Jazan University
Mechanical Engineering Department

CHAPTER 6

GEAR TRAINS

EngM271 Theory of Machines

DR. Amr Assie

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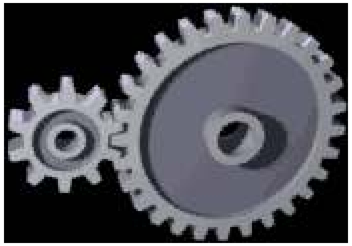


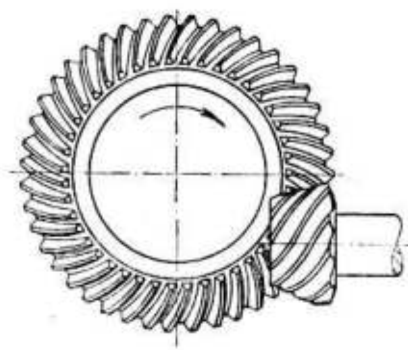
Spur gear

Internal gear

Rack and pinion gearing

Worm gear

Bevel gear



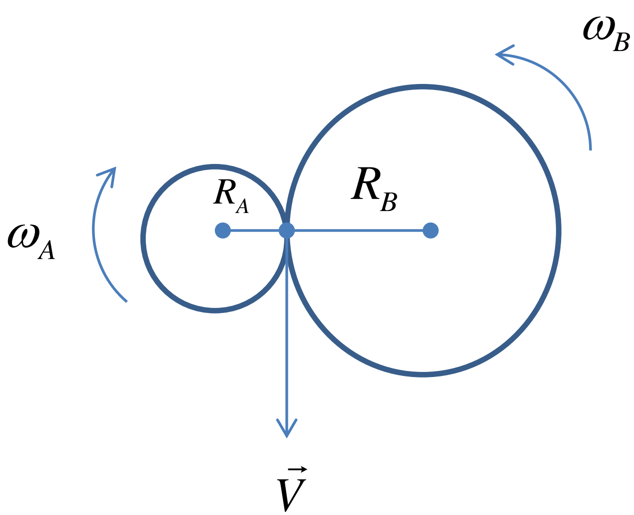
Hypoid gear

Helical gear

Gear Trains

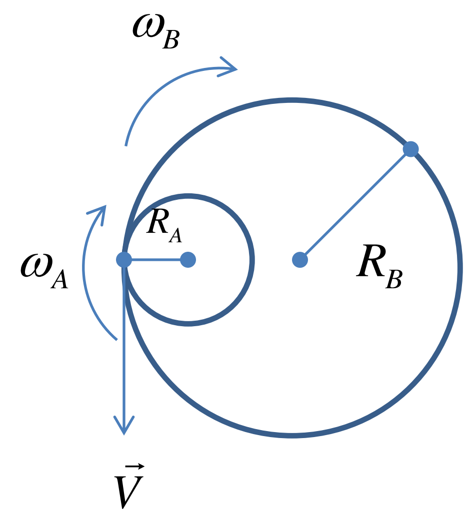
- A **gear train** is a set or system of **gears** arranged to transfer **torque** from one part of a mechanical system to another.
- Gear trains may consist of:
- Driving gears - attached to the input shaft
- Driven gears - attached to the output shaft
- Idler gears - interposed between the driving and driven gear in order to maintain the direction of the output shaft the same as the input shaft or to increase the distance between the drive and driven gears.

Basic relations



$$V = R_A\omega_A = R_B\omega_B$$

$$\frac{\omega_B}{\omega_A} = \frac{R_A}{R_B} = \frac{N_A}{N_B}$$



$$D = 2R = mN \quad (m \text{ is called module})$$

$$\frac{\omega_B}{\omega_A} = \frac{R_A}{R_B} = \frac{N_A}{N_B}$$

$$sr = \frac{\text{output gear speed}}{\text{input gear speed}} = \pm\frac{\omega_B}{\omega_A} = \pm\frac{R_A}{R_B} = \pm\frac{N_A}{N_B} \text{ speed ratio}$$

$$gr = \frac{\text{output gear teeth number}}{\text{input gear teeth number}} = \frac{N_B}{N_A} \text{ gear ratio}$$

$$sr = \frac{1}{gr}$$

R_A : pitch radius of gear A

R_B : pitch radius of gear B

N_A : number of teeth of gear A

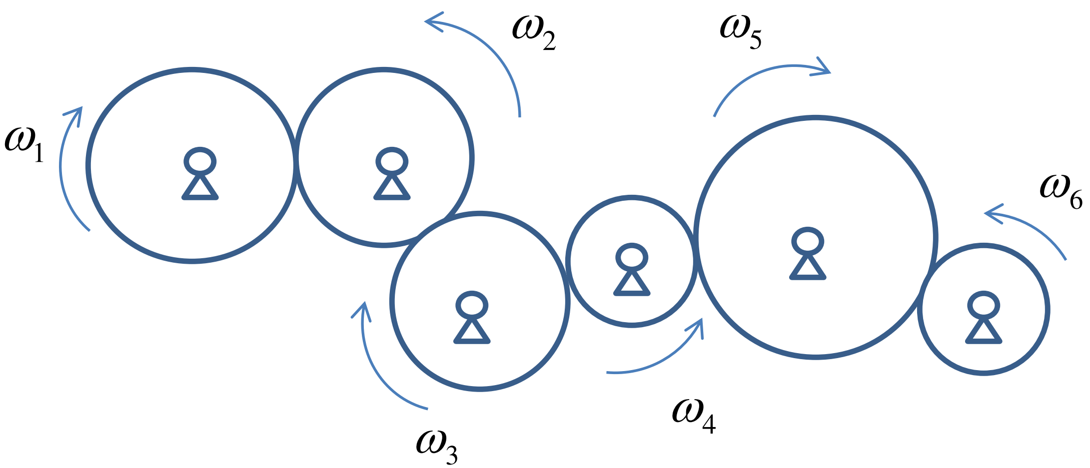
N_B : number of teeth of gear B

ω_A : angular speed of gear A

ω_B : angular speed of gear B

Simple gear train:

A simple gear train is a series of gear which are capable of rotating about independent axes and capable of receiving and transmitting motion from one gear to another.



$$sr_{12} = \frac{\omega_2}{\omega_1}$$

$$sr_{23} = \frac{\omega_3}{\omega_2}$$

$$sr_{34} = \frac{\omega_4}{\omega_3}$$

$$sr_{45} = \frac{\omega_5}{\omega_4}$$

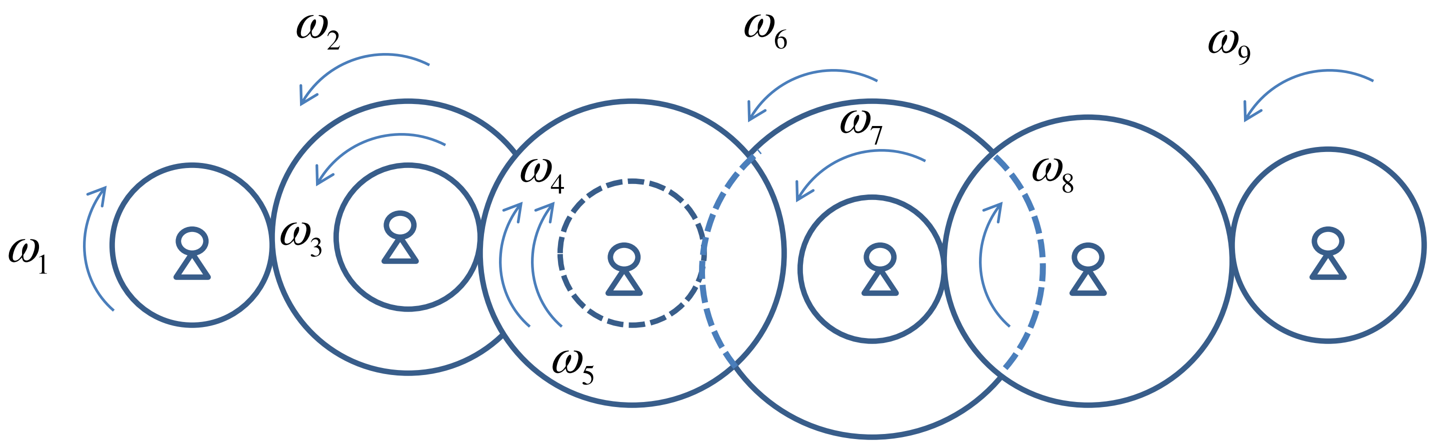
$$sr_{56} = \frac{\omega_6}{\omega_5}$$

$$sr_{16} = \frac{\omega_6}{\omega_1} = \frac{\omega_6}{\omega_5}\frac{\omega_5}{\omega_4}\frac{\omega_4}{\omega_3}\frac{\omega_3}{\omega_2}\frac{\omega_2}{\omega_1} = sr_{56} \times sr_{45} \times sr_{34} \times sr_{23} \times sr_{12}$$

$$gr_{16} = \frac{N_6}{N_1} = \frac{N_6}{N_5}\frac{N_5}{N_4}\frac{N_4}{N_3}\frac{N_3}{N_2}\frac{N_2}{N_1} = gr_{56} \times gr_{45} \times gr_{34} \times gr_{23} \times gr_{12}$$

Compound gear train:

A compound gear train is a series of gears connected in such a way that the train has one or more gears rotating about an axis with the same angular velocity.



$$sr_{12} = \frac{\omega_2}{\omega_1} \qquad sr_{34} = \frac{\omega_4}{\omega_3} \qquad sr_{56} = \frac{\omega_6}{\omega_5} \qquad sr_{78} = \frac{\omega_8}{\omega_7} \qquad sr_{89} = \frac{\omega_9}{\omega_8}$$

