FAHAD BIN SULTAN UNIVERSITY
College of Engineering
Department of Mechanical Engineering

Mechanical Design
MECH 330

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Chapter 6

Fatigue Failure Resulting from Variable Loading

Part: II

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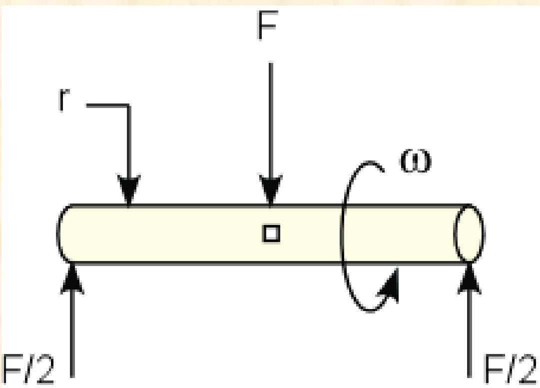
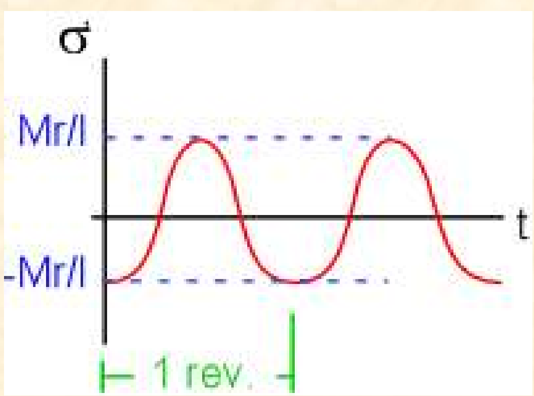
Fluctuating Fatigue Failures

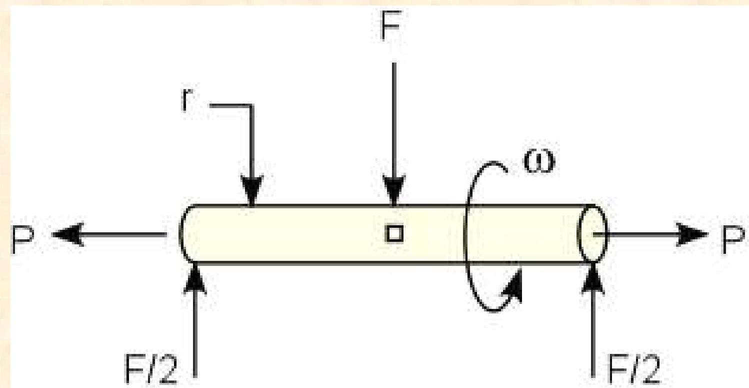
Alternating vs. fluctuating

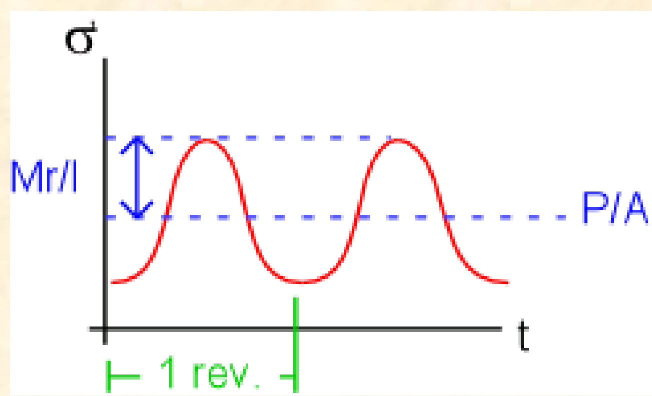
Alternating (Reversing)





Fluctuating





$$\sigma_m = \frac{P}{A}$$

$$\sigma_a = \frac{Mr}{I}$$

Fluctuating stresses

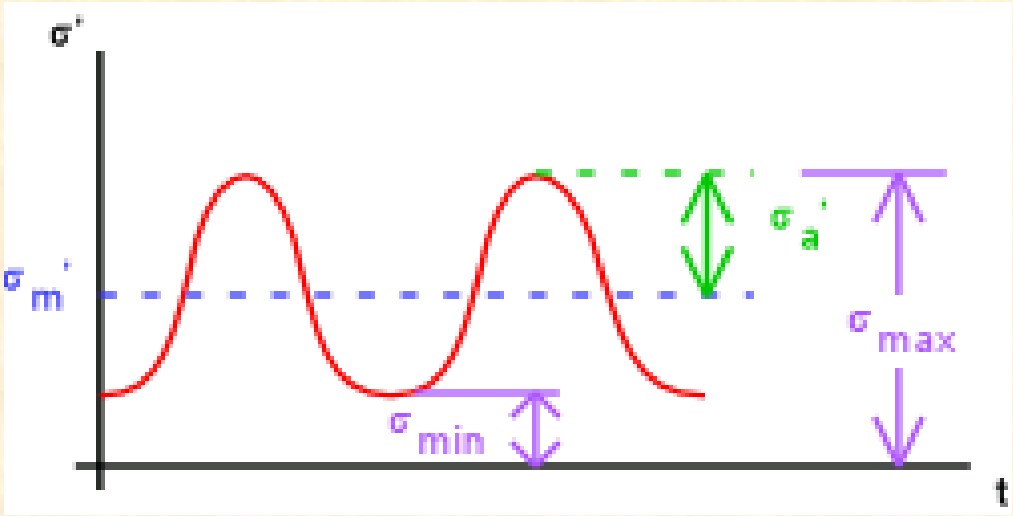
- Mean Stress

$$\sigma'_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

- Stress amplitude

$$\sigma'_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

- Together, σ_m and σ_a characterize fluctuating stress



Plot of Fatigue Failures for Midrange Stresses in both Tensile and Compressive Regions.

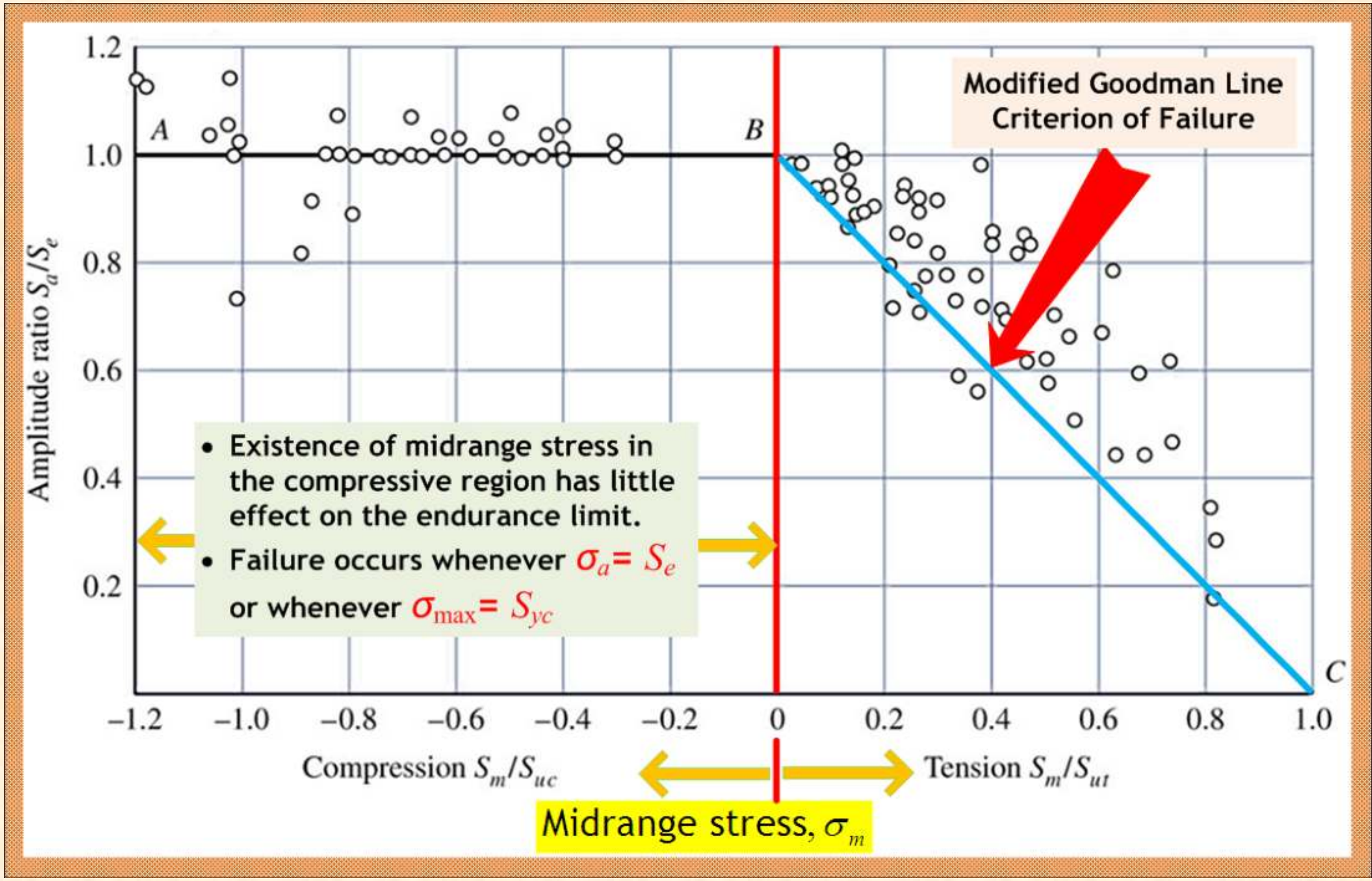


Figure 6-25
Plot of fatigue failures for midrange stresses in both tensile and compressive regions. Normalizing the data by using the ratio of steady strength components to tensile strength S_m/S_{ut}, steady strength component to compressive strength S_m/S_{uc}, and strength amplitude component to endurance limit S_a/S'_e enables a plot of experimental results for a variety of steels.

