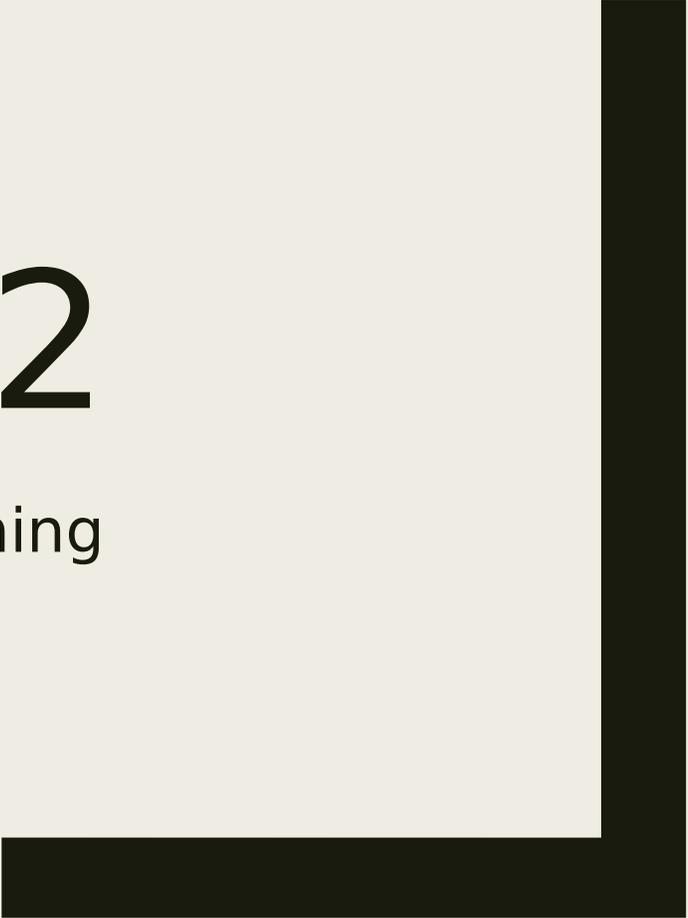


CHAPTER 2

Modeling with Linear Programming



Linear Programming (LP)

- Linear programming (LP) is a technique for optimizing a linear objective function, subject to linear equality and linear inequality constraints.
- Linear programming determines the best outcome (such as maximum profit or lowest cost) in a given mathematical model.

LP Mathematical formulation

1. Understand the problem well
2. Identify decision variables
3. Formulate the objective function
4. Formulate the constraints of the problem
5. Add non-negativity constraints

Example – Reddy Mikks Company

	Tons of raw material per ton of		Maximum daily availability (tons)
	<i>Exterior paint</i>	<i>Interior paint</i>	
Raw material, <i>M1</i>	6	4	24
Raw material, <i>M2</i>	1	2	6
Profit per ton (\$1000)	5	4	

- Daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton
- Maximum daily demand of interior paint is 2 tons

Reddy Mikks LP Model

x_1 = tons produced daily of exterior paint

x_2 = tons produced daily of interior paint

Maximize

$$\text{Profit} = 5 * x_1 + 4 * x_2 \quad (\text{total daily profit in thousands of dollars})$$

subject to

$$6 * x_1 + 4 * x_2 \leq 24 \quad (\text{Constraint1: Raw material M1})$$

$$x_1 + 2 * x_2 \leq 6 \quad (\text{Constraint2: Raw material M2})$$

$$-x_1 + x_2 \leq 1 \quad (\text{Constraint3: } x_2 \text{ cannot exceed } x_1 \text{ by more than 1 ton})$$