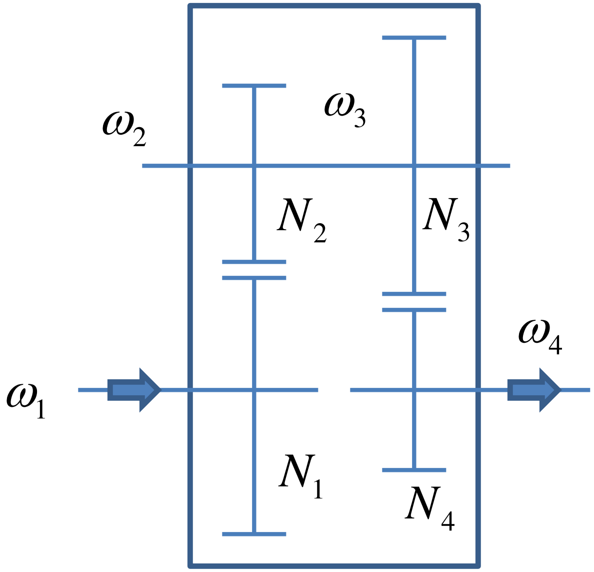
$$sr_{19} = \frac{\omega_9}{\omega_1} = \frac{\omega_9}{\omega_8}\frac{\omega_8}{\omega_7}\frac{\omega_7}{\omega_6}\frac{\omega_6}{\omega_5}\frac{\omega_5}{\omega_4}\frac{\omega_4}{\omega_3}\frac{\omega_3}{\omega_2}\frac{\omega_2}{\omega_1} = \frac{\omega_9}{\omega_8}\frac{\omega_8}{\omega_7}\frac{\omega_6}{\omega_5}\frac{\omega_4}{\omega_3}\frac{\omega_2}{\omega_1} = \frac{\omega_9}{\omega_7}\frac{\omega_6}{\omega_5}\frac{\omega_4}{\omega_3}\frac{\omega_2}{\omega_1}$$

$$\frac{\omega_7}{\omega_6} = \frac{\omega_5}{\omega_4} = \frac{\omega_3}{\omega_2} = 1$$

Gear #8 is idler gear

$$gr_{19} = \frac{N_7}{N_9}\frac{N_5}{N_6}\frac{N_3}{N_4}\frac{N_1}{N_2} = \frac{1}{sr_{19}}$$

Reverted compound gear train is a special case of a compound gear train. The input and the output gears in the reverted compound gear train however has same axis of rotation.



$$sr_{12} = \frac{\omega_2}{\omega_1} \qquad sr_{34} = \frac{\omega_4}{\omega_3}$$

$$sr_{14} = \frac{\omega_4}{\omega_1} = \frac{\omega_4}{\omega_3}\frac{\omega_3}{\omega_2}\frac{\omega_2}{\omega_1} = \frac{\omega_4}{\omega_3}\frac{\omega_2}{\omega_1} = \frac{N_3}{N_4}\frac{N_1}{N_2} \qquad \frac{\omega_3}{\omega_2} = 1$$

$$gr_{14} = \frac{1}{sr_{14}} = \frac{N_4}{N_3}\frac{N_2}{N_1}$$

Center distance between gears 1, 2 and 3, 4 are related

$$R_1 + R_2 = R_3 + R_4$$

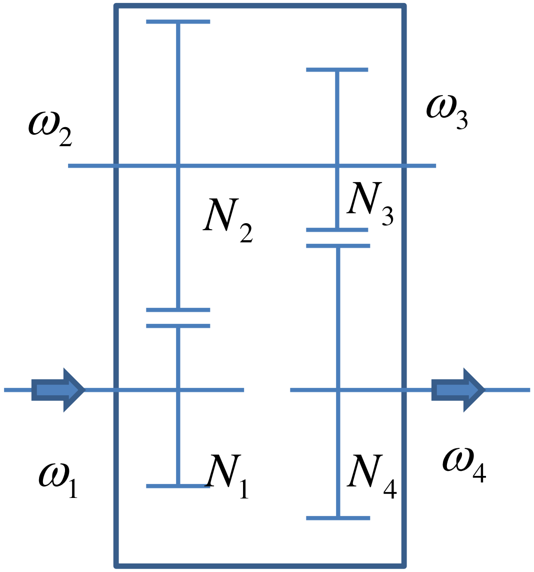
$$D = 2R = mN$$

If the module is same for all gears

$$N_1 + N_2 = N_3 + N_4$$

Example

We wish to use two pairs of gears in a train to obtain a speed ratio of 1/12. Tooth number should not be less than 15 and speed reduction obtained in the first pair of gears should be about twice that obtained for the second pair.



$$sr_{14} = \frac{\omega_4}{\omega_1} = \frac{\omega_4}{\omega_3}\frac{\omega_3}{\omega_2}\frac{\omega_2}{\omega_1} = \frac{\omega_4}{\omega_3}\frac{\omega_2}{\omega_1} = \frac{N_3}{N_4}\frac{N_1}{N_2} = \left(\frac{N_3}{N_4}\right)\left(\frac{1}{2}\frac{N_3}{N_4}\right) = \frac{1}{12}$$

$$\frac{1}{2}\left(\frac{N_3}{N_4}\right)^2 = \frac{1}{12} \qquad \frac{N_3}{N_4} = \sqrt{\frac{1}{6}} = 0.4082$$

Possible solutions

$$\frac{N_3}{N_4} = \frac{15}{37} = 0.4054 \quad \frac{N_3}{N_4} = \frac{16}{39} = 0.4102 \quad \frac{N_3}{N_4} = \frac{17}{42} = 0.4047 \quad \frac{N_3}{N_4} = \frac{18}{44} = 0.4091$$

$$\frac{N_3}{N_4} = \frac{19}{46} = 0.4130 \quad \frac{N_3}{N_4} = \frac{20}{49} = 0.4081 \quad \frac{N_3}{N_4} = \frac{21}{51} = 0.4117 \quad \frac{N_3}{N_4} = \frac{22}{54} = 0.4074$$

$$\text{good choice is } \frac{N_3}{N_4} = \frac{20}{49} = 0.4081$$

$$\frac{N_1}{N_2} = \frac{1}{2}\frac{N_3}{N_4} = \frac{1}{2}\frac{20}{49} = \frac{20}{98}$$

$$sr = \frac{20}{98} \times \frac{20}{49} = \frac{400}{4802} = \frac{1}{12.005} \text{ not exact}$$

Second solution

$$\frac{N_1}{N_2} = \frac{1}{2}\frac{N_3}{N_4} = \frac{1}{2}\frac{22}{54} = \frac{22}{108}$$

$$\frac{N_3}{N_4} = \frac{18}{44}$$

$$sr = \frac{22}{108} \times \frac{18}{44} = \frac{396}{4752} = \frac{1}{12} \text{ exact solution}$$

$$\frac{N_1}{N_2} = \frac{1}{2}\frac{N_3}{N_4}$$

$$\frac{22}{108} \neq \frac{1}{2}\left(\frac{18}{44}\right) = \frac{18}{88} \quad \text{second condition is not satisfied}$$

Example

The speed ratio of the reverted gear train is 1/12. Module of gears is same. Calculate suitable number of teeth for the gears. No gear should be less than 24 teeth.

$$sr_{14} = \frac{\omega_4}{\omega_1} = \frac{\omega_4}{\omega_3}\frac{\omega_3}{\omega_2}\frac{\omega_2}{\omega_1} = \frac{\omega_4}{\omega_3}\frac{\omega_2}{\omega_1} = \frac{N_3}{N_4}\frac{N_1}{N_2} = \frac{1}{12} = \left(\frac{1}{4}\right)\left(\frac{1}{3}\right)$$

$$N_1 + N_2 = N_3 + N_4$$

$$\frac{N_1}{N_2} = \frac{1}{3} \qquad \frac{N_3}{N_4} = \frac{1}{4}$$

$$N_2 = 3N_1 \qquad N_4 = 4N_3$$

$$N_1 + 3N_1 = N_3 + 4N_3$$

$$4N_1 = 5N_3$$

$$N_3 = \frac{4}{5}N_1$$

Possible solution #1

$$N_1 = 30$$
$$N_2 = 3N_1 = 3(30) = 90$$
$$N_3 = \frac{4}{5}30 = 24$$
$$N_4 = 4N_3 = 4(24) = 96$$

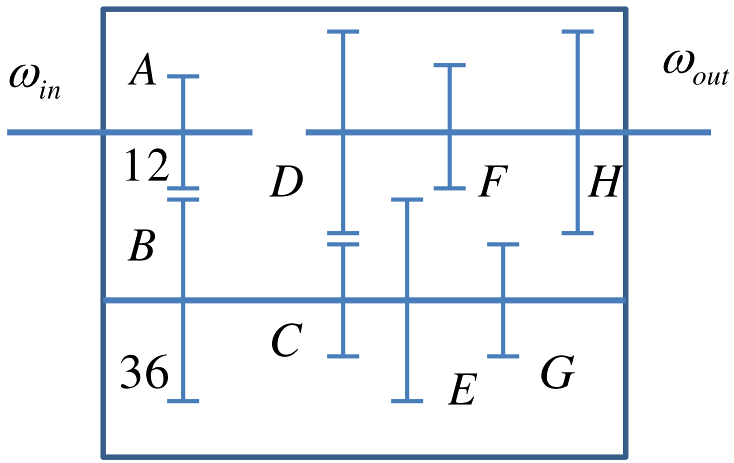
$$\frac{N_3}{N_4}\frac{N_1}{N_2} = \left(\frac{24}{96}\right)\left(\frac{30}{90}\right) = \frac{1}{12}$$

Possible solution #2

$$N_1 = 35$$
$$N_2 = 3N_1 = 3(35) = 105$$
$$N_3 = \frac{4}{5}35 = 28$$
$$N_4 = 4N_3 = 4(28) = 112$$

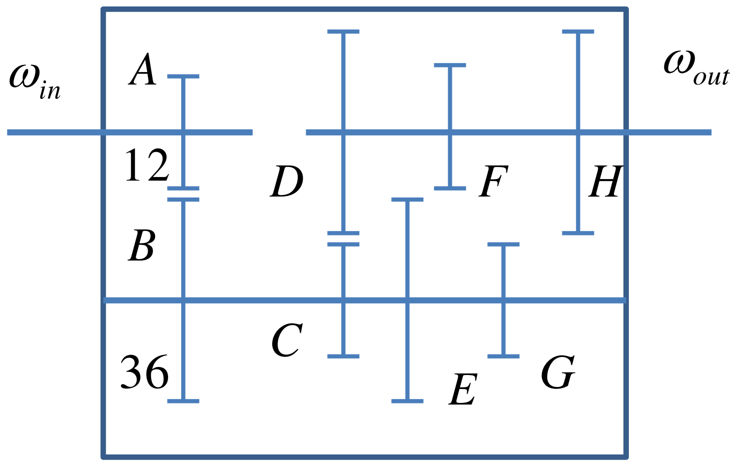
$$\frac{N_3}{N_4}\frac{N_1}{N_2} = \left(\frac{28}{112}\right)\left(\frac{35}{105}\right) = \frac{1}{12}$$

example



Required speed ratios for A-B-C-D is 0.2 for A-B-E-F is 0.43, for A-B-G-H is 0.24. Module is same for all gears, find suitable teeth numbers for gears C, D, E, F, G, H.