Fluctuating Stresses

2. Representing mean stress effect using **modified Goodman Diagram**

S is for strength

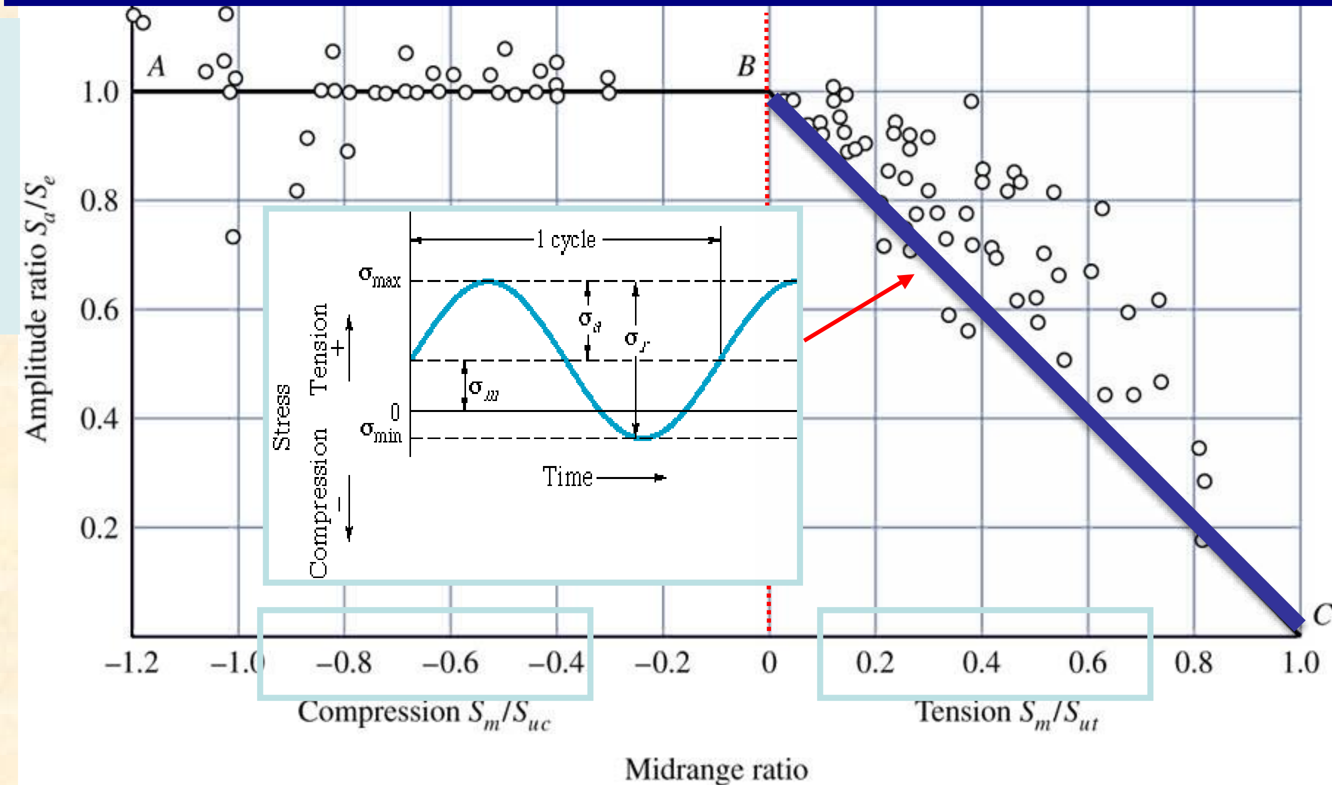


Fig. 6-25

Failure data for S_m in tension and in compression

COMPRESSIVE mean stresses are BENEFICIAL (or have no effect) in fatigue

TENSILE mean stresses are DETRIMENTAL for fatigue behavior

- In Fig. 6-27, the tensile side of Fig. 6-25 has been redrawn in terms of strengths, instead of strength ratios, with the same modified Goodman criterion together with four additional criteria of failure.
- Such diagrams are often constructed for analysis and design purposes; they are easy to use and the results can be scaled off directly.
- The early viewpoint expressed on a diagram was that *there existed a locus (s_a, s_m) diagram was that there existed a locus which divided safe from unsafe combinations of (s_a, s_m)*.
- Ensuing proposals included:
  1. The parabola of Gerber (1874),
  2. The The modified Goodman (1890) (straight) line,
  3. The Soderberg (1930) (straight) line.
  4. The ASME-elliptic

