Jazan University
Mechanical Engineering Department

CHAPTER 7

CAM DESIGN

EngM271 Theory of Machines

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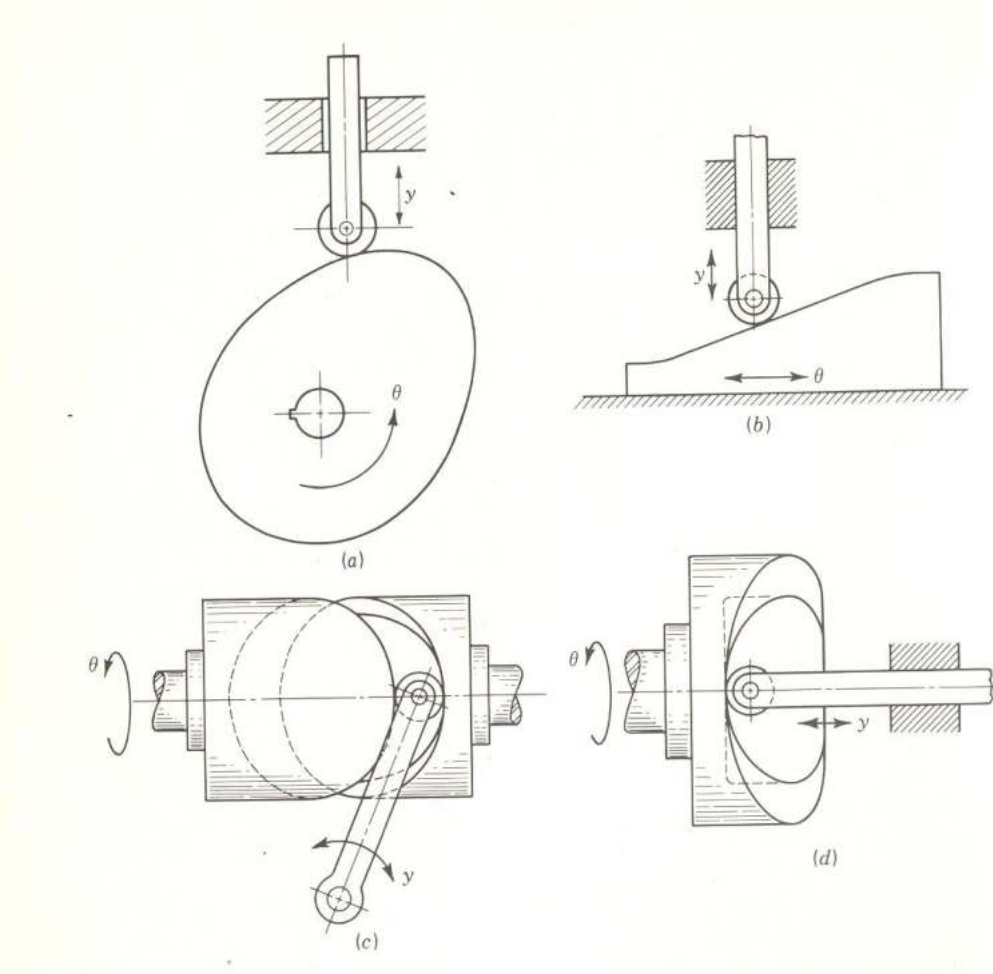
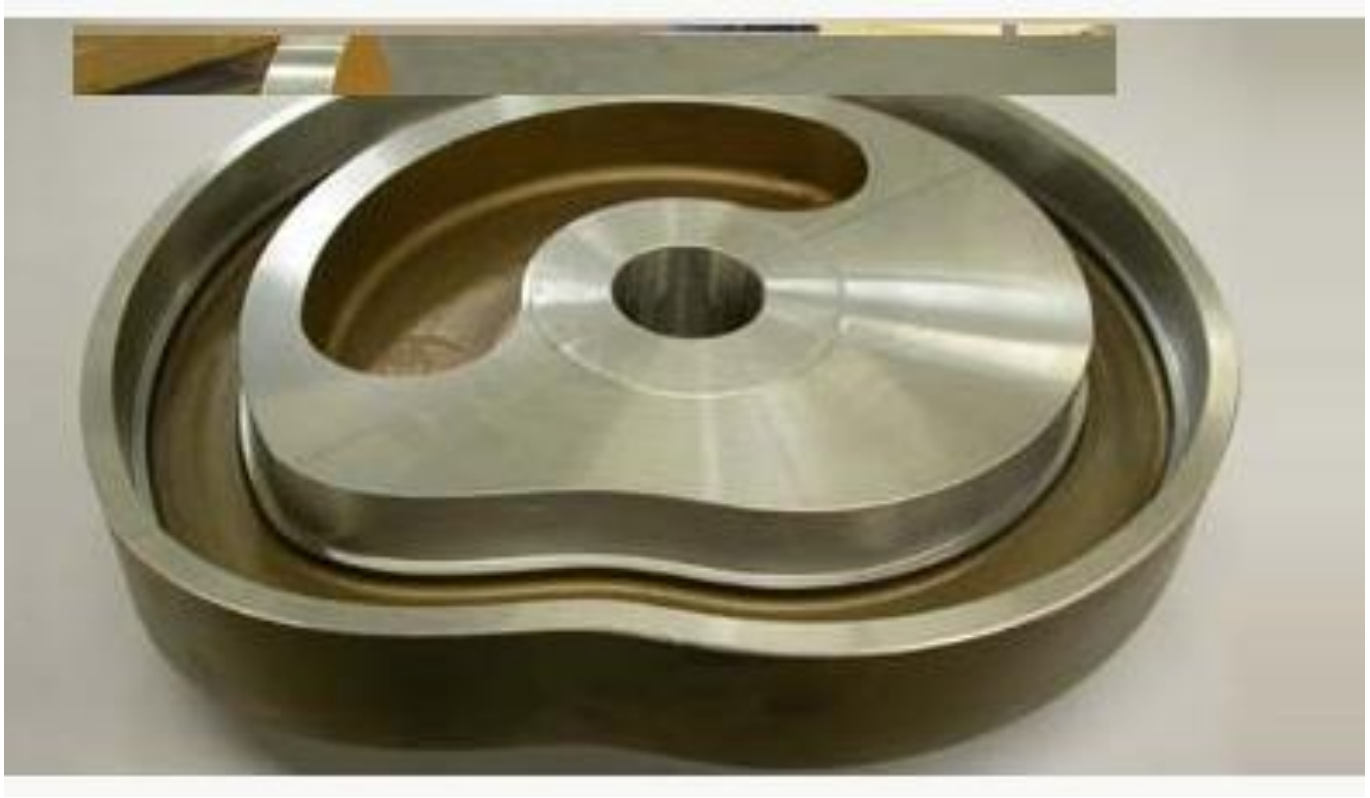
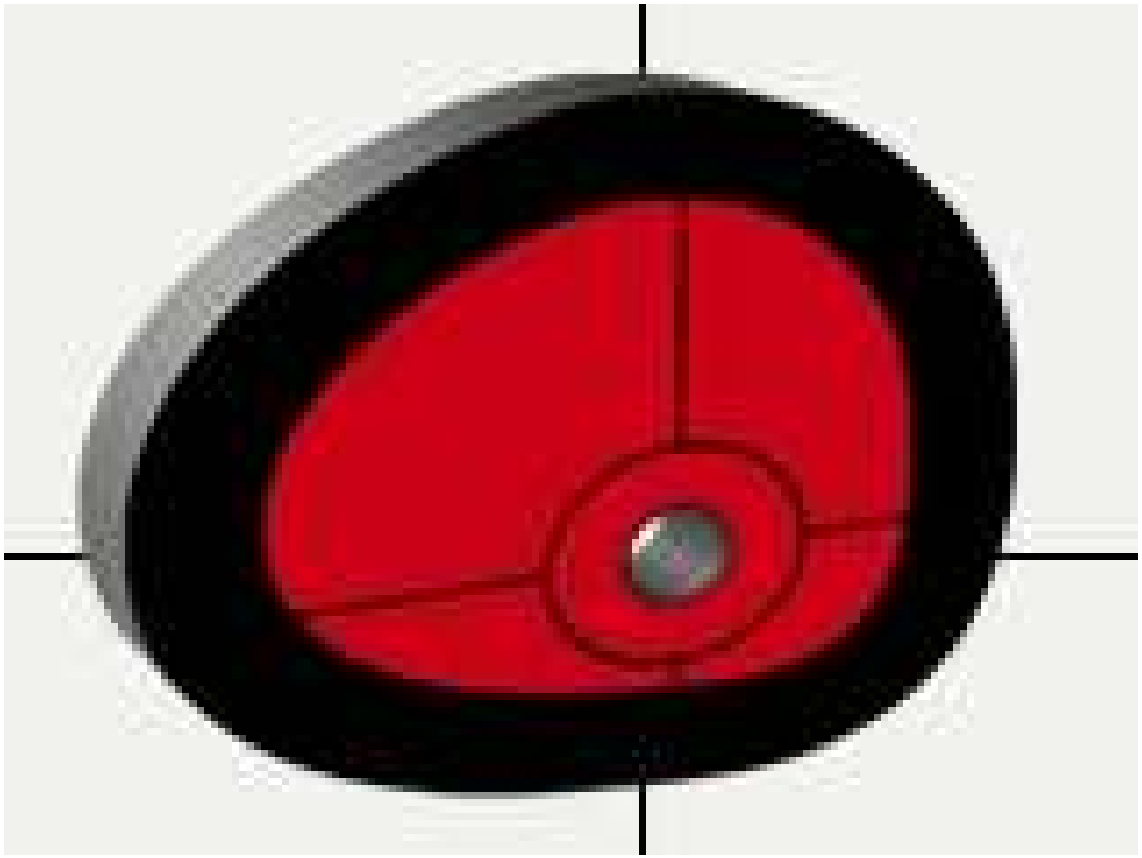
A **cam** is a mechanical element used to derive another element, called the follower, through a specified motion by direct contact. **Cam** and **follower** mechanisms are simple and inexpensive, have few moving parts, and occupy a very small space. Furthermore, follower motions having almost and desired characteristics are not difficult to design. For these reasons cam mechanisms are used extensively in modern machinery.

Cams are classified according to their basic shapes

1. A plate cam, also called a disk cam or a radial cam (a)
2. A wedge cam (b)
3. A cylindrical cam or barrel cam (c)
4. An end cam or face cam (d)

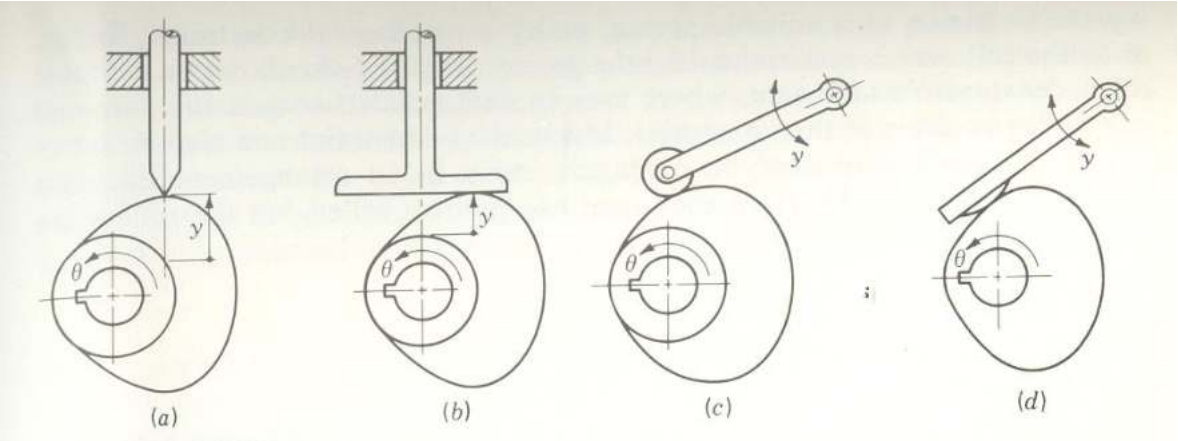

y
θ
(a)
y
θ
(b)
θ
y
(c)
θ
y
(d)



