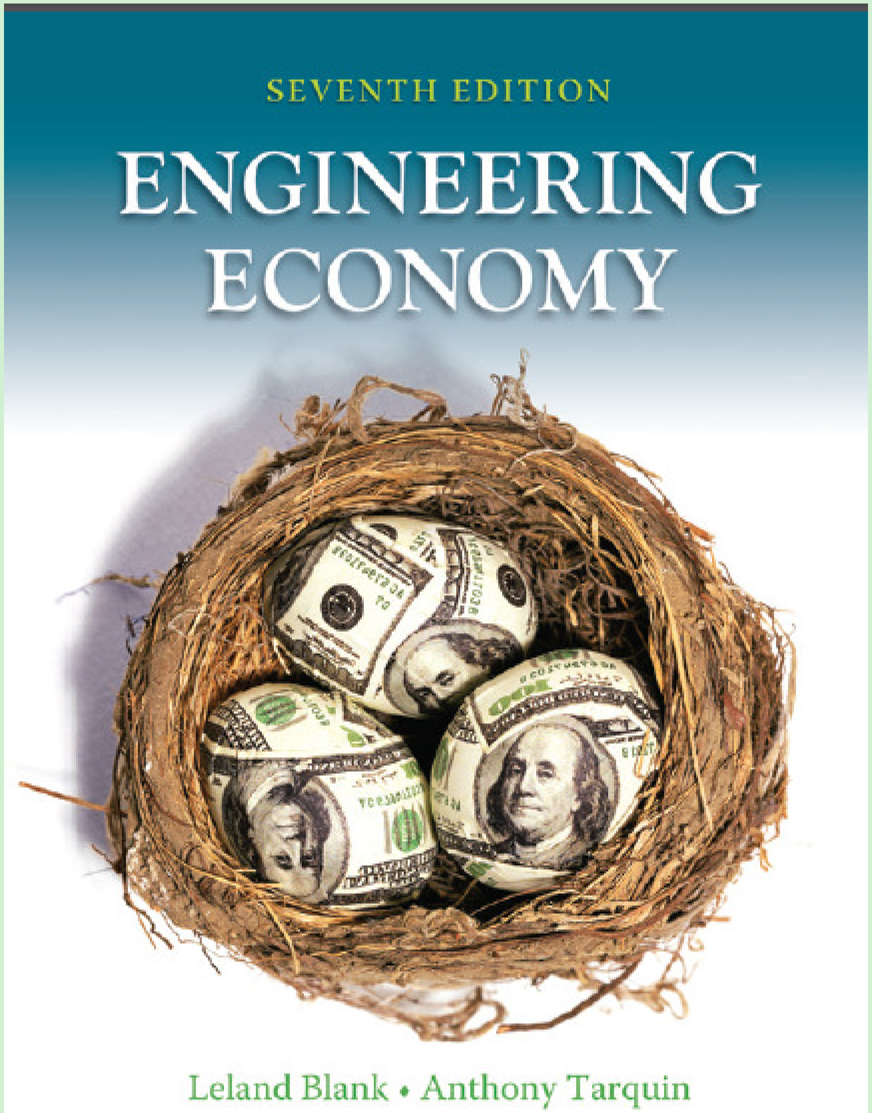

Chapter 3

Combining Factors and Spreadsheet Functions

Lecture slides to accompany

Engineering Economy

7th edition

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McGraw Hill Higher Education