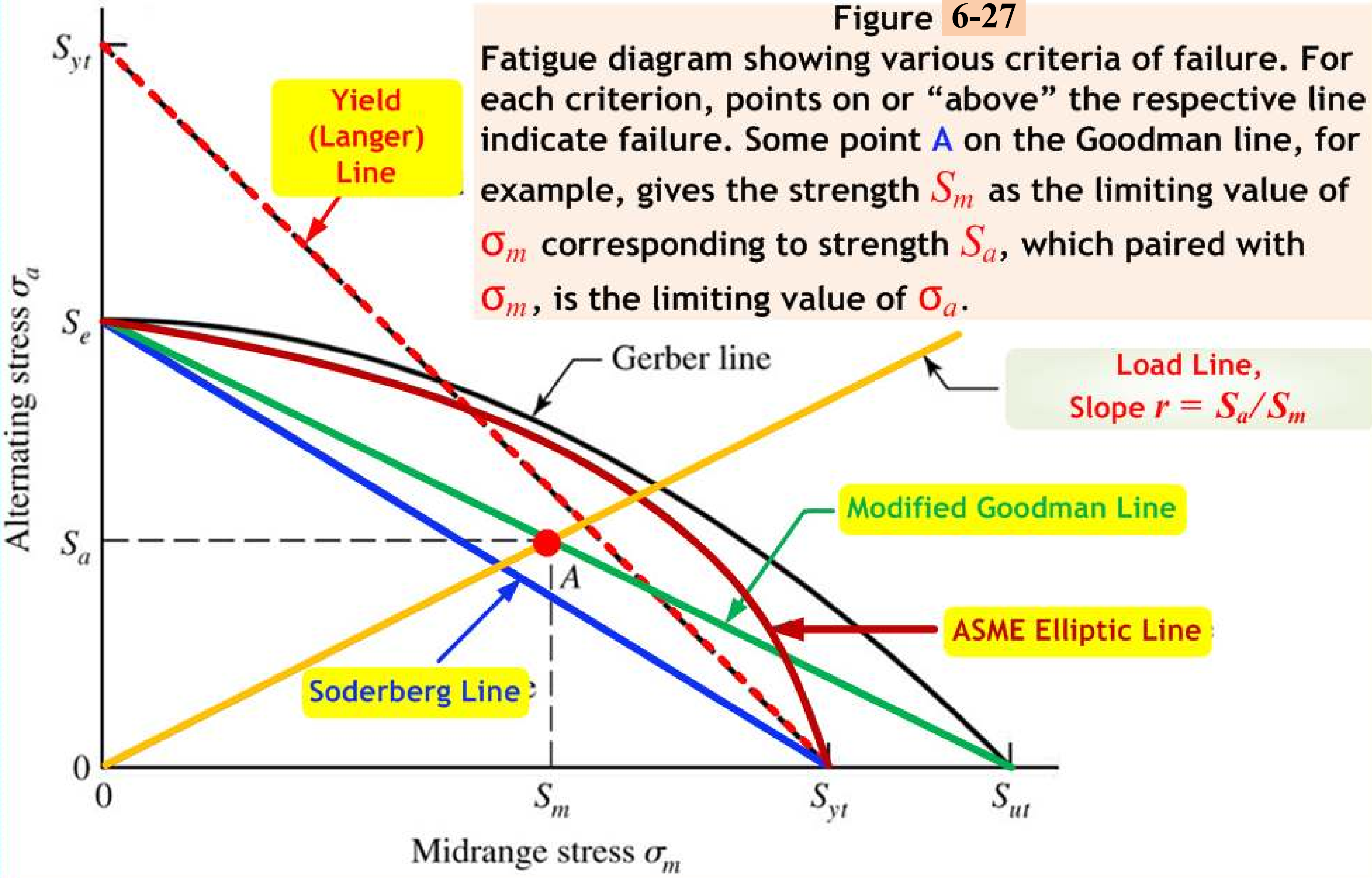


Figure 6-27

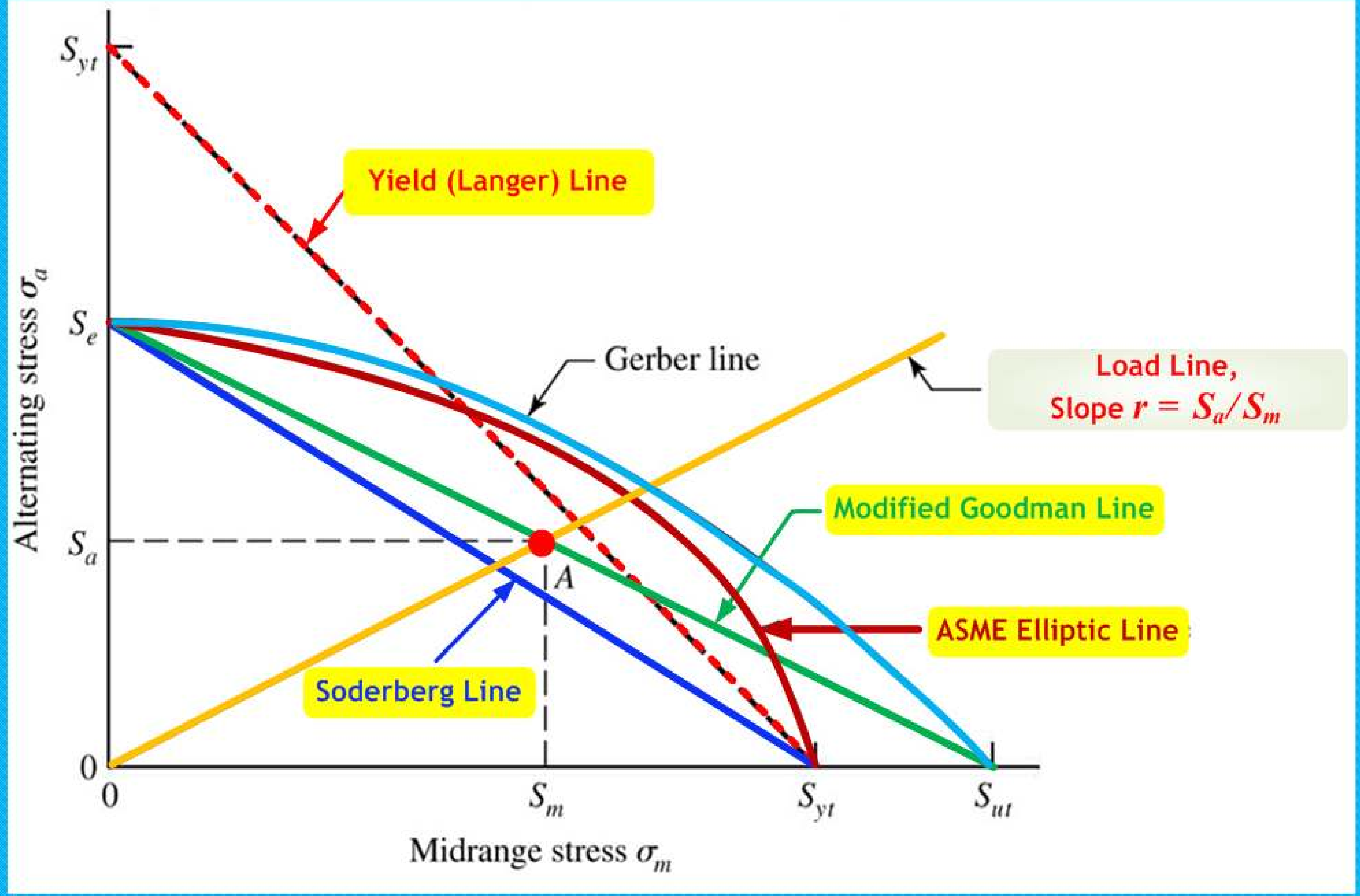
Fatigue diagram showing various criteria of failure. For each criterion, points on or "above" the respective line indicate failure. Some point A on the Goodman line, for example, gives the strength S_m as the limiting value of σ_m corresponding to strength S_a, which paired with σ_m, is the limiting value of σ_a.

- As more data were generated it became clear that a fatigue criterion, rather than being a “fence”, was more like a zone or band wherein the probability of failure could be estimated. We include the failure criterion of Goodman because
  - It is a straight line and the algebra is linear and easy.
  - It is easily graphed, every time for every problem.
  - It reveals subtleties of insight into fatigue problems.
  - Answers can be scaled from the diagrams as a check on the algebra.

- Either the fatigue limit S_e or the finite-life strength S_f is plotted on the ordinate of Fig. 6-27.
- These values will have already been corrected using the Marin factors of Eq.(6-17).
- Note that the yield strength is plotted on the ordinate too.
- This serves as a reminder that first-cycle yielding rather than fatigue might be the criterion of failure.
- The midrange-stress axis of Fig. 6-27 has the yield strength S_{yt} and the tensile strength plotted along it.

- The criteria of failure are diagrammed in Fig.6-27:
  1. The Soderberg,
  2. The modified Goodman
  3. The Gerber
  4. The ASME-elliptic
  5. Yielding
- The diagram shows that only the Soderberg criterion guards against any yielding, but is biased low.
- Considering the modified Goodman line as a criterion, point A represents a limiting point with an alternating strength S_a and midrange strength S_m . The slope of the load line shown is defined as .

$$r = S_a / S_m$$


Yield (Langer) Line
Gerber line
Load Line,
Slope $r = S_a/S_m$
Modified Goodman Line
ASME Elliptic Line
Soderberg Line
Alternating stress σ_a
S_{yt}
S_e
S_a
0
A
0
S_m
S_{yt}
S_{ut}
Midrange stress σ_m

FAILURE CRITERIA (mean stress)

1- Modified Goodman Theory (Germany, 1899)

For infinite life Failure Occurs When:

$$\frac{S_a}{S_e} + \frac{S_m}{S_u} = 1$$

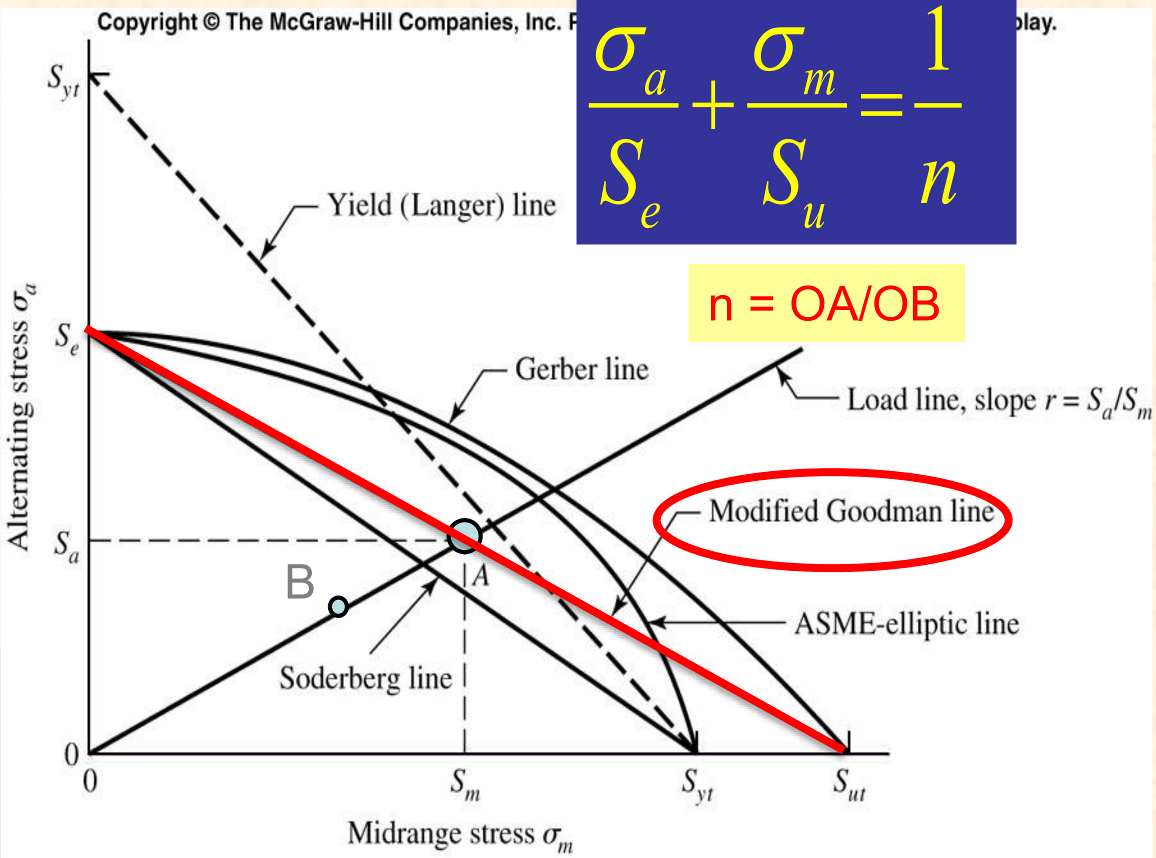
Factor of Safety

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = \frac{1}{n}$$

n = OA/OB

Load Line slope

$$r = \frac{S_a}{S_m}$$



For finite life fatigue strength S_{rev} (equavilant reversed) ***replaces Se and solve for*** S_{rev}

