 0.35e^{j105.5}}{0.76e^{-j69} \cdot 0.61e^{-j56.2} - 0.35e^{j105.5} \cdot 0.29e^{j80}} = 10.76e^{j146.6}$$

$$\vec{U}_R = \frac{\vec{X}_B \cdot \vec{A}_L - \vec{X}_A \cdot \vec{B}_L}{\vec{A}_L \cdot \vec{B}_R - \vec{A}_R \cdot \vec{B}_L} = \frac{6.5e^{j206} \cdot 0.76e^{-j69} - 8.6e^{j63} \cdot 0.29e^{j80}}{0.76e^{-j69} \cdot 0.61e^{-j56.2} - 0.35e^{j105.5} \cdot 0.29e^{j80}} = 6.2e^{-j114}$$

To balance the system we need equal and opposite balancing

$$\vec{U}_L = 10.76e^{j146.6} \text{ unbalance}$$
$$\vec{B}_L = 10.76e^{j(146.6+180)} \text{ balance}$$
$$B_L = m_{BL} r_{BL} = 10.76$$
$$\Phi_{BL} = 146.6^\circ + 180^\circ = 326.6^\circ$$

$$\vec{U}_R = 6.2e^{-j114} \text{ unbalance}$$
$$\vec{B}_L = 6.26e^{j(-114+180)} \text{ balance}$$
$$B_L = m_{BL} r_{BL} = 6.2$$
$$\Phi_{BL} = -114^\circ + 180^\circ = 66^\circ$$