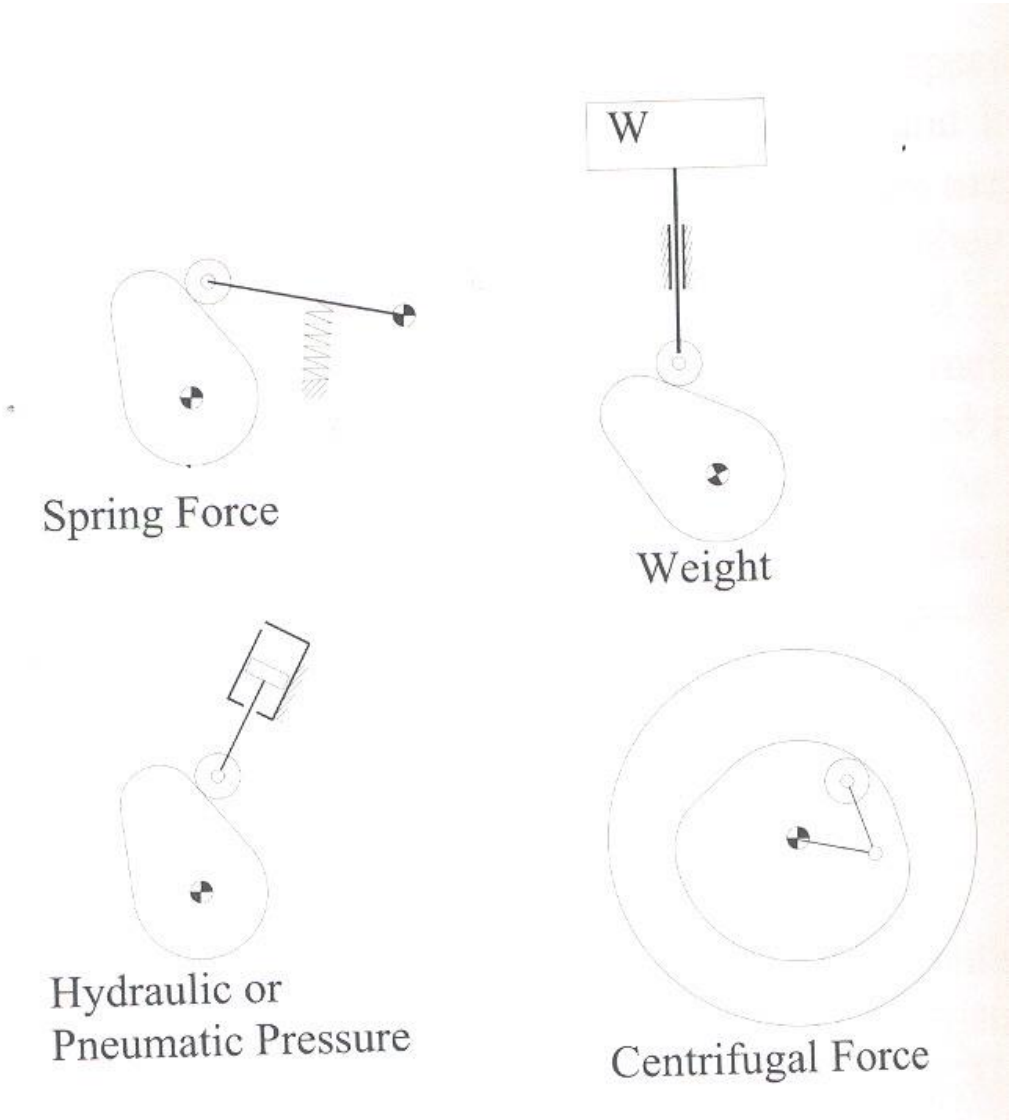


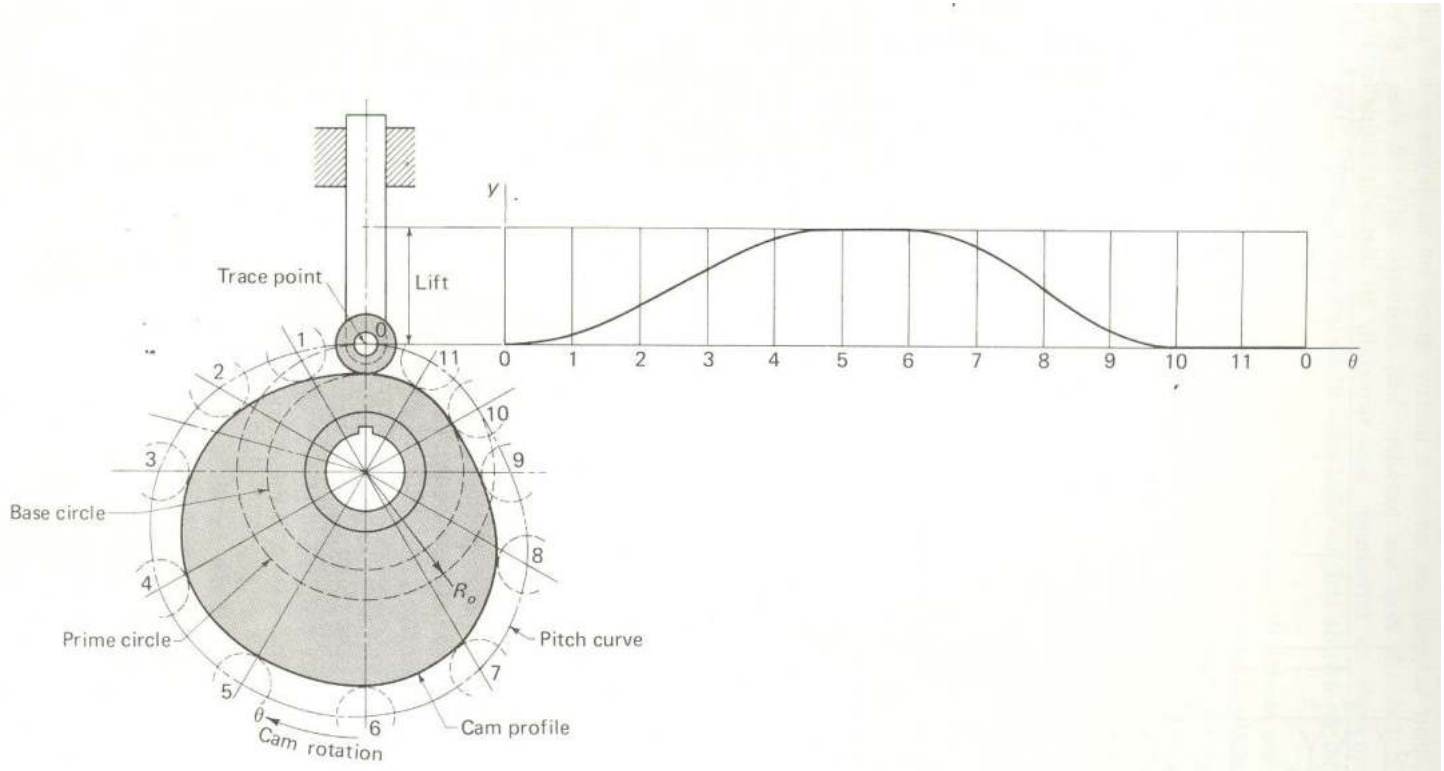
Cam systems can also be classified according to the basic shape of the follower

1. A knife edge follower (a)
2. A flat face follower (b)
3. A roller follower (c)
4. A spherical face or curved shoe follower (d)



a) An offset reciprocating knife-edge follower
b) Reciprocating flat-face follower
c) An oscillating roller follower
d) An oscillating curved-shoe follower


W
Spring Force
Weight
Hydraulic or
Pneumatic Pressure
Centrifugal Force

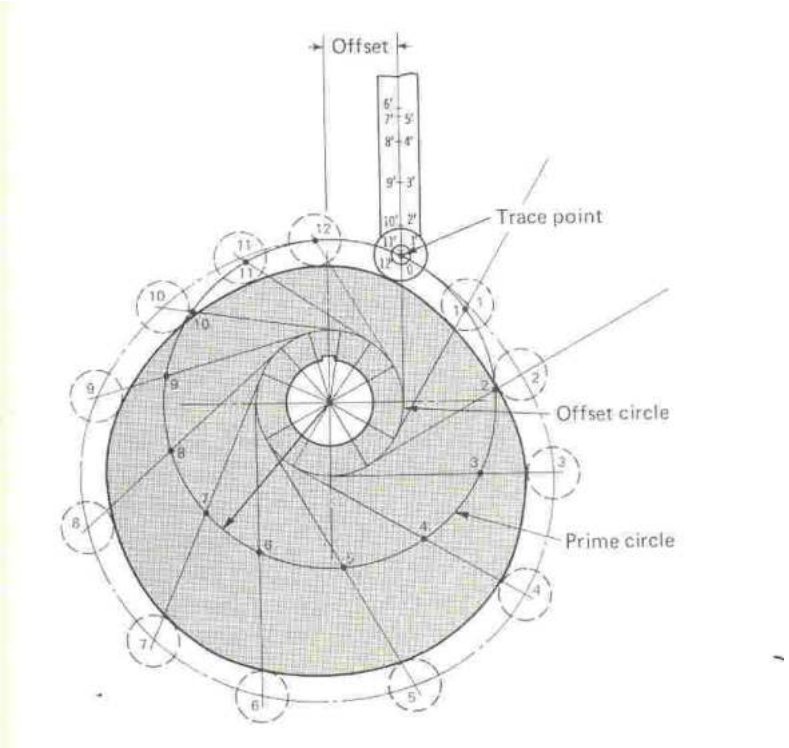

y
Trace point
Lift
0
1
2
3
4
5
6
7
8
9
10
11
0 1 2 3 4 5 6 7 8 9 10 11 0 θ
Base circle
Prime circle
Pitch curve
Cam profile
R_o
θ
Cam rotation

The Trace point is a theoretical point of the follower; it corresponds to the point of a fictitious knife-edge follower. It is chosen at the center of a roller follower or on the surface of a flat-face follower.

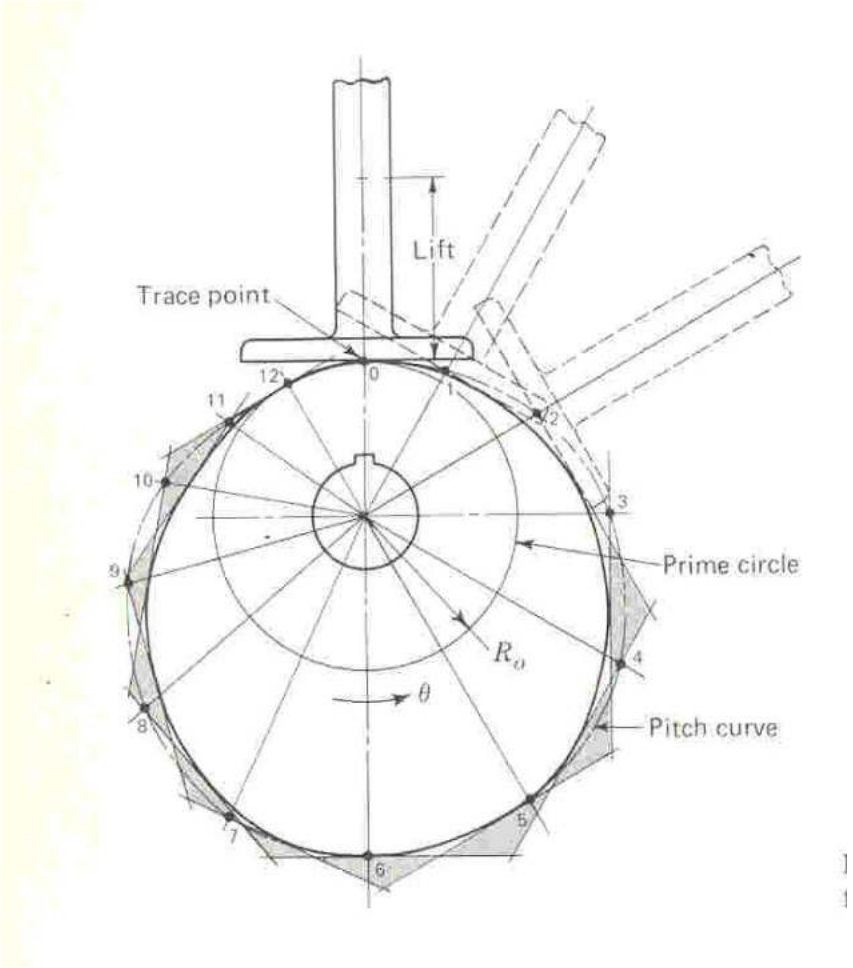
The pitch curve is the locus generated by the trace point as the follower moves relative to the cam. For a knife-edge follower the pitch curve and cam surface are identical. For a roller follower they are separated by the radius of the roller.

The prime circle is the smallest circle which can be drawn with center at the cam rotation axis and tangent to the pitch curve. The radius of this circle is R_0.

The base circle is the smallest circle centered on the cam rotation axis and tangent to the cam surface. For a roller follower it is smaller than the prime circle by the radius of the roller, and for a flat-face follower it is identical with the prime circle.

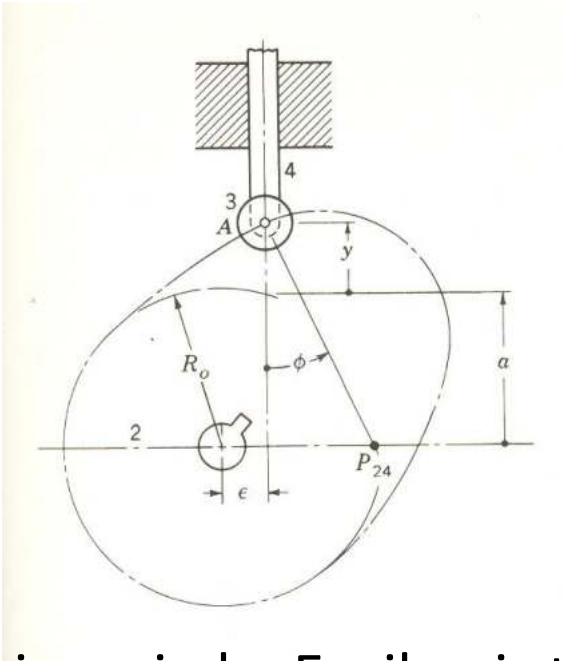


Offset reciprocating roller follower



Reciprocating flat-face follower

Plate cam with reciprocating roller follower



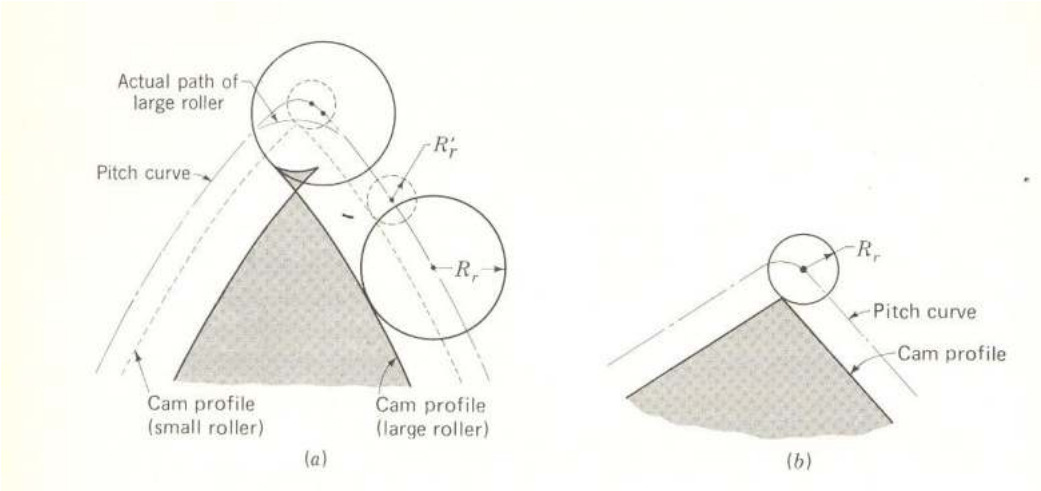
R_0 is the radius of prime circle, Epsilon is the eccentricity and Phi is the pressure angle.