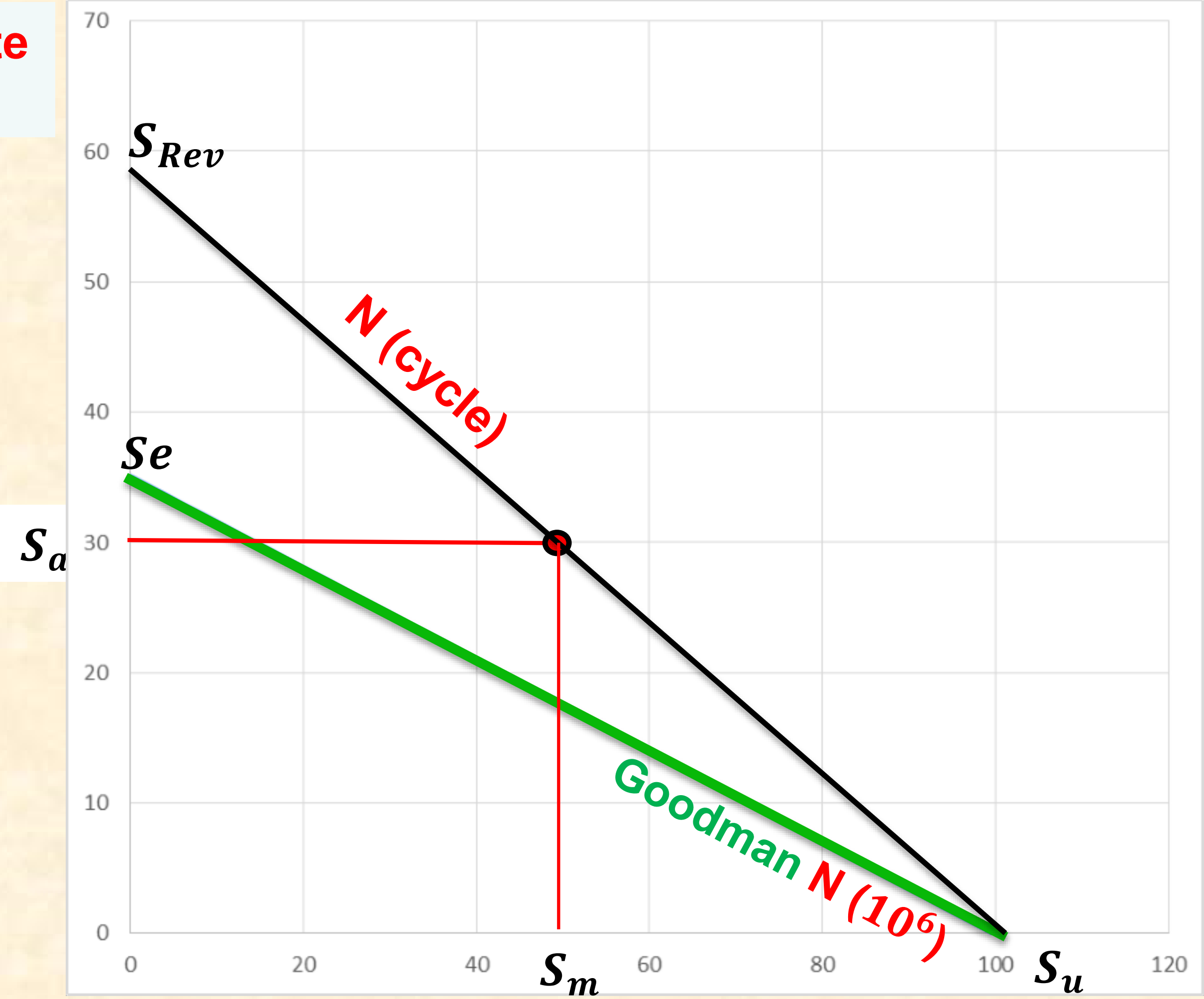
For finite life : How to calculate The life N (cycle)

$$\frac{S_a}{S_e} + \frac{S_m}{S_u} = 1$$

$$\frac{S_a}{S_{Rev}} + \frac{S_m}{S_u} = 1$$

$$S_f = S_{Rev}$$

if: $S_f \geq f S_u$



$$S_f = S_{ut} N^{(\log f)/3} \qquad 1 \leq N \leq 10^3$$

if: $S_e < S_f \leq f S_u$

$$S_f = a\, N^b$$

$$a = \frac{(f\, S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3} \log\left(\frac{f\, S_{ut}}{S_e}\right)$$

FAILURE CRITERIA (mean stress)

2- The Soderberg Theory (USA, 1933)

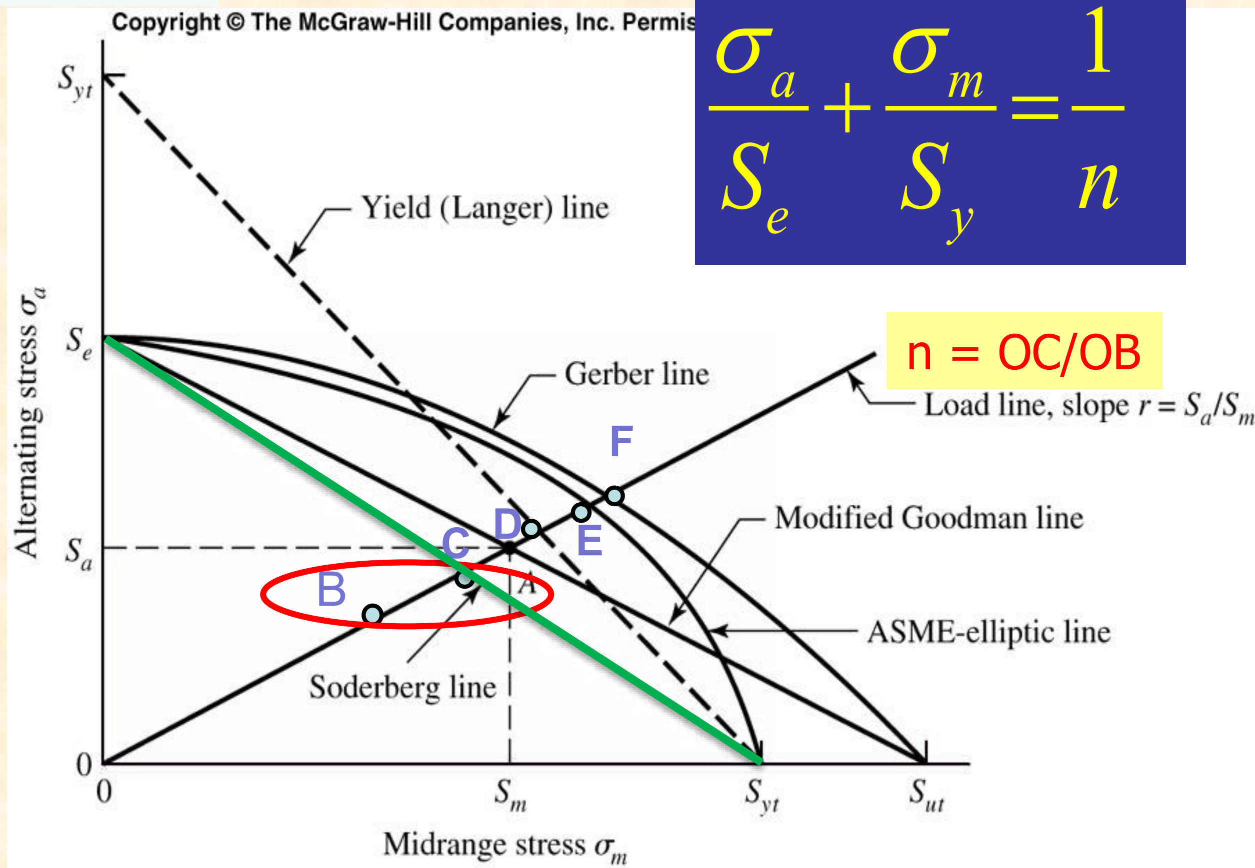
For infinite life Failure Occurs When:

$$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1$$

Factor of Safety

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

n = OC/OB



For finite life fatigue strength S_{rev} (equavilant reversed) ***replaces Se***

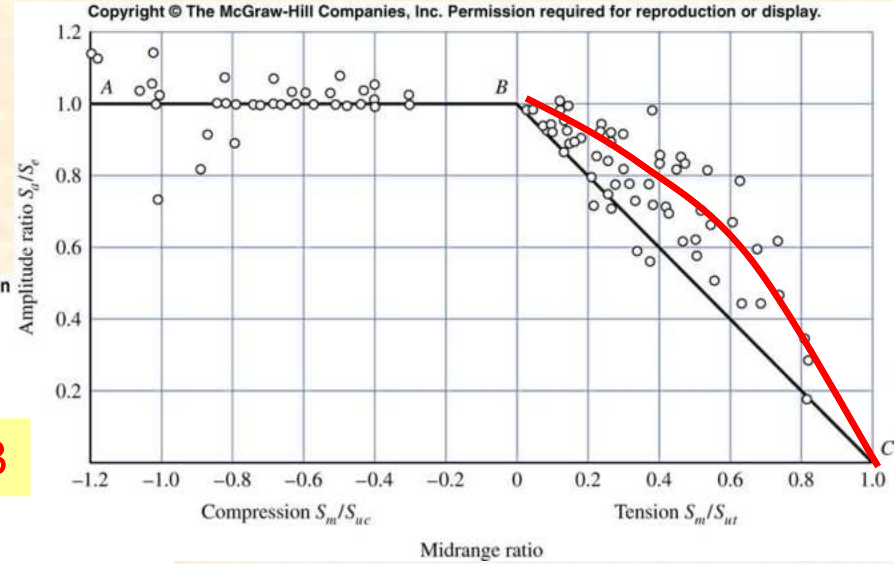
and solve for S_{rev}

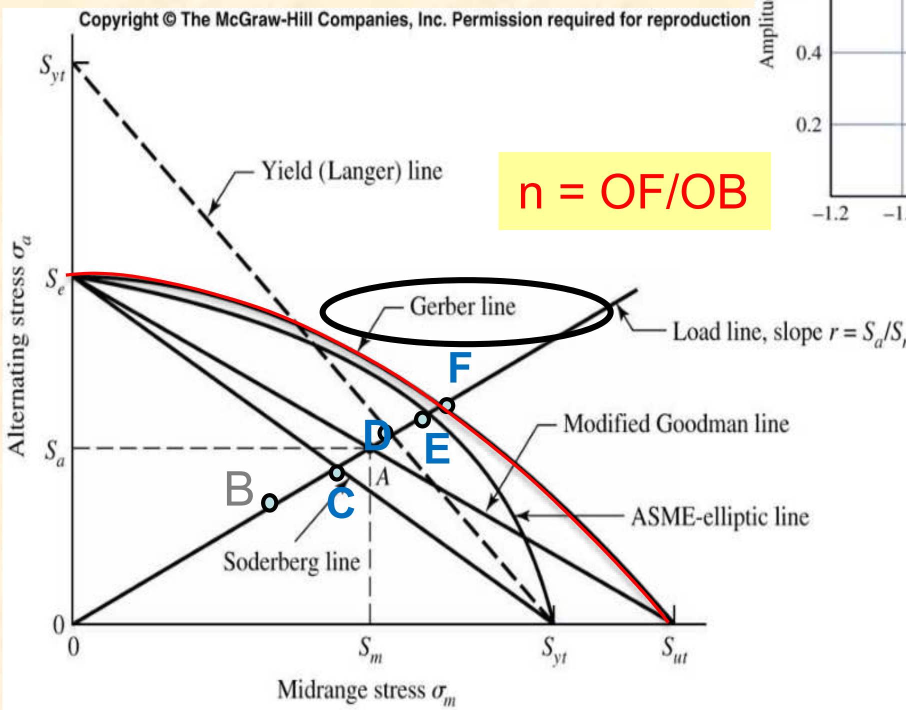
FAILURE CRITERIA (mean stress)

3- The Gerber Theory (Germany, 1874)

Failure Occurs When:

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_u}\right)^2 = 1$$





Factor of Safety

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_u}\right)^2 = 1$$

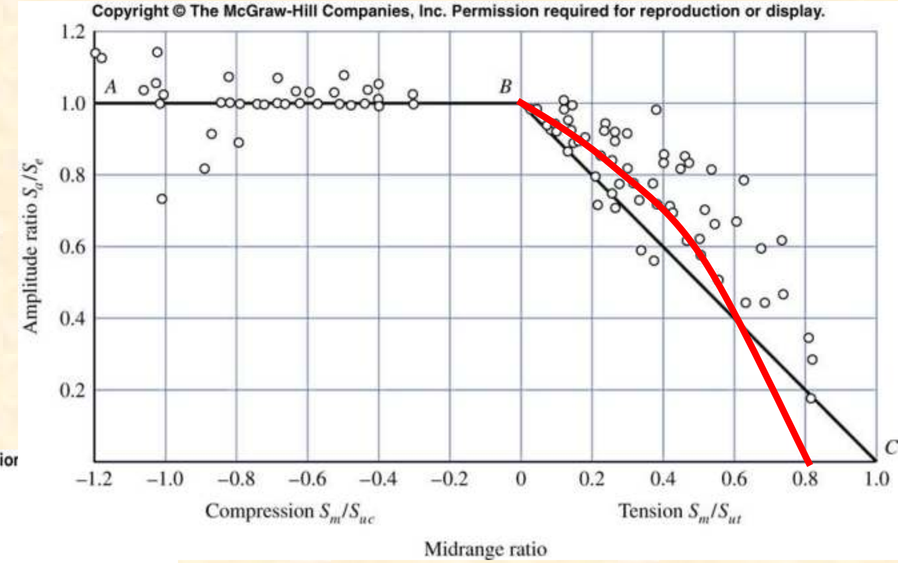
For finite life S_{rev} (equavilant reversed) *replaces* Se *and solve for* S_{rev}

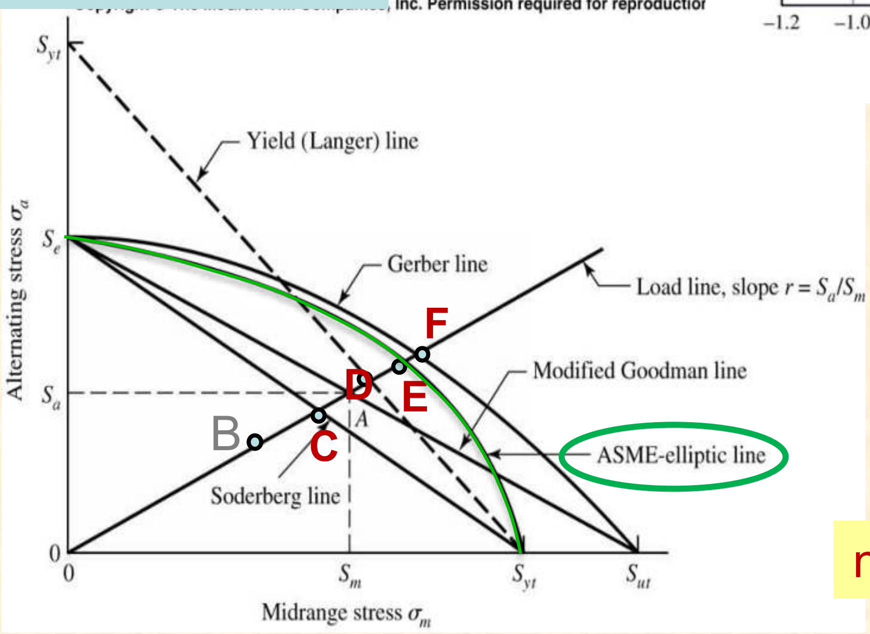
FAILURE CRITERIA (mean stress)

4- The ASME Elliptic

Failure Occurs When

$$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$$





Factor of Safety

$$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$$

n = OE/OB

For finite life S_{rev} (equavilant reversed) ***replaces Se and solve for*** S_{rev}

