Chapter 2

The three-phase electrical circuits in sinusoidal mode

1. Three phase system

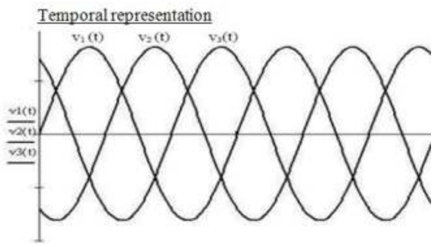
1.1. Introduction

- Three-phase circuits are of great importance in electrical engineering.
- Electric energy is produced, transported and, to a large extent, consumed in three-phase form (three cables share power), hence the importance of three-phase currents. Moreover, during the transport of electrical energy, the losses are less in three-phase.
- Three-phase machines have powers of more than 50% higher than single-phase machines of the same mass and therefore their prices are lower.
- The distribution of energy through the electrical network is composed of four terminals:
  - ♦ Three phase terminals marked by *1, 2, 3* or *A, B, C* or *R, S, T*.
  - ♦ A neutral terminal *N*.

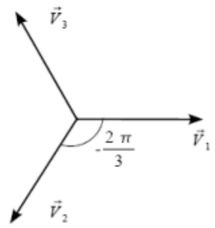
1.2. Definitions

- A three-phase system is a set of three sinusoidal magnitudes (voltages or currents) of the same frequency, out of phase with each other.
- A three-phase system is said <u>balanced</u> when the three sinusoidal magnitudes (voltages or currents) have the <u>same effective value</u> and are <u>out of phase</u> with each other by $\frac{2\pi}{3}$.
- A balanced three-phase system is said <u>direct</u> if its phase diagram is ordered in <u>the negative trigonometric direction</u> (clockwise direction).

$$\begin{cases} v_1(t) = \sqrt{2}V\cos(\omega t) \\ v_2(t) = \sqrt{2}V\cos(\omega t - \frac{2\pi}{3}) \\ v_3(t) = \sqrt{2}V\cos(\omega t - \frac{4\pi}{3}) \end{cases}$$



$$\Rightarrow \begin{cases} \overline{V_1} = V \\ \overline{V_2} = Ve^{-j\frac{2\pi}{3}} \\ \overline{V_3} = Ve^{-j\frac{4\pi}{3}} \end{cases} \Rightarrow \overline{V_1} + \overline{V_2} + \overline{V_3} = V\,(1 + e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}}) = 0$$



1.3. Three phase circuit

The usual simplified model of a source of three-phase voltages is composed by three single-phase sources in a wye (Y) connection, with a common point called neutral (Fig. 2.1). Each source corresponds to a phase.

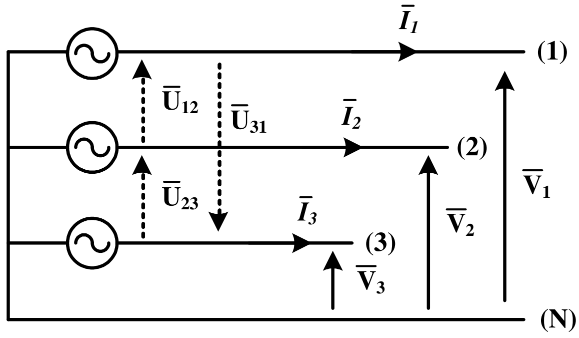


Fig 2.1. Three phase circuit

- We call <u>simple voltages</u>, the three voltages, $\overline{V_1}$, $\overline{V_2}$ and $\overline{V_3}$ of module V, measured between each phase and the neutral of the three-phase source.

$$\begin{cases} \overline{V_1} = V \\ \overline{V_2} = Ve^{-j\frac{2\pi}{3}} \\ \overline{V_3} = Ve^{-j\frac{4\pi}{3}} \end{cases}$$

- <u>Composite voltages</u> are the three voltages measured between two phases: $\overline{u}_{12}$, $\overline{u}_{23}$ et $\overline{u}_{31}$.

$$\begin{cases} \overline{u}_{12} = v_1 - v_2 \\ \overline{u}_{23} = v_2 - v_3 \\ \overline{u}_{31} = v_3 - v_1 \end{cases} \Rightarrow \begin{cases} \overline{U}_{12} = \overline{V_1} - \overline{V_2} = \sqrt{3}Ve^{j\frac{\pi}{6}} \\ \overline{U}_{23} = \overline{V_2} - \overline{V_3} = \sqrt{3}Ve^{-j\frac{\pi}{2}} \\ \overline{U}_{31} = \overline{V_3} - \overline{V_1} = \sqrt{3}Ve^{j\frac{5\pi}{6}} \end{cases} \Rightarrow \begin{cases} \overline{U}_{12} + \overline{U}_{23} + \overline{U}_{31} = 0 \\ \Rightarrow (\overline{U}_{12}, \overline{U}_{23}, \overline{U}_{31}) \text{ form a balanced three phase system} \end{cases}$$