The **pressure angle** is the angle between the axis of the follower stem and the line of the force exerted by the cam into the roller follower, the normal to the pitch curve through the trace point.

Only the component of force along the line of motion of the follower is useful in overcoming the output load; perpendicular component should be kept low to reduce sliding friction between the follower and its guide way. Too high a pressure angle will increase the effect of friction and may cause the translating follower to chatter or perhaps even to jam. Cam pressure angles of up to about 30 to 35 degrees are about the largest that can be used without causing difficulties.

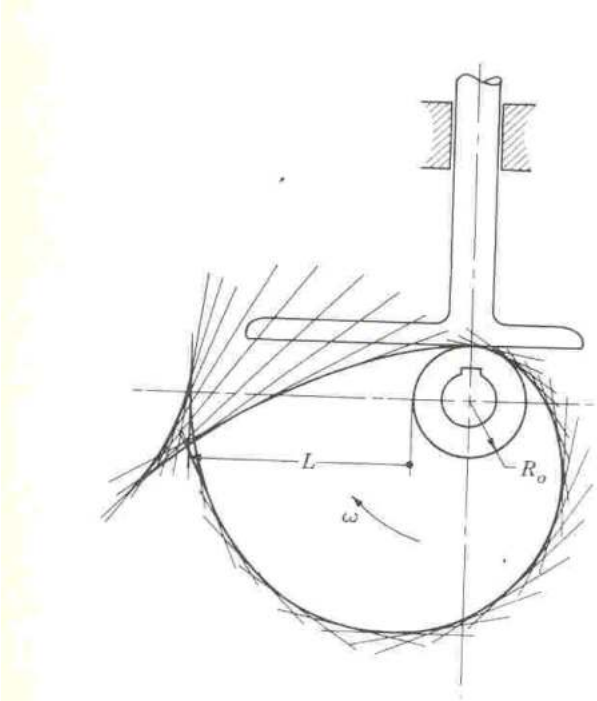
$$\phi = \tan^{-1} \frac{y' - \epsilon}{\sqrt{R_0^2 - \epsilon^2 + y}}$$

We see that increasing epsilon either increases or decreases the magnitude of the numerator depending on the sign of y'. Thus a small eccentricity can be used to reduce the pressure angle Phi during the rise motion when y' is positive but only at the expense of an increased pressure angle during the return motion when y' is negative. A much more significant effect can be made in reducing the pressure angle by increasing the prime circle radius R_0.



Even though the prime circle has been proportioned to give a satisfactory pressure angle, the follower may still not complete the desired motion; if the curvature of the pitch curve is too sharp, the cam profile may be **undercut**. The cam profile generated by the larger roller is undercut and doubles over itself. The result after machining is a pointed cam which does not produce the desired motion. It is also clear from the same figure that a smaller roller moving on the same pitch curve generates a satisfactory cam profile. Similarly, if the prime circle and thus the cam size is increased enough, the larger roller will operate satisfactorily.

Plate cam with reciprocating flat-face follower

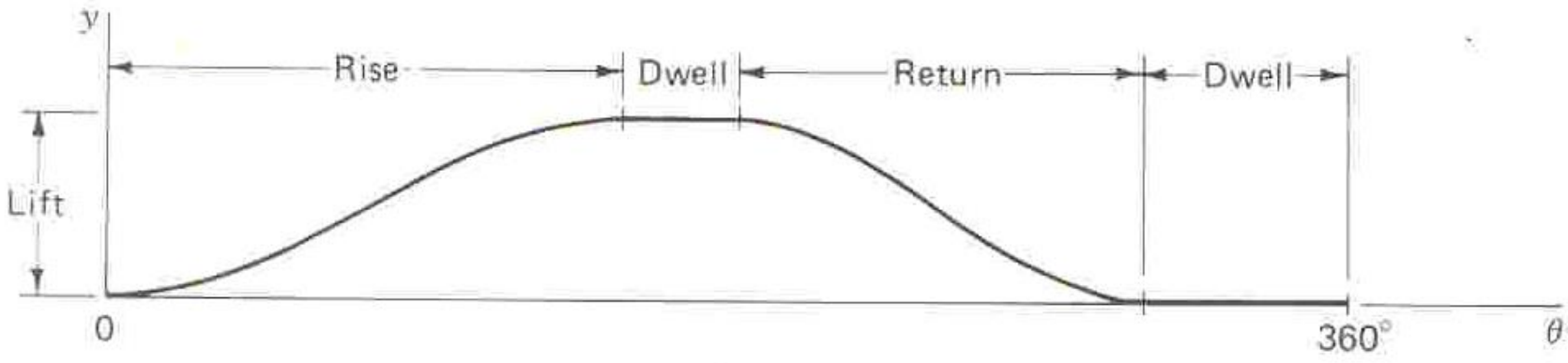


Cam profile crosses over itself. In machining, part of the cam shape would be lost and thereafter the intended motion would not be achieved. Such a cam is said to be **undercut.**

Undercutting may be resulted from attempting to achieve too great a lift in too little cam rotation with too small a cam. One possibility to decrease the desired lift L or to increases the cam rotation to avoid the problem. However, this may not be possible while still achieving the design objectives. Another solution is to use the same displacement characteristics but increase the prime circle radius R_0. This will produce a larger cam but a sufficient increase will overcome the undercutting problem.

The minimum face width for the flat-face follower must extend at least y'_{max} to the right and $-y'_{min}$ to the left of the cam center to maintain contact. That is,

$$Face\ width > y'_{max} - y'_{min}$$



Displacement diagram

SOME STANDARD CAM MOTIONS

1. Constant velocity
2. Constant acceleration
3. Cycloidal
4. Modified harmonic
5. Modified trapezoid
6. Modified sine
7. 345-polynomial
8. 4567-polynomial

displacement $y = y(\theta)$ $\theta = \theta(t)$

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velocity $\dot{y} = \frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt} = y'\omega$ $\frac{d\theta}{dt} = \omega$

acceleration $\ddot{y} = \frac{d^2y}{dt^2} = y''\omega^2 + y'\alpha$ $\frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \alpha$

jerk $\dddot{y} = \frac{d^3y}{dt^3} = y'''\omega^3 + 3y''\omega\alpha + y'\dot{\alpha}$

When the camshaft speed is constant $\alpha = \dot{\alpha} = 0$

$\dot{y} = y'\omega$ $\ddot{y} = y''\omega^2$ $\dddot{y} = y'''\omega^3$

Jerk is often used in engineering, especially when building roller coasters. Some precision or fragile objects—such as passengers, who need time to sense stress changes and adjust their muscle tension or suffer conditions such as whiplash—can be safely subjected not only to a maximum acceleration, but also to a maximum jerk. Jerk may be considered when the excitation of vibrations is a concern. A device that measures jerk is called a "jerk meter".

Jerk is also important to consider in manufacturing processes. Rapid changes in acceleration of a cutting tool can lead to premature tool wear and result in uneven cuts. This is why modern motion controllers include jerk limitation features.

In mechanical engineering, jerk is considered, in addition to velocity and acceleration, in the development of cam profiles because of tribological implications and the ability of the actuated body to follow the cam profile without chatter.