$$x_2 \leq 2 \quad (\text{Constraint4 : Maximum demand of } x_2)$$

$$x_1 \geq 0 \quad (\text{Constraint4: non-negativity})$$

$$x_2 \geq 0 \quad (\text{Constraint4: non-negativity})$$

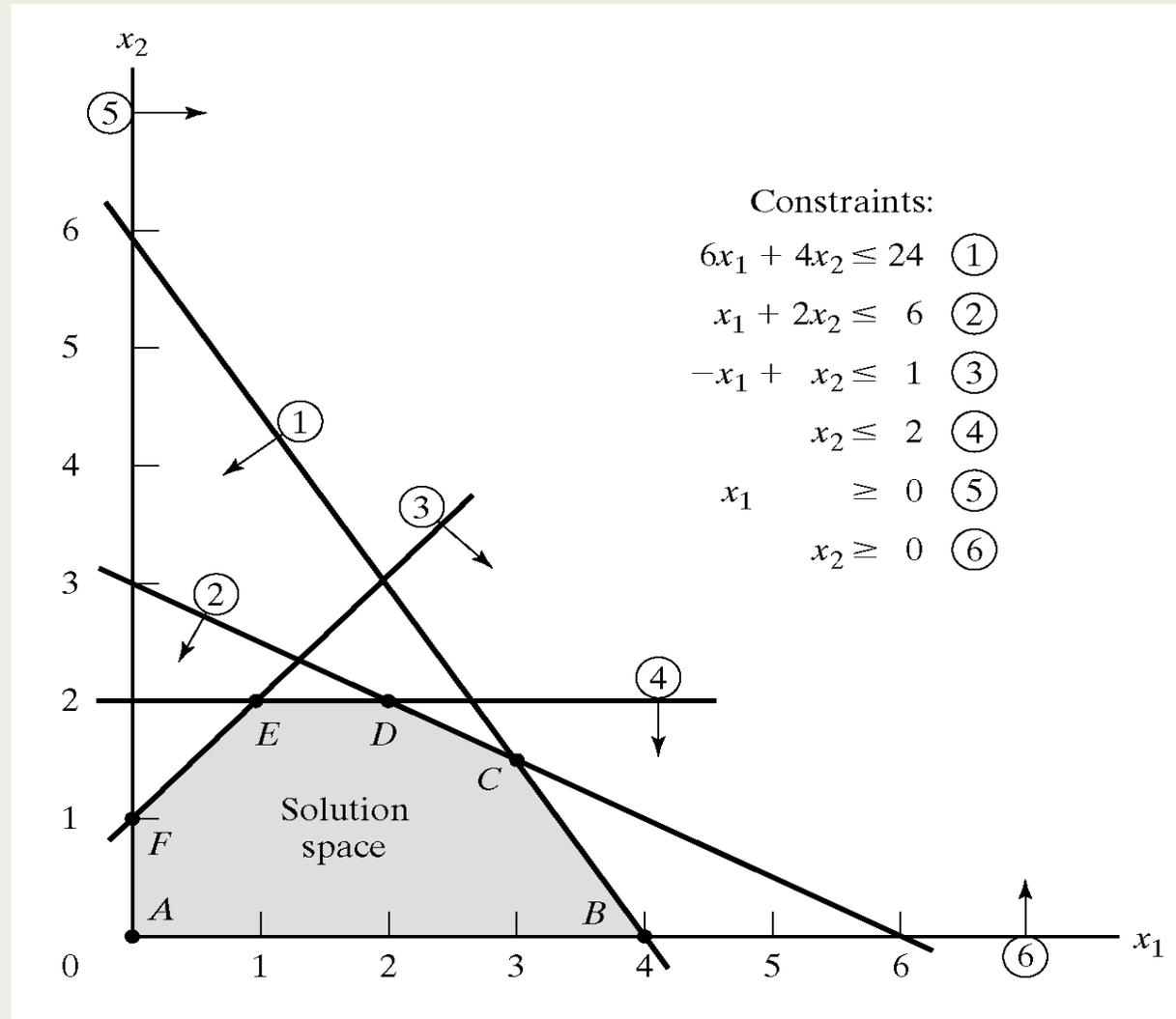
Practice and Exercises

- Work out Problem set 2.1A on Page 15

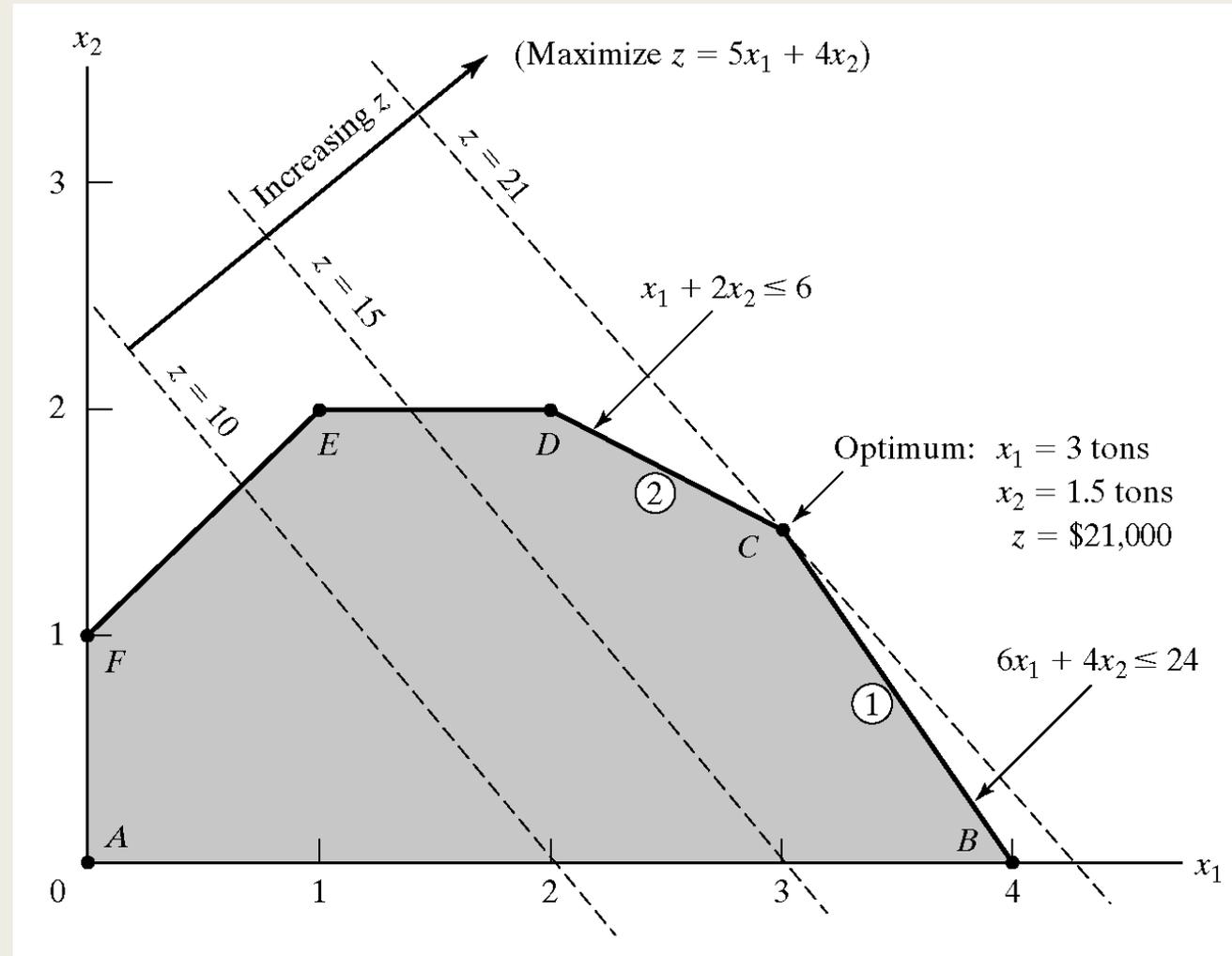
Graphical LP Solution

1. Determination of the feasible solution space.
 - *For each constraint, replace the inequality by equal sign*
 - *Draw the straight line that represent the equation*
 - *Check which side of the straight line satisfies the inequality*
2. Determination of the optimum solution from among all the feasible points in the solution space.
 - *Identify the direction in which profit increases*
 - *Move the objective function till it meets the last point in the feasible solution space ☾ This point represents the optimum*

Graphical LP Solution – Step 1



Graphical LP Solution – Step 2



Graphical LP Solution – Step 2

- The optimum LP solution is always associated with a corner point of the solution space, so the optimum solution can be found by enumerating all the corner points as below:

<u>Corner point</u>	<u>(X_1, X_2)</u>	<u>z</u>
A	(0,0)	0
B	(4,0)	20
C (optimum solution)	(3,1.5)	21
