CHAPTER 1

Introduction to Operations Research (OR), Optimization, and Mathematical Modeling

What is Operations Research (OR)

- Definition
  - *Operations research (OR) is a discipline that adapts the scientific approach to problem solving and decision making.*
  - *Operations research (OR) is a discipline that deals with the application of advanced analytical methods to help make better decisions*

- History: In World War II, British scientists aim to use scientific approach to best utilize war materials

What is Operations Research (OR)

- Other used names: Management science, Decision science, “maybe Optimization”…etc

- Goal:
  - *Optimize (use of available resources, scheduling of tasks, … etc.)*
  - *Maximize (profit, throughput, performance, … etc.)*
  - *Minimize (cost, delay, power consumption, … etc.)*

OR Basic Questions

- What are decision alternatives?
- What is an appropriate objective criterion “function” to evaluate alternatives
- Under what restrictions “constraints” is the decision made?

Simple Example (1)

Imagine that you have a 5-week business commitment between Fayetteville (FYV) and Denver (DEN). You fly out of Fayetteville on Mondays and return on Wednesdays. A regular round-trip ticket costs $400, but a 20% discount is granted if the dates of the ticket span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should you buy the tickets for the 5-week period?

- Let's work it out
  - *What are decision alternatives?*
  - *Under what restrictions "constraints" is the decision made?*
  - *What is an appropriate objective criterion "function" to evaluate alternative*

- Small number of discrete "finite" alternatives
  - *Solved by a simple Ranking Technique*

Simple Example (2)

Amy, Jim, John, and Kelly are standing on the east bank of a river and wish to cross to the west side using a canoe. The canoe can hold at most two people at a time. Amy, being the most athletic, can row across the river in 1 minute. Jim, John, and Kelly would take 2, 5, and 10 minutes, respectively. If two people are in the canoe, the slower person dictates the crossing time. The objective is for all four people to be on the other side of the river in the shortest time possible.

(a) Identify at least two feasible plans for crossing the river (remember, the canoe is the only mode of transportation and it cannot be shuttled empty).

(b) Define the criterion for evaluating the alternatives.

(c)[1] What is the smallest time for moving all four people to the other side of the river?

- What are decision alternatives?
- Under what restrictions “constraints” is the decision made?
- What is an appropriate objective criterion “function” to evaluate alternative

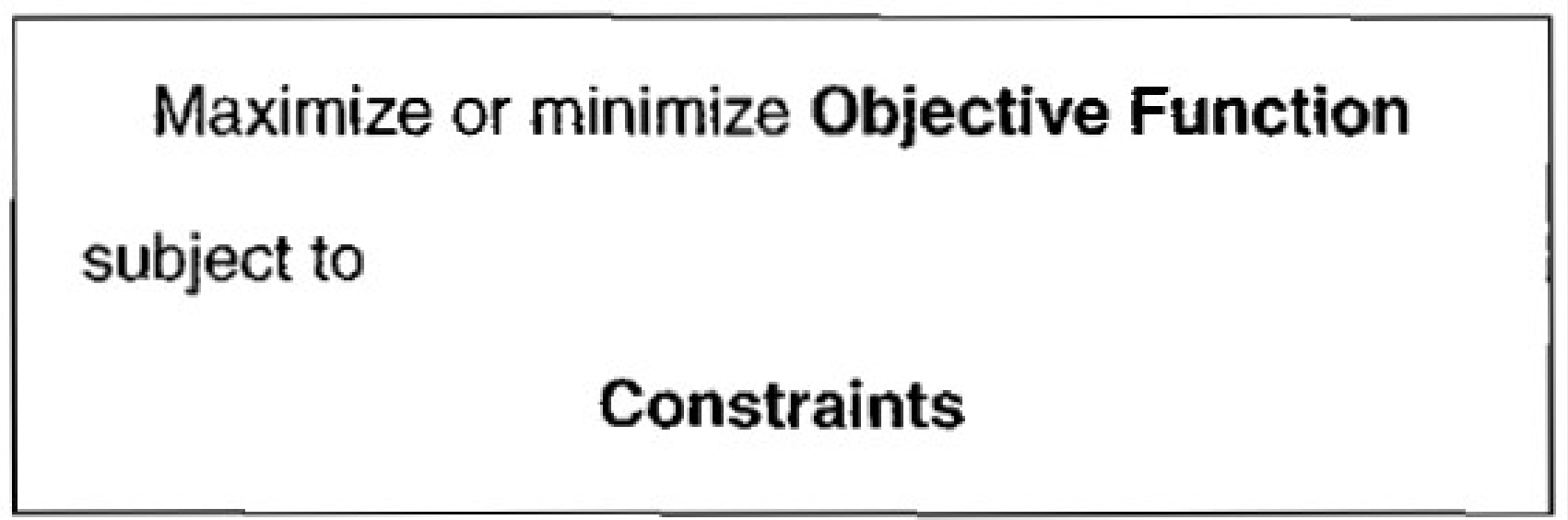
Simple Example (3)

- For a wire of length P, What should be the length and width to form the maximum-area rectangle?