example



Required speed ratios for A-B-C-D is 0.2 for A-B-E-F is 0.43, for A-B-G-H is 0.24. Module is same for all gears, find suitable teeth numbers for gears C, D, E, F, G, H.

$$sr_{AD} = \frac{\omega_D}{\omega_A} = \frac{\omega_D}{\omega_C}\frac{\omega_C}{\omega_B}\frac{\omega_B}{\omega_A} = \frac{\omega_D}{\omega_C}\frac{\omega_B}{\omega_A} = \frac{N_C}{N_D}\frac{N_A}{N_B} = \frac{N_C}{N_D}\frac{12}{36} = 0.2$$

$$\frac{N_C}{N_D} = \frac{36}{12}0.2 = 0.6$$

$$N_C + N_D = N_E + N_F = N_G + N_H = N_A + N_B = 12 + 36 = 48$$

$$N_C + 0.6N_C = 1.6N_C = 48$$

$$N_C = \frac{48}{1.6} = 30$$

$$N_D = 48 - 30 = 18$$

$$sr_{AF} = \frac{\omega_F}{\omega_A} = \frac{N_E}{N_F}\frac{N_A}{N_B} = \frac{N_E}{N_F}\frac{12}{36} = 0.43$$

$$\frac{N_E}{N_F} = \frac{36}{12}0.43 = 1.29$$

$$N_F + 1.29N_F = 2.29N_F = 48$$

$$N_F = \frac{48}{2.29} = 21$$

$$N_E = 48 - 21 = 27$$

$$sr_{AH} = \frac{\omega_H}{\omega_A} = \frac{N_G}{N_H}\frac{N_A}{N_B} = \frac{N_G}{N_H}\frac{12}{36} = 0.24$$

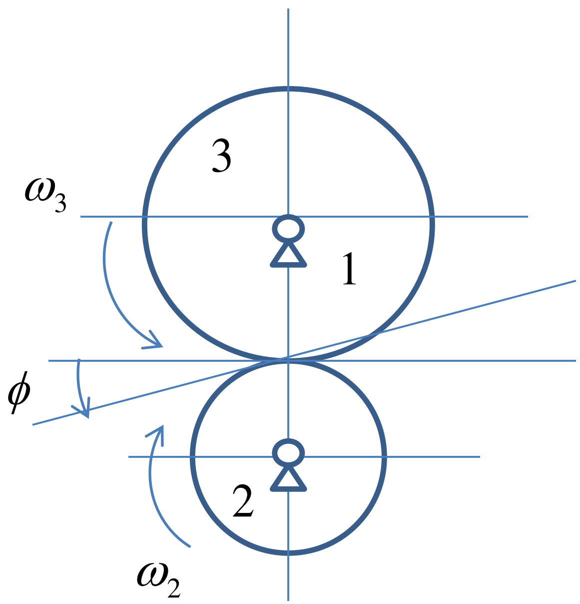
$$\frac{N_G}{N_H} = \frac{36}{12}0.24 = 0.72$$

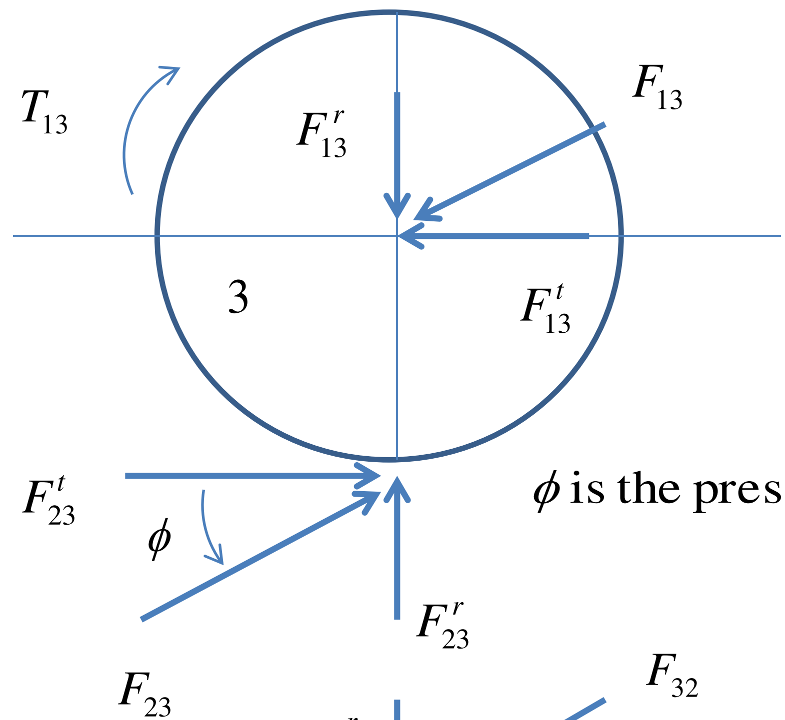
$$N_H + 0.72N_H = 1.72N_H = 48$$

$$N_H = \frac{48}{1.72} = 28$$

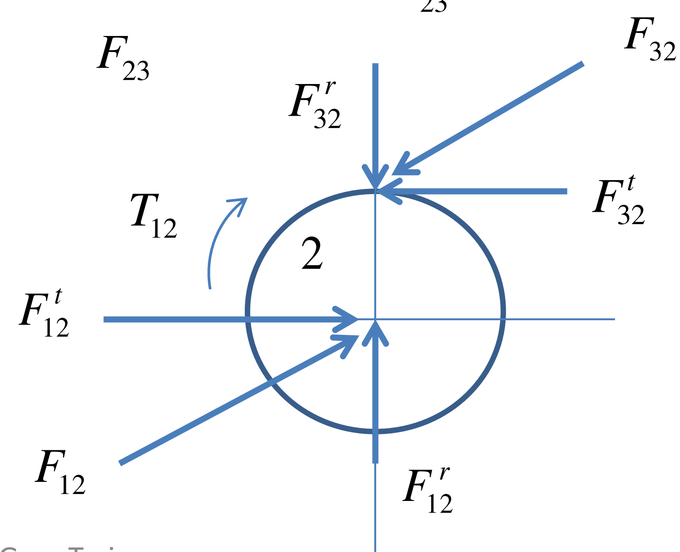
$$N_G = 48 - 28 = 20$$

Force analysis of spur gears





ϕ is the pressure angle



$$Power = T_{12}\omega_2 = T_{13}\omega_3$$

$P = F_{32}^t R_2 \omega_2$ if gear 2 is driver

$F_{32}^t = \dfrac{P}{R_2\omega_2}$ transmitted force

$F_{32}^r = F_{32}^t \tan\phi$ radial force $\quad F_{32} = \dfrac{F_{32}^t}{\cos\phi}$

$$F_{12} = F_{32}$$
$$F_{23} = F_{13}$$

Example

A gearbox having one pair of spur gears is attached to an electric motor of having 2 kW power and 1800 rpm speed. Pressure angle of the gears is 20 degree. Pitch radius of the driver gear is 15 cm, pitch radius of driven gear is 25 cm. Find forces acting on the gears and shafts of the gearbox. What are the torques on each shaft?

$$\omega_2 = 1800\frac{2\pi}{60} = 188.4 \text{ rad/s}$$

$$F_{32}^t = \frac{P}{R_2\omega_2} = \frac{2000 \text{ Nm/s}}{(0.15m)(188.4 rad/s)} = 70.77 \text{ N}$$