D	(2,2)	18
E	(1,2)	13
F	(0,1)	4

Practice and Exercises

- Work out Problem set 2.2A on Pages 19-23:

LP: Solution of Minimization Problem

- Follow the same steps, as the maximization problem
- While calculating the objective function value of corner points, select the point with the minimum value. This point represents the optimal solution

Example - Ozark Farms

Feedstuff	lb per lb of feedstuff		Cost (\$/lb)
	<i>Protein</i>	<i>Fiber</i>	
Corn	.09	.02	.30
Soybean meal	.60	.06	.90

- Use at least 800 lb of feed daily (Corn + Soybean meal)
- The feed should have at least 30% protein
- The feed should have at most 5% fiber

Ozark Farms LP Model

x_1 = lb of corn in the daily feed

x_2 = lb of soybean meal in the daily feed

Minimize

$$\text{Cost} = 0.3x_1 + 0.9x_2 \quad (\text{total daily cost in dollars})$$

subject to

$$x_1 + x_2 \geq 800 \quad (\text{Constraint1: Minimum daily feed})$$

$$0.09x_1 + 0.6x_2 \geq 0.3(x_1 + x_2) \quad (\text{Constraint2: Minimum amount of protein})$$

$$0.02x_1 + 0.06x_2 \leq 0.05(x_1 + x_2) \quad (\text{Constraint3: Maximum amount of fiber})$$

$$x_1 \geq 0 \quad (\text{Constraint4: non-negativity})$$

$$x_2 \geq 0 \quad (\text{Constraint4: non-negativity})$$

Real Examples: Urban Planning (Ohio)

