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Mechanical Design
MECH 330

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Chapter 6

Fatigue Failure Resulting from Variable Loading

Solved - Problems

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Solved Problems

Example 6-7 (Textbook)

Consider an unnotched specimen with an endurance limit of 55 kpsi. If the specimen was notched such that $K_f = 1.6$, what would be the factor of safety against failure for $N > 10^6$ cycles at a reversing stress of 30 kpsi?

(a) Solve by reducing S'_e.

(b) Solve by increasing the applied stress.

SOLUTION

(a) The endurance limit of the notched specimen is given by

$$S_e = \frac{S'_e}{K_f} = \frac{55}{1.6} = 34.4\ \text{kpsi}$$

and the factor of safety is

$$n = \frac{S_e}{\sigma_a} = \frac{34.4}{30} = 1.15$$

(b) The maximum stress can be written as

$$(\sigma_a)_{\text{max}} = K_f \sigma_a = 1.6(30) = 48.0\ \text{kpsi}$$

and the factor of safety is

$$n = \frac{S'_e}{K_f \sigma_a} = \frac{55}{48} = 1.15$$

EXAMPLE 6-8 (Textbook)

A 1015 hot-rolled steel bar has been machined to a diameter of 1 in. It is to be placed in reversed axial loading for 70 000 cycles to failure in an operating environment of 550°F. Using ASTM minimum properties, and a reliability of 99 percent, estimate the endurance limit and fatigue strength at 70 000 cycles.

SOLUTION

From Table A–20, $S_{ut} = 50$ kpsi at 70°F. Since the rotating-beam specimen endurance limit is not known at room temperature, we determine the ultimate strength at the elevated temperature first, using Table 6–4. From Table 6–4,

$$\left(\frac{S_T}{S_{RT}}\right)_{550^\circ} = \frac{0.995 + 0.963}{2} = 0.979$$

The ultimate strength at 550°F is then

$$(S_{ut})_{550^\circ} = (S_T/S_{RT})_{550^\circ}\,(S_{ut})_{70^\circ} = 0.979(50) = 49.0 \text{ kpsi}$$

The rotating-beam specimen endurance limit at 550°F is then estimated from Eq. (6–8) as

$$S'_e = 0.5(49) = 24.5 \text{ kpsi}$$

Next, we determine the Marin factors. For the machined surface, Eq. (6–19) with Table 6–2 gives

$$k_a = aS_{ut}^b = 2.70(49^{-0.265}) = 0.963$$

For axial loading, from Eq. (6–21), the size factor $k_b = 1$, and from Eq. (6–26) the loading factor is $k_c = 0.85$. The temperature factor $k_d = 1$, since we accounted for the temperature in modifying the ultimate strength and consequently the endurance limit. For 99 percent reliability, from Table 6–5, $k_e = 0.814$. Finally, since no other conditions were given, the miscellaneous factor is $k_f = 1$. The endurance limit for the part is estimated by Eq. (6–18) as

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

$$= 0.963(1)(0.85)(1)(0.814)(1)24.5 = 16.3 \text{ kpsi}$$

For the fatigue strength at 70 000 cycles we need to construct the *S-N* equation. From p. 277, since $S_{ut} = 49 < 70$ kpsi, then $f = 0.9$. From Eq. (6–14)

$$a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.9(49)]^2}{16.3} = 119.3 \text{ kpsi}$$

and Eq. (6–15)

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.9(49)}{16.3}\right] = -0.1441$$