$$F_{32}^r = F_{32}^t \tan\phi = 70.77 \times \tan 20 = 25.76 \text{N}$$

$$F_{32} = \frac{F_{32}^t}{\cos\phi} = \frac{70.77}{\cos 20} = 75.31N$$

$$T_2 = F_{32}^t R_2 = 70.77N \times 0.15m = 10.6Nm$$

$$T_3 = F_{32}^t R_3 = 70.77N \times 0.25m = 17.69Nm$$

Force analysis of helical gears

example

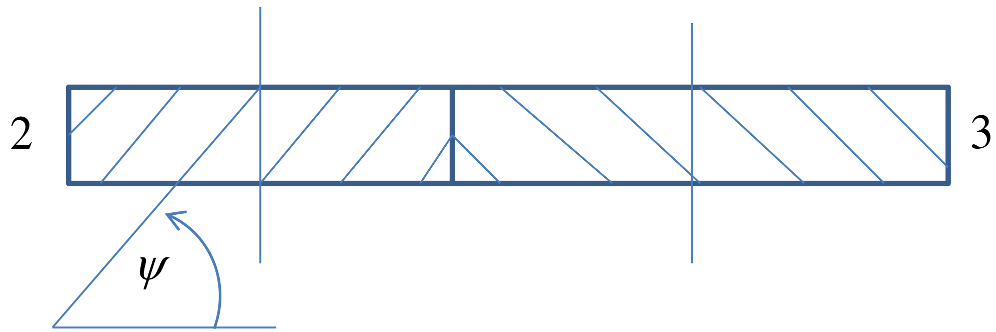
$D_2 = 60mm$

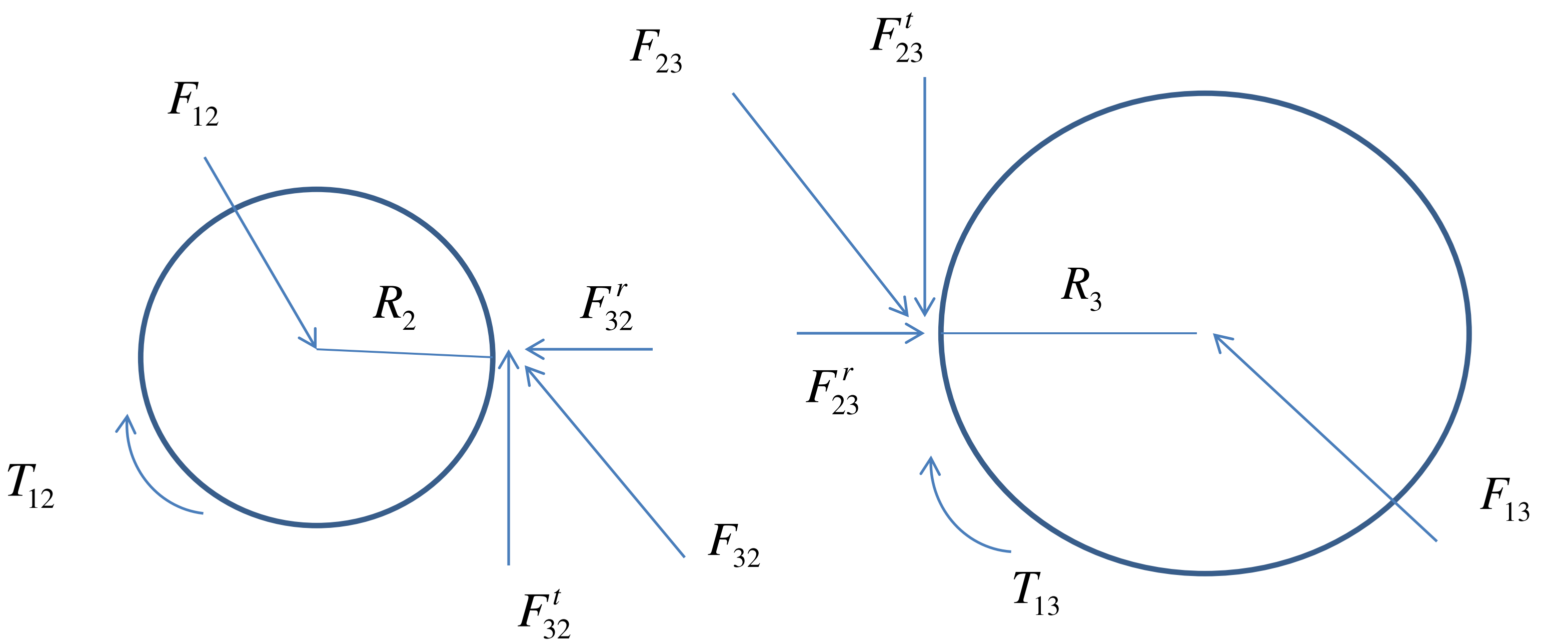
$D_3 = 90mm$

$T_2 = 5Nm$

$\phi = 20°$ pressure angle

$\psi = 30°$ helix angle





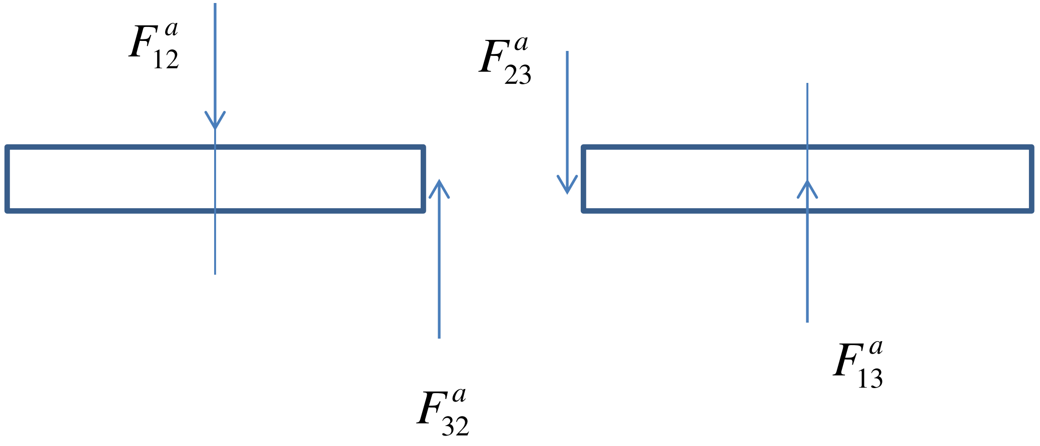
$$F_{32}^{t} = \frac{T_{12}}{R_2} = \frac{5\text{Nm}}{0.03\text{m}} = 166.7\text{N} \quad \text{transmitted force}$$

$$F_{32}^{r} = F_{32}^{t} \tan\phi = 166.7 \times \tan 20 = 60.6\text{N} \quad \text{radial force}$$

$$F_{32} = \frac{F_{32}^{t}}{\cos\phi} = \frac{166.7}{\cos 20} = 177.4N$$

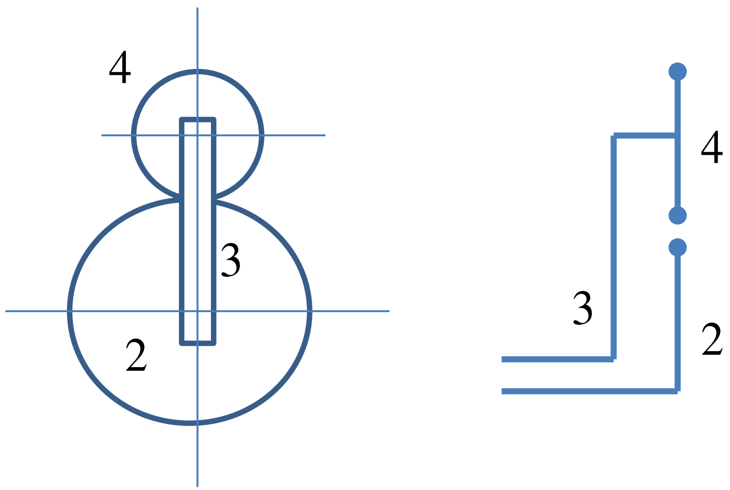
$$T_{13} = F_{23}^{t} R_3 = 166.7N \times 0.045m = 7.5Nm$$

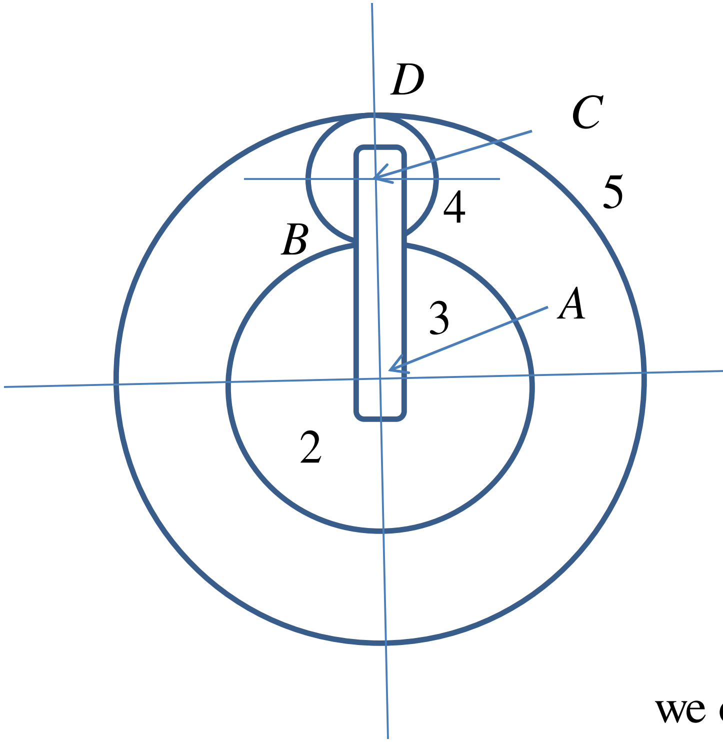
$$F_{32}^{a} = F_{32}^{t} \tan\psi = 166.7N \times \tan 30 = 96.2N \quad \text{axial force}$$



Planetary or Epicyclic Gear Trains

Figure shows the elementary epicyclic gear train together with the simplified designation for these used by Levai. The train consists of a central gear 2 and a epicyclic gear 4, which produces epicyclic motion by rolling around the periphery of the central gear. The crank arm 3 contains the bearings for the epicyclic gear to maintain the two gear wheel in mesh. These trains are also called *planetary* or *sun-and-planet* gear trains. In this nomenclature gear 2 is the *sun gear*, gear 4 is the *planet gear*, and crank 3 is called the *planet carrier*.





$$V_D = \omega_5 R_5$$

$$V_B = \omega_2 R_2$$

$$V_C = \omega_3 (R_2 + R_4)$$

$$\omega_2 R_2 + \omega_5 R_5 = 2\omega_3 (R_2 + R_4)$$

$$R_2 + 2R_4 = R_5$$

$$\omega_2 R_2 + \omega_5 R_5 = \omega_3 (2R_2 + 2R_4)$$

$$\omega_2 R_2 + \omega_5 R_5 = \omega_3 (R_2 + R_5)$$

we can use tooth number instead of pitch radius

$$\omega_2 N_2 + \omega_5 N_5 = \omega_3 (N_2 + N_5)$$

if arm is fixed $\omega_3 = 0$

$$\omega_2 N_2 + \omega_5 N_5 = 0$$

$$\omega_5 = -\frac{N_2}{N_5}\omega_2 \quad \text{or}$$

$$\omega_2 = -\frac{N_5}{N_2}\omega_5$$

$$V_B = V_C + V_{B/C}$$

$$V_D = V_C + V_{D/C}$$

$$V_{B/C} = -V_{D/C}$$

$$V_B - V_C = -(V_D - V_C)$$

$$V_B + V_D = 2V_C$$

$$\text{if sun is fixed } \omega_2 = 0$$

$$\omega_5 N_5 = \omega_3 (N_2 + N_5)$$

$$\omega_5 = \left(\frac{N_2 + N_5}{N_5} \right) \omega_3 = \left(1 + \frac{N_2}{N_5} \right) \omega_3$$

$$\text{if ring is fixed } \omega_5 = 0$$

$$\omega_2 N_2 = \omega_3 (N_2 + N_5)$$

$$\omega_2 = \left(\frac{N_2 + N_5}{N_2} \right) \omega_3 = \left(1 + \frac{N_5}{N_2} \right) \omega_3$$