LEARNING OUTCOMES

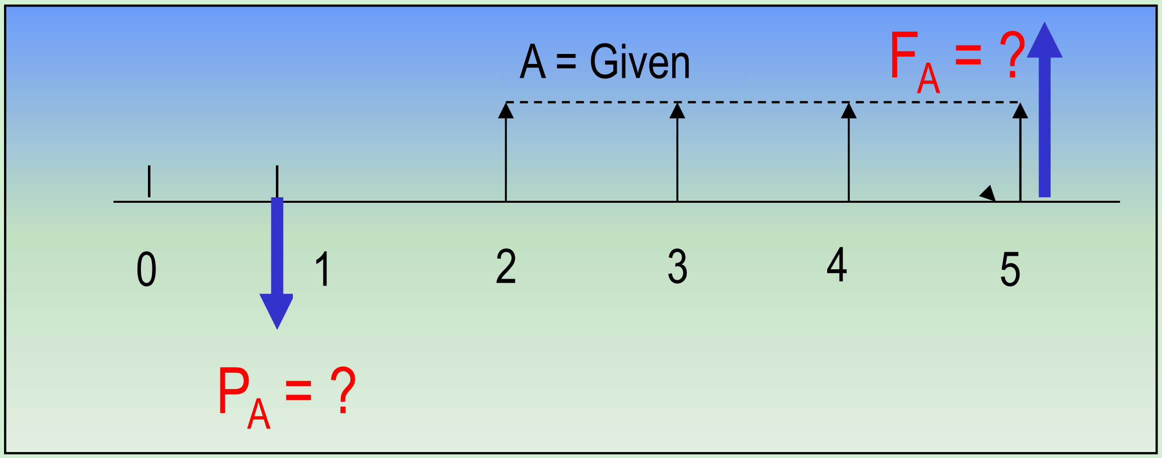
1. **Shifted uniform series**
2. **Shifted series and single cash flows**
3. **Shifted gradients**

Shifted Uniform Series

A shifted uniform series starts at a time *other than period 1*

The cash flow diagram below is an example of a shifted series

Series starts in period 2, not period 1



Shifted series usually require the use of *multiple factors*

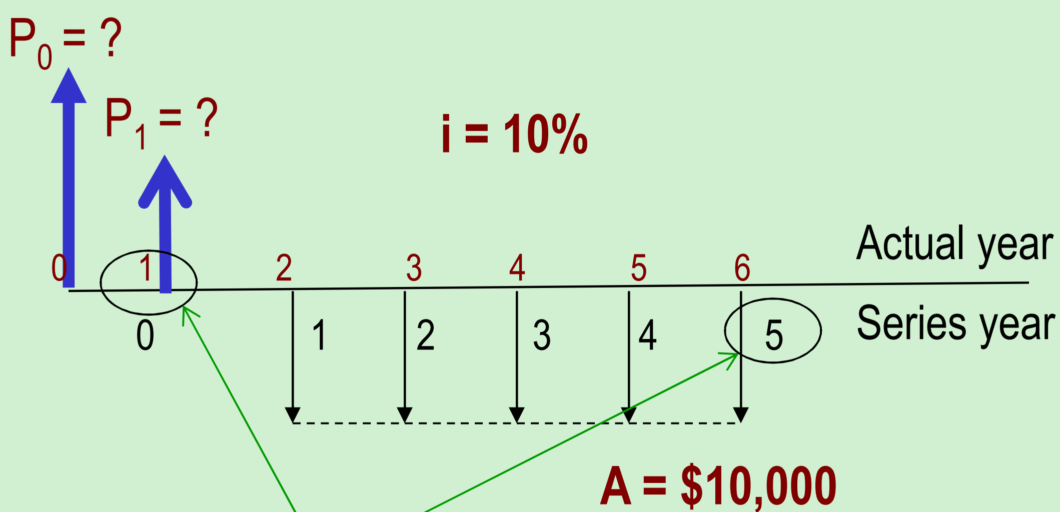
Remember: **When using P/A or A/P factor, P_A is always *one year ahead* of first A**

When using F/A or A/F factor, F_A is in *same year as last A*

Example Using P/A Factor: Shifted Uniform Series

The present worth of the cash flow shown below at i = 10% is:

(a) $25,304 (b) $29,562 (c) $34,462 (d) $37,908



Solution: (1) Use P/A factor with n = 5 (for 5 arrows) to get P_1 **in year 1**

(2) Use P/F factor with n = 1 to move P_1 back for P_0 **in year 0**

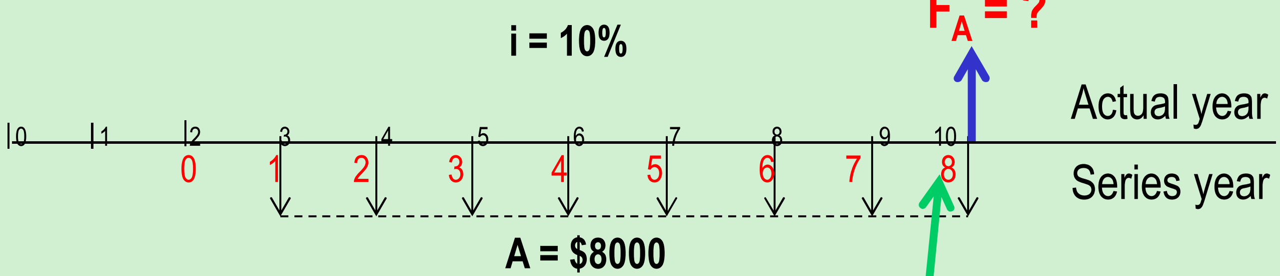
$$P_0 = P_1(P/F,10\%,1) = A(P/A,10\%,5)(P/F,10\%,1) = 10{,}000(3.7908)(0.9091) = \$34{,}462$$

Answer is (c)

Example Using F/A Factor: Shifted Uniform Series

How much money would be available in year 10 if $8000 is deposited each year in years 3 through 10 at an interest rate of 10% per year?

Cash flow diagram is:



Solution: Re-number diagram to determine n = 8 (number of arrows)

$F_A = 8000(F/A,10\%,8)$
$= 8000(11.4359)$
$= \mathbf{\$91,487}$

Shifted Series and Random Single Amounts

For cash flows that include *uniform series* **and** *randomly placed single amounts*:

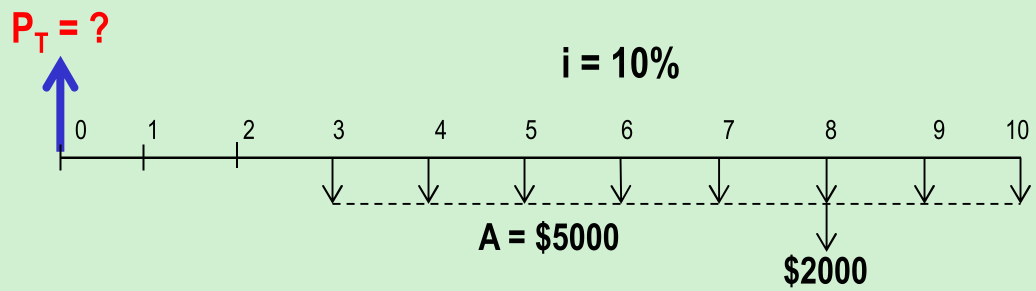
- ***Uniform series procedures*** **are applied to the** ***series amounts***
- ***Single amount formulas*** **are applied to the** ***one-time cash flows***

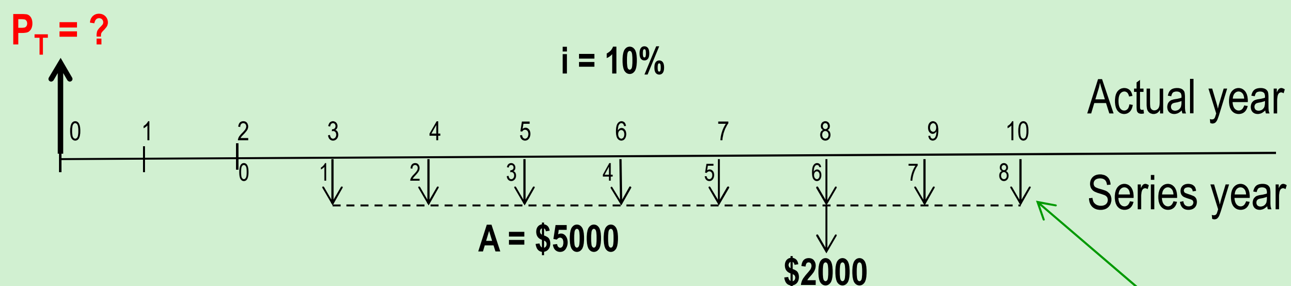
The resulting values are then *combined* per the problem statement