Finally, for the fatigue strength at 70 000 cycles, Eq. (6–13) gives

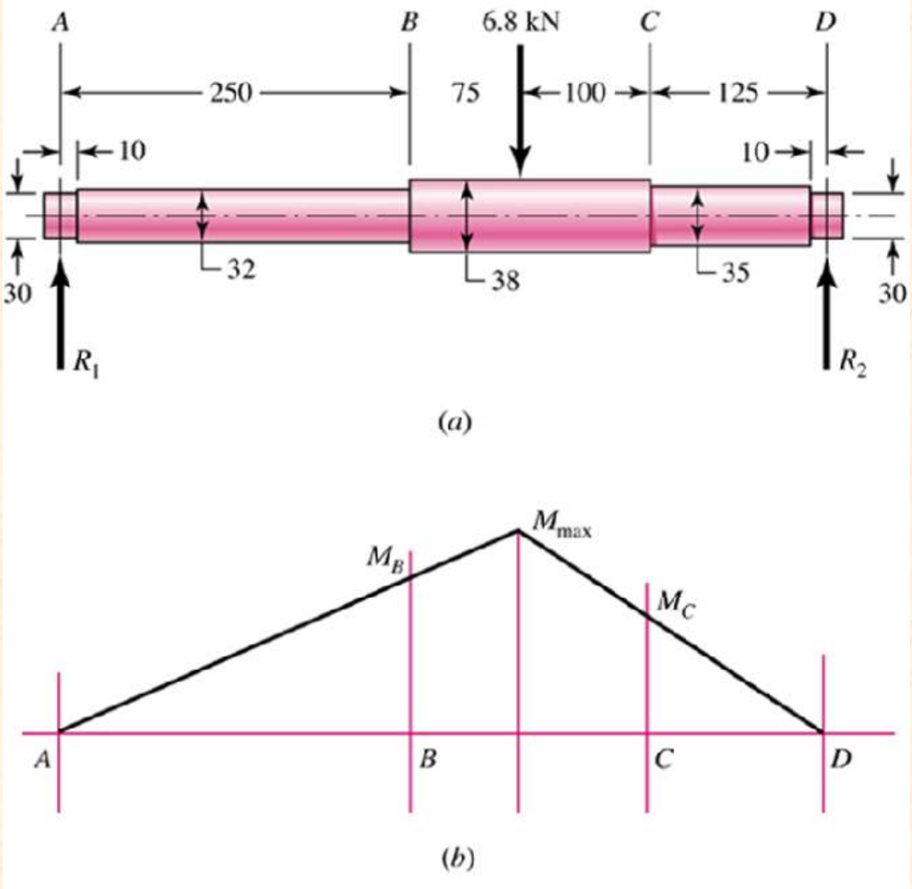
$$S_f = a\,N^b = 119.3(70\,000)^{-0.1441} = 23.9\ \text{kpsi}$$

EXAMPLE 6-9 (Textbook)

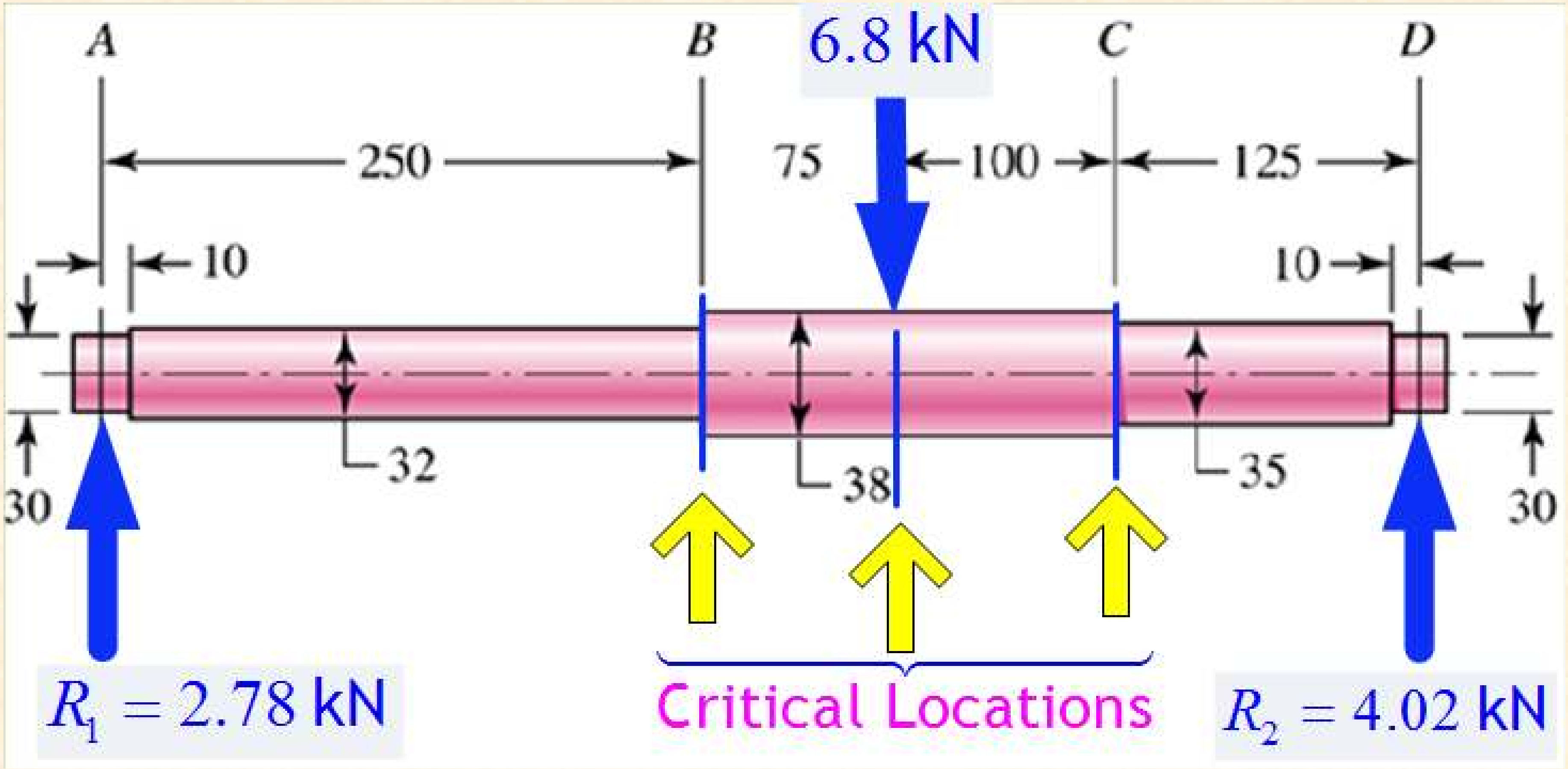
Figure 6–22*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