- The city of Erstville has a project of two phases: demolishing old houses, building new ones (single, double, triple, quadruple units)
- A maximum of 300 old houses could be demolished, area/house=0.25 arce, demolishing cost/house=\$2000
- Areas of new units: 0.18, 0.28, 0.4, 0.5 acre, respectively
- Streets, open area and utilities take 15% of the available acreage
- Single units must be at least 20%, double units 10%, both triple and quadruple 25%
- Tax/unit: \$1000, \$1900, \$2700, and \$3400, respectively
- Construction cost of new units: \$50000, \$70000, \$130000, and \$160000
- Maximum Financing: \$15 million

Real Examples: Urban Planning (Ohio)

x_1 = Number of single units

x_2 = Number of double units

x_3 = Number of triple units

x_4 = Number of quadruple units

x_5 = Number of old houses to be demolished

Maximize

$$\text{Tax} = 1000x_1 + 1900x_2 + 2700x_3 + 3400x_4 \quad (\text{Income from Tax})$$

subject to

$$0.18x_1 + 0.28x_2 + 0.4x_3 + 0.5x_4 \leq 0.25x_5(1 - 0.15) \quad (\text{C1: Available Acreage})$$

$$x_5 \leq 300 \quad (\text{C2: Demolished houses})$$

$$x_1 \geq 0.2(x_1 + x_2 + x_3 + x_4) \quad (\text{C3: Single units})$$

$$x_2 \geq 0.1(x_1 + x_2 + x_3 + x_4) \quad (\text{C4: Double units})$$

$$x_3 + x_4 \geq 0.25(x_1 + x_2 + x_3 + x_4) \quad (\text{C5: Triple and quadruple units})$$

$$50x_1 + 70x_2 + 130x_3 + 160x_4 + 2x_5 \leq 15000 \quad (\text{C6: Cost})$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0 \quad (\text{C7: non-negativity})$$

Real Examples: Production Model

- A clothing company manufactures parka, goose overcoats, insulated pants, and gloves
- The company uses four processes: cutting, insulating, sewing, and packaging
- The contract has penalty on undelivered items

Department	Time per units (hr)				Capacity (hr)
	<i>Parka</i>	<i>Goose</i>	<i>Pants</i>	<i>Gloves</i>	
Cutting	.30	.30	.25	.15	1000
Insulating	.25	.35	.30	.10	1000
Sewing	.45	.50	.40	.22	1000
Packaging	.15	.15	.1	.05	1000
Demand	800	750	600	500	
Unit profit	\$30	\$40	\$20	\$10	
Unit penalty	\$15	\$20	\$10	\$8	

Real Examples: Urban Planning (Ohio)

x_1 = Number of parka jackets

x_2 = Number of goose overcoats

x_3 = Number of insulated pants

x_4 = Number of gloves

s_1 = Number shortage items of parka jackets

s_2 = Number shortage items of goose overcoats

s_3 = Number shortage items of insulated pants

s_4 = Number shortage items of gloves

Maximize

$$\text{Net Profit} = 30*x_1 + 40*x_2 + 20*x_3 + 10*x_4 - (15*s_1 + 20*s_2 + 10*s_3 + 8*s_4) \quad (\text{Profit} - \text{Penalty})$$

subject to

$$0.3*x_1 + 0.3*x_2 + 0.25*x_3 + 0.15*x_4 \leq 1000 \quad (\text{C1: Cutting capacity})$$

$$0.25*x_1 + 0.35*x_2 + 0.3*x_3 + 0.1*x_4 \leq 1000 \quad (\text{C2: Insulating capacity})$$

$$0.45*x_1 + 0.5*x_2 + 0.4*x_3 + 0.22*x_4 \leq 1000 \quad (\text{C3: Sewing capacity})$$

$$0.15*x_1 + 0.15*x_2 + 0.1*x_3 + 0.05*x_4 \leq 1000 \quad (\text{C4: Packaging capacity})$$

$$x_1 + s_1 = 800 \quad (\text{C5: Parka demand})$$

$$x_2 + s_2 = 750 \quad (\text{C6: Goose overcoat demand})$$

$$x_3 + s_3 = 600 \quad (\text{C7: Insulated pants demand})$$

$$x_4 + s_4 = 500 \quad (\text{C8: Gloves demand})$$

$$x_1, x_2, x_3, x_4, s_1, s_2, s_3, s_4 \geq 0 \quad (\text{C9: non-negativity})$$