The following slides illustrate the procedure

Example: Series and Random Single Amounts

Find the present worth in year 0 for the cash flows shown using an interest rate of 10% per year.

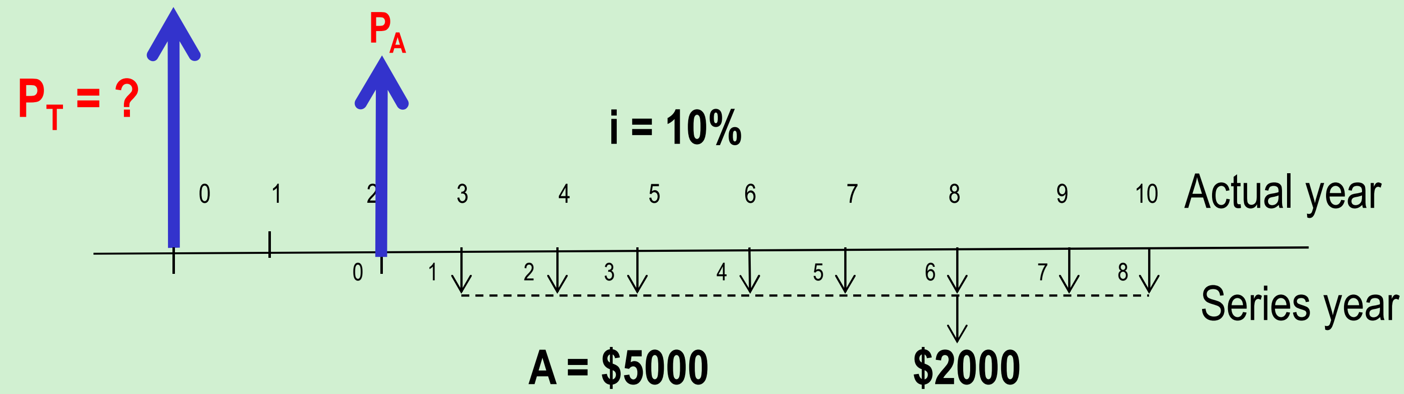




Solution:

First, re-number cash flow diagram to get n for uniform series: n = 8

Example: Series and Random Single Amounts



Use P/A to get P_A in year 2: $P_A = 5000(P/A,10\%,8) = 5000(5.3349) = \$26,675$

Move P_A back to year 0 using P/F: $P_0 = 26,675(P/F,10\%,2) = 26,675(0.8264) = \$22,044$

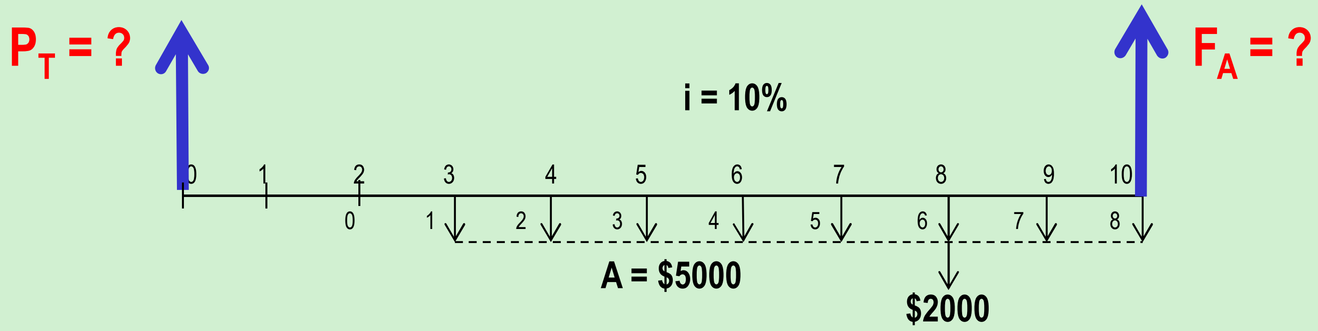
Move $2000 single amount back to year 0: $P_{2000} = 2000(P/F,10\%,8) = 2000(0.4665) = \$933$