SOLUTION

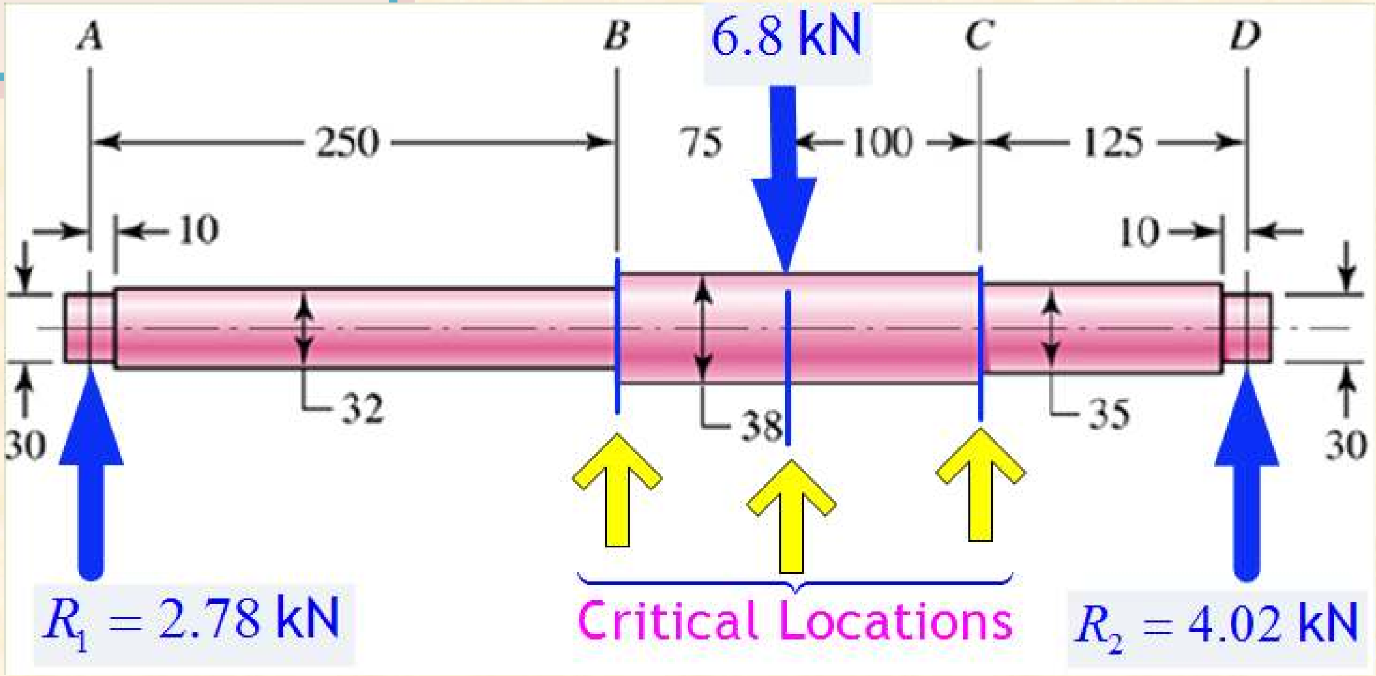
1 UNS No.	2 SAE and/or AISI No.	3 Process-ing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)
G10500	1050	HR	620 (90)	340 (49.5)
		CD	690 (100)	580 (84)



Example 6-9 (Textbook)



- Failure will probably occur at B rather than C or at the point of maximum bending moment.
- Point B has:
  - a smaller cross-section
  - a higher bending moment
  - a higher stress concentration factor than C.
- Location of the maximum bending moment has a larger size and no stress concentration.