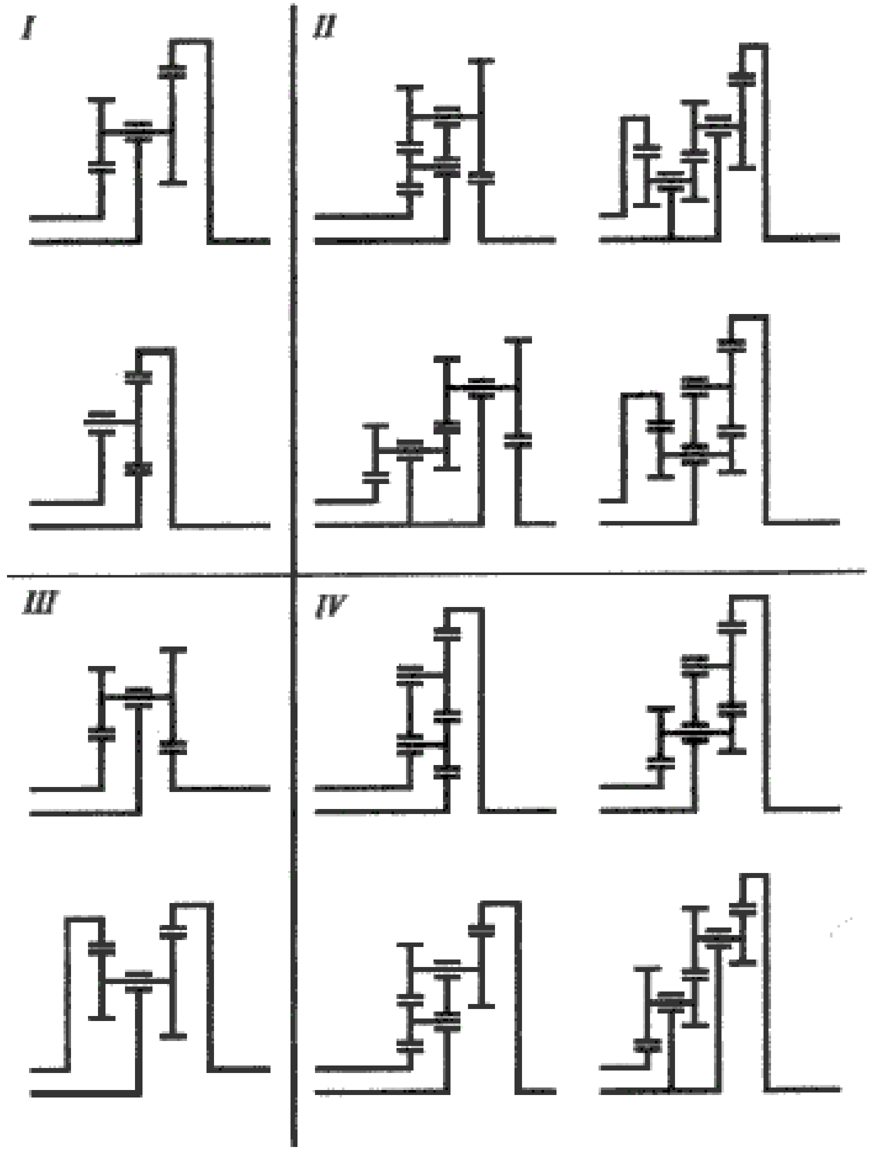
Example

Sun is rotating with the speed of 100 rpm ccw, arm is rotating with the speed of 200 rpm cw. What is the speed of ring? Teeth number for ring is 80, for sun is 40 for planet is 20.

$$\omega_2 N_2 + \omega_5 N_5 = \omega_3 (N_2 + N_5)$$

$$(100)(40) + \omega_5 (80) = (-200)(40 + 80)$$

$$\omega_5 = -350\, rpm\, (\text{cw})$$



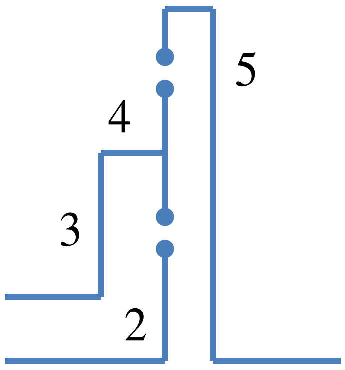
Possible combinations of epicyclic gear trains

Calculation method using table

Once the epicyclic elements of a simple epicyclic gear train have been identified, analysis is done as follows:

- Lock the train (freeze the elements so that no relative movement is possible) and rotate the entire train one positive turn.
- Hold (fix) the arm, and rotate the stationary (fixed gear by one negative turn. Note down the number of rotations of relevant gears.
- Add the resultant revolutions.

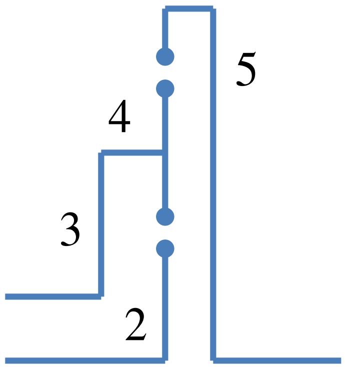
Example #1



#2 (sun) is fixed

	Sun	Arm	Planetary	Ring
Steps	#2	#3	#4	#5
#1 Gears locked	+1	+1	+1	+1
#2 Arm fixed	-1	0	$+\frac{N_2}{N_4}$	$+\frac{N_2}{N_4}\frac{N_4}{N_5}$
#3 Results	0	+1	$1+\frac{N_2}{N_4}$	$1+\frac{N_2}{N_5}$

$$\omega_5 = \left(1+\frac{N_2}{N_5}\right)\omega_3$$



#5 (ring) is fixed

	Sun	Arm	Planetary	Ring
Steps	#2	#3	#4	#5
#1 Gears locked	+1	+1	+1	+1
#2 Arm fixed	$+\frac{N_5}{N_4}\frac{N_4}{N_2}$	0	$-\frac{N_5}{N_4}$	-1
#3 Results	$1+\frac{N_5}{N_2}$	+1	$1-\frac{N_5}{N_4}$	0

$$\omega_2 = \left(1+\frac{N_5}{N_2}\right)\omega_3$$

#3 (arm) is fixed

$$sr_{25} = \frac{\omega_5}{\omega_2} = \frac{\omega_5}{\omega_4}\frac{\omega_4}{\omega_2} = \frac{N_4}{N_5}\frac{N_2}{N_4} = \frac{N_2}{N_5}$$

$$\omega_5 = -\frac{N_2}{N_5}\omega_2$$

Ring speed is opposite to the sun speed that is why (–) sign is needed

If #5 is output

$$\omega_5 = \left(1 + \frac{N_2}{N_5}\right)\omega_3 - \frac{N_2}{N_5}\omega_2$$

If #2 is output

$$\omega_2 = \left(1 + \frac{N_5}{N_2}\right)\omega_3 - \frac{N_5}{N_2}\omega_5$$

If #3 is output

$$\omega_3 = \frac{N_5}{N_2 + N_5}\omega_5 + \frac{N_2}{N_2 + N_5}\omega_2$$

Constraint between gear tooth numbers

$d = 2r = mN$ assume module m is same

$$r_2 + 2r_4 = r_5$$

$$\frac{N_2}{2} + N_4 = \frac{N_5}{2}$$

Example

$$N_2 = 40T$$
$$N_4 = 15T$$
$$N_5 = 70T$$

a) $\omega_3 = 100 \text{ rad/s ccw}$

$\omega_2 = 200 \text{ rad/s ccw}$

$\omega_5 = ?$

$$\omega_5 = \left(1 + \frac{N_2}{N_5}\right)\omega_3 - \frac{N_2}{N_5}\omega_2$$

$$\omega_5 = (1 + \frac{40}{70})(100) - \frac{40}{70}(200) = (1.5714)(100) - (0.5714)(200) = 42.86 \text{ rad/s}$$

b)

$$\omega_3 = 100 \text{ rad/s ccw}$$
$$\omega_2 = 200 \text{ rad/s cw}$$
$$\omega_5 = ?$$

$$\omega_5 = (1+\frac{40}{70})(100) - \frac{40}{70}(-200) = (1.5714)(100) - (0.5714)(-200) = 271.42 \text{ rad/s}$$

c)

$$\omega_3 = 100 \text{ rad/s cw}$$
$$\omega_2 = 200 \text{ rad/s cw}$$
$$\omega_5 = ?$$

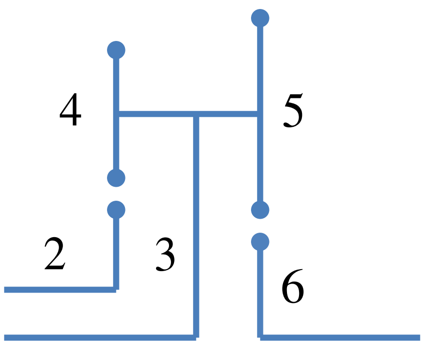
$$\omega_5 = (1+\frac{40}{70})(-100) - \frac{40}{70}(-200) = (1.5714)(-100) - (0.5714)(-200) = -42.86 \text{ rad/s}$$

d)

$$\omega_3 = 100 \text{ rad/s cw}$$
$$\omega_2 = 200 \text{ rad/s ccw}$$
$$\omega_5 = ?$$

$$\omega_5 = (1+\frac{40}{70})(-100) - \frac{40}{70}(200) = (1.5714)(-100) - (0.5714)(200) = -271.42 \text{ rad/s}$$