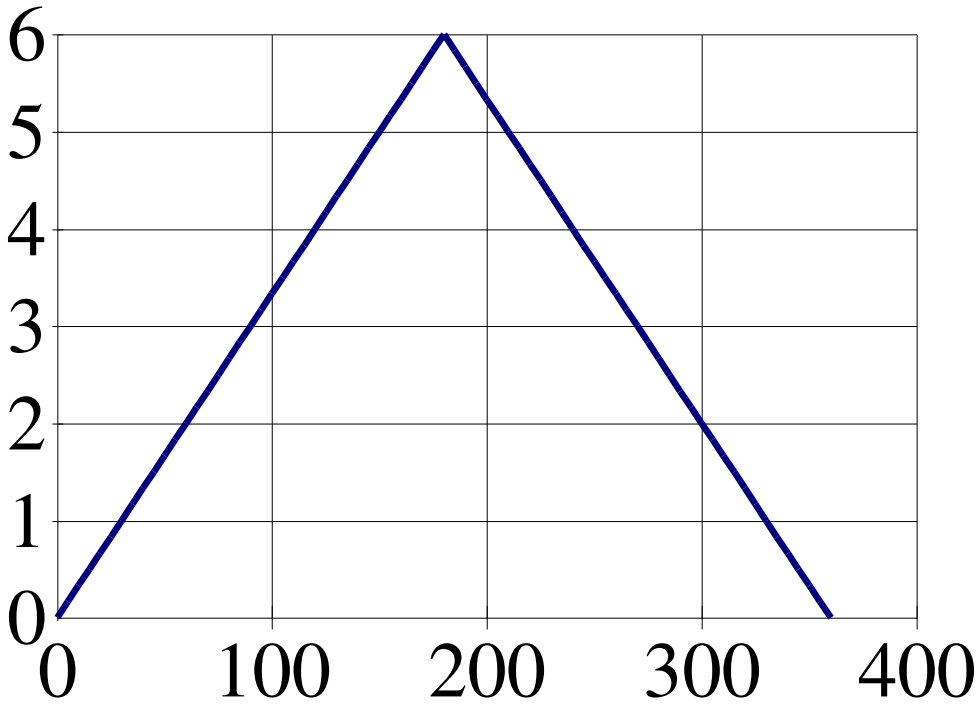
Constant velocity full rise motion

$$y = L\frac{\theta}{\beta}$$

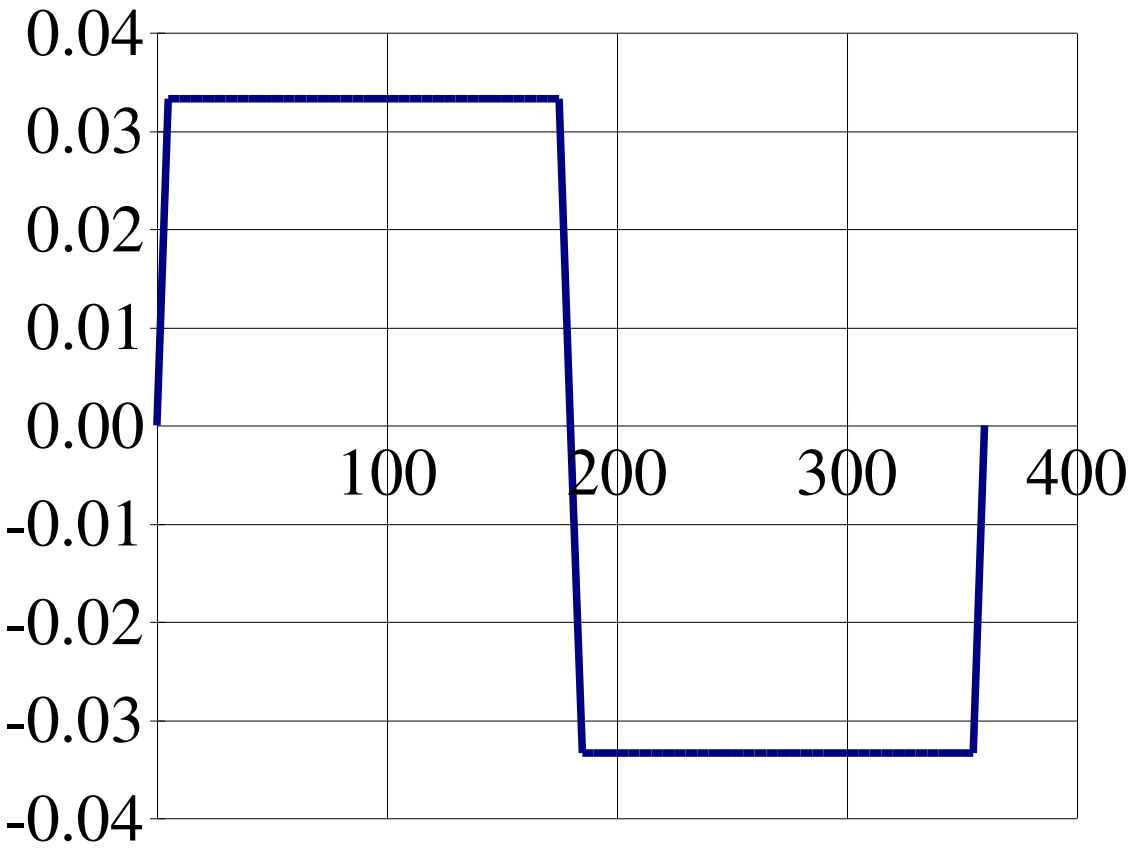
$$y' = \frac{L}{\beta}$$

$$y'' = 0$$

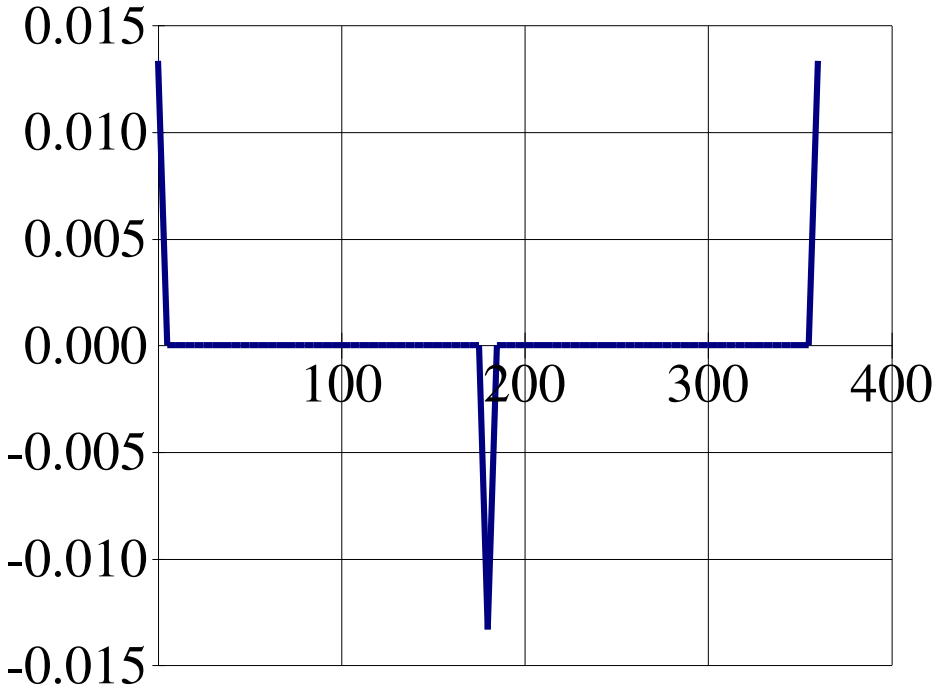
Constant velocity full rise motion



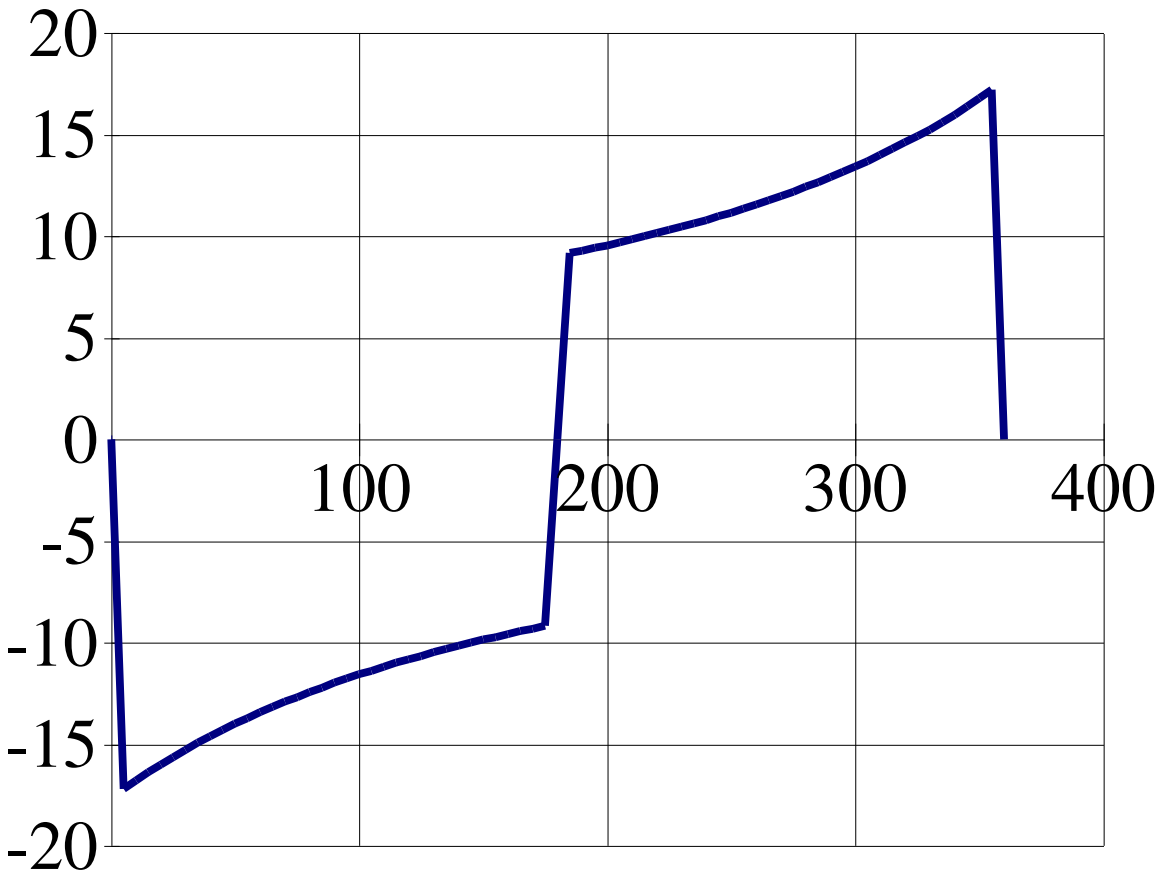
Displacement diagram for Constant Velocity rise-return motion



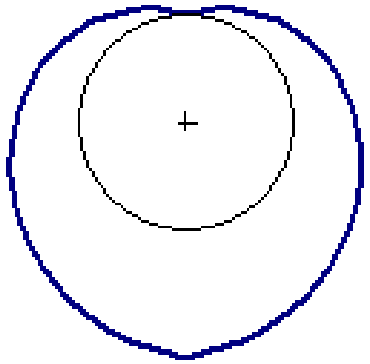
Velocity diagram for Constant Velocity rise-return motion



Acceleration diagram for Constant Velocity rise-return motion



Pressure angle diagram for Constant Velocity rise-return motion



Cam profile for Constant Velocity rise-return motion

Parabolic (Constant acceleration) full rise motion

First half rise

$$y = 2L\left(\frac{\theta}{\beta}\right)^2$$

Second half rise

$$y = L\left[1 - 2\left(1 - \frac{\theta}{\beta}\right)^2\right]$$

Parabolic (Constant acceleration) full rise motion

First half rise

$$y = 2L\left(\frac{\theta}{\beta}\right)^2$$

$$y' = \frac{4L}{\beta}\left(\frac{\theta}{\beta}\right)$$

$$y'' = \frac{4L}{\beta^2}$$

$$y''' = 0$$

Second half rise

$$y = L\left[1 - 2\left(1 - \frac{\theta}{\beta}\right)^2\right]$$

$$y' = \frac{4L}{\beta}\left(1 - \frac{\theta}{\beta}\right)$$

$$y'' = -\frac{4L}{\beta^2}$$

$$y''' = 0$$

Cycloidal full rise motion

$$y = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right)$$

Beta is total rise angle

Cycloidal full rise motion

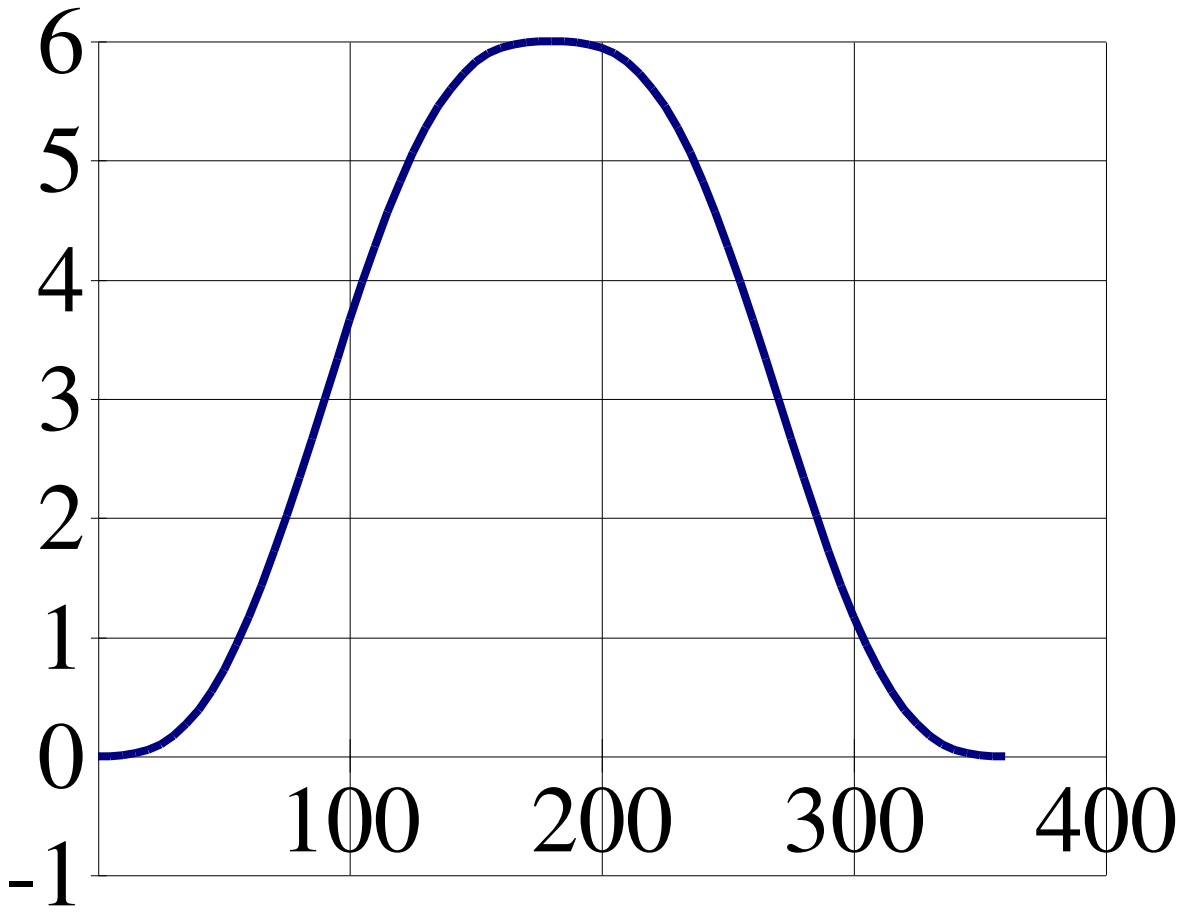
$$y = L\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right)$$

Beta is total rise angle

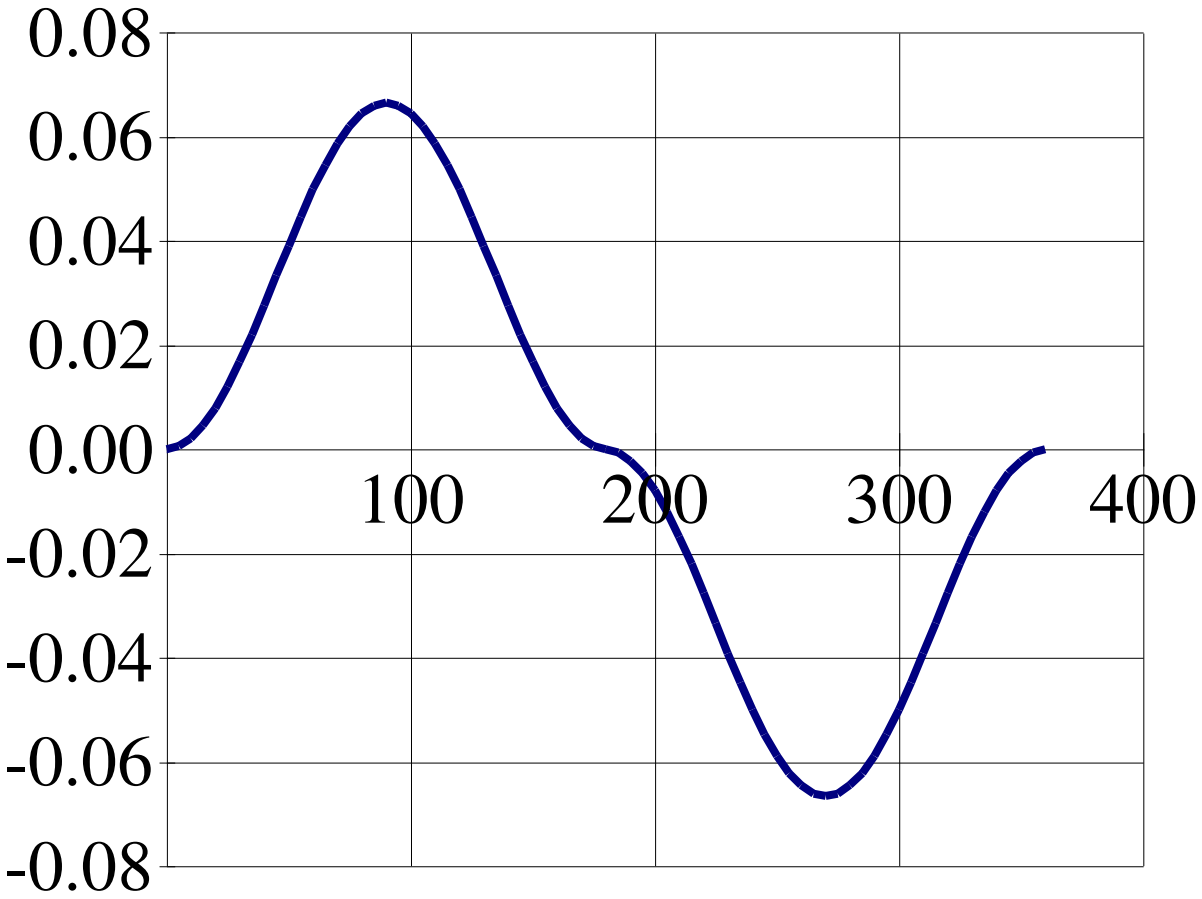
$$y' = \frac{L}{\beta}\left(1 - \cos\frac{2\pi\theta}{\beta}\right)$$