We shall solve the problem by first estimating the strength at point B, since the strength will be different elsewhere, and comparing this strength with the stress at the same point.

From Table A–20 we find $S_{ut} = 690$ MPa and $S_y = 580$ MPa. The endurance limit S'_e is estimated as

$$S'_e = 0.5(690) = 345 \text{ MPa}$$

From Eq. (6–19) and Table 6–2,

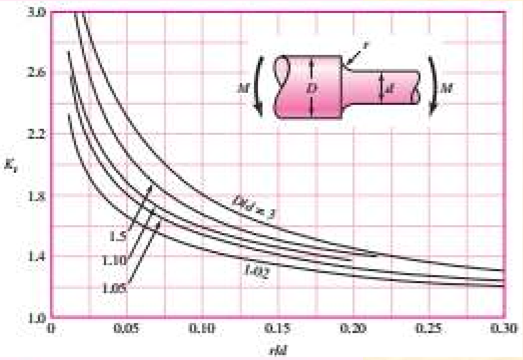
$$k_a = 4.51(690)^{-0.265} = 0.798$$

From Eq. (6–20),

$$k_b = (32/7.62)^{-0.107} = 0.858$$

Since $k_c = k_d = k_e = k_f = 1$,

$$S_e = 0.798(0.858)345 = 236 \text{ MPa}$$



To find the geometric stress-concentration factor K_t we enter Fig. A–15–9 with $D/d = 38/32 = 1.1875$ and $r/d = 3/32 = 0.093\,75$ and read $K_t \doteq 1.65$. Substituting $S_{ut} = 690/6.89 = 100$ kpsi into Eq. (6–35) yields $\sqrt{a} = 0.0622\sqrt{\text{in}} = 0.313\sqrt{\text{mm}}$. Substituting this into Eq. (6–33) gives

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} = 1 + \frac{1.65 - 1}{1 + 0.313/\sqrt{3}} = 1.55$$

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}}$$

Bending or axial: $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$

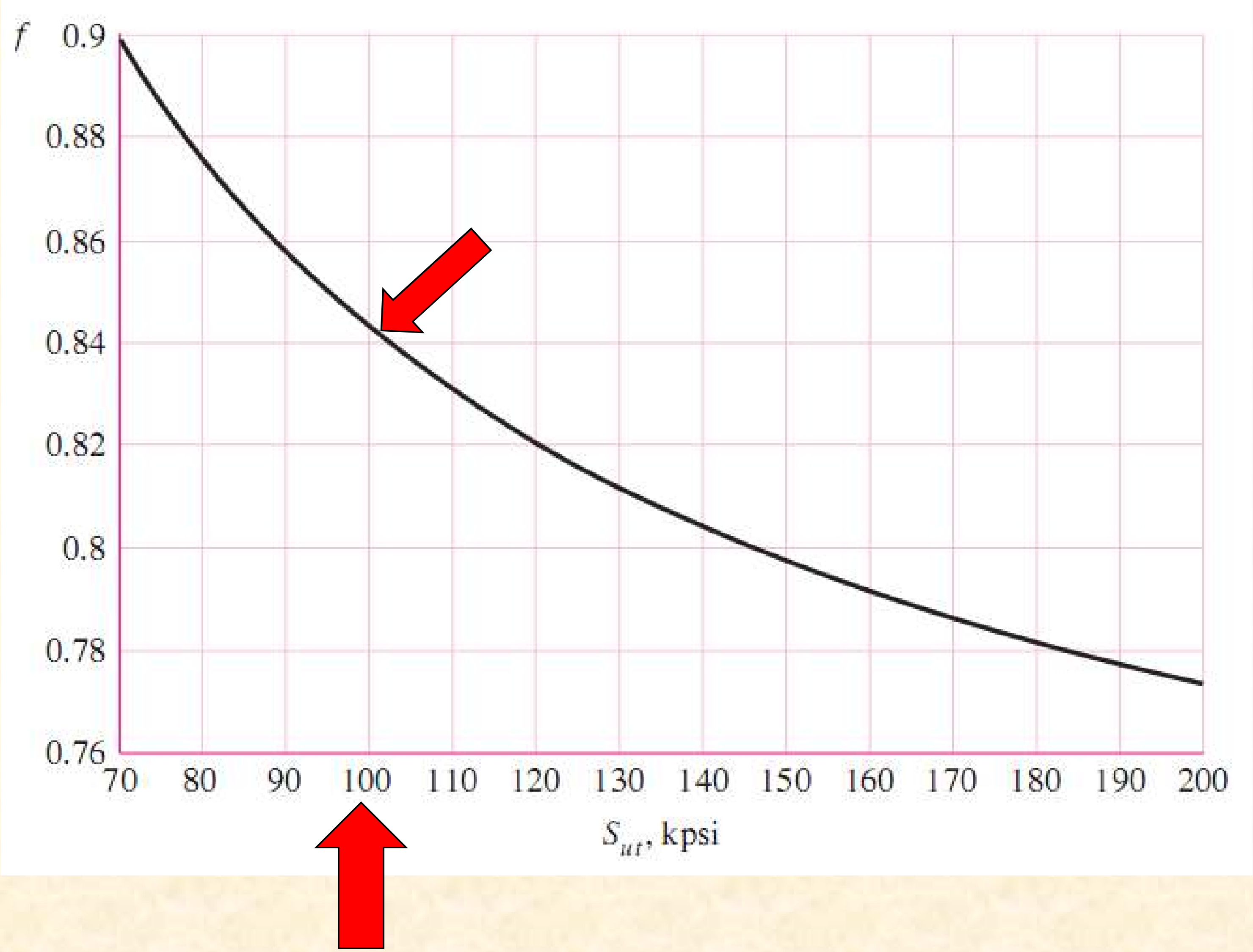
The next step is to estimate the bending stress at point B. The bending moment is