Now, add P_0 and P_{2000} to get P_T: $P_T = 22,044 + 933 = \$22,977$

Example Worked a Different Way

(Using F/A instead of P/A for uniform series)

The same re-numbered diagram from the previous slide is used



Solution: Use F/A to get F_A in actual year 10: F_A = 5000(F/A,10%,8) = 5000(11.4359) = $57,180

Move F_A back to year 0 using P/F: P_0 = 57,180(P/F,10%,10) = 57,180(0.3855) = $22,043

Move $2000 single amount back to year 0: P_{2000} = 2000(P/F,10%,8) = 2000(0.4665) = $933

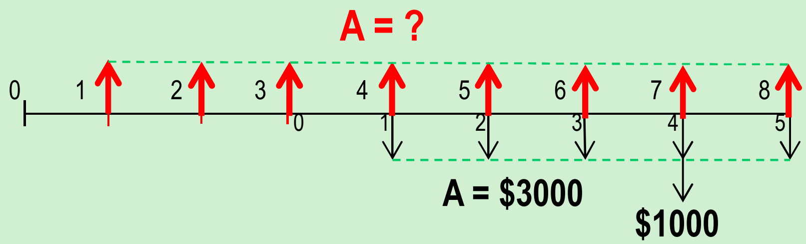
Now, add two P values to get P_T: P_T = 22,043 + 933 = $22,976

Same as before

As shown, there are usually multiple ways to work equivalency problems

Example: Series and Random Amounts

Convert the cash flows shown below (black arrows) into an equivalent annual worth A in years 1 through 8 (red arrows) at i = 10% per year.



i = 10%

Approaches:

1. **Convert all cash flows into P in year 0** and use A/P with n = 8
2. **Find F in year 8** and use A/F with n = 8

Solution:

Solve for F: **F = 3000(F/A,10%,5) + 1000(F/P,10%,1)**
= 3000(6.1051) + 1000(1.1000)
= $19,415

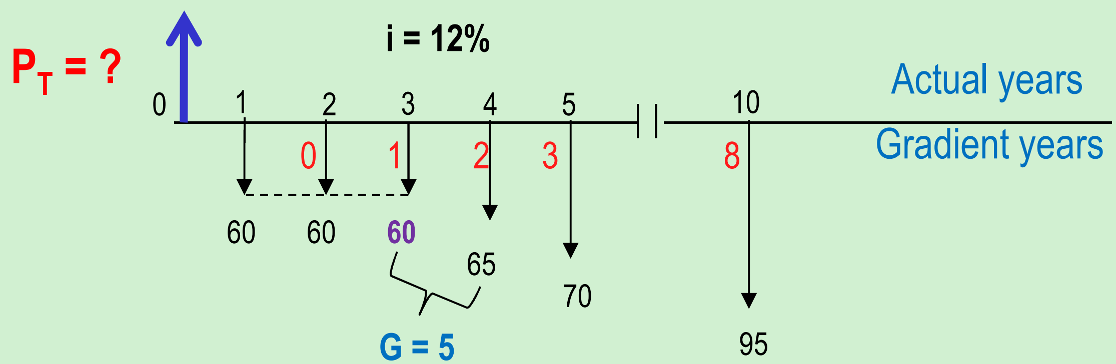
Find A: **A = 19,415(A/F,10%,8)**
= 19,415(0.08744)
= $1698

Shifted Arithmetic Gradients

- Shifted gradient begins at a time other than between periods 1 and 2
- Present worth P_G is located 2 periods before gradient starts
- Must use multiple factors to find P_T in actual year 0
- To find equivalent A series, find P_T at actual time 0 and apply (A/P,i,n)

Example: Shifted Arithmetic Gradient

John Deere expects the cost of a tractor part to increase by $5 per year beginning 4 years from now. If the cost in years 1-3 is $60, determine the *present worth in year 0* of the cost through year 10 at an interest rate of 12% per year.