Example #2



#2 (sun) is fixed

	Sun	Arm	Sun
Steps	#2	#3	#6
#1 Gears locked	+1	+1	+1
#2 Arm fixed	-1	0	$-\frac{N_2}{N_4}\frac{N_5}{N_6}$
#3 Results	0	+1	$1-\frac{N_2}{N_4}\frac{N_5}{N_6}$

$$\omega_6 = \left(1-\frac{N_2}{N_4}\frac{N_5}{N_6}\right)\omega_3$$

#6 (sun) is fixed

	Sun	Arm	Sun
Steps	#2	#3	#6
#1 Gears locked	+1	+1	+1
#2 Arm fixed	$-\frac{N_4}{N_2}\frac{N_6}{N_5}$	0	-1
#3 Results	$1-\frac{N_4}{N_2}\frac{N_6}{N_5}$	+1	0

$$\omega_2 = \left(1-\frac{N_4}{N_2}\frac{N_6}{N_5}\right)\omega_3$$

#3 (arm) is fixed

$$sr_{26} = \frac{\omega_6}{\omega_2} = \frac{\omega_6}{\omega_5}\frac{\omega_5}{\omega_4}\frac{\omega_4}{\omega_2}$$

$$= \frac{\omega_6}{\omega_5}\frac{\omega_4}{\omega_2} = \frac{N_5}{N_6}\frac{N_2}{N_4}$$

$$\omega_6 = \frac{N_5}{N_6}\frac{N_2}{N_4}\omega_2$$

If #6 is output

$$\omega_6 = \left(1 - \frac{N_2 N_5}{N_4 N_6}\right)\omega_3 + \frac{N_2 N_5}{N_4 N_6}\omega_2$$

If #2 is output

$$\omega_2 = \left(1 - \frac{N_4 N_6}{N_2 N_5}\right)\omega_3 + \frac{N_4 N_6}{N_2 N_5}\omega_6$$

If #3 is output

$$\omega_3 = \frac{N_2 N_5}{N_2 N_5 - N_4 N_6}\omega_2 + \frac{N_4 N_6}{N_4 N_6 - N_2 N_5}\omega_6$$

Constraint between gear tooth numbers

$$d = 2r = mN \quad \text{assume module m is same}$$

$$r_2 + r_4 = r_5 + r_6$$

$$N_2 + N_4 = N_5 + N_6$$

Example

$$N_2 = 50T$$

$$N_4 = 15T$$

$$N_5 = 25T$$

$$N_6 = 40T$$

a)

$$\omega_3 = 100 \text{ rad/s cw}$$

$$\omega_2 = 100 \text{ rad/s cw}$$

$$\omega_6 = ?$$

$$\omega_6 = \left(1 - \frac{N_2 N_5}{N_4 N_6}\right)\omega_3 + \frac{N_2 N_5}{N_4 N_6}\omega_2$$

$$\omega_6 = (1 - \frac{50\times 25}{15\times 40})(-100) - \frac{50\times 25}{15\times 40}(-100) = (-1.0833)(-100) - (2.0833)(-100) = -100 \,\text{rad/s}$$

b) $\omega_3 = 100$ rad/s cw

$\omega_2 = 100$ rad/s ccw

$\omega_6 = ?$

$$\omega_6 = \left(1 - \frac{N_2 N_5}{N_4 N_6}\right)\omega_3 + \frac{N_2 N_5}{N_4 N_6}\omega_2$$

$$\omega_6 = (1 - \frac{50\times 25}{15\times 40})(-100) - \frac{50\times 25}{15\times 40}(100) = (-1.0833)(-100) - (2.0833)(100) = 316.66 \text{ rad/s}$$

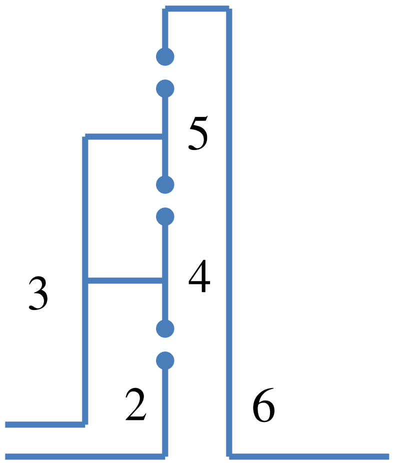
c) $\omega_3 = 100$ rad/s ccw

$\omega_2 = 100$ rad/s cw

$\omega_6 = ?$

$$\omega_6 = (1 - \frac{50\times 25}{15\times 40})(100) - \frac{50\times 25}{15\times 40}(-100) = (-1.0833)(100) - (2.0833)(-100) = -316.66 \text{ rad/s}$$

Example #3



#2 (sun) is fixed

	Sun	Arm	Ring
Steps	#2	#3	#6
#1 Gears locked	+1	+1	+1
#2 Arm fixed	-1	0	$-\frac{N_2}{N_4}\frac{N_4}{N_5}\frac{N_5}{N_6}$
#3 Results	0	+1	$1-\frac{N_2}{N_6}$

$$\omega_6 = \left(1-\frac{N_2}{N_6}\right)\omega_3$$

#6 (sun) is fixed

	Sun	Arm	Ring
Steps	#2	#3	#6
#1 Gears locked	+1	+1	+1
#2 Arm fixed	$-\frac{N_6}{N_5}\frac{N_5}{N_4}\frac{N_4}{N_2}$	0	-1
#3 Results	$1-\frac{N_6}{N_2}$	+1	0

$$\omega_2 = \left(1-\frac{N_6}{N_2}\right)\omega_3$$

#3 (arm) is fixed

$$sr_{26} = \frac{\omega_6}{\omega_2} = \frac{\omega_6}{\omega_5}\frac{\omega_5}{\omega_4}\frac{\omega_4}{\omega_2}$$

$$= \frac{N_5}{N_6}\frac{N_4}{N_5}\frac{N_2}{N_4} = \frac{N_2}{N_6}$$

$$\omega_6 = \frac{N_2}{N_6}\omega_2$$

If #6 is output

$$\omega_6 = \left(1 - \frac{N_2}{N_6}\right)\omega_3 + \frac{N_2}{N_6}\omega_2$$

If #2 is output

$$\omega_2 = \left(1 - \frac{N_6}{N_2}\right)\omega_3 + \frac{N_6}{N_2}\omega_6$$

If #3 is output

$$\omega_3 = \frac{N_2}{N_2 - N_6}\omega_2 + \frac{N_6}{N_6 - N_2}\omega_6$$

or

$$\omega_3 = \frac{N_6\omega_6 - N_2\omega_2}{N_6 - N_2}$$

Constraint between gear tooth numbers

$$d = 2r = mN \quad \text{assume module m is same}$$

$$r_2 + 2r_4 + 2r_5 = r_6$$

$$\frac{N_2}{2} + N_4 + N_5 = \frac{N_6}{2}$$

Example

$$N_2 = 60T$$
$$N_4 = 20T$$
$$N_5 = 20T$$
$$N_6 = 140T$$

a)

$$\omega_3 = 100 \text{ rad/s cw}$$
$$\omega_2 = 100 \text{ rad/s cw}$$
$$\omega_6 = ?$$

$$\omega_6 = \left(1 - \frac{N_2}{N_6}\right)\omega_3 + \frac{N_2}{N_6}\omega_2$$

$$\omega_6 = (1 - \frac{60}{140})(-100) - \frac{60}{140}(-100) = (0.5714)(-100) - (0.4285)(-100) = -100 \text{ rad/s}$$

b) $\omega_3 = 100 \text{ rad/s cw}$

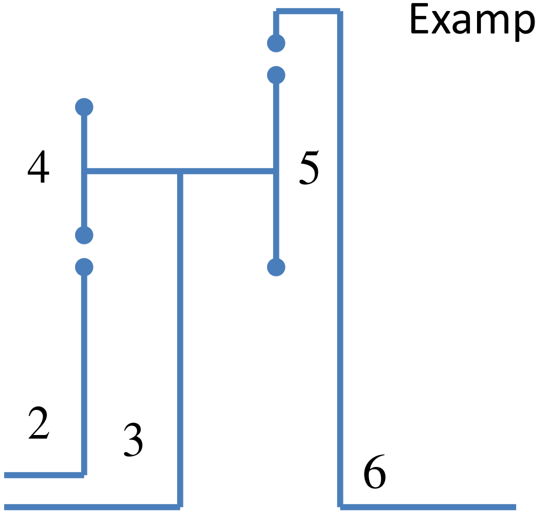
$\omega_2 = 100 \text{ rad/s ccw}$

$\omega_6 = ?$

$$\omega_6 = \left(1 - \frac{N_2}{N_6}\right)\omega_3 + \frac{N_2}{N_6}\omega_2$$

$$\omega_6 = (1 - \frac{60}{140})(-100) - \frac{60}{140}(100) = (0.5714)(-100) - (0.4285)(100) = -14.29 \text{ rad/s}$$

Example #4



#2 (sun) is fixed

	Sun	Arm	Ring
Steps	#2	#3	#6
#1 Gears locked	+1	+1	+1
#2 Arm fixed	-1	0	$+\frac{N_2}{N_4}\frac{N_5}{N_6}$
#3 Results	0	+1	$1+\frac{N_2}{N_4}\frac{N_5}{N_6}$

$$\omega_6 = \left(1 + \frac{N_2}{N_4}\frac{N_5}{N_6}\right)\omega_3$$

#6 (sun) is fixed

	Sun	Arm	Ring
Steps	#2	#3	#6
#1 Gears locked	+1	+1	+1
#2 Arm fixed	$+\frac{N_6}{N_5}\frac{N_4}{N_2}$	0	-1
#3 Results	$1+\frac{N_6}{N_5}\frac{N_4}{N_2}$	+1	0

$$\omega_2 = \left(1 + \frac{N_6}{N_5}\frac{N_4}{N_2}\right)\omega_3$$

#3 (arm) is fixed

$$sr_{26} = \frac{\omega_6}{\omega_2} = \frac{\omega_6}{\omega_5}\frac{\omega_5}{\omega_4}\frac{\omega_4}{\omega_2}$$

$$= -\frac{N_5}{N_6}\frac{N_2}{N_4}$$

$$\omega_6 = -\frac{N_5}{N_6}\frac{N_2}{N_4}\omega_2$$

If #6 is output

$$\omega_6 = \left(1 + \frac{N_2 N_5}{N_4 N_6}\right)\omega_3 - \frac{N_2 N_5}{N_4 N_6}\omega_2$$

If #2 is output

$$\omega_2 = \left(1 + \frac{N_4 N_6}{N_2 N_5}\right)\omega_3 - \frac{N_4 N_6}{N_2 N_5}\omega_6$$