$$M_B = R_1 x = \frac{225F}{550}250 = \frac{225(6.8)}{550}250 = 695.5 \text{ N} \cdot \text{m}$$

Just to the left of B the section modulus is $I/c = \pi d^3/32 = \pi 32^3/32 = 3.217\,(10^3)\text{mm}^3$. The reversing bending stress is, assuming infinite life,

$$\sigma = K_f \frac{M_B}{I/c} = 1.55\frac{695.5}{3.217}(10)^{-6} = 335.1(10^6)\text{ Pa} = 335.1 \text{ MPa}$$

This stress is greater than S_e and less than S_y. This means we have both finite life and no yielding on the first cycle.



f
0.9
0.88
0.86
0.84
0.82
0.8
0.78
0.76
70
80
90
100
110
120
130
140
150
160
170
180
190
200
S_{ut}, kpsi

For finite life, we will need to use Eq. (6–16). The ultimate strength, $S_{ut} = 690$ MPa = 100 kpsi. From Fig. 6–18, $f = 0.844$. From Eq. (6–14)

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{[0.844(690)]^2}{236} = 1437 \text{ MPa}$$

and from Eq. (6–15)

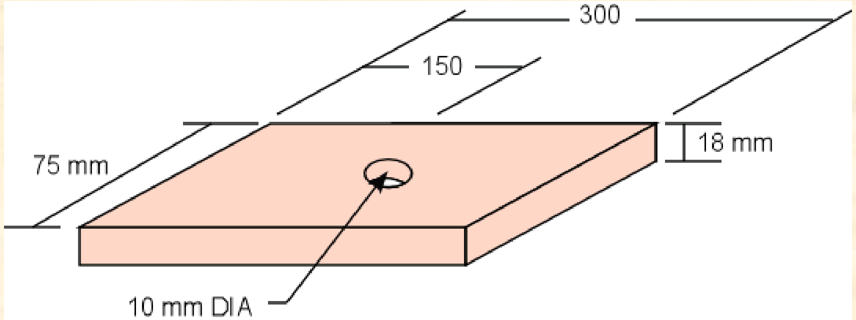
$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.844(690)}{236}\right] = -0.1308$$

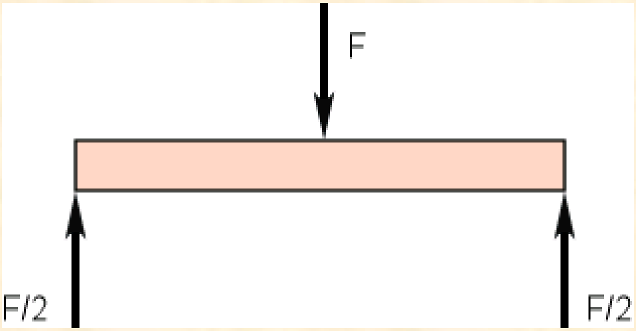
From Eq. (6–16),

$$N = \left(\frac{\sigma_a}{a}\right)^{1/b} = \left(\frac{335.1}{1437}\right)^{-1/0.1308} = 68(10^3) \text{ cycles}$$