- Let’s work it out
  - *What are decision alternatives?*
  - *Under what restrictions “constraints” is the decision made?*
  - *What is an appropriate objective criterion “function” to evaluate alternative*

- Very simple continuous “infinite” alternatives
  - *Solved by Differential Calculus*

General OR Model



- Feasible Solution: The one that satisfies all the constraints

- Optimal Solution: The one that yield the best (maximum or minimum) objective function value

Example

- Wilson company produces three products: desks, chairs, and molded steel. It is trying to decide on the number of desks (D), chairs (C), and pounds of molded steels (M) to produce. If the profit on each desk is $50, on each chair is $30, and on each pound of molded steel is $6. 7 pounds of raw steel are needed to manufacture a desk, 3 pounds to manufacture a chair, and 1.15 pounds to produce a pound of molded steel. Wilson has only 2000 pounds of raw steel available. At least 100 desks must be produced to satisfy the contract commitment. Work out the mathematical model of this problem

Example – Solution

Maximize

Profit = 50*D + 30*C + 6*M

Subject to

7*D + 3*C + 1.15*M <= 2000 (Raw steel constraint)

D >= 100 (Desks/contract constraint)

C >= 0

M >= 0

Phases of OR Process

1. Definition of the Problem
2. Construction of the Model
3. Solution of the Model
4. Validation of the Model
5. Implementation of the Solution

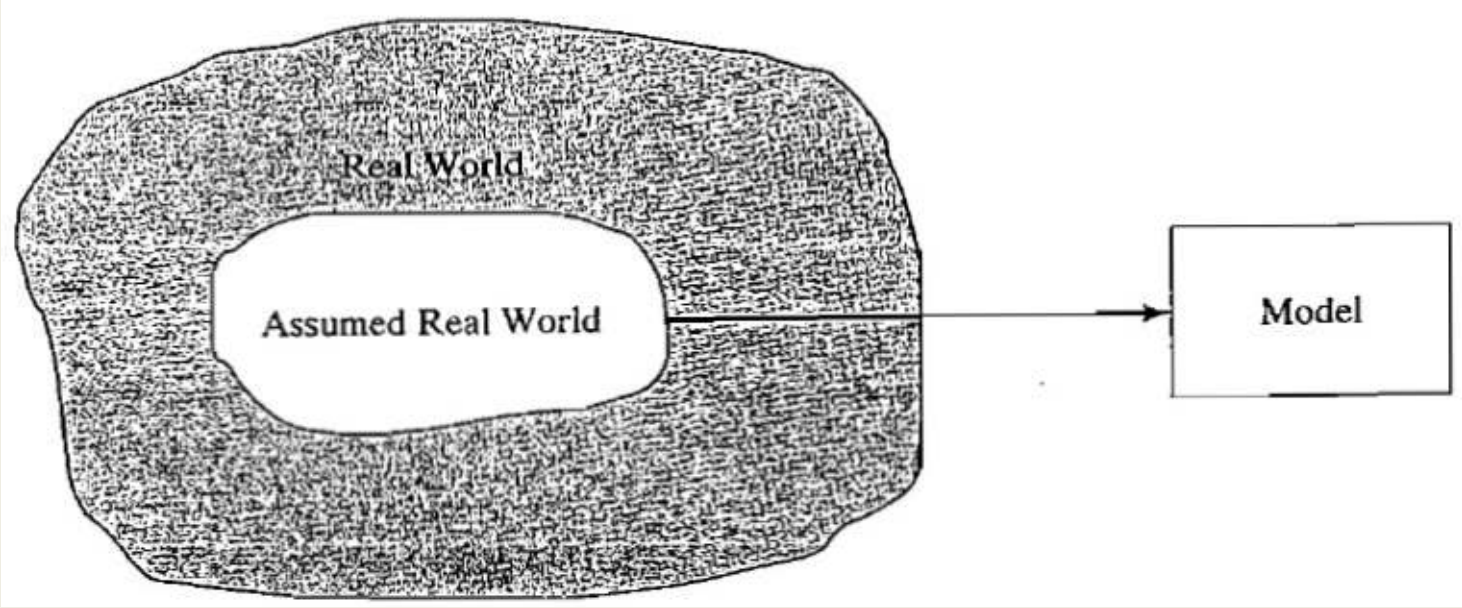
Phases of OR Process – Problem Definition

- The problem should be completely understood by the whole OR team

- The output of this understanding
  - *Decision variables*
  - *Decision alternatives*
  - *Objective function*
  - *Constraints*

Phases of OR Process – Modeling

- Mathematical Modeling: A process that translates the problem definition into mathematical expressions.
- To keep things manageable, you might use simplified assumptions



Phases of OR Process – Model Solution

- Simplest OR phase, and the heart of our course
- Use of a well-defined optimization technique
  - *Alternatives Ranking*
  - *Differential Calculus*
  - *Linear Programming: The most prominent OR technique, linear objective function and constraints*
  - *Integer Programming: Integer decision variables*
  - *…etc*
- Complex problems might be solved using iterative algorithms: computational rules applied repeatedly moving the solution closer to the optimum
- For problems that are impossible to be solved, use heuristics to seek a good solution not the optimum one

Phases of OR Process – Simulation vs Modeling

- Modeling is purely mathematical

- Simulation – Pros
  - *Imitates the behavior of real systems*
  - *Does not use simplified assumptions, such as modeling*
  - *Could be used to practically analyze OR problems*

- Simulation – Cons
  - *Costly in both time and resources*

Phases of OR Process – Model Validation

- Does the proposed model accurately describe the underlying problem
- Compare the output with historical data
- If no historical data are available, use simulation to verify the output of the model
- Unacceptable results ☾ Go back to modeling

OR – More than just Mathematics

- Mathematical modeling is a cornerstone of OR. However, many other factors affect the final decision
- In other words, mathematics suggest certain decisions to the manager, but it don’t necessitate taking these decisions
- OR is rooted in a teamwork. It is a science and an art