Solution: **First find P_2 for G = $5 *and* base amount ($60) in actual year 2**

P_2 = 60(P/A,12%,8) + 5(P/G,12%,8) = $370.41

Next, move P_2 back to year 0

P_0 = P_2(P/F,12%,2) = $295.29

Next, find P_A for the $60 amounts of years 1 and 2

P_A = 60(P/A,12%,2) = $101.41

Finally, add P_0 and P_A to get P_T in year 0

P_T = P_0 + P_A = $396.70

Shifted Geometric Gradients

Shifted gradient begins at a time other than between periods 1 and 2

Equation yields P_g for *all* cash flows (base amount A_1 is included)

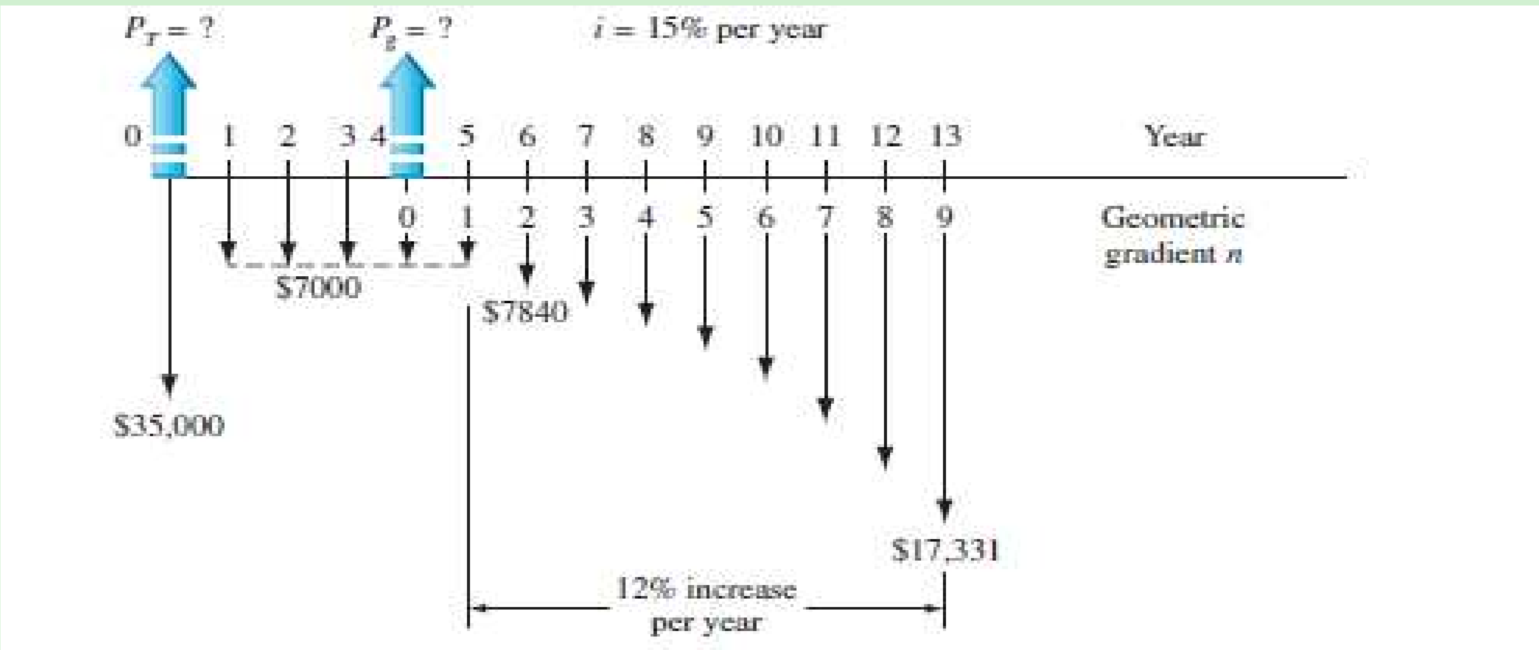
Equation ($i \neq g$): $P_g = A_1\{1 - [(1+g)/(1+i)]^n/(i-g)\}$