Example #04

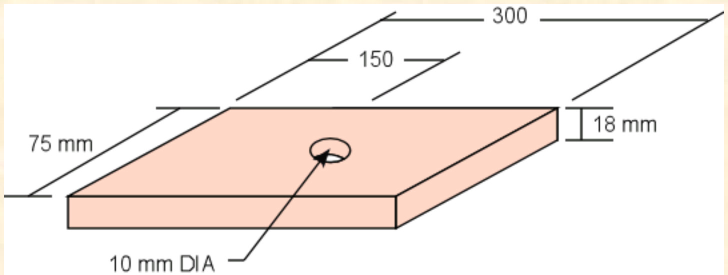
- Given:
  - S_{ut} = 1400 MPa
  - S_{yt} = 950 MPa
  - Heat-treated (as-forged)
  - F_{mean} = 9.36 kN
  - F_{max} = 10.67 kN
  - d/w = 0.133; d/h = 0.55
- Find:
  - N_{cycle} for finite life, or factor of safety for infinite life, assuming F_{mean} is constant

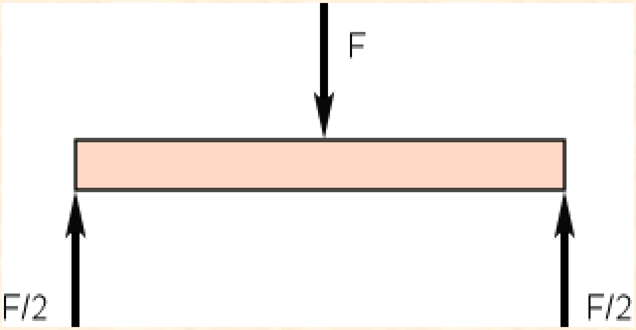




Example, cont.

- Find σ_m and σ_a





$$\sigma = \frac{My}{I}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(w-d)h^3 = \frac{1}{12}(75-10)18^3 = 3.16\text{x}10^{-8}\,\text{m}^4$$

$$y_{\max} = \frac{h}{2} = 0.009\text{ m}$$

$$M_m = \left(\frac{F_m}{2}\right)\left(\frac{L}{2}\right) = \frac{1}{4}F_m L = \frac{1}{4}\left(9.36\text{x}10^3\right)(0.3) = 702\text{ Nm}$$

$$M_{\max} = \left(\frac{F_{\max}}{2}\right)\left(\frac{L}{2}\right) = \frac{1}{4}F_{\max} L = \frac{1}{4}\left(10.67\text{x}10^3\right)(0.3) = 800\text{ Nm}$$

$$\sigma_m = \frac{M_m y_{\max}}{I} = 200\text{ MPa}$$

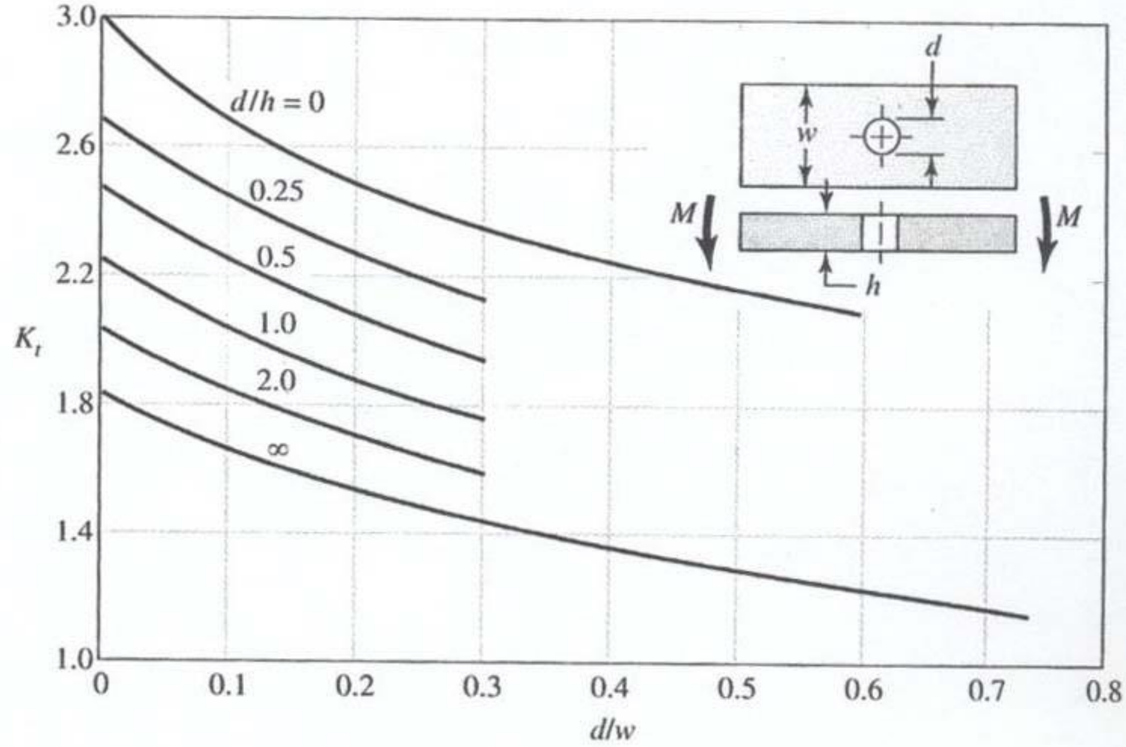
$$\sigma_{\max} = \frac{M_{\max} y_{\max}}{I} = 228\text{ MPa}$$