FAILURE CRITERIA

5- The Langer (1st Cycle) Yield Line

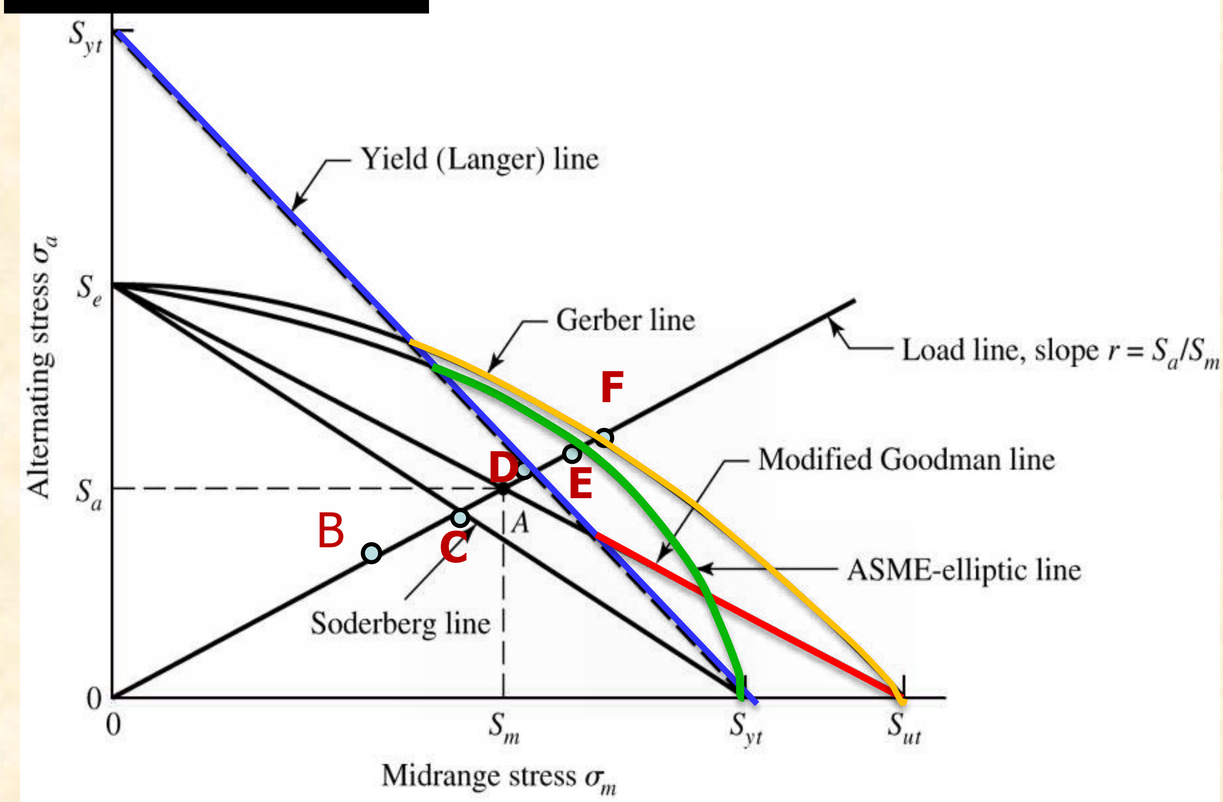
Failure Occurs When

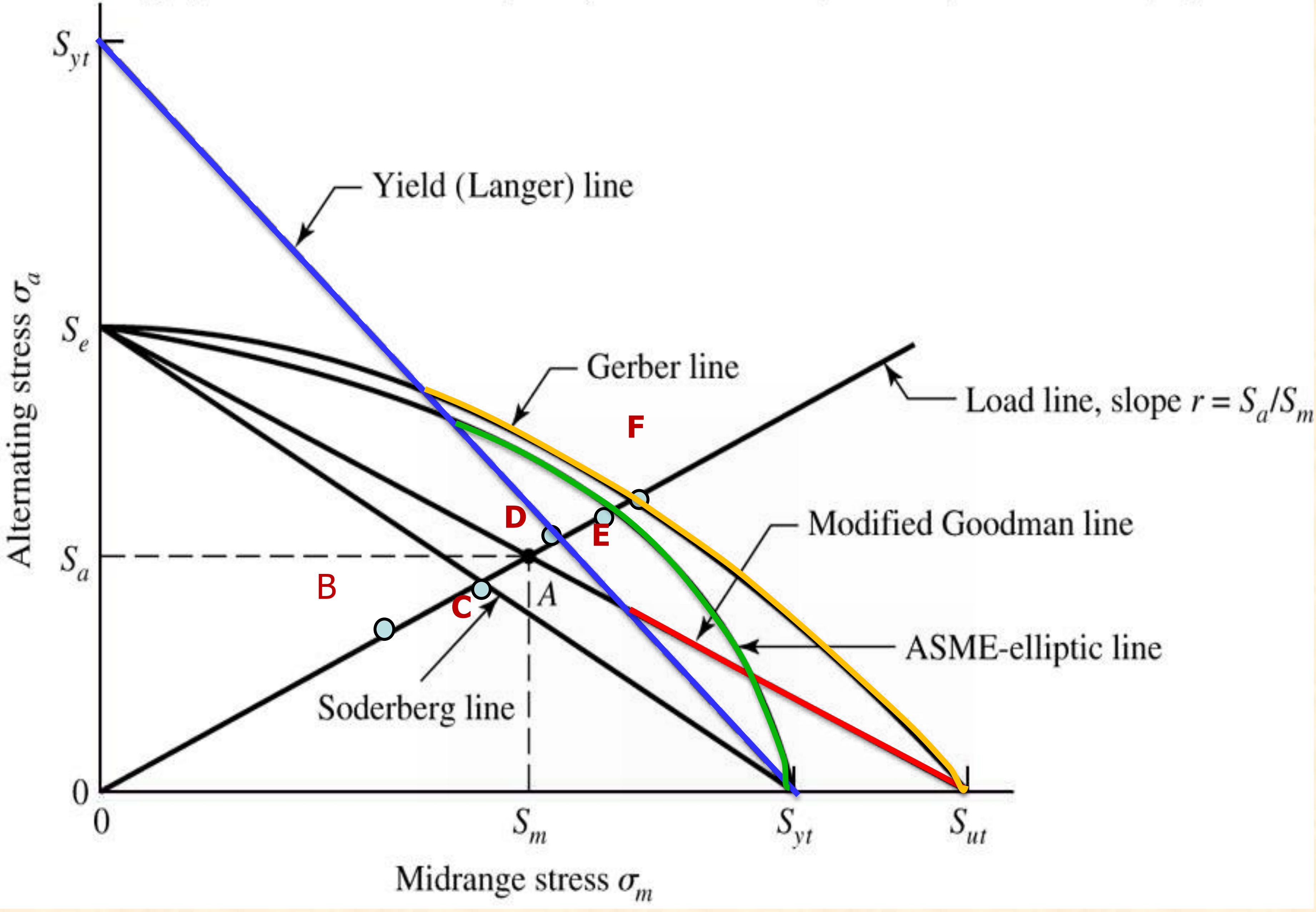
$$\frac{S_a}{S_{yt}} + \frac{S_m}{S_{yt}} = 1$$

Factor of Safety

$$\frac{\sigma_a}{S_{yt}} + \frac{\sigma_m}{S_{yt}} = \frac{1}{n}$$

n = OD/OB





S_{yt}
Yield (Langer) line
S_e
Gerber line
Load line, slope $r = S_a/S_m$
F
D
E
Modified Goodman line
S_a
B
A
C
ASME-elliptic line
Soderberg line
0
0
S_m
S_{yt}
S_{ut}
Alternating stress σ_a
Midrange stress σ_m

Criteria Equations

Solderberg Line $$\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1$$ (6-40)

Modified Goodman Relation $$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$ (6-41)

Gerber Failure Criterion $$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$ (6-42)

ASME Elliptic Line $$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$$ (6-43)

Langer First cycle yielding criterion $$\frac{S_a}{S_{yt}} + \frac{S_m}{S_{yt}} = 1$$ (6-44)

- The stresses $n\sigma_a$ and $n\sigma_m$ can replace S_a and S_m, where n is the design factor or factor of safety. Then, Eqs. (6-40) to (6-43) become:

Solderberg Line $$\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_y}=\frac{1}{n} \quad \text{(6-45)}$$

Modified Goodman Relation $$\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_{ut}}=\frac{1}{n} \quad \text{(6-46)}$$

Gerber Failure Criterion $$\frac{n\sigma_a}{S_e}+\left(\frac{n\sigma_m}{S_{ut}}\right)^2=1 \quad \text{(6-47)}$$

ASME Elliptic Line $$\left(\frac{n\sigma_a}{S_e}\right)^2+\left(\frac{n\sigma_m}{S_y}\right)^2=1 \quad \text{(6-48)}$$

We will emphasize the **Gerber** and **ASME-elliptic** for fatigue failure criterion and the **Langer** for first-cycle yielding. However, conservative designers often use the modified Goodman criterion. The design equation for the Langer first -cycle-yielding is

$$\textbf{Langer static yield} \quad \sigma_a + \sigma_m = \frac{S_y}{n} \qquad (6\text{-}49)$$

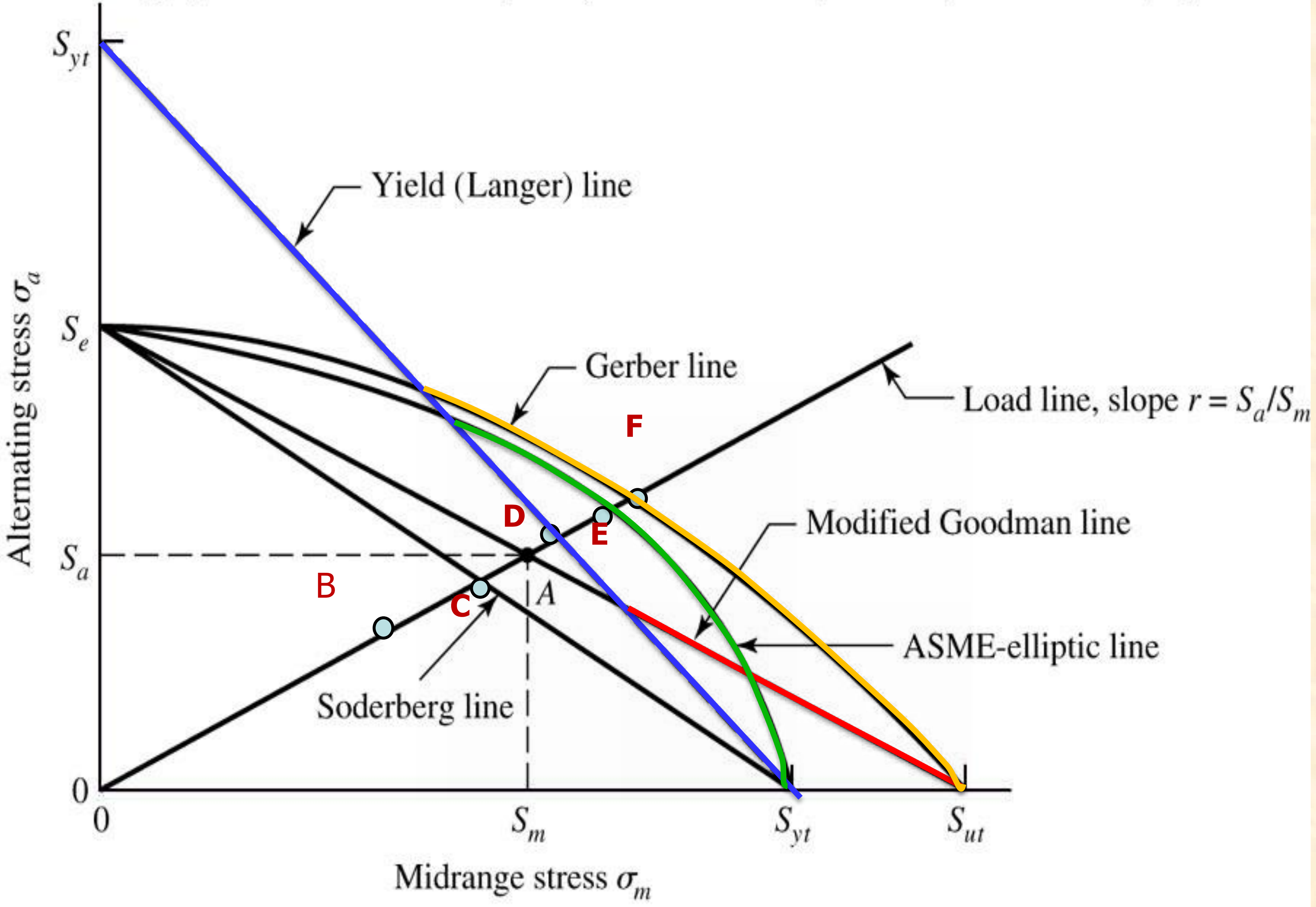
The failure criteria are used in conjunction with a load line, $r = S_a/S_m$ Principal intersections are tabulated in Tables 6-6 to 6-8. Formal expressions for fatigue factor of safety are given in the lower panel of Tables 6-6 to 6-8. The first row of each table corresponds to the fatigue criterion, the second row is the static Langer criterion, and the third row corresponds to the intersection of the static and fatigue criteria.