For negative gradient, change signs on both g values

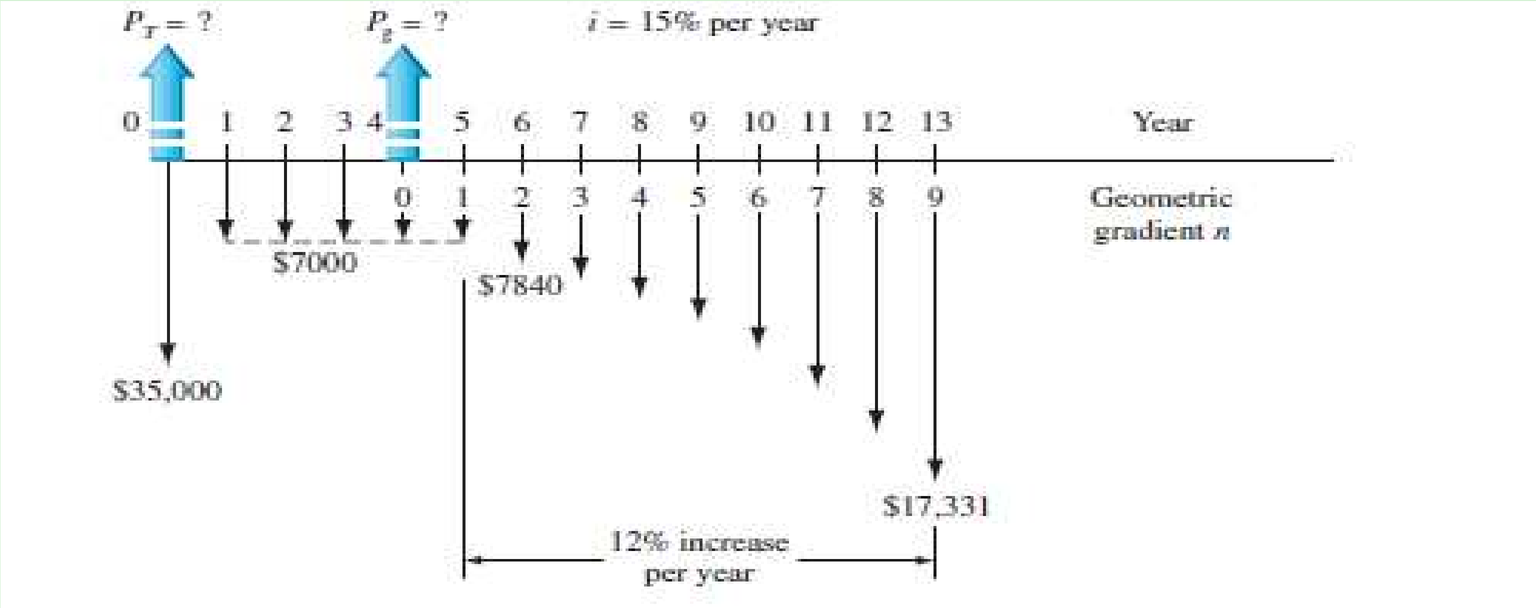
There are no tables for geometric gradient factors

Example: Shifted Geometric Gradient

Weirton Steel signed a 5-year contract to purchase water treatment chemicals from a local distributor for $7000 per year. When the contract ends, the cost of the chemicals is expected to increase by 12% per year for the next 8 years. If an initial investment in storage tanks is $35,000, determine the equivalent present worth in year 0 of all of the cash flows at i = 15% per year.