$$\sigma_a = \sigma_{\max} - \sigma_m = 28\text{ MPa}$$

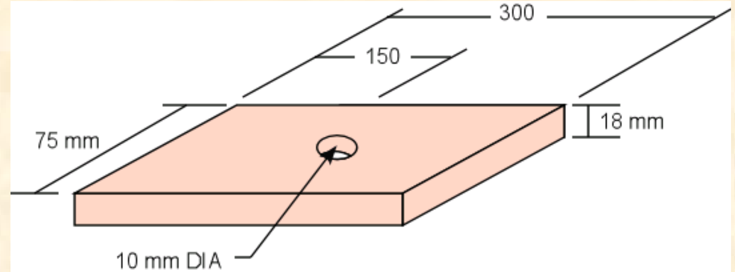
Stress Concentration Factor

Figure A–15–2

Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.



Example, cont.

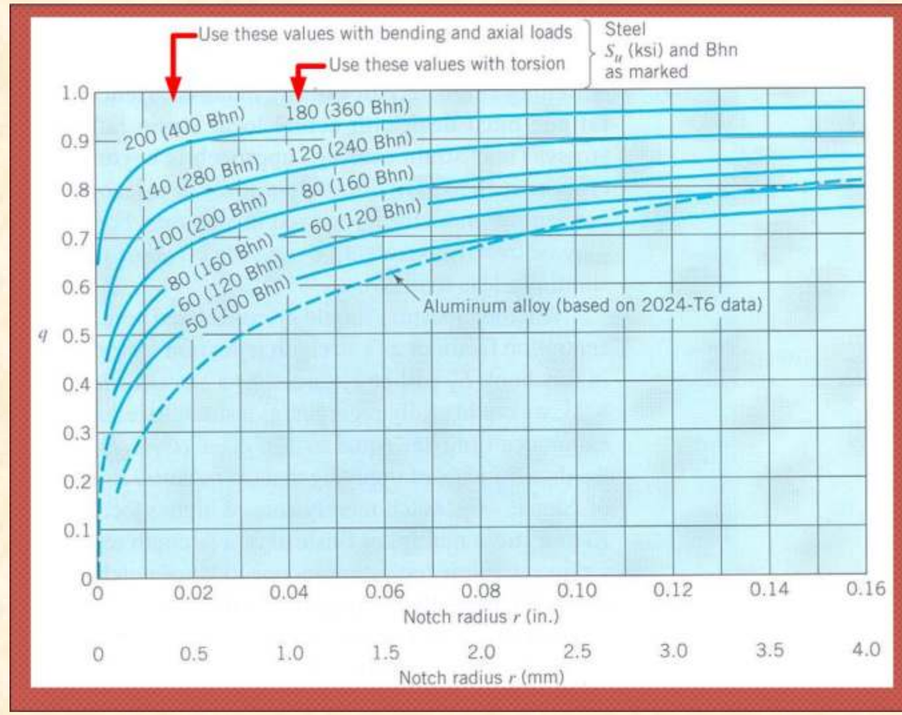


- Since this is uniaxial loading,
  - σ_m = 200 MPa
  - σ_a = 28 MPa

  } nominal
- We need to take care of the SC factors
  - Su = 1400Mpa

    $$k_f = 1 + q(k_t - 1)$$

    k_t ~ 2.2 (Figure A15-2)
  - q ~ 0.95 (Figure 6-20)
  - k_f = 2.14



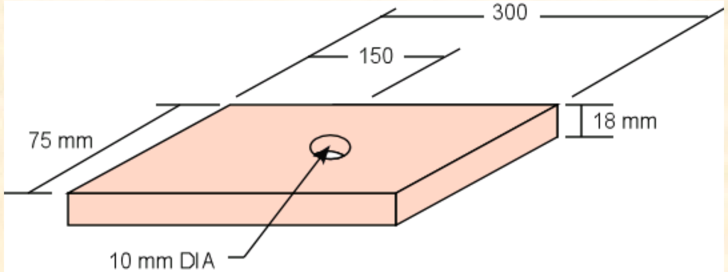
$$\sigma_a = \sigma'_a = k_f \sigma_{a_{nom}} = (2.14)(28) = 60\ \text{MPa}$$

$$\sigma_m = \sigma'_m = k_f \sigma_{m_{nom}} = (2.14)(200) = 428\ \text{MPa}$$

Example, cont.