$$y'' = \frac{2\pi L}{\beta^2}\sin\frac{2\pi\theta}{\beta}$$

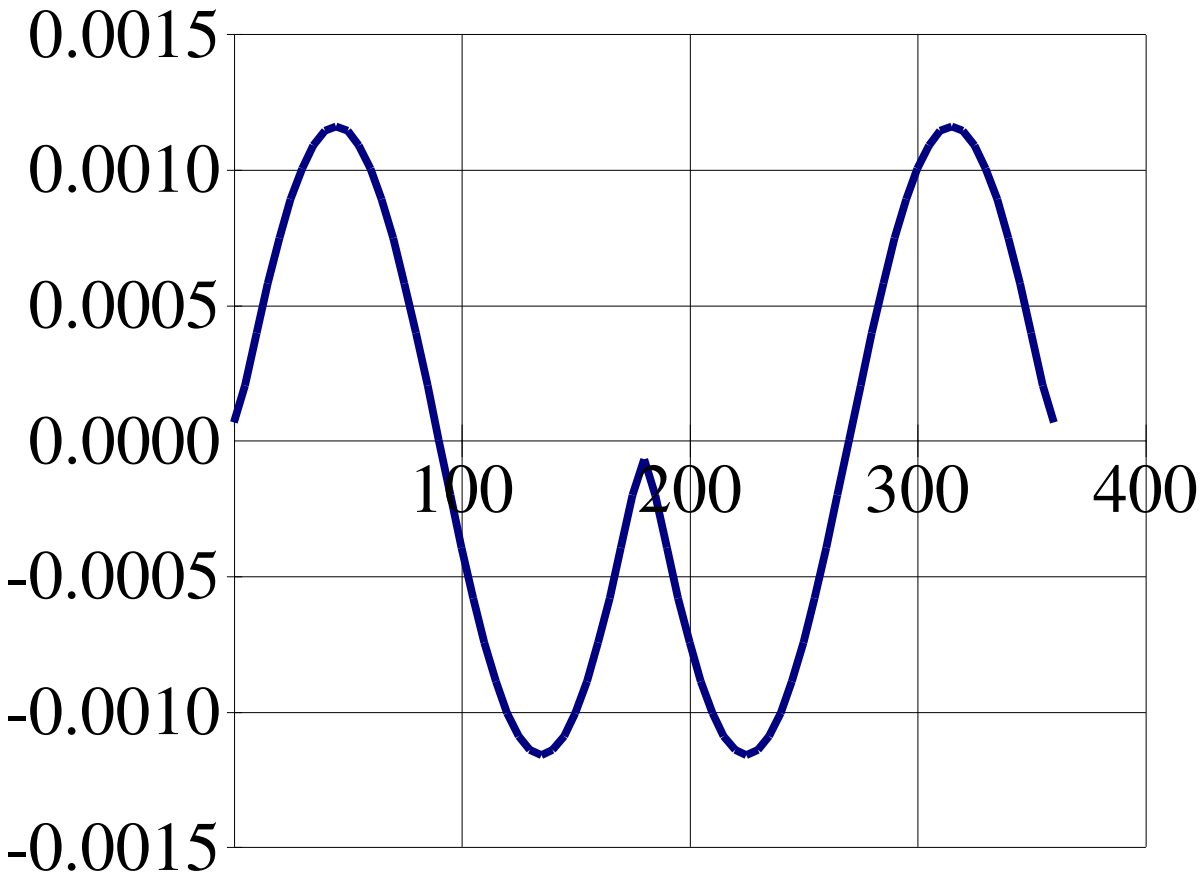
$$y''' = \frac{4\pi^2 L}{\beta^3}\cos\frac{2\pi\theta}{\beta}$$



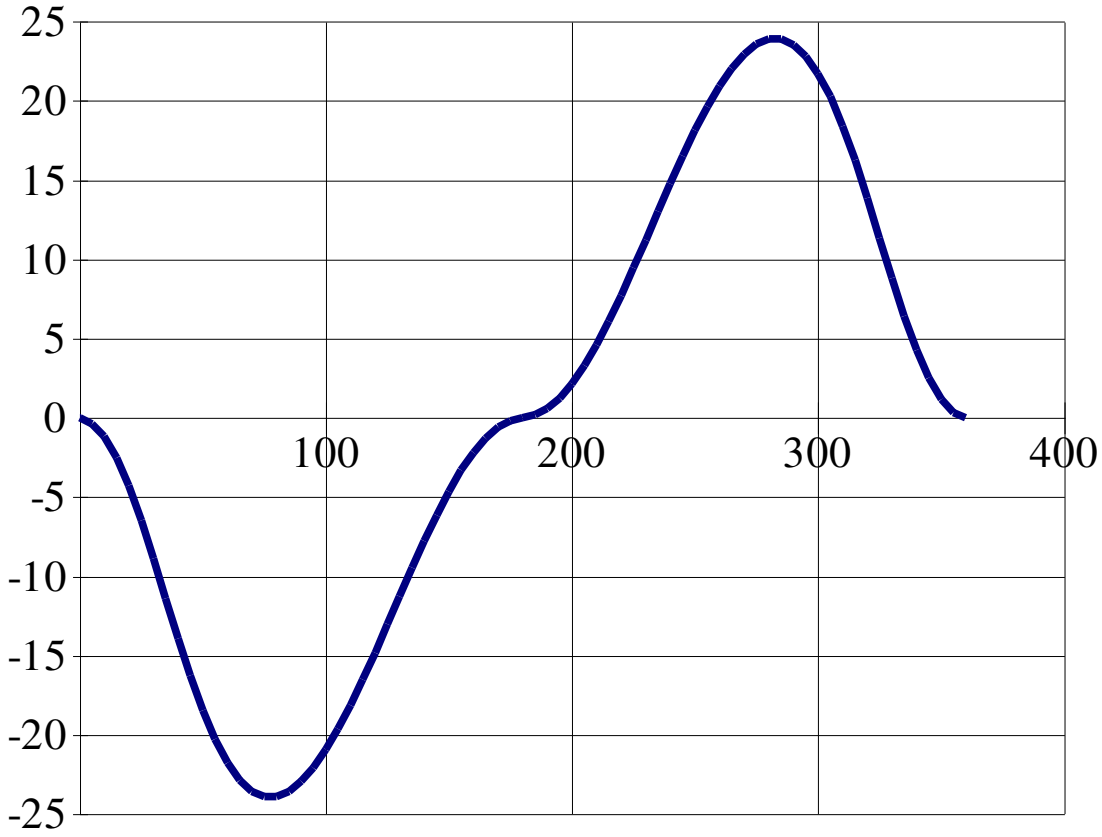
Displacement diagram for cycloidal full rise motion



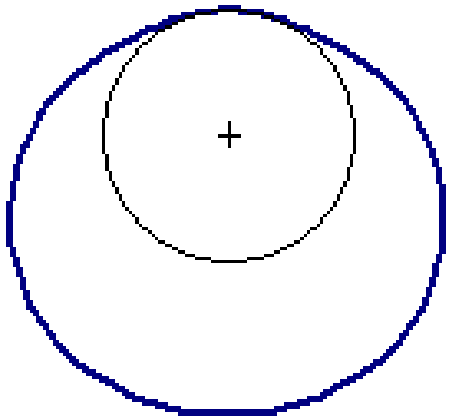
Velocity diagram for cycloidal full rise motion



Acceleration for full rise cycloidal motion



Pressure angle for full rise cycloidal motion



Cam profile for full rise cycloidal motion

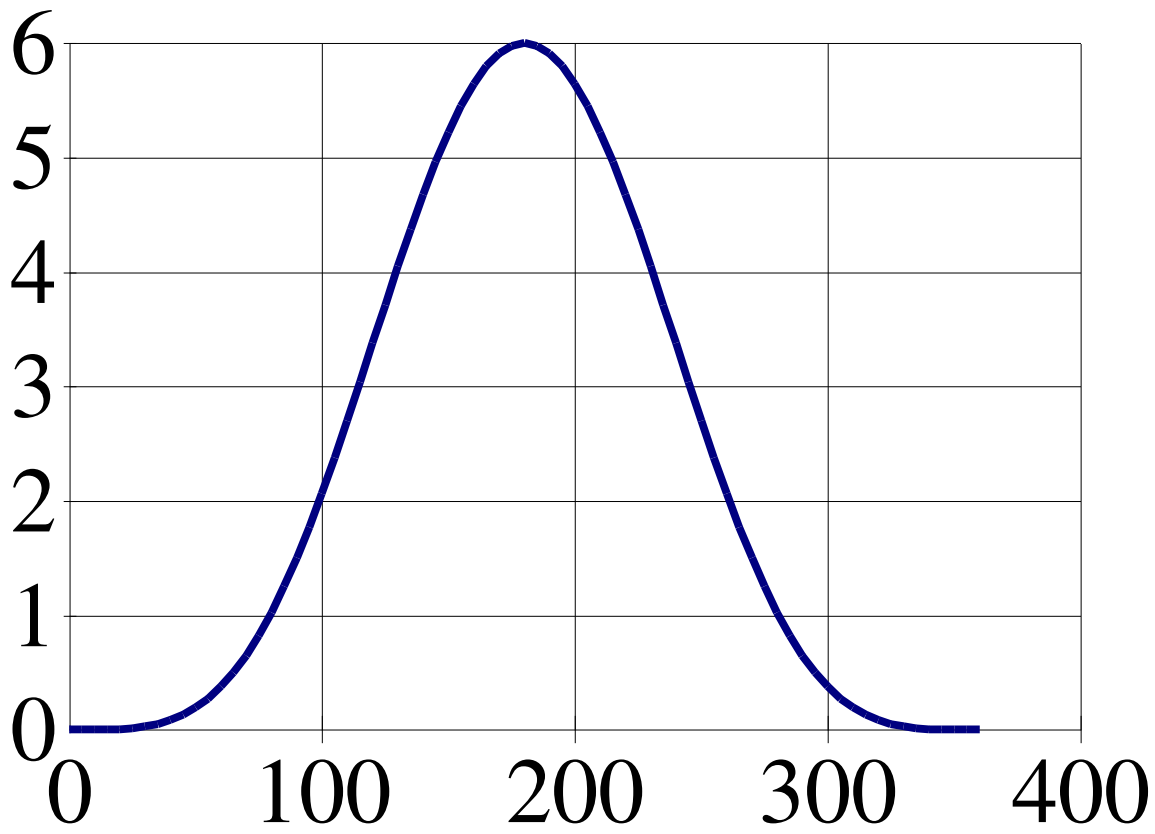
Modified Harmonic full rise motion

$$y = \frac{L}{2}\left(1 - \cos\frac{\pi\theta}{\beta}\right) - \frac{1}{4}\left(1 - \cos\frac{2\pi\theta}{\beta}\right)$$

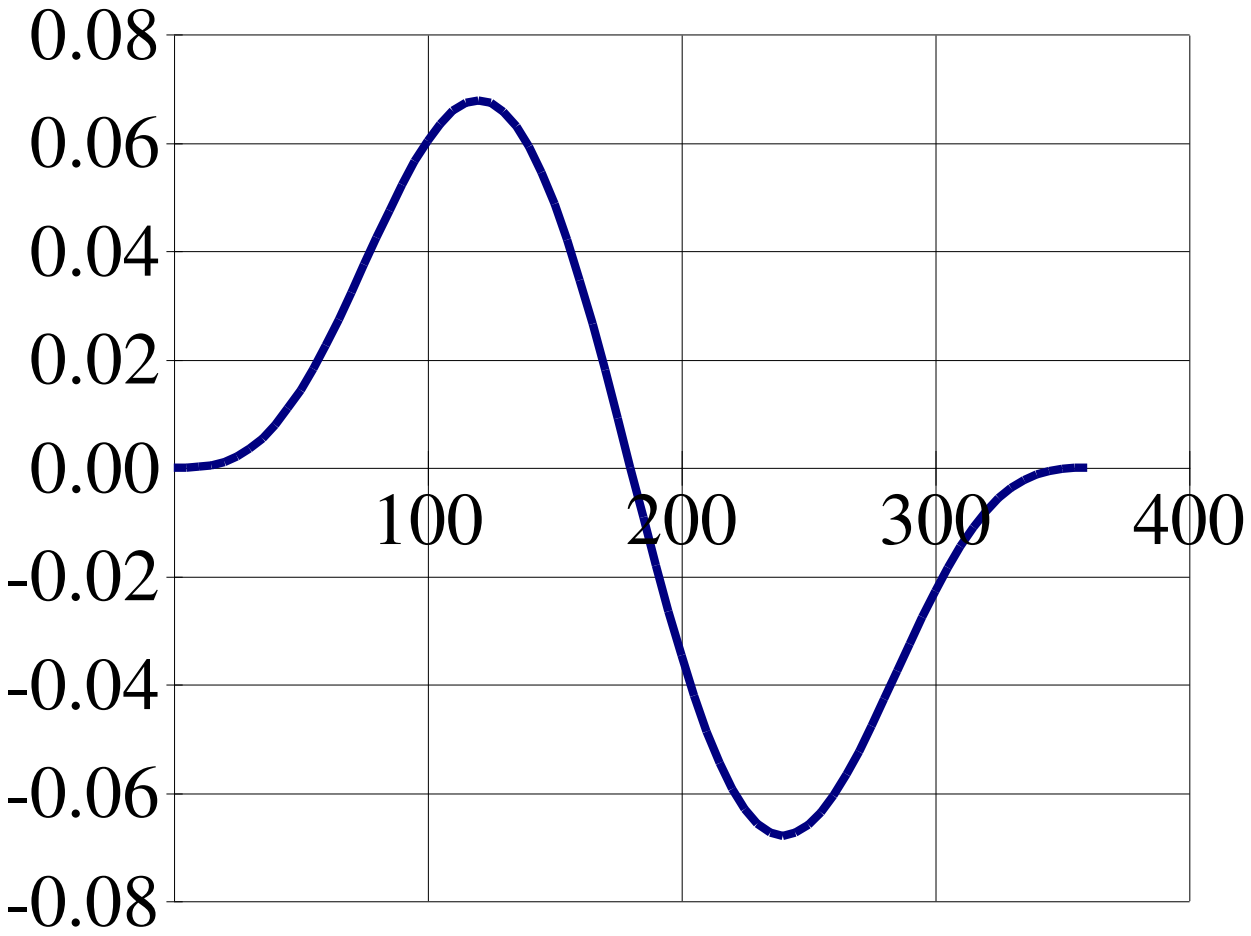
$$y' = \frac{\pi L}{2\beta}\left(\sin\frac{\pi\theta}{\beta} - \frac{1}{2}\sin\frac{2\pi\theta}{\beta}\right)$$

$$y'' = \frac{\pi^2 L}{2\beta^2}\left(\cos\frac{\pi\theta}{\beta} - \cos\frac{2\pi\theta}{\beta}\right)$$