Finally, we obtain $U = \sqrt{3}V$

<u>Example 2.1</u>

In the Kingdom AS, the Saudi Electricity Company distributes a three-phase network:

$$\begin{cases} \text{frequency} = 50Hz \\ \text{effective value betweeen two phases : } U = 400V \end{cases}$$

$$\Rightarrow \text{Effective value of simple voltage : } V = \frac{U}{\sqrt{3}} = \frac{400}{\sqrt{3}} \approx 220V$$

The single-phase voltage comes from a three-phase network where the neutral is used with one of the three phases.

2. The three phase's coupling

A balanced three-phase load is characterized by three identical impedances (same module and same argument) that are called the three phases of the user (or load). These three impedances can be independently connected in either a **wye** (Y) or **delta**(Δ).

$$\overline{Z}_1 = \overline{Z}_2 = \overline{Z}_3 = \overline{Z} = Ze^{j\varphi}$$

- The three line currents are the currents crossing the wires of the three-phase network: $\overline{I}_1, \overline{I}_2$ and $\overline{I}_3$.
- The three phase currents of the load are the currents crossing each impedance: $\overline{J}_1, \overline{J}_2$ and $\overline{J}_3$.

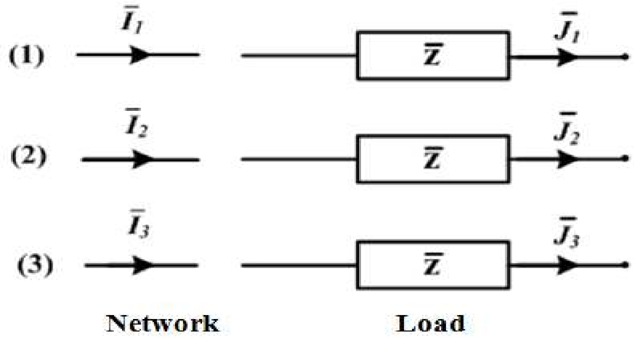


Fig 2.2. Three phases's coupling

2.1. The wye connection (Y):

In the wye connection (symbolized by the sign Y), the three impedances of the three-phase load have a common point called the neutral point of the load, and are supplied by the three simple voltages (Fig 2.3).

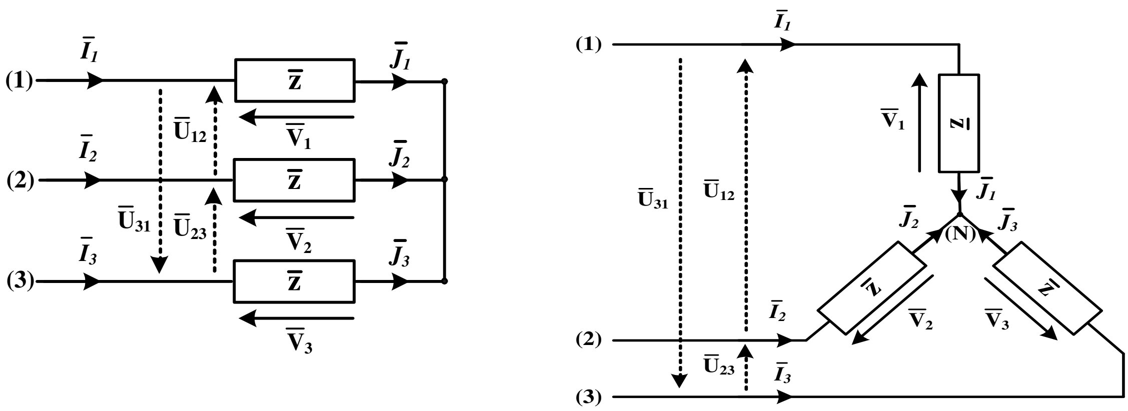


Fig 2.3. Wye coupling without neutral

We have the same impedances $\overline{Z}$:

$\Rightarrow \overline{I}_1 + \overline{I}_2 + \overline{I}_3 = \overline{J}_1 + \overline{J}_2 + \overline{J}_3 = 0$

$\Rightarrow \overline{I}_N = 0$

$\Rightarrow$ The neutral is not necessary

$\overline{V_Z}$: The impedance voltage.