Example: Shifted Geometric Gradient



Gradient starts between actual years 5 and 6; these are gradient years 1 and 2. P_g is located in gradient year 0, which is actual year 4

P_g = 7000{1-[(1+0.12)/(1+0.15)]^9/(0.15-0.12)} = $49,401

Move P_g and other cash flows to year 0 to calculate P_T

P_T = 35,000 + 7000(P/A,15%,4) + 49,401(P/F,15%,4) = $83,232

Negative Shifted Gradients

For negative arithmetic gradients, change sign on G term from + to -

General equation for determining P: **P = present worth of base amount - P_G**

Changed from + to -

For negative geometric gradients, change signs on both g values

Changed from + to -

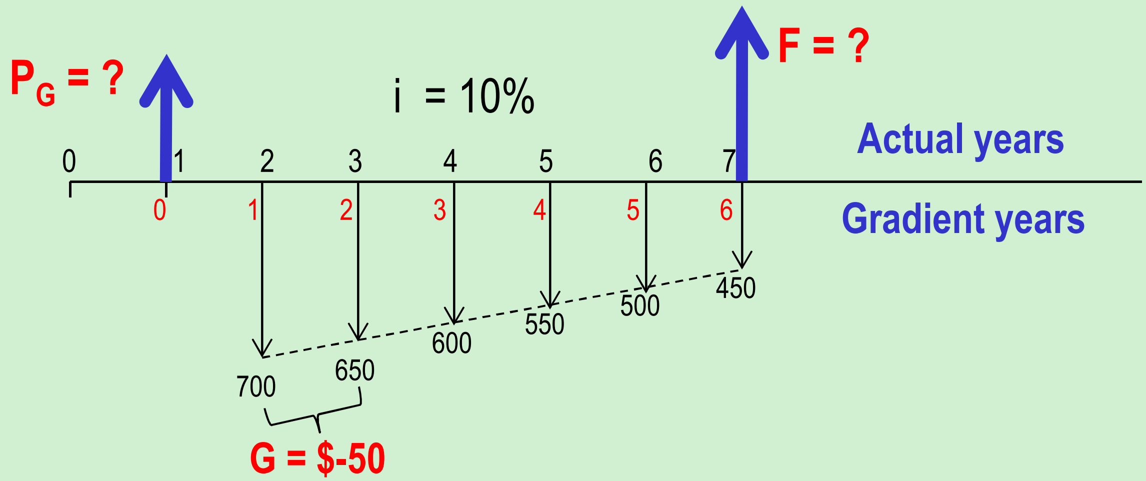
$$P_g = A_1\{1-[(1-g)/(1+i)]^n/(i+g)\}$$

Changed from - to +

All other procedures are the same as for positive gradients

Example: Negative Shifted Arithmetic Gradient

For the cash flows shown, find the future worth in year 7 at i = 10% per year



Solution: Gradient G first occurs between actual years 2 and 3; these are gradient years 1 and 2

P_G is located in gradient year 0 (actual year 1); base amount of $700 is in gradient years 1-6

$$P_G = 700(P/A,10\%,6) – 50(P/G,10\%,6) = 700(4.3553) – 50(9.6842) = \$2565$$

$$F = P_G(F/P,10\%,6) = 2565(1.7716) = \$4544$$

Summary of Important Points

P for shifted uniform series is ***one period ahead*** of first A; n is equal to number of A values

F for shifted uniform series is in ***same period*** as last A; n is equal to number of A values

For gradients, ***first change*** equal to G or g occurs between **gradient years 1 and 2**

For **negative arithmetic** gradients, change sign on G from + to -

For **negative geometric** gradients, change sign on g from + to -