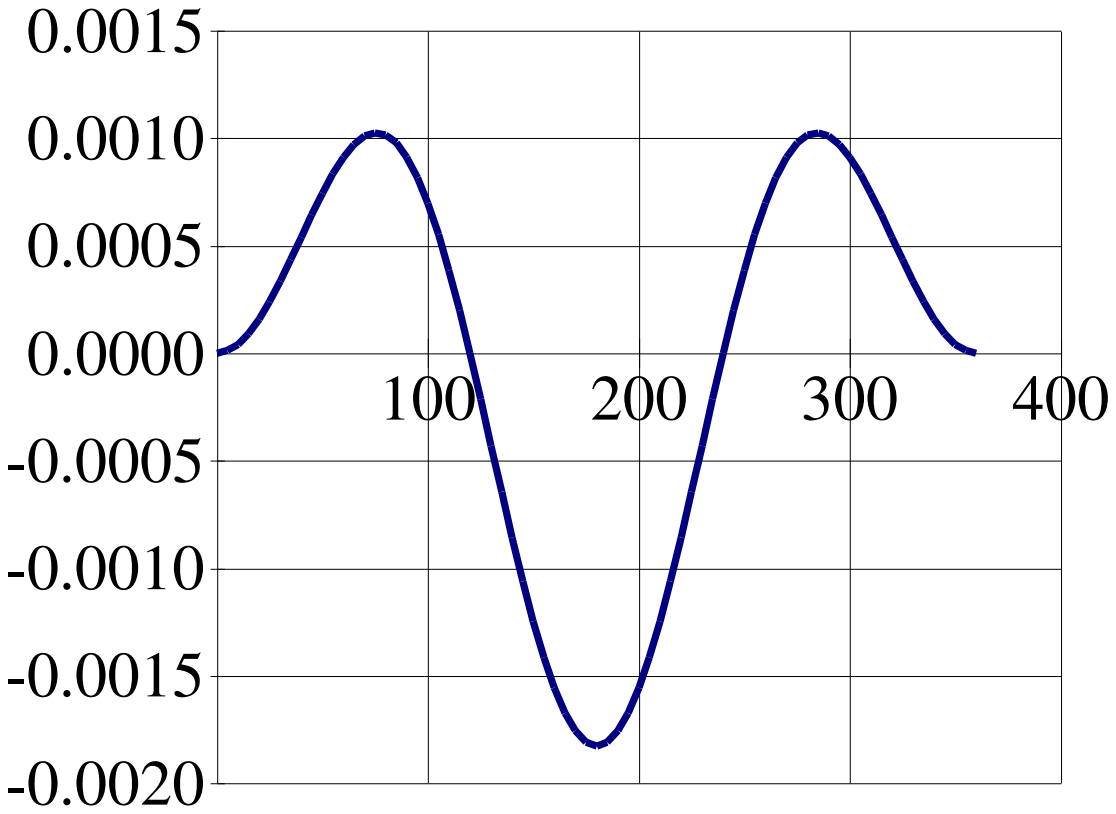
$$y''' = \frac{\pi^3 L}{2\beta^3}\left(\sin\frac{\pi\theta}{\beta} - 2\sin\frac{2\pi\theta}{\beta}\right)$$



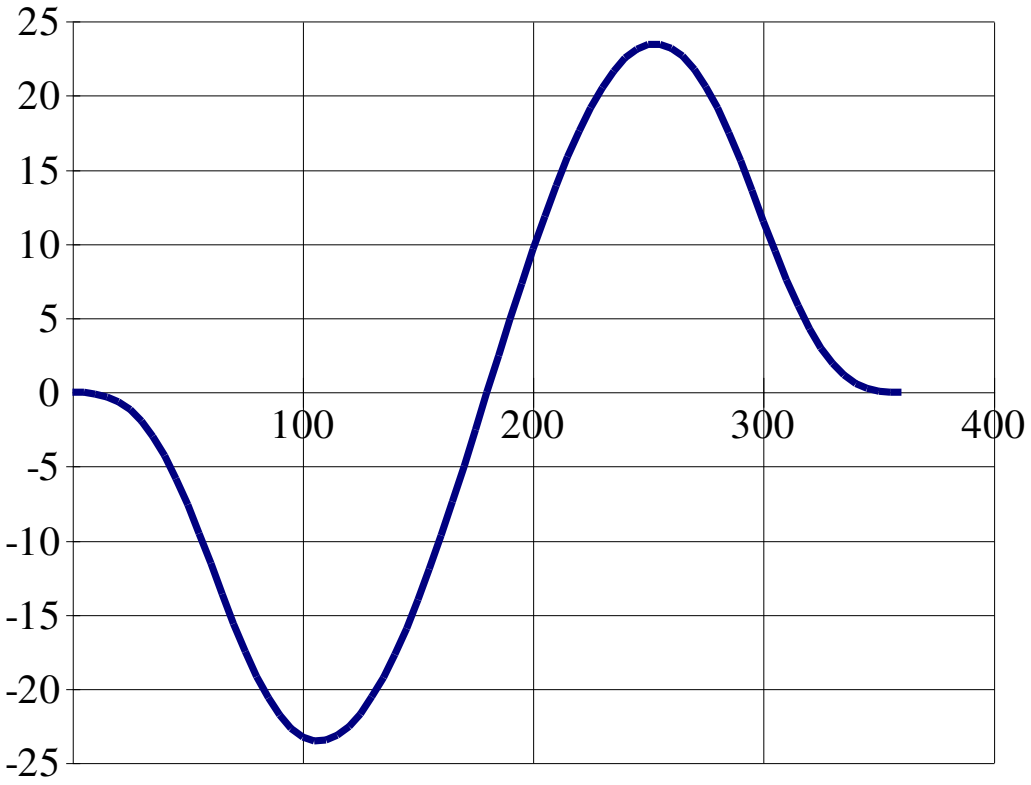
Displacement diagram for modified harmonic full-rise motion



Velocity diagram for modified harmonic full-rise motion



Acceleration diagram for modified harmonic full-rise motion



Pressure angle for modified harmonic full-rise motion

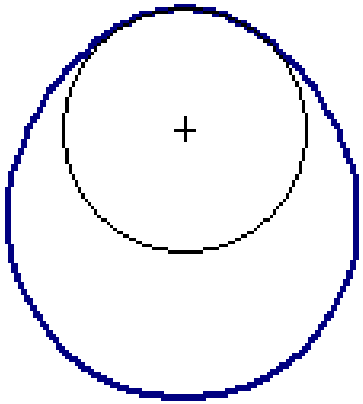
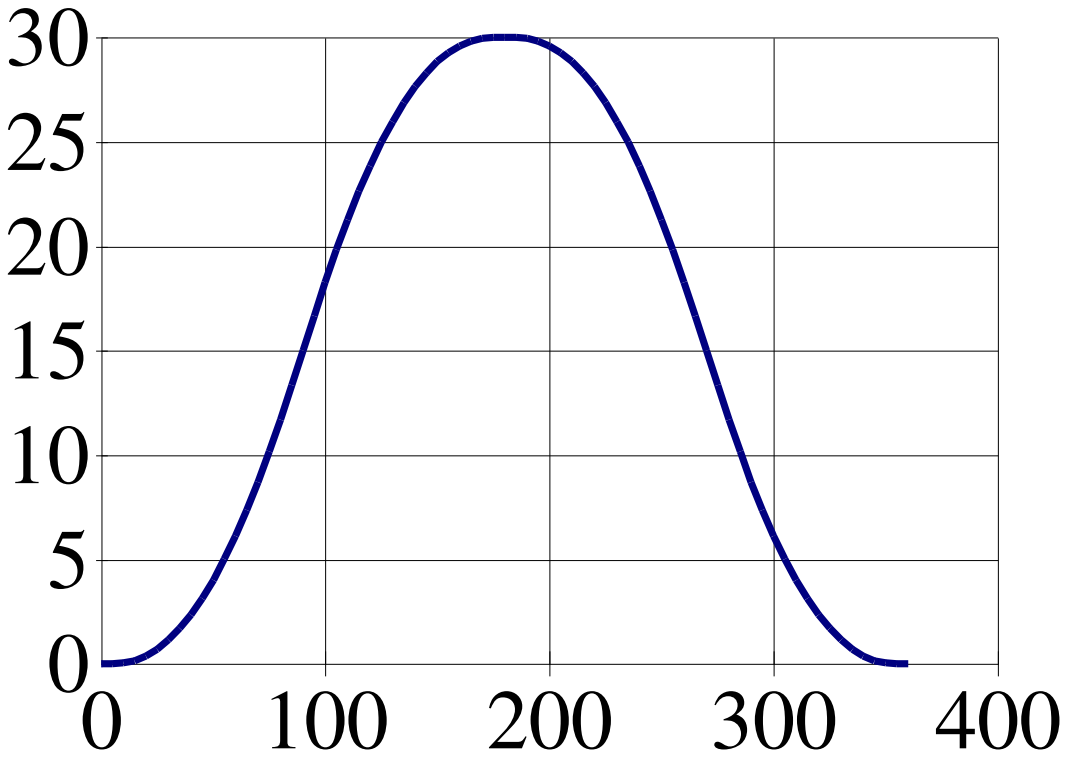
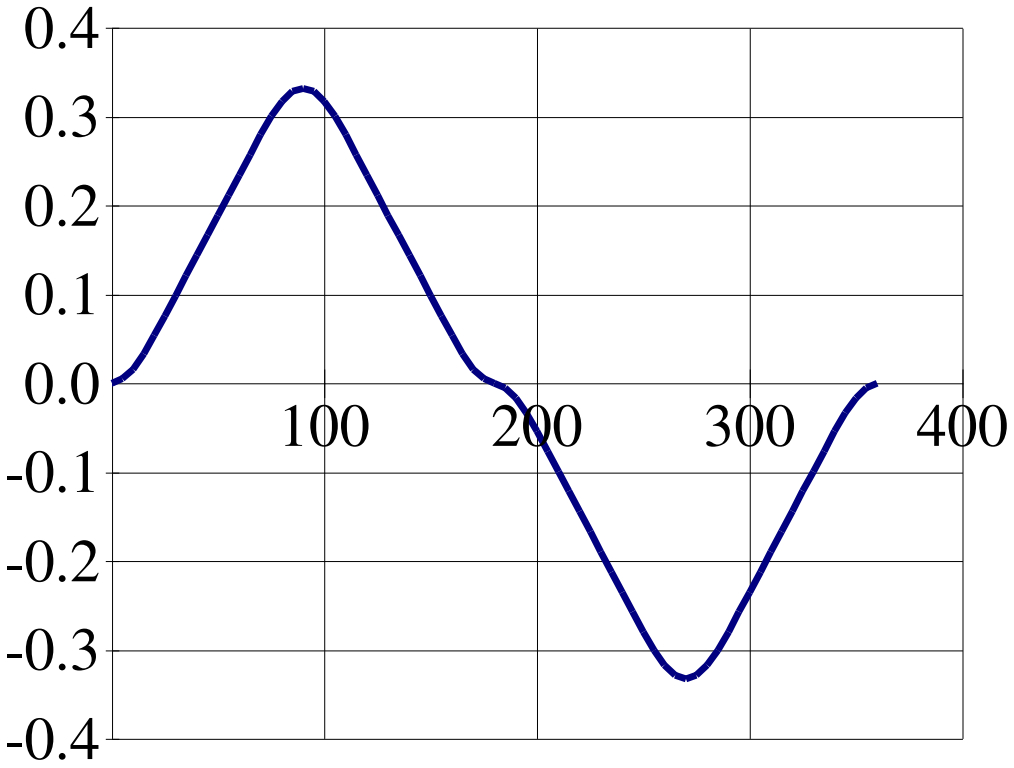