We have :

$$\left.\begin{array}{l}\overline{I_1}=\overline{J_1}\\ \overline{I_2}=\overline{J_2}\\ \overline{I_3}=\overline{J_3}\end{array}\right\}\Rightarrow\begin{cases}I_1=I_2=I_3=I\\ J_1=J_2=J_3=J\end{cases}\Rightarrow I=J$$

$$\begin{cases}V_1=V_2=V_3=V\\ U_{12}=U_{23}=U_{31}=U\end{cases}\Rightarrow V_Z=V=\frac{U}{\sqrt{3}}$$

2.2. The delta coupling Δ :

In the delta connection (symbolized by Δ), the three impedances of the three-phase load are supplied by the three voltages composed of the three-phase source and form a closed circuit on itself. The delta connection load has no neutral point (Fig 2.4).

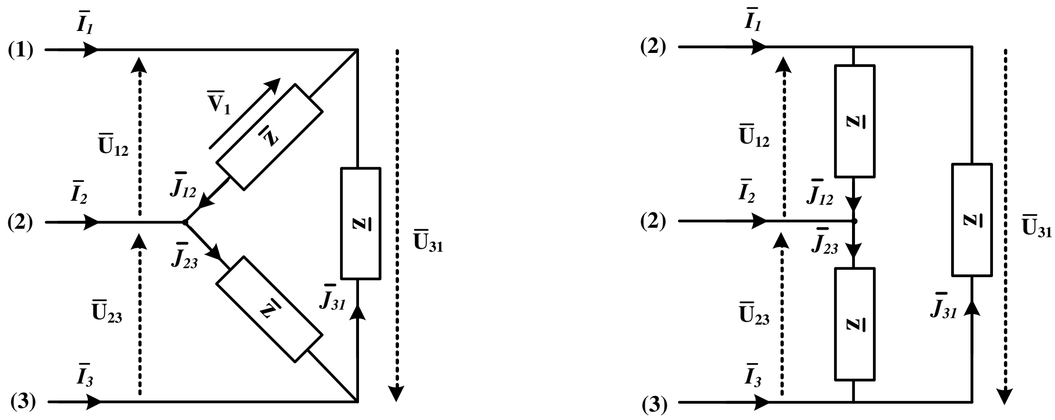


Fig 2.4. Delta coupling

We have the same impedances $\overline{Z}$ ($\overline{V_Z}$: impedance's voltage) :

$$\Rightarrow\begin{cases}\overline{I_1}+\overline{I_2}+\overline{I_3}=0\\ \overline{J_{12}}+\overline{J_{23}}+\overline{J_{31}}=0\end{cases}$$

So :

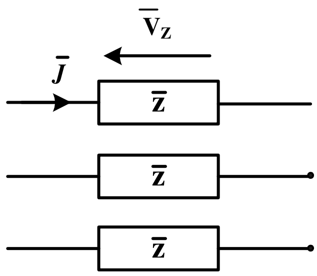
$$\left.\begin{array}{l}I_1=I_2=I_3=I\\ J_{12}=J_{23}=J_{31}=J\end{array}\right\}\Rightarrow I=\sqrt{3}J$$

$$\left.\begin{array}{l}V_1=V_2=V_3=V\\ U_{12}=U_{23}=U_{31}=U\end{array}\right\}\Rightarrow V_Z=U=\sqrt{3}V$$

3. Power in three-phase system

The active power ***P*** absorbed by a three-phase load is the sum of the powers absorbed by each impedance:

$P=3V_Z J\cos\varphi$



- For the reactive power, we get:

$$Q = 3V_Z J \sin\varphi$$

- The total apparent power :

$$S = 3V_Z J$$

- For a wye coupling (Y), we have:

$$\left.\begin{array}{l} I = J \\ V_Z = \dfrac{U}{\sqrt{3}} \end{array}\right\} \Rightarrow \mathrm{P} = 3V_Z J \cos\varphi = 3\frac{U}{\sqrt{3}} I \cos\varphi = \sqrt{3}UI \cos\varphi \Rightarrow \begin{cases} \mathrm{P} = \sqrt{3}UI \cos\varphi \\ \varphi = \sqrt{3}UI \sin\varphi \\ S = \sqrt{3}UI \end{cases}$$

- For a delta coupling (Δ), we have:

$$\left.\begin{array}{l} J = \dfrac{I}{\sqrt{3}} \\ V_Z = U \end{array}\right\} \Rightarrow \mathrm{P} = 3V_Z J \cos\varphi = 3U \frac{I}{\sqrt{3}} \cos\varphi = \sqrt{3}UI \cos\varphi \Rightarrow \begin{cases} \mathrm{P} = \sqrt{3}UI \cos\varphi \\ \varphi = \sqrt{3}UI \sin\varphi \\ S = \sqrt{3}UI \end{cases}$$

$\Rightarrow$ Hence, whatever the coupling, we have the following power relationships:

$$\begin{cases} \mathrm{P} = \sqrt{3}UI \cos\varphi \\ \varphi = \sqrt{3}UI \sin\varphi \\ S = \sqrt{3}UI \end{cases}$$