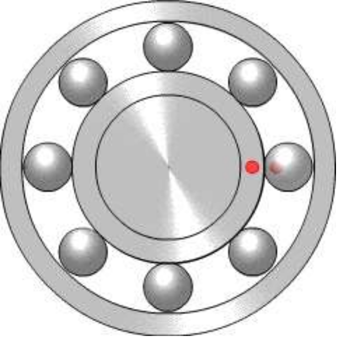


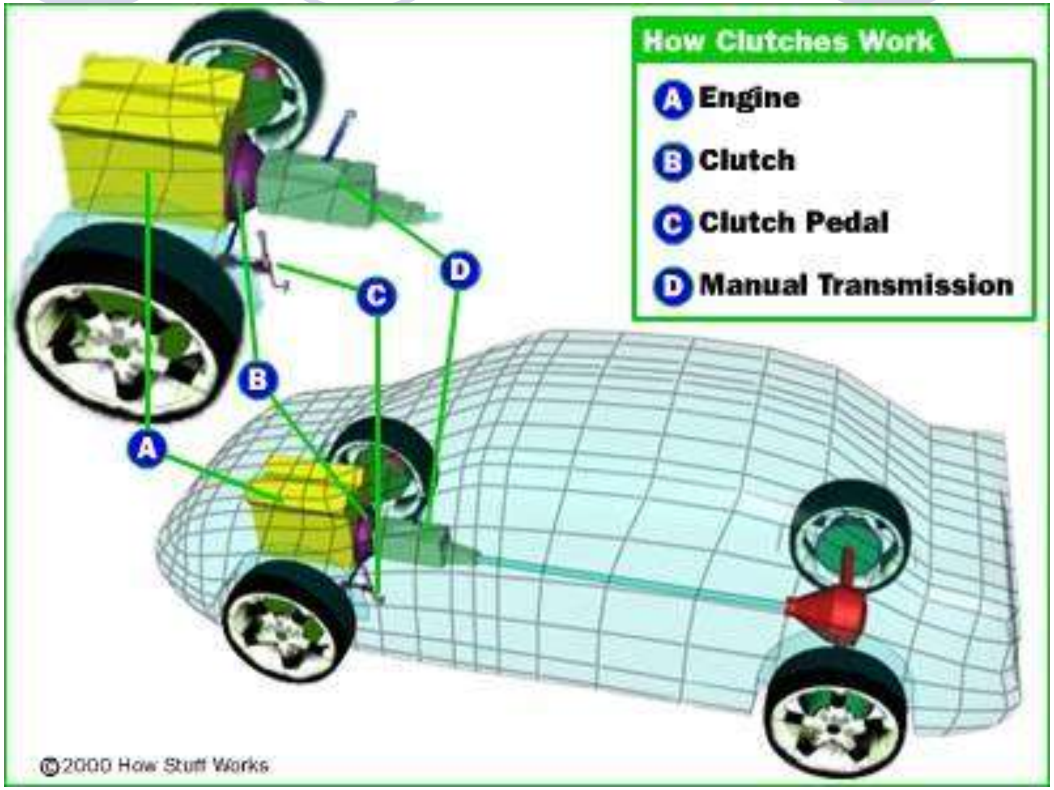
Mechanical Design II

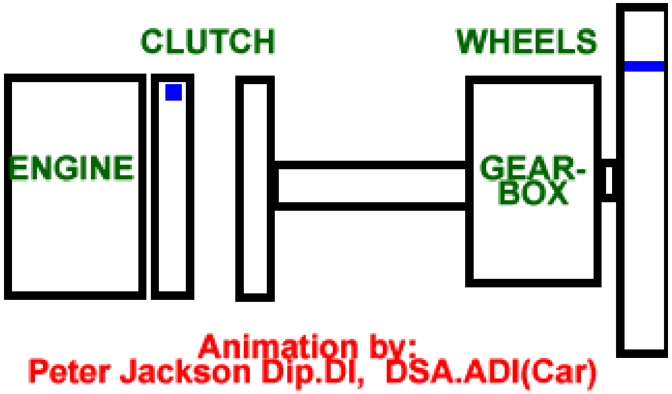
By S.G.Khan

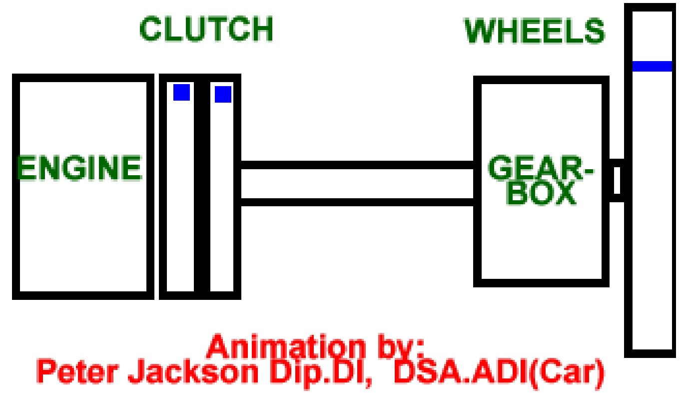
Chapter 16 Shigley

Brakes and Clutches

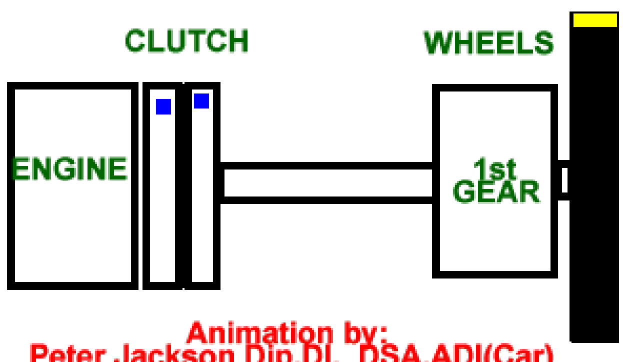
Clutch


How Clutches Work
A Engine
B Clutch
C Clutch Pedal
D Manual Transmission
A
B
C
D
©2000 How Stuff Works





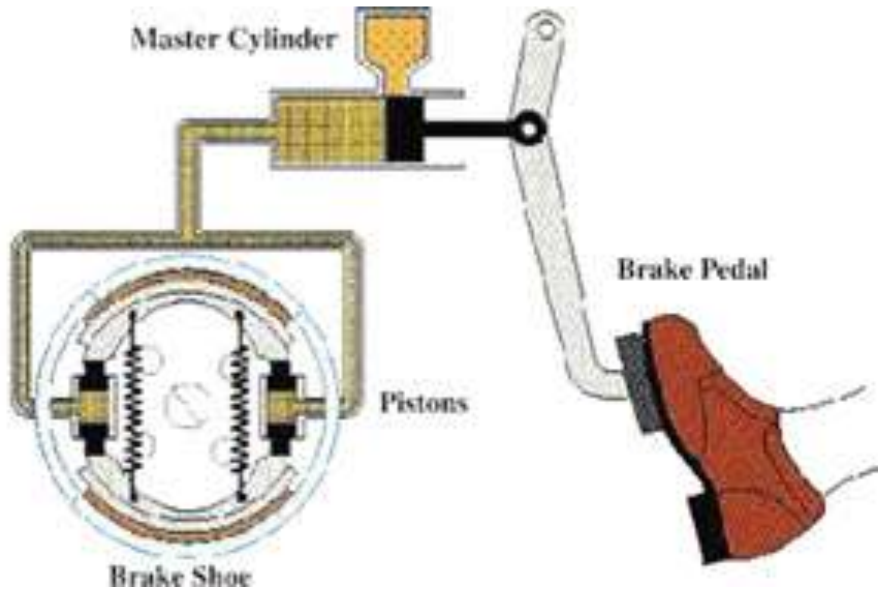
- The clutch is a device that transfers power from the engine to the gearbox, then to the wheels that drive the car. The animation above (left) shows the clutch disengaged (driver pressing clutch pedal down) thereby preventing any power from the engine being used to drive the car.
- The animation above (right) shows the clutch engaged (driver is not pressing the clutch down) thereby allowing power from the engine to drive the car, as it would be when driving along the road normally.
- The animation below shows the clutch only partially engaged (driver has the clutch pedal about half way up) and is known as 'clutch control'. The driver can let the clutch pedal up a little more to power the car, or push it down to remove that power, which results in the car slowing down. This technique is used where power from the engine is needed, but only very low speeds are required, for example, when maneuvering.