- Find strength
- Eqn. 6-8: $S'_e = .5 \times S_{ut}$

$S'_e \sim 700\ \text{MPa}\ \ \text{since}\ \ S_{ut} = 1400\ \text{MPa}$



- Modification factors

Surface:

$k_a = aS_{ut}^{\ b}$

$a = 271$

$b = -0.995$

$k_a = 0.201$

Size:

Equation (7-19):

$2.8 \le d_{eq} \le 51\,\text{mm}$

$d_{eq} = 0.808(hb)^{\frac{1}{2}}$

$k_b = 1.24 d_{eq}^{\ -0.107}$

$k_b = 0.86$

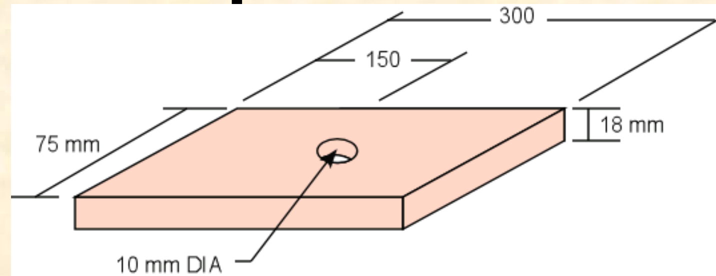
Load:

Bending

$k_c = 1 \quad (\text{Eq. } 7\text{-}25)$

$$S_e = (0.201)(0.86)(700) = 121\ \text{MPa}$$

Example,



- Design criteria
  - Goodman line:

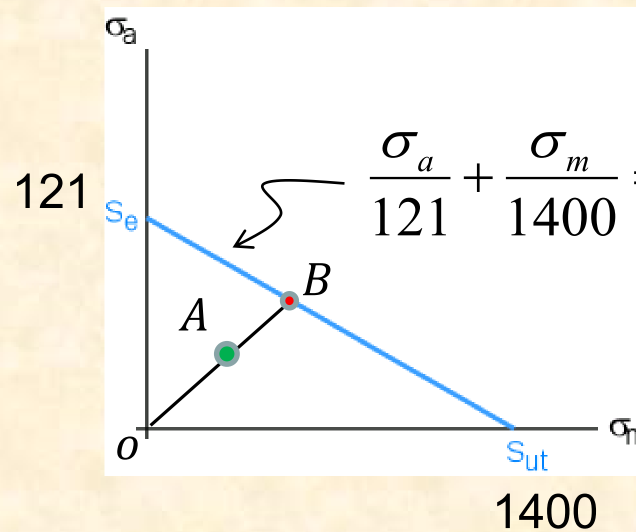
$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1/n$$

- For arbitrary variation in σ_a and σ_m,

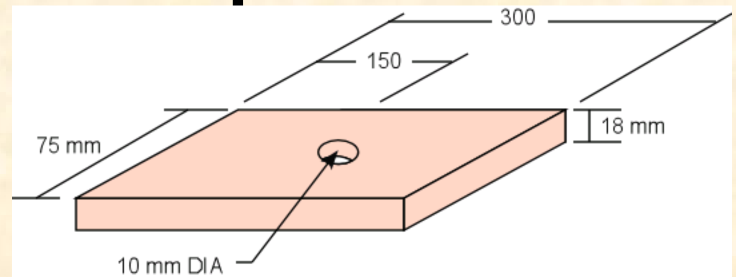
$$\frac{\sigma_a}{121} + \frac{\sigma_m}{1400} = \frac{1}{\eta}$$

$$\frac{1}{\eta} = \frac{60}{121} + \frac{428}{1400}$$

$$\eta = 1.25$$



Example,



- However, we know that F_{mean} = constant from problem statement
  - σ_m = constant

$$\frac{S_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1$$

$$\frac{S_a}{121} + \frac{428}{1400} = 1$$

$$S_a = 84 \text{ MPa}$$

$$\eta = \frac{S_a}{\sigma_a} = \frac{84}{60} = 1.4$$

Less conservative!