Plate-reciprocating roller type cam profile for modified harmonic motion
e=0 mm

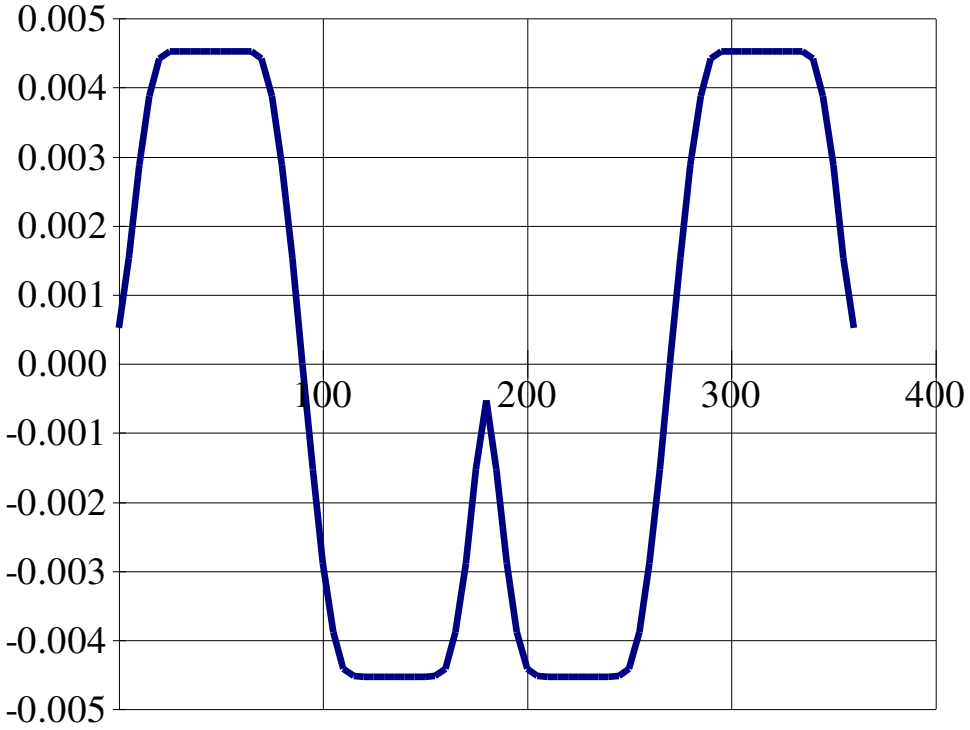
Modified trapezoid (or trapezoidal acceleration) full rise-return motion



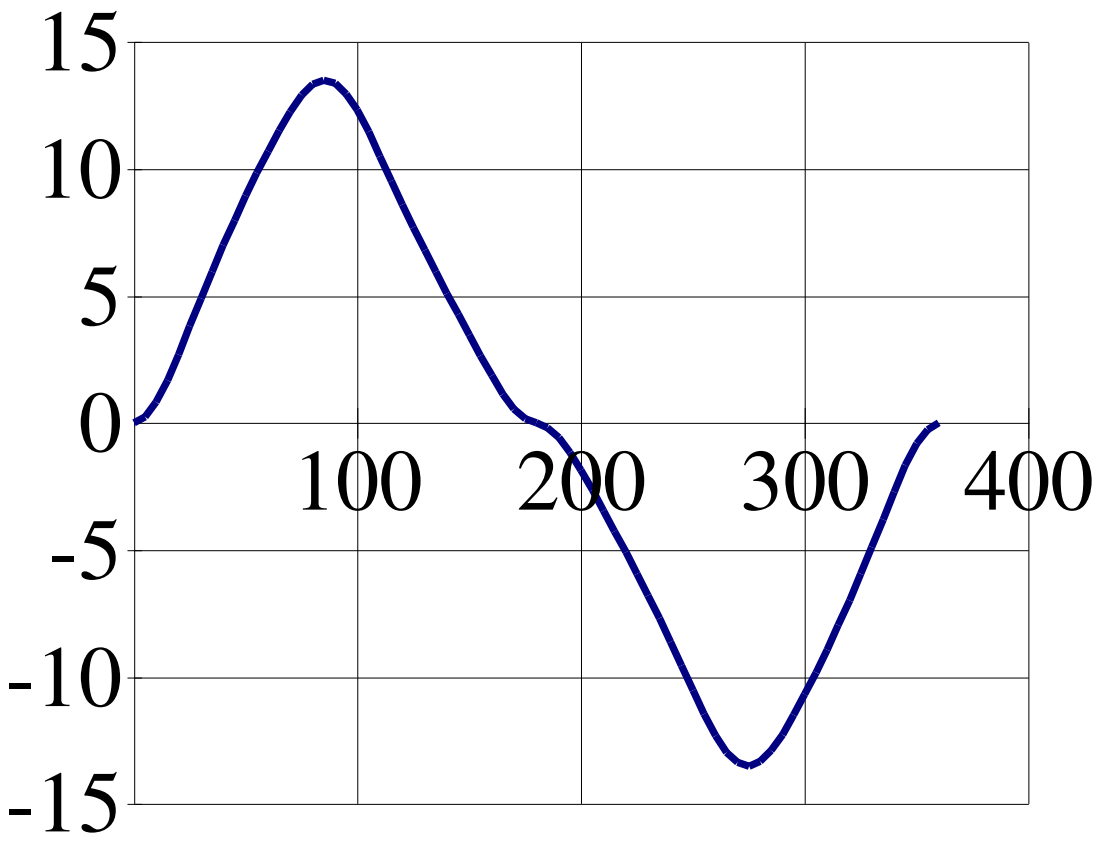
Displacement diagram for modified trapezoid motion



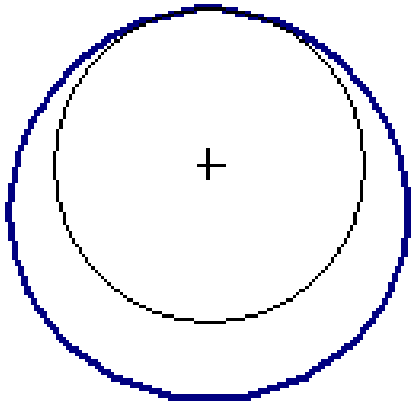
Velocity diagram for modified trapezoid motion



Acceleration diagram for modified trapezoid motion



Pressure angle diagram for modified trapezoid motion



Modified trapezoid full rise-return cam profile
R_0=60 mm, e= 0 mm, r=5 mm

345 Polynomial full rise motion

$$y = L\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right]$$

$$y' = \frac{L}{\beta}\left[30\left(\frac{\theta}{\beta}\right)^2 - 60\left(\frac{\theta}{\beta}\right)^3 + 30\left(\frac{\theta}{\beta}\right)^4\right]$$

$$y'' = \frac{L}{\beta^2}\left[60\frac{\theta}{\beta} - 180\left(\frac{\theta}{\beta}\right)^2 + 120\left(\frac{\theta}{\beta}\right)^3\right]$$

$$y''' = \frac{L}{\beta^3}\left[60 - 360\frac{\theta}{\beta} + 360\left(\frac{\theta}{\beta}\right)^2\right]$$