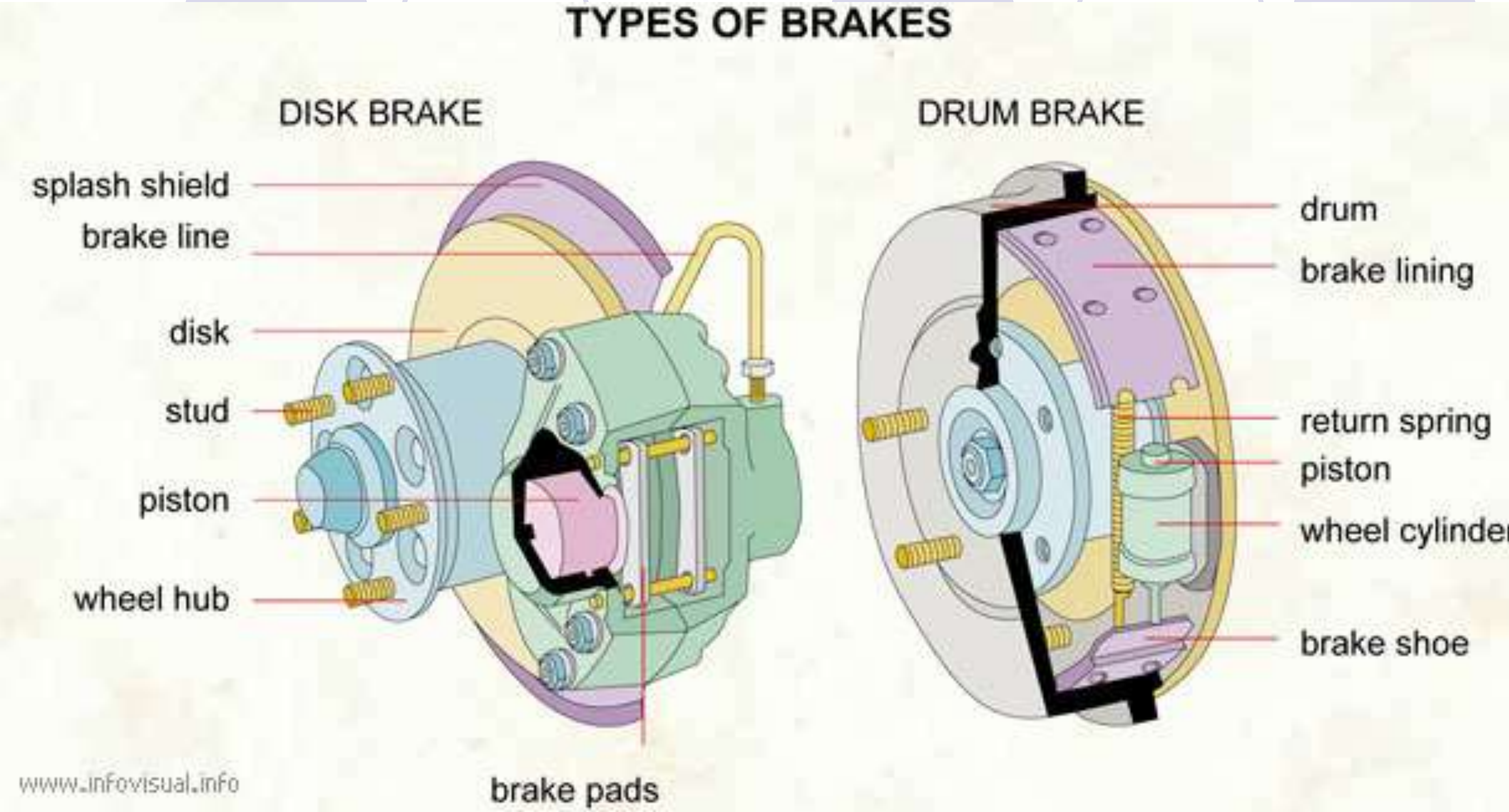


What is Brake?

restraint used to slow or stop a vehicle.

A brake is a device for slowing or stopping the motion of a machine or vehicle, and to keep it from starting to move again. The kinetic energy lost by the moving part is usually translated to heat by friction.




TYPES OF BRAKES
DISK BRAKE
DRUM BRAKE
splash shield
brake line
disk
stud
piston
wheel hub
brake pads
drum
brake lining
return spring
piston
wheel cylinder
brake shoe
www.infovisual.info

Some General Definitions

Brakes: apparatuses used to slow or stop a moving vehicle.

Drum brake: mechanism that slows and stops a car by fiction, by pressing brake shoes against a drum.

Drum: cylindrical part attached to the wheel, against which the brake shoes are pressed to stop the car.

Brake lining: frictional part on the outside edges of the brake shoes.

Return spring: part of the brake mechanism that returns the brake shoes to their initial position.

Piston: cylindrical part that transmits the pressure to and receives pressure from the brake shoes.

Some General Definitions

Brake shoe: part on which the brake lining is mounted.

Brake pads: part activated by the piston.

Wheel hub: central part crossed by the axel.

Stud: metal pin.