If #3 is output

$$\omega_3 = \frac{N_2 N_5}{N_2 N_5 + N_4 N_6}\omega_2 + \frac{N_4 N_6}{N_2 N_5 + N_4 N_6}\omega_6$$

Constraint between gear tooth numbers

$$d = 2r = mN \quad \text{assume module m is same}$$

$$r_2 + r_4 = r_6 - r_5$$

$$N_2 + N_4 = N_6 - N_5$$

Example

$$N_2 = 40T$$

$$N_4 = 20T$$

$$N_5 = 30T$$

$$N_6 = 90T$$

a) $\omega_3 = 200 \text{ rad/s ccw}$

$\omega_2 = 100 \text{ rad/s cw}$

$\omega_6 = ?$

$$\omega_6 = \left(1 + \frac{N_2 N_5}{N_4 N_6}\right)\omega_3 - \frac{N_2 N_5}{N_4 N_6}\omega_2$$

$$\omega_6 = (1 + \frac{40\times 30}{20\times 90})(200) - \frac{40\times 30}{20\times 90}(-100) = (1.6666)(200) - (0.6666)(-100) = 399.98 \text{ rad/s}$$

b)

$$\omega_3 = 200 \text{ rad/s cw}$$
$$\omega_2 = 100 \text{ rad/s cw}$$
$$\omega_6 = ?$$

$$\omega_6 = \left(1 + \frac{N_2 N_5}{N_4 N_6}\right)\omega_3 - \frac{N_2 N_5}{N_4 N_6}\omega_2$$

$$\omega_6 = (1 + \frac{40\times 30}{20\times 90})(-200) - \frac{40\times 30}{20\times 90}(-100) = (1.6666)(-200) - (0.6666)(-100) = -266.66\,\text{rad/s}$$

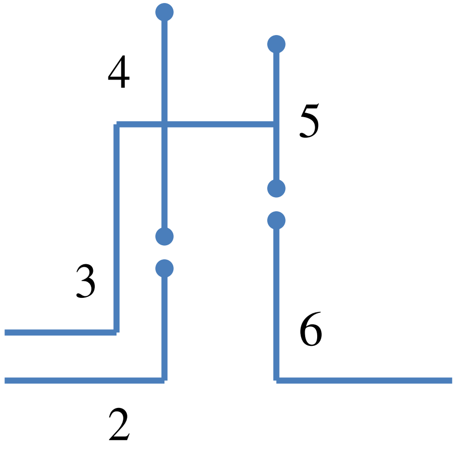
c)

$$\omega_3 = 100 \text{ rad/s cw}$$
$$\omega_6 = 100 \text{ rad/s cw}$$
$$\omega_2 = ?$$

$$\omega_2 = \left(1 + \frac{N_4 N_6}{N_2 N_5}\right)\omega_3 - \frac{N_4 N_6}{N_2 N_5}\omega_6$$

$$\omega_2 = (1 + \frac{20\times 90}{40\times 30})(-100) - \frac{20\times 90}{40\times 30}(-100) = (2.5)(-200) - (1.5)(-100) = -100\,\text{rad/s}$$

Force analysis



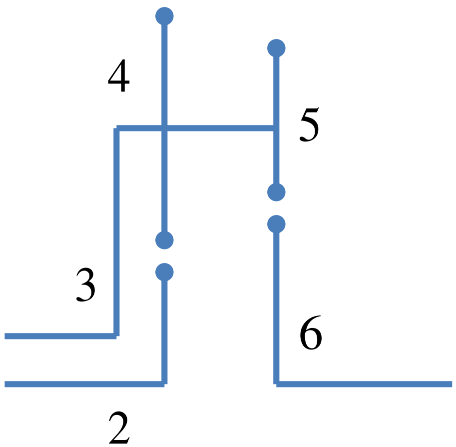
$$D_2 = 40mm \qquad N_2 = 20T$$

$$D_4 = 60mm \qquad N_4 = 30T \qquad \phi = 20°$$

$$D_5 = 32mm \qquad N_5 = 16T \qquad T_{12} = 10Nm$$

$$D_6 = 68mm \qquad N_6 = 34T$$

Force analysis

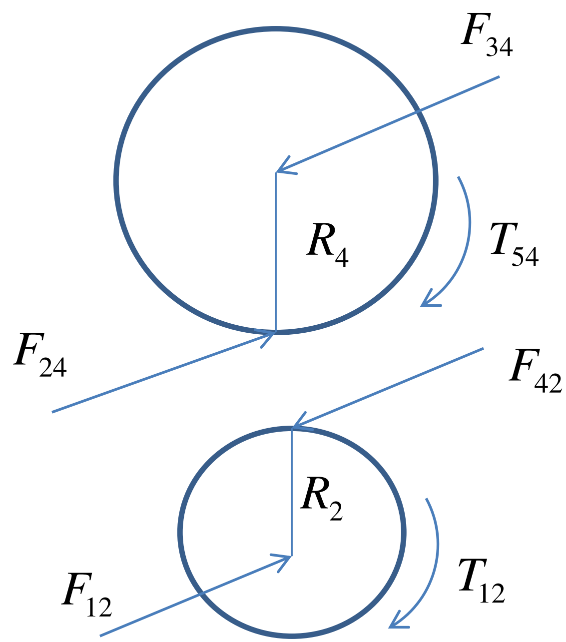


$$D_2 = 40mm \qquad N_2 = 20T$$

$$D_4 = 60mm \qquad N_4 = 30T \qquad \phi = 20°$$

$$D_5 = 32mm \qquad N_5 = 16T \qquad T_{12} = 10Nm$$

$$D_6 = 68mm \qquad N_6 = 34T$$

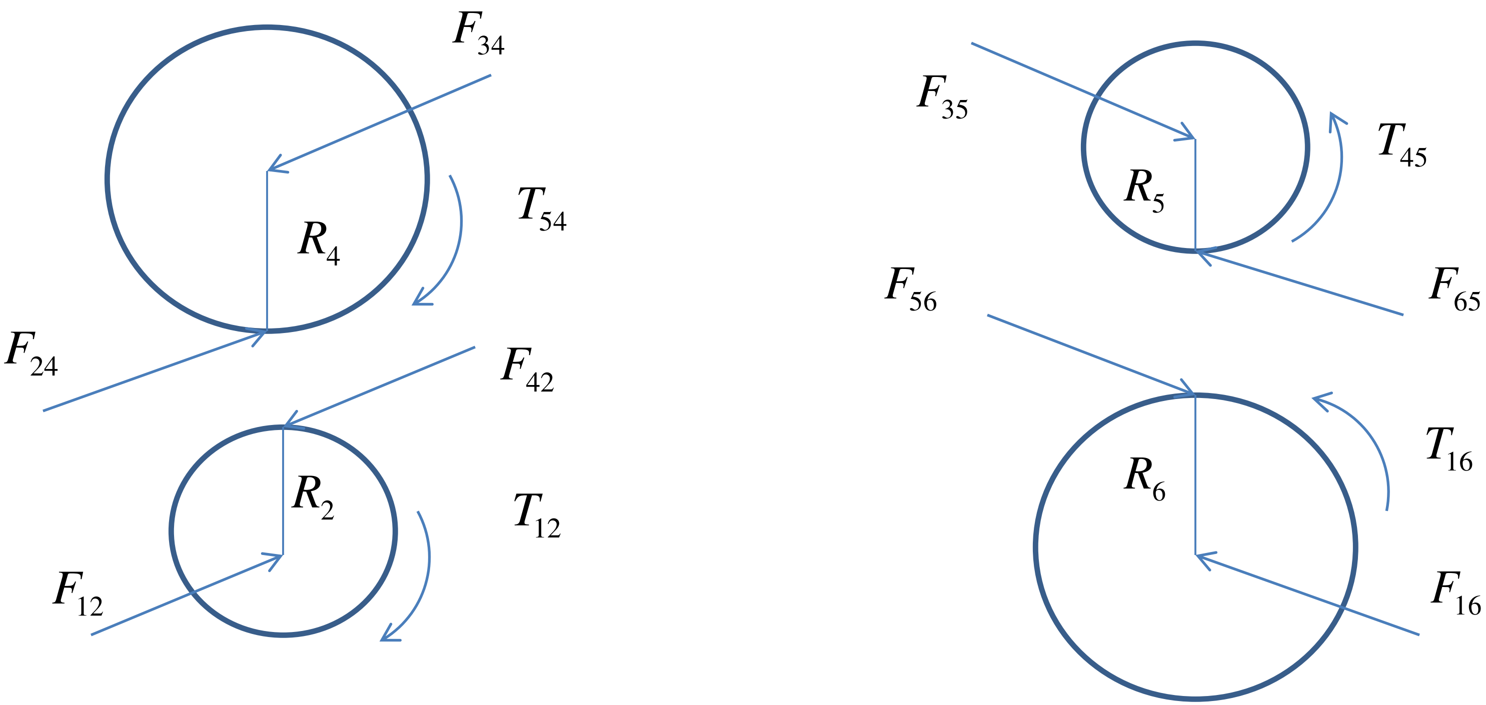


$$F_{42}^t = \frac{T_{12}}{R_2} = \frac{10Nm}{0.02m} = 500N$$

$$F_{42}^r = F_{43}^t \tan\phi = 500 \tan 20 = 182N$$

$$F_{42} = \frac{F_{42}^t}{\cos\phi} = \frac{500Nm}{\cos 20} = 532N$$

$$F_{24} = -F_{42} \qquad F_{24}^t = -F_{42}^t \qquad F_{24}^r = -F_{42}^r$$



$$T_{54} = F_{24}^t R_4 = 500N \times 0.03m = 15Nm$$

$$T_{45} = -T_{54}$$

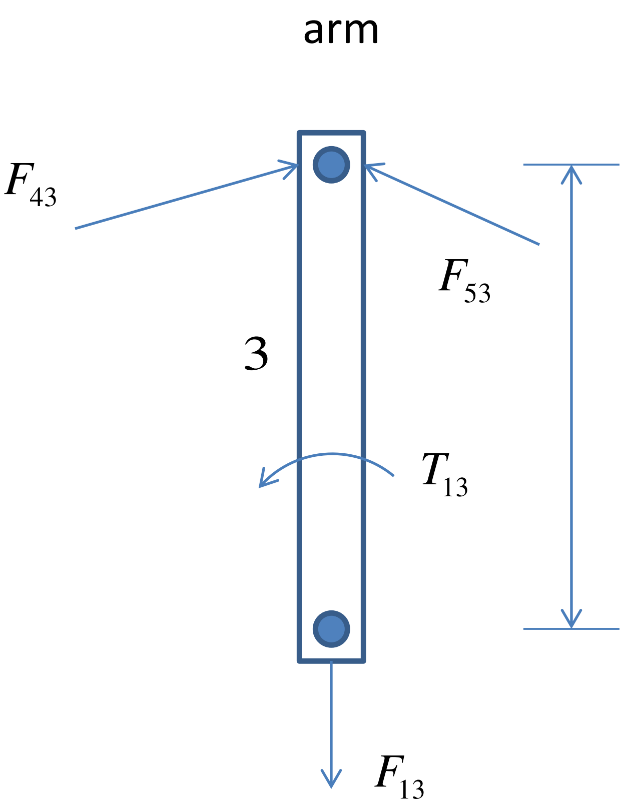
$$F_{65}^t = \frac{T_{45}}{R_5} = \frac{15Nm}{0.016m} = 937.5N$$