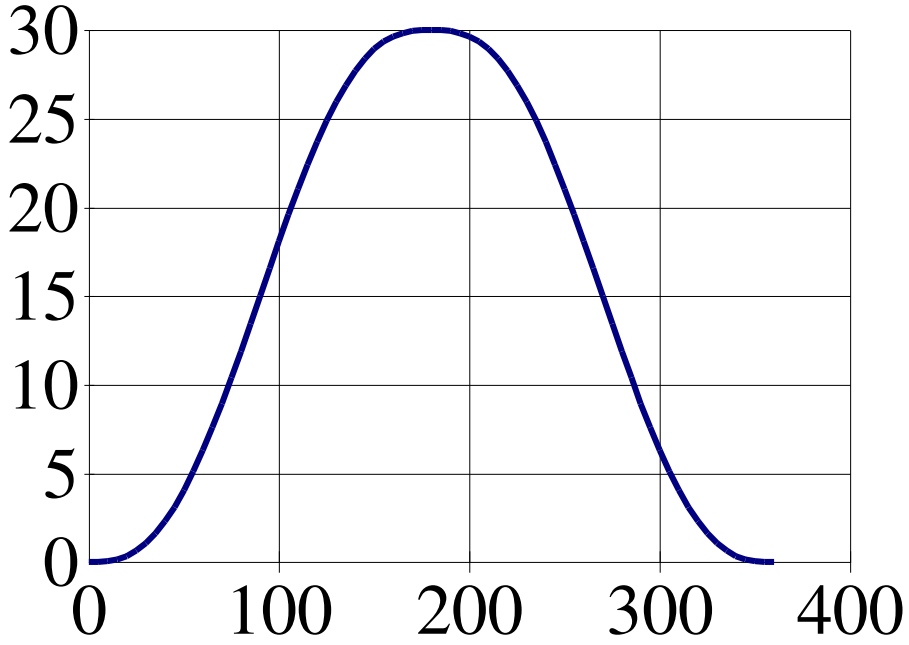


Plate-reciprocating follower cam displacement diagram, 345 polynomial

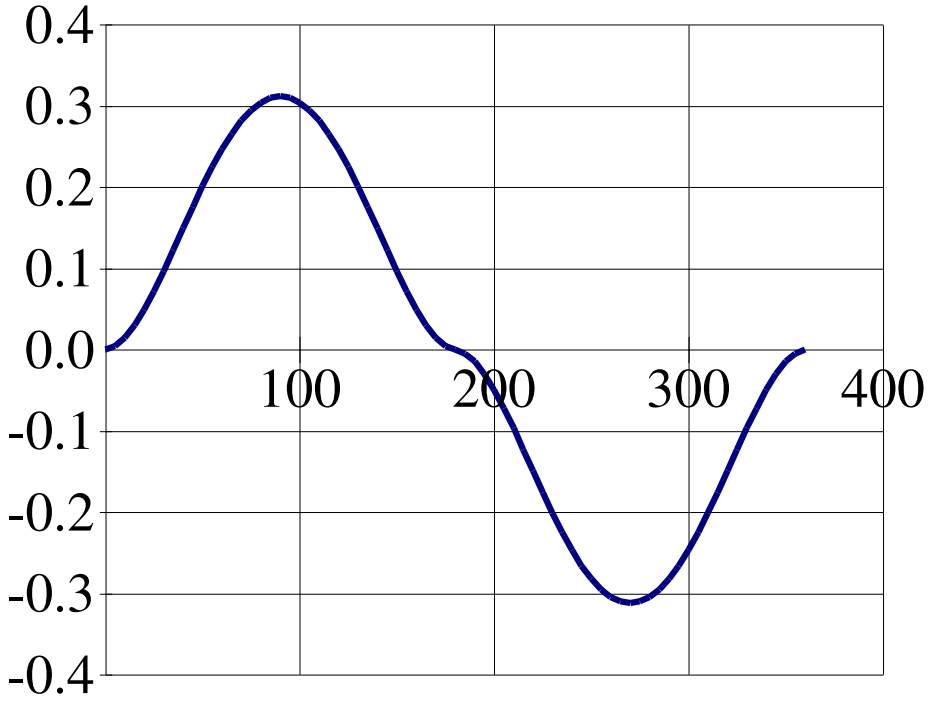


Plate-reciprocating follower cam velocity diagram, 345 polynomial

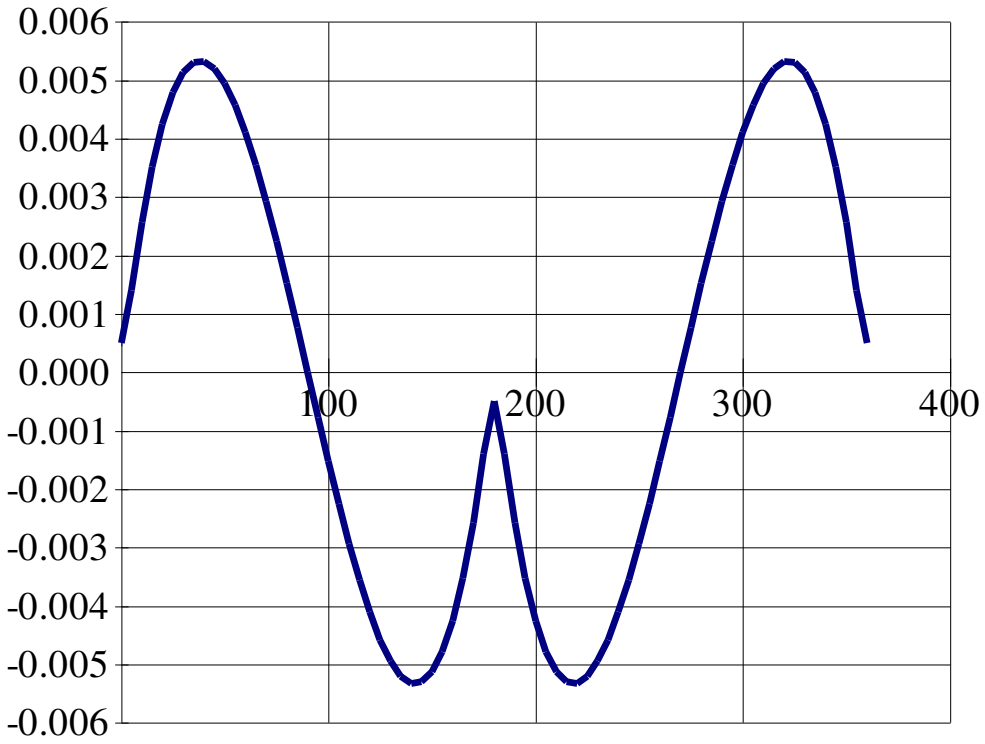


Plate-reciprocating follower cam acceleration diagram, 345 polynomial

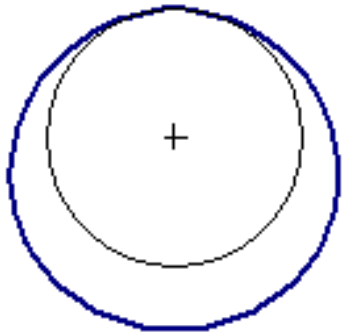


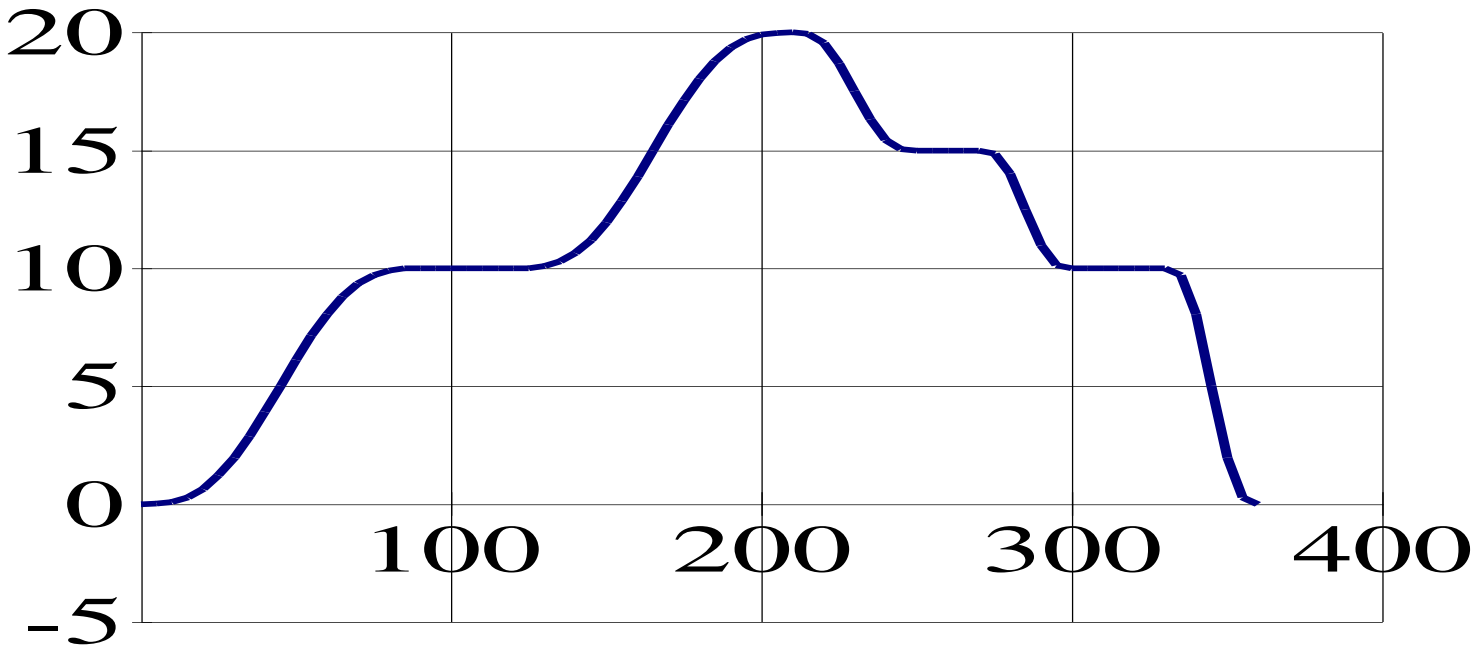
Plate-reciprocating follower cam profile, 345 polynomial
Base circle radius 60 mm, roller radius 5 mm.

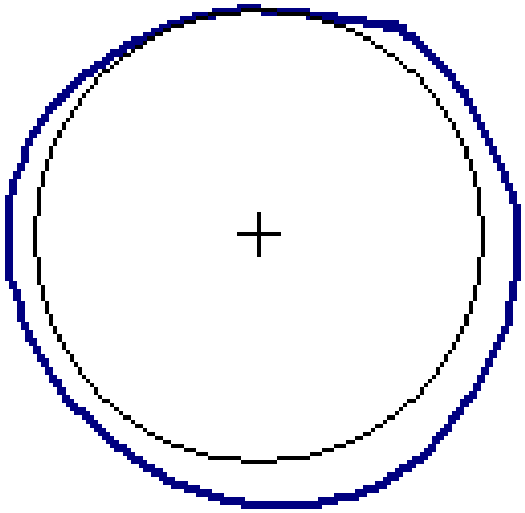
Example

Design two cams with three dwells; one dwell during the rise and two dwells during the return. Total rise is to be 20 mm in each cam. Use the smallest reasonable basic circle radius.

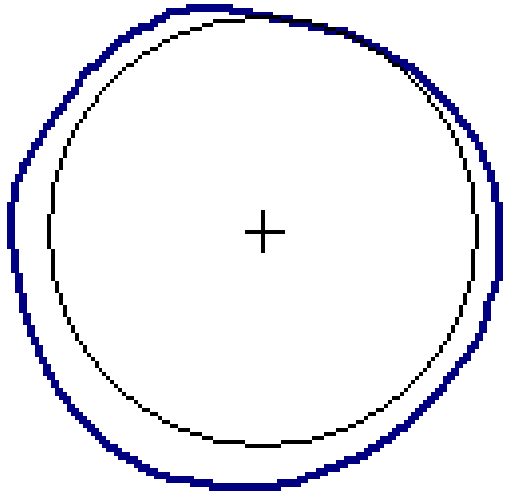
a) A disk cam (or plate cam) with a flat-faced reciprocating follower, rotating cw.
b) A disk cam with a roller follower having an offset of 12 mm to the right and the cam rotating ccw. Check if the pressure angle is within the safe limits.

Assume
0-90 degrees rise cycloidal 10 mm
90-120 (30) degrees dwell
120-210 (90) degrees rise cycloidal 10 mm
210-250 (60) degrees return cycloidal 5 mm
250-270 (20) degrees dwell
270-300 (30) degrees return cycloidal 5 mm
300-330 (30) degrees dwell
330-360 (30) degrees return cycloidal 10 mm





Disk cam profile for with flat-face follower, cw rotation



Disk cam profile for reciprocating roller follower, e=12 mm right, ccw rotation.

EXAMPLE

It is required to design a plate cam with a reciprocating roller follower such that

a) The follower has an offset of 11 mm
b) The radius of roller is 7 mm
c) Cam rotates cw
d) The follower is to rise to maximum of 35 mm
e) There are two returns
f) Design the cam such that the value of the pressure angle remains < + or – 25 degrees.
g) Try to keep the size of the cam as small as possible.

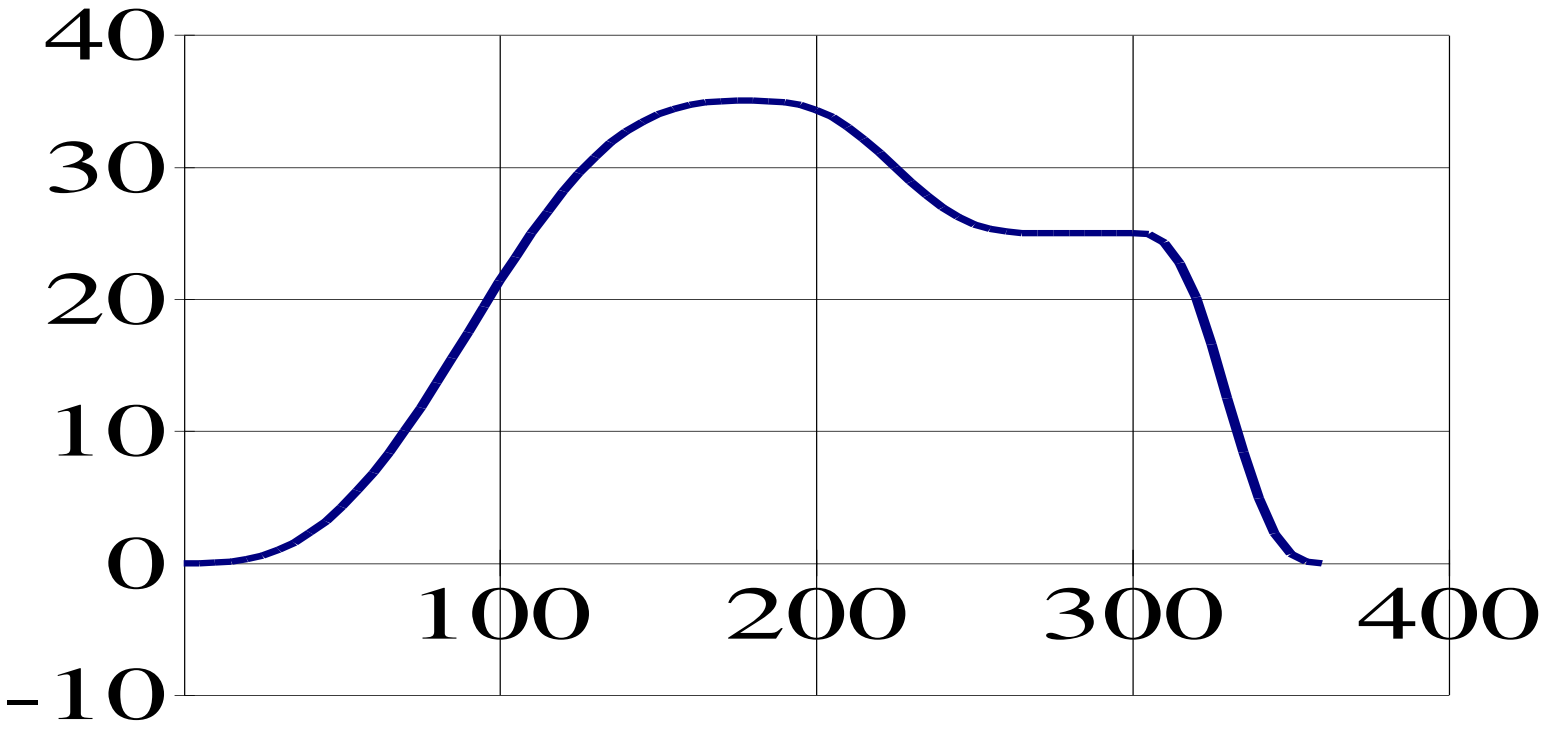
Assume

0-180 (180) cycloidal rise 35 mm

180-270 (90) cycloidal return 10 mm

270-300 (30) dwell

300-360 (60) cycloidal return 15 mm



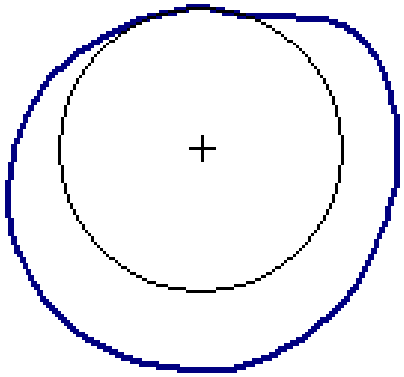
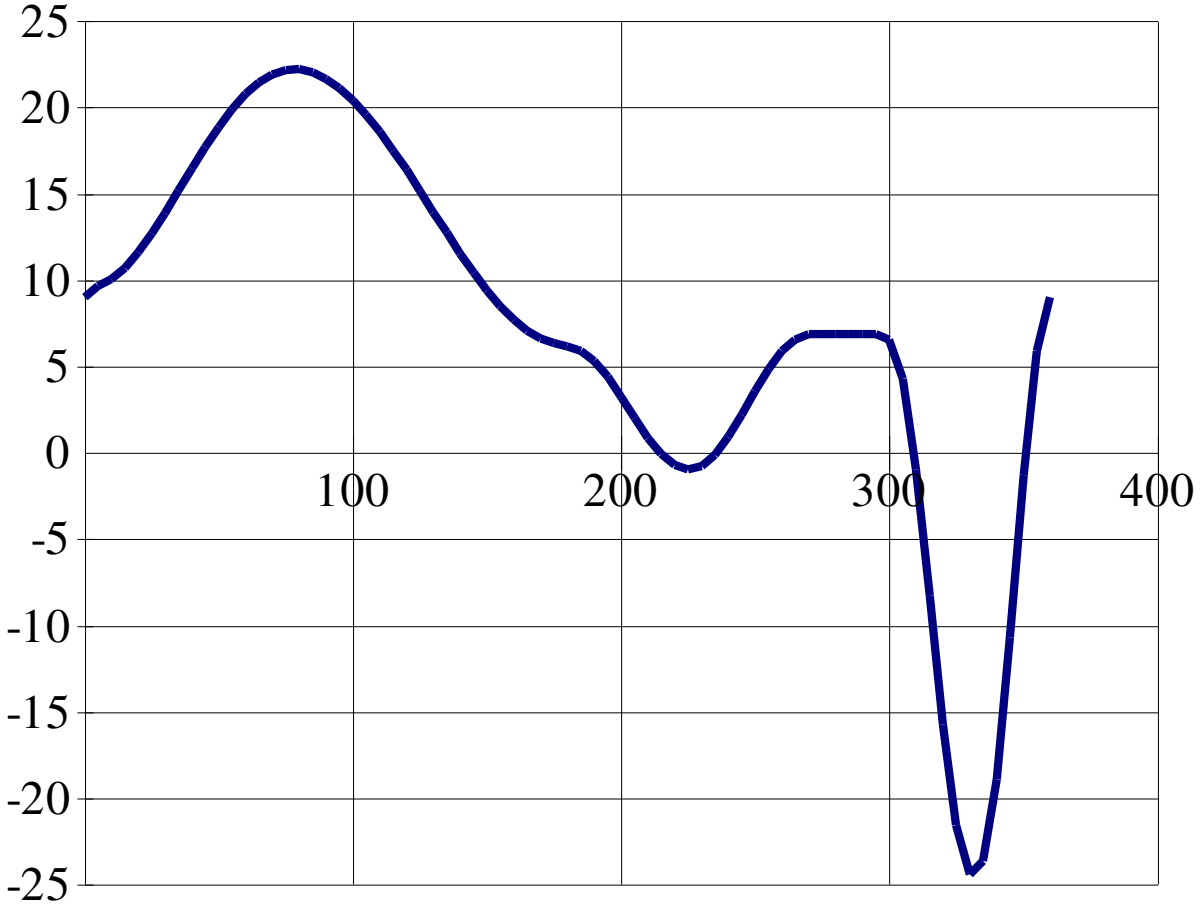


Plate cam profile, cw rotation, 11 mm offset right, 7 mm roller radius.
Base circle radius is 60 mm



Pressure angle diagram, PA < = 25 degrees