- The first column gives the intersecting equations and the second column the intersection coordinates.

- There are two ways to proceed with a typical analysis:

1. One method is to assume that fatigue occurs first and use one of Eqs. (6-45) to (6-49) to determine n or size, depending on the task. Most often fatigue is the governing failure mode. Then follow with a static check. If static failure governs then the analysis is repeated using Langer Static yield equation.

2. Alternatively, one could use the tables. Determine the load line and establish which criterion the load line intersects first and use the corresponding equations in the tables.



S_{yt}
Yield (Langer) line
S_e
Gerber line
Load line, slope $r = S_a/S_m$
F
D
E
Modified Goodman line
S_a
B
C
A
ASME-elliptic line
Soderberg line
0
0
S_m
S_{yt}
S_{ut}
Alternating stress σ_a
Midrange stress σ_m

Modified Goodman

Criterion	Intersecting Equations	Intersection Coordinates
Fatigue Criterion	$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$ Load line $r = \frac{S_a}{S_m}$	$S_a = \frac{r S_e S_{ut}}{r S_{ut} + S_e}$ $S_m = \frac{S_a}{r}$
Static Langer Criterion	$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$ Load line $r = \frac{S_a}{S_m}$	$S_a = \frac{r S_y}{1 + r}$ $S_m = \frac{S_y}{1 + r}$
Intersection of the Static and Fatigue Criteria	$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$ $\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_m = \frac{(S_y - S_e)\, S_{ut}}{S_{ut} - S_e}$ $S_a = S_y - S_m,\ r_{crit} = S_a / S_m$

Fatigue factor of safety

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

TABLE (7-9)

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Modified Goodman and Langer Failure Criteria.

Gerber

Langer

Intersection of Gerber and Langer

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2 S_e}\left[-1 + \sqrt{1 + \left(\frac{2 S_e}{r S_{ut}}\right)^2}\right]$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2 S_e}\left[1 - \sqrt{1 + \left(\frac{2 S_e}{S_{ut}}\right)^2\left(1 - \frac{S_y}{S_e}\right)}\right]$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{crit} = S_a / S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

TABLE (7-10)

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Gerber and Langer Failure Criteria.

	Intersecting Equations	Intersection Coordinates
ASME Elliptic	$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$ Load line $r = S_a/S_m$	$S_a = \sqrt{\frac{r^2 S_e^2 S_y^2}{S_e^2 + r^2 S_y^2}}$ $S_m = \frac{S_a}{r}$
Langer	$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$ Load line $r = S_a/S_m$	$S_a = \frac{r S_y}{1 + r}$ $S_m = \frac{S_y}{1 + r}$
Intersection of ASME Elliptic and Langer	$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$ $\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = 0, \ \frac{2 S_y S_e^2}{S_e^2 + S_y^2}$ $S_m = S_y - S_a, \ r_{crit} = S_a/S_m$

Fatigue factor of safety

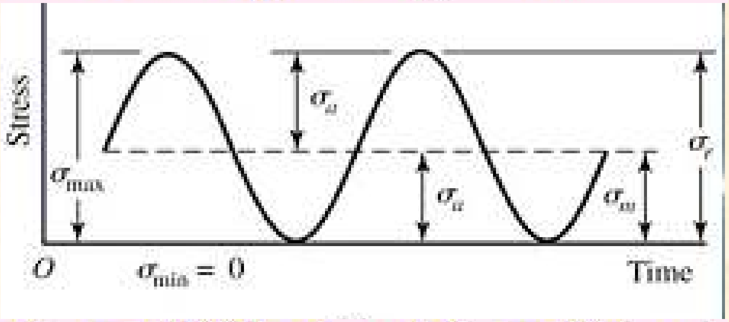
$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$

TABLE (7-11)

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for ASME Elliptic and Langer Failure Criteria.

EXAMPLE 6-10 (Textbook)

A 1.5-in-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.



Solution

We begin with some preliminaries. From Table A–20, $S_{ut} = 100$ kpsi and $S_y = 84$ kpsi. Note that $F_a = F_m = 8$ kip. The Marin factors are, deterministically,

$k_a = 2.70(100)^{-0.265} = 0.797$: Eq. (6–19), Table 6–2, p. 279

$k_b = 1$ (axial loading, see k_c)

EXAMPLE 6-10 (Textbook)

$k_c = 0.85$: Eq. (6–26), p. 282

$k_d = k_e = k_f = 1$

$S_e = 0.797(1)0.850(1)(1)(1)0.5(100) = 33.9\,\text{kpsi}$: Eqs. (6–8), (6–18), p. 274, p. 279

The nominal axial stress components σ_{ao} and σ_{mo} are

$$\sigma_{ao} = \frac{4F_a}{\pi d^2} = \frac{4(8)}{\pi 1.5^2} = 4.53\text{ kpsi} \qquad \sigma_{mo} = \frac{4F_m}{\pi d^2} = \frac{4(8)}{\pi 1.5^2} = 4.53\text{ kpsi}$$

Applying K_f to both components σ_{ao} and σ_{mo} constitutes a prescription of no notch yielding:

$$\sigma_a = K_f \sigma_{ao} = 1.85(4.53) = 8.38\text{ kpsi} = \sigma_m$$

(*a*) Let us calculate the factors of safety first. From the bottom panel from Table 6–7 the factor of safety for fatigue is

Gerber

$$n_f = \frac{1}{2}\left(\frac{100}{8.38}\right)^2\left(\frac{8.38}{33.9}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(8.38)33.9}{100(8.38)}\right]^2}\right\} = 3.66$$

From Eq. (6–49) the factor of safety guarding against first-cycle yield is

$$n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{84}{8.38 + 8.38} = 5.01$$

Thus, we see that fatigue will occur first and the factor of safety is 3.68. This can be seen in Fig. 6–28 where the load line intersects the Gerber fatigue curve first at point B. If the plots are created to true scale it would be seen that $n_f = OB/OA$.

From the first panel of Table 6–7, $r = \sigma_a/\sigma_m = 1$,

$$S_a = \frac{(1)^2 100^2}{2(33.9)}\left\{-1 + \sqrt{1 + \left[\frac{2(33.9)}{(1)100}\right]^2}\right\} = 30.7 \text{ kpsi}$$

$$S_m = \frac{S_a}{r} = \frac{30.7}{1} = 30.7 \text{ kpsi}$$

As a check on the previous result, $n_f = OB/OA = S_a/\sigma_a = S_m/\sigma_m = 30.7/8.38 = 3.66$ and we see total agreement.

TABLE (7-10)

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Gerber and Langer Failure Criteria.