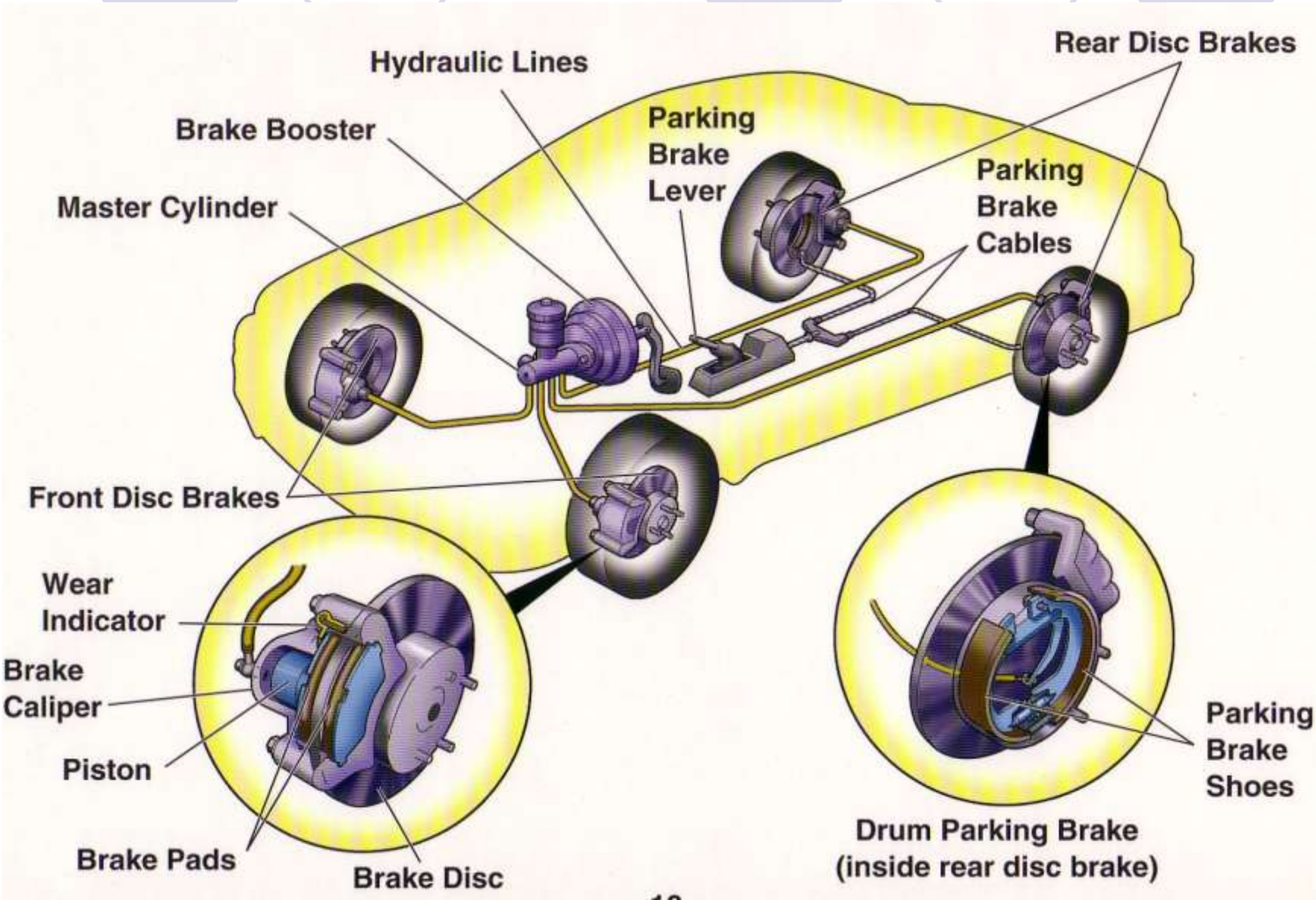
Disk: round, flat, piece of metal, pressed against the wheel to slow or stop the car.

Brake line: system liquid-transporting tubes.

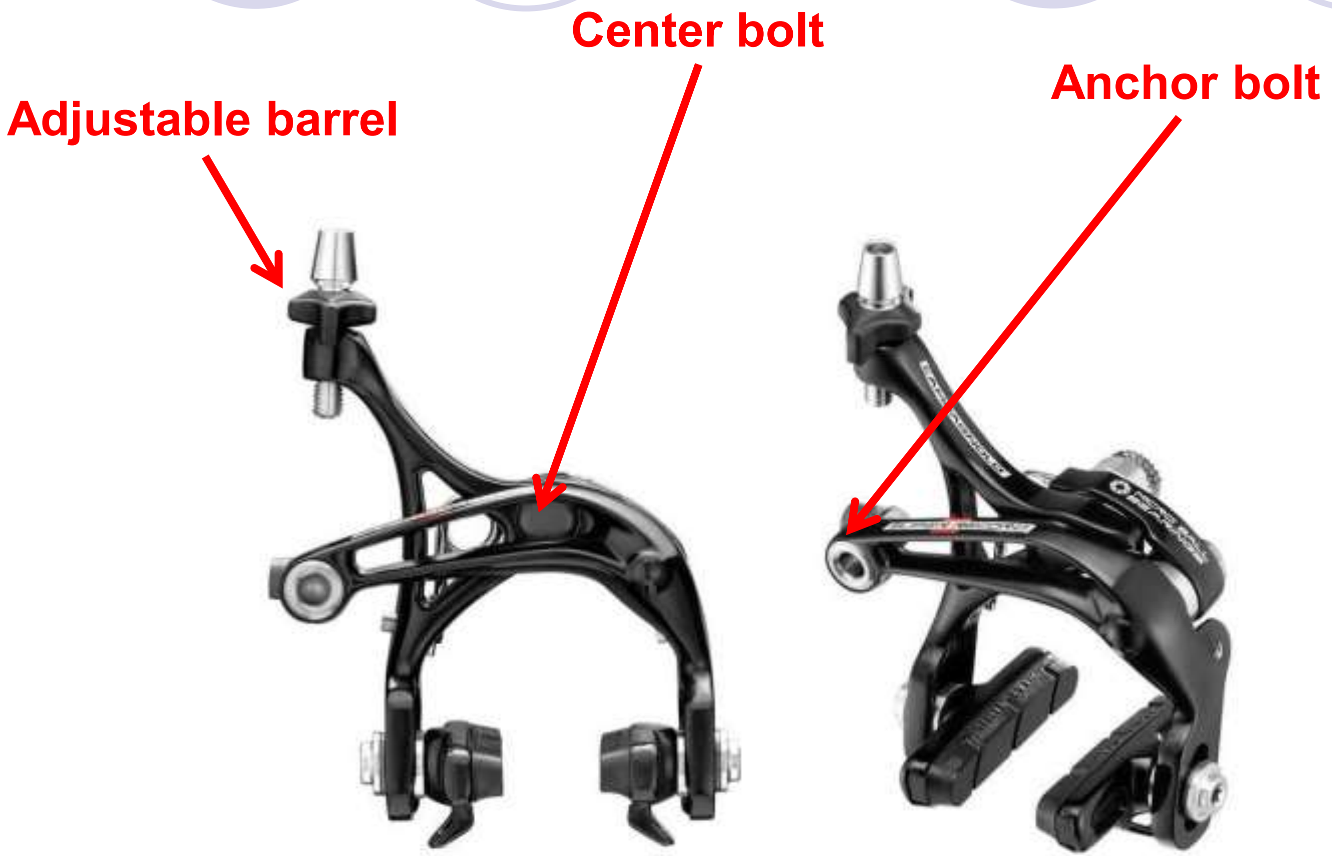
Splash shield: protector that prevents dirt from fouling the braking system.

Disk brake: mechanism that slows and stops a car by friction, by pressing a disk against the wheel axel.

Automotive Braking System

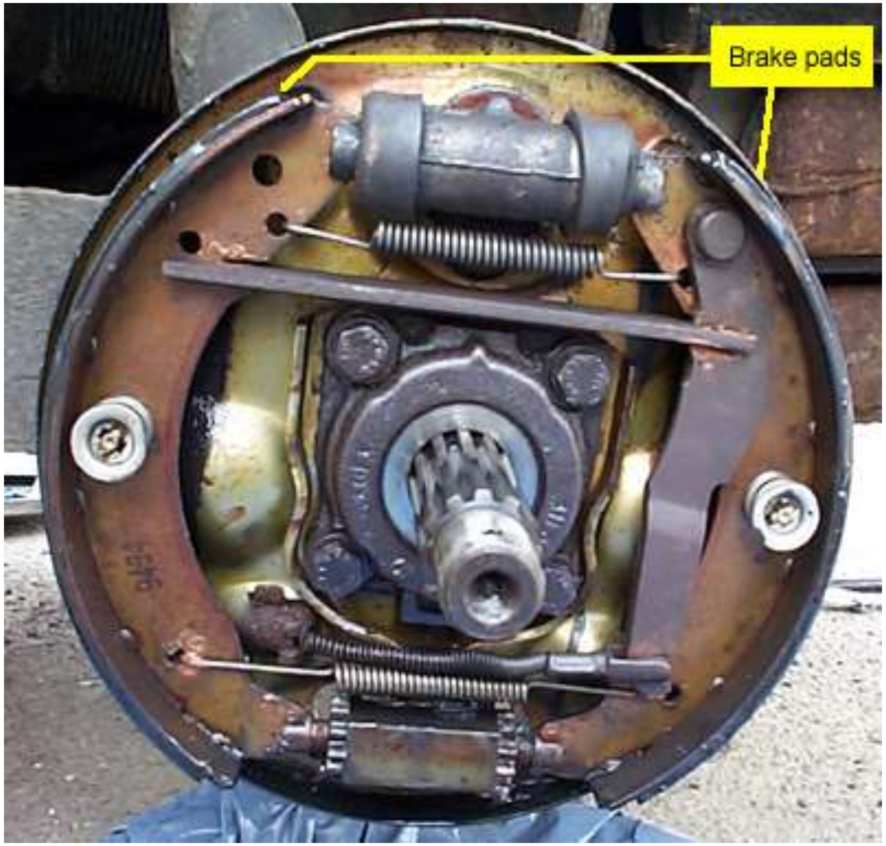


Bike Brakes



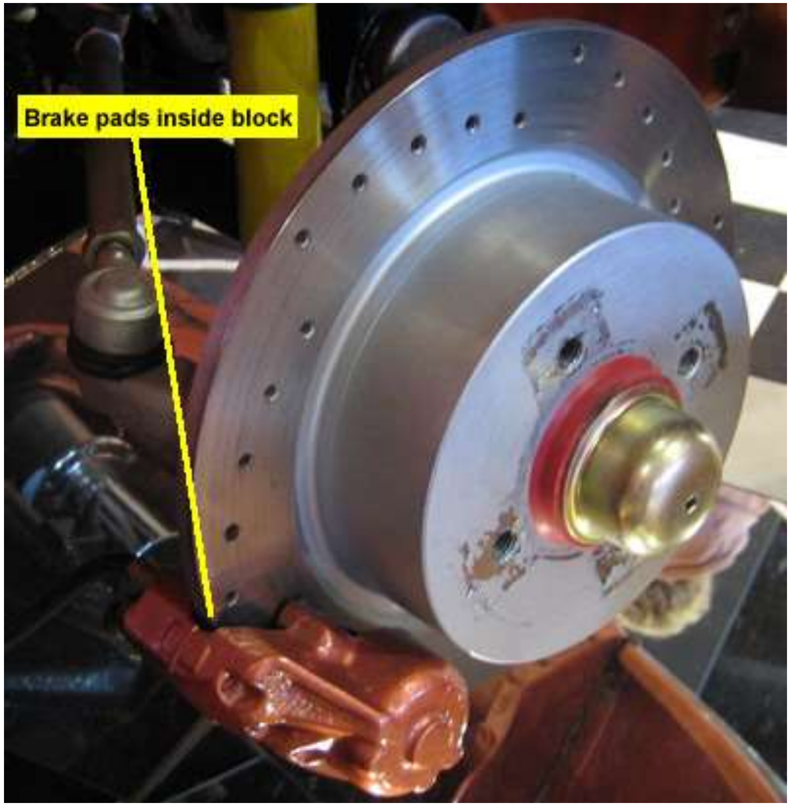
Drum Brakes

- So called because the braking part is drum-shaped. Inside the drum, two large, curved brake pads push outwards, slowing you down

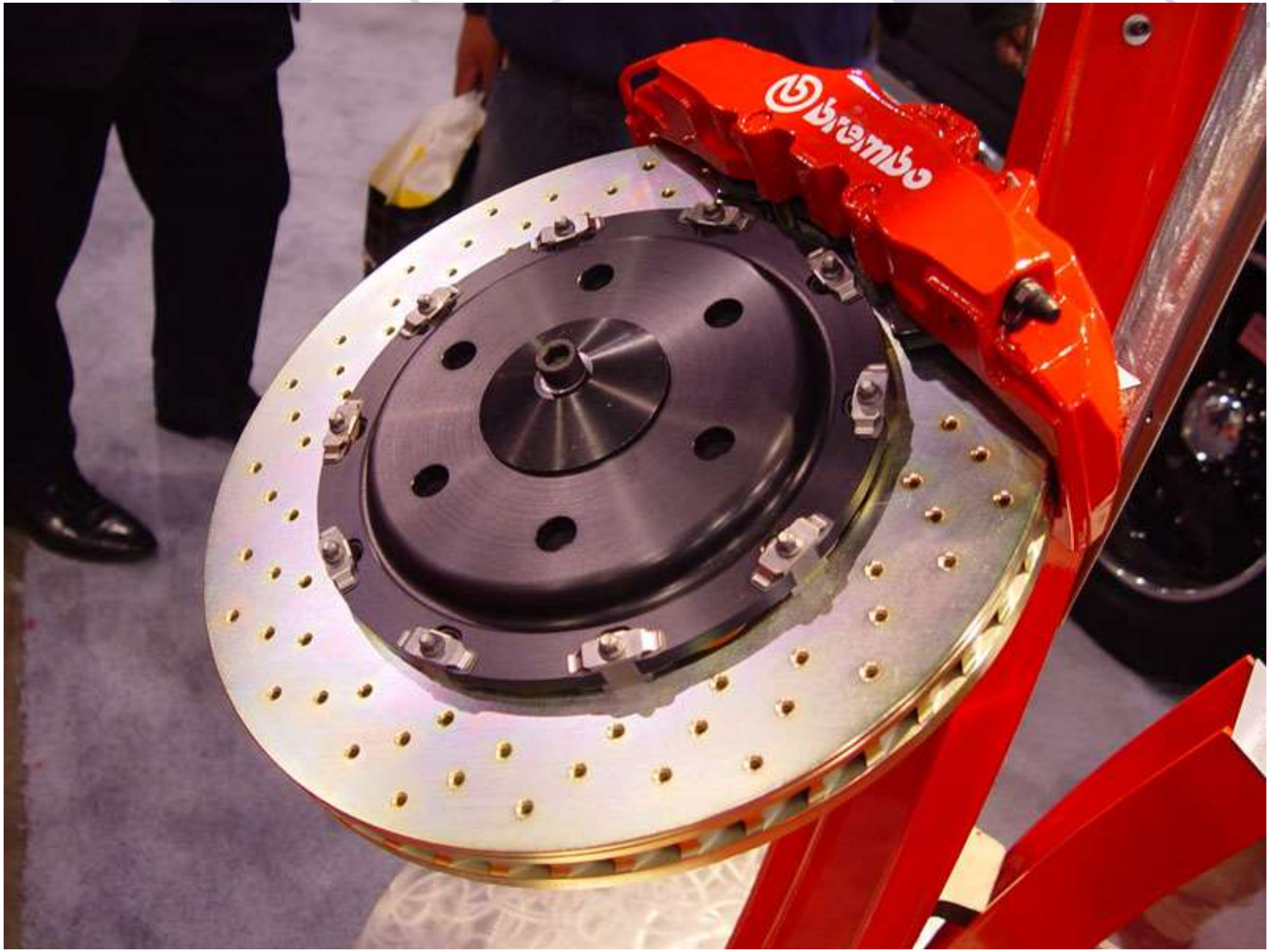


Disk Brakes

- *Working somewhat like bicycle brakes, disc brakes work by using calipers to clamp brake pads against either side of the spinning disc.*

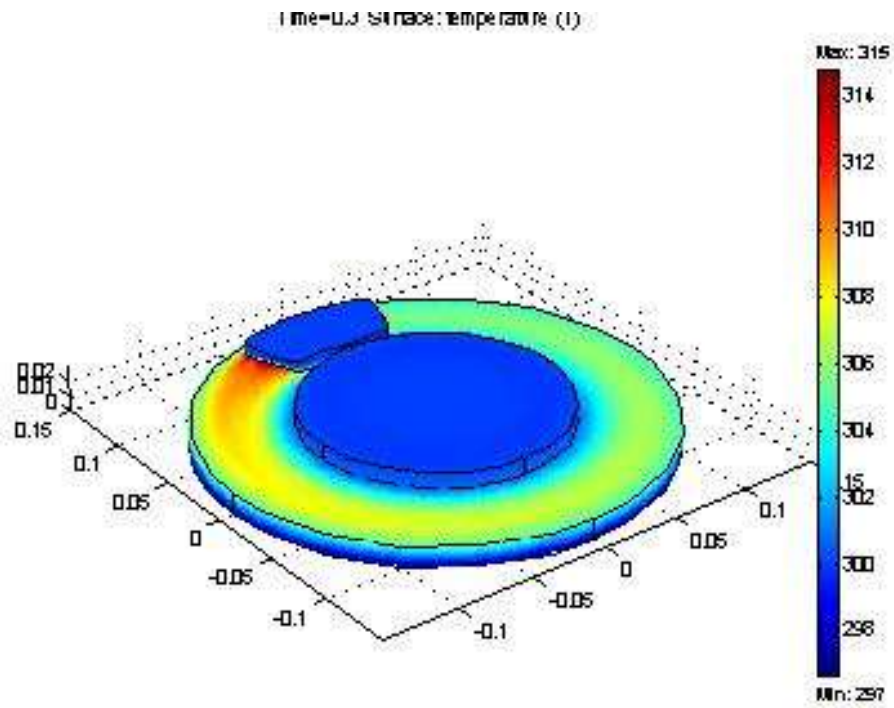


Disk Brakes



Drum Brakes (Hand Brake)

Disk Brakes


Max: 315
314
312
310
308
306
304
302
300
298
Min: 297
0.15
0.1
0.05
0
-0.05
-0.1
0.1
0.05
0
-0.05
-0.1

Why are Drums still around?

So if disc brakes are so marvelous, why are drum brakes still around?

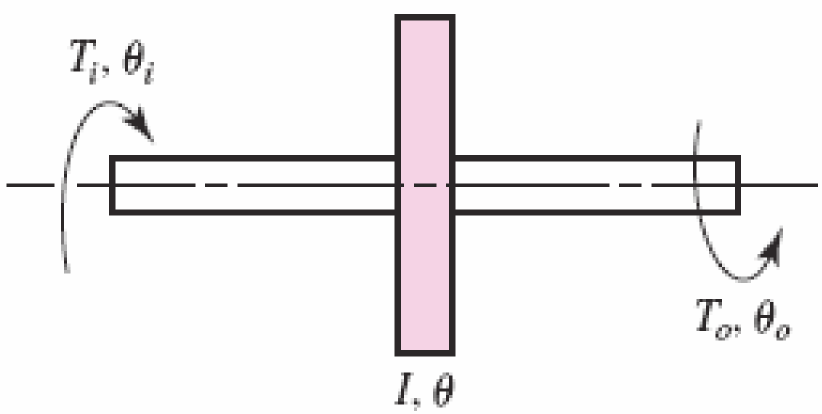
Well, disc brakes work only through hydraulics, which make them very tricky to use for hand brakes (that's 'emergency' brakes to Americans.) If you try to clamp a disc brake, as it cools down after use the disc gets slightly smaller, which means the car has to gradually increase the amount of force on the brakes to keep the car still.

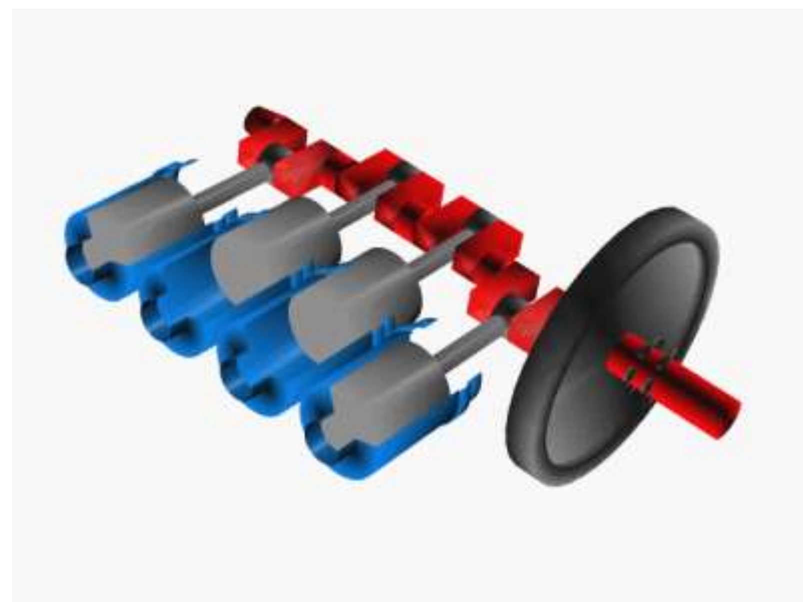
The Drum brakes on the other hand shrinks inward as they cool, therefore, further increasing the pressure on the brake pads.

The cars with all four wheel disk brakes, still have two small drum brakes mounted in the rear wheel for the emergency brakes.

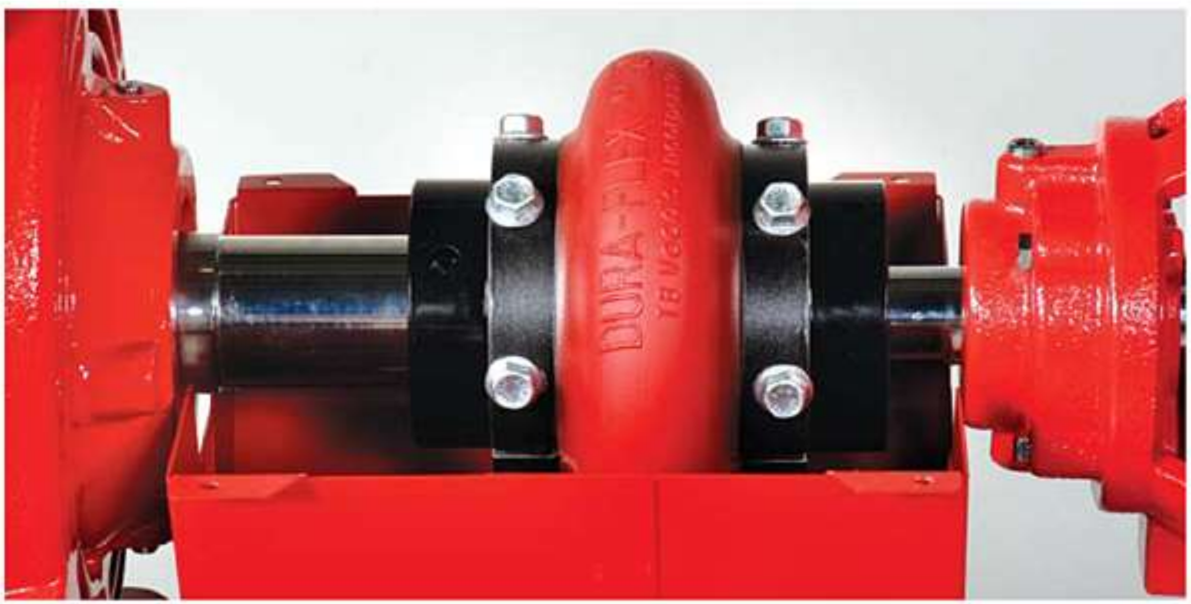
Flywheels

A flywheel is an inertial energy-storage device. It absorbs mechanical energy by increasing its angular velocity and delivers energy by decreasing its velocity. Figure 16–1*b* is a mathematical representation of a flywheel. An input torque T_i, corresponding to a coordinate θ_i, will cause the flywheel speed to increase. And a load or output torque T_o, with coordinate θ_o, will absorb energy from the flywheel and cause it to slow down. We shall be interested in designing flywheels so as to obtain a specified amount of speed regulation.

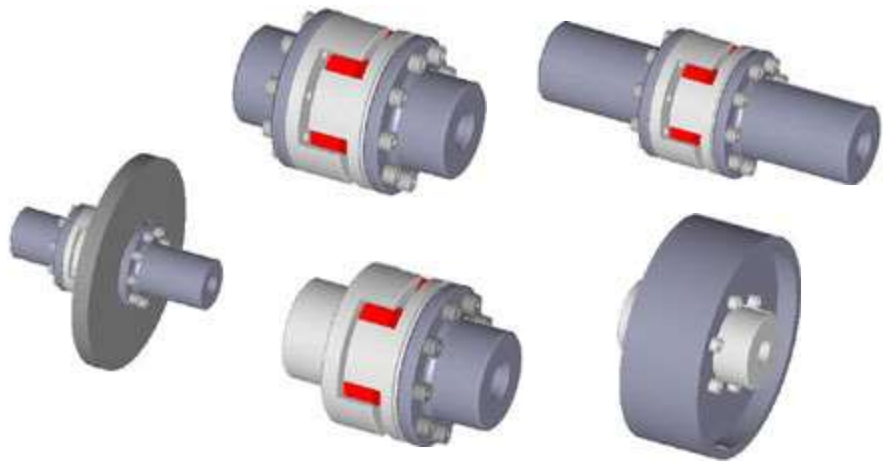