$$F_{65}^r = F_{65}^t \tan\phi = 937.5 \tan 20 = 341.2N$$

$$F_{56} = -F_{65} \qquad F_{16} = -F_{56}$$

$$T_{16} = F_{56}^t R_6 = 937.5N \times 0.034m = 31.87Nm$$

$$F_{65} = \frac{F_{65}^t}{\cos\phi} = \frac{937.5Nm}{\cos 20} = 997.6N$$



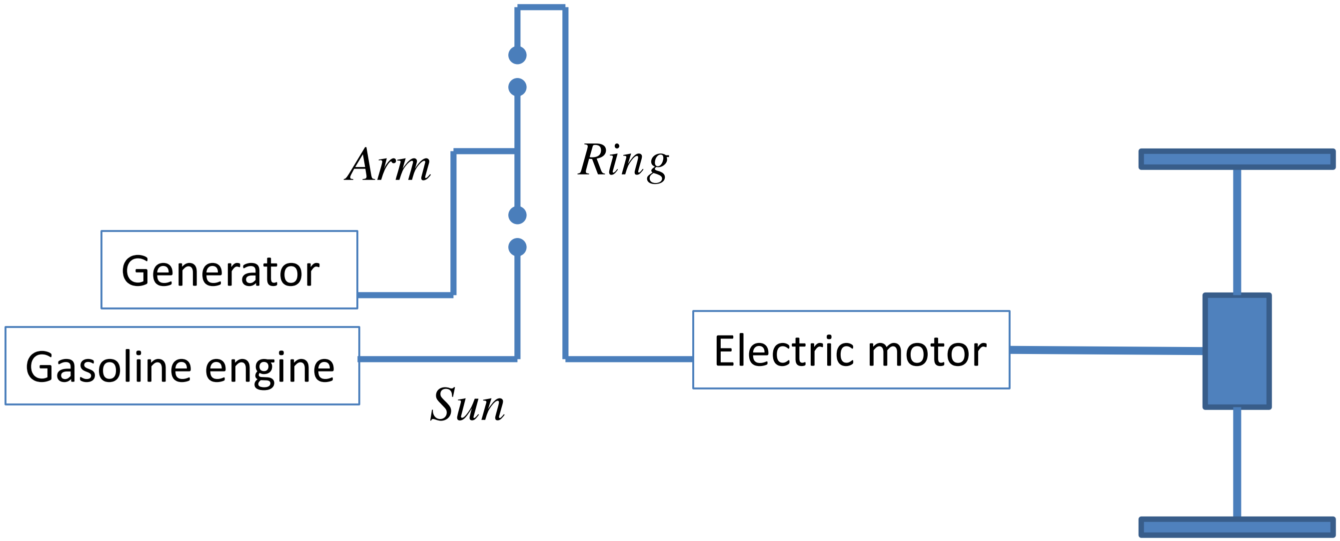
$$L = R_2 + R_4 = 20 + 30 = 50mm$$

$$T_{13} = F_{53}\cos\phi \times L - F_{43}\cos\phi \times L = 997.6 \times \cos 20 \times 0.05 - 532 \times \cos 20 \times 0.05 = 21.87Nm$$

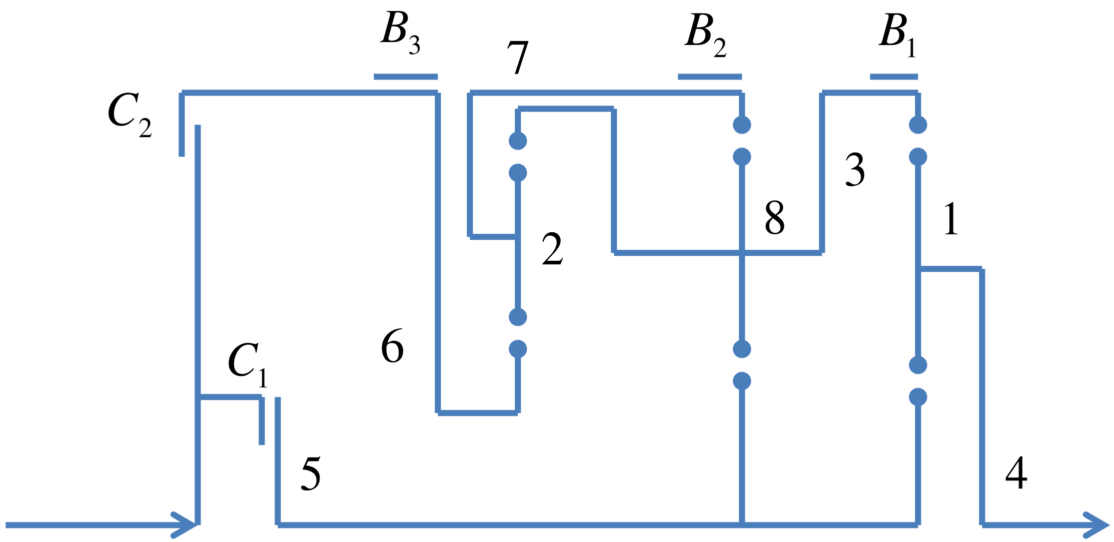
$$F_{13} = F_{53}\sin\phi + F_{43}\sin\phi = 997.6 \times \sin 20 + 532 \times \sin 20 = 523N$$

Example from real applications:

The power split device is the heart of the Toyota Prius. This is a clever gearbox that hooks the gasoline engine, generator and electric motor together. It allows the car to operate like a parallel hybrid. The electric motor can power the car by itself, the gas engine can power the car by itself or they can power the car together. The power split device also allows the car to operate like a series hybrid. The gasoline engine can operate independently of the vehicle speed, charging the batteries or providing power to the wheels as needed. It also acts as a continuously variable transmission (CVT) eliminating the need for a manual or automatic transmission. Finally because the power split device allows the generator to start the engine, the car does not need a starter.



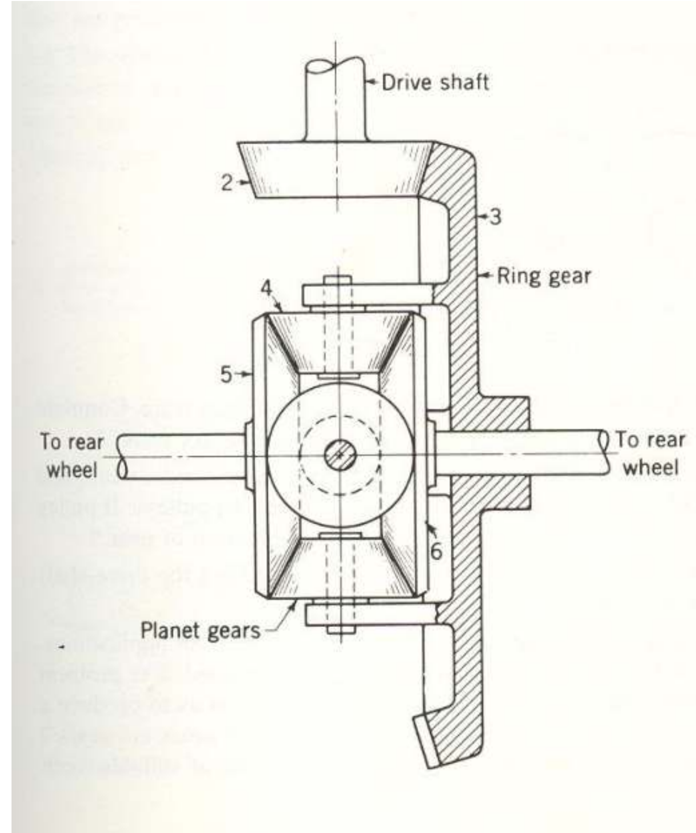
Example from real applications



Schematic of 4-speed automatic transmission

Range	Clutch/Brake activation				
	C_1	C_2	B_1	B_2	B_3
First	X		X		
Second	X			X	
Third	X				X
Fourth	X	X			
Reverse		X	X		

Example from real applications



bevel-gear automotive differential.

Automotive differential

The class of planetary gear trains known as differentials is used so widely that it deserves special attention. The operation of a differential is illustrated by the schematic drawing. The drive shaft pinion and the ring gear are normally hypoid gears. The ring gear acts as the planet carrier, and its speed can be calculated as for a simple gear train when the speed of the drive shaft is given. Gears 5 and 6 are connected, respectively, to each rear wheel, and when the car is travelling in a straight line, these two gears rotate in the same direction with exactly the same speed. Thus for straight line motion of the car, there is no relative motion between the planet gears and gears 5 and 6. The planet gears, in effect, serve only as keys to transmit motion from the planet carrier to both wheels.

When the vehicle is making a turn, the wheel on the inside of the turn makes fewer revolutions than the wheel with a longer turning radius. Unless this difference in speed is accommodated in some manner, one or both of the tires would have to slide in order to make the turn. The differential permits each wheel to rotate at different velocities while at the same time delivering power to both. During a turn, the planet gears rotate about their own axes, thus permitting gears 5 and 6 to revolve at different velocities.