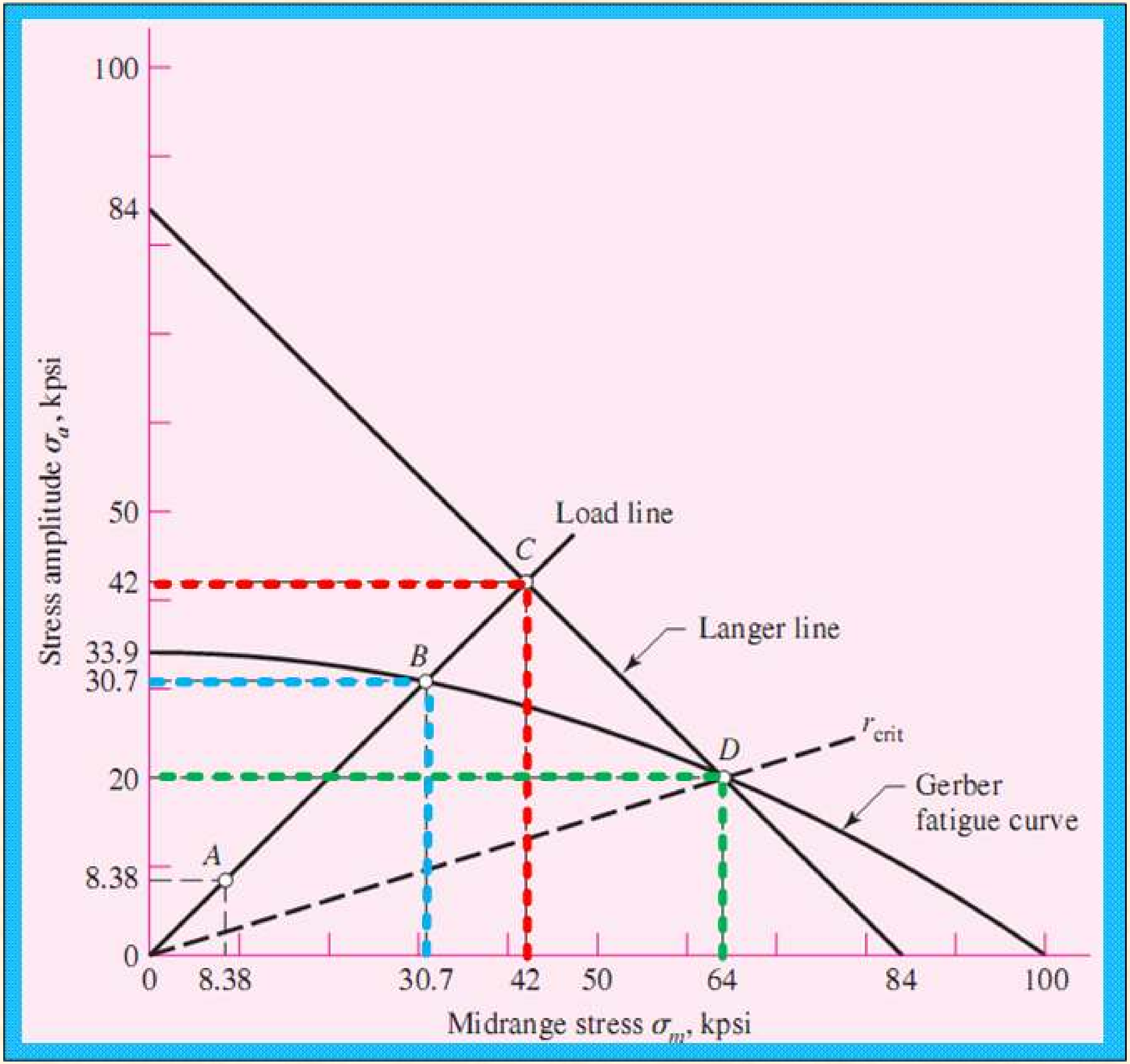
	Intersecting Equations	Intersection Coordinates
Gerber	$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2 S_e}\left[-1 + \sqrt{1 + \left(\frac{2 S_e}{r S_{ut}}\right)^2}\right]$
	Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
Langer	$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
	Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
Intersection of Gerber and Langer	$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2 S_e}\left[1 - \sqrt{1 + \left(\frac{2 S_e}{S_{ut}}\right)^2 \left(1 - \frac{S_y}{S_e}\right)}\right]$
	$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{crit} = S_a / S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

Figure 6-28

Principal points A, B, C, and Don the designer's diagram drawn for Gerber, Langer and load line.



We could have detected that fatigue failure would occur first without drawing Fig. 6–28 by calculating r_{crit}. From the third row third column panel of Table 6–7, the intersection point between fatigue and first-cycle yield is

$$S_m = \frac{100^2}{2(33.9)}\left[1 - \sqrt{1 + \left(\frac{2(33.9)}{100}\right)^2\left(1 - \frac{84}{33.9}\right)}\right] = 64.0 \text{ kpsi}$$

$$S_a = S_y - S_m = 84 - 64 = 20 \text{ kpsi}$$

The critical slope is thus

$$r_{\text{crit}} = \frac{S_a}{S_m} = \frac{20}{64} = 0.312$$

which is less than the actual load line of $r = 1$. This indicates that fatigue occurs before first-cycle-yield.

(b) Repeating the same procedure for the ASME-elliptic line, for fatigue

$$n_f = \sqrt{\frac{1}{(8.38/33.9)^2 + (8.38/84)^2}} = 3.75$$

Again, this is less than $n_y = 5.01$ and fatigue is predicted to occur first. From the first row second column panel of Table 6–8, with $r = 1$, we obtain the coordinates S_a and S_m of point B in Fig. 6–29 as

$$S_a = \sqrt{\frac{(1)^2 33.9^2 (84)^2}{33.9^2 + (1)^2 84^2}} = 31.4 \text{ kpsi}, \quad S_m = \frac{S_a}{r} = \frac{31.4}{1} = 31.4 \text{ kpsi}$$

To verify the fatigue factor of safety, $n_f = S_a/\sigma_a = 31.4/8.38 = 3.75$.

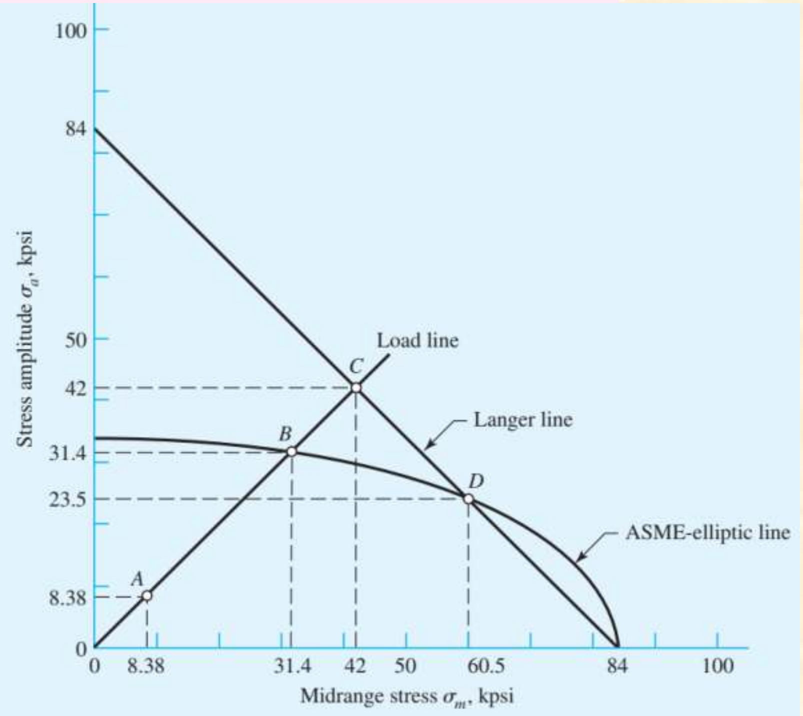
As before, let us calculate r_{crit}. From the third row second column panel of Table 6–8,

$$S_a = \frac{2(84)33.9^2}{33.9^2 + 84^2} = 23.5 \text{ kpsi}, \quad S_m = S_y - S_a = 84 - 23.5 = 60.5 \text{ kpsi}$$

$$r_{\text{crit}} = \frac{S_a}{S_m} = \frac{23.5}{60.5} = 0.388$$

which again is less than $r = 1$, verifying that fatigue occurs first with $n_f = 3.75$.

The Gerber and the ASME-elliptic fatigue failure criteria are very close to each other and are used interchangeably. The ANSI/ASME Standard B106.1M–1985 uses ASME-elliptic for shafting.



Combinations of Loading Modes

- Completely reversing simple loads
- Fluctuating simple loads
- ***Combinations of loading modes*** ➔ axial, bending, and torsional loads ➔ different load factor *kc* and concentration factor K_f

The procedures

- ❑ For combined loading, use **Distortion Energy theory to combine** them.
- ❑ Obtain Von Mises stresses, $\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{\frac{1}{2}}$, for both midrange σ'_m and alternating components σ'_a.
- ❑ Apply appropriate K_f to each type of stress.
- ❑ For load factor, use **k_c = 1 for bending & k_c = 0.85 for axial**.
- ❑ **The torsional load factor (k_c = 0.59) is inherently included** in the von Mises equations.

Combinations of Loading Modes

- If needed, axial load factor can be divided into the axial stress.

$$\sigma'_a = \left\{ \left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}} \frac{(\sigma_a)_{\text{axial}}}{0.85} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}} \right]^2 \right\}^{1/2} \quad (6\text{–}55)$$

$$\sigma'_m = \left\{ \left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}} \right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}} \right]^2 \right\}^{1/2} \quad (6\text{–}56)$$

Static Check for Combination Loading

- Distortion Energy theory still applies for check of static yielding
- Obtain Von Mises stress for maximum stresses
- Stress concentration factors are not necessary to check for yielding at first cycle

Static Check for Combination Loading

- Alternate simple check is to obtain conservative estimate of σ'_{max} by summing σ'_a and σ'_m

$$\sigma'_{\max} = \left[\left(\sigma_a + \sigma_m \right)^2 + 3\left(\tau_a + \tau_m \right)^2 \right]^{1/2}$$

$$n_y = \frac{S_y}{\sigma'_{\max}}$$

- A simpler and more conservative method is $\boldsymbol{\sigma'_{max} = \sigma'_m + \sigma'_a}$ Equation 6-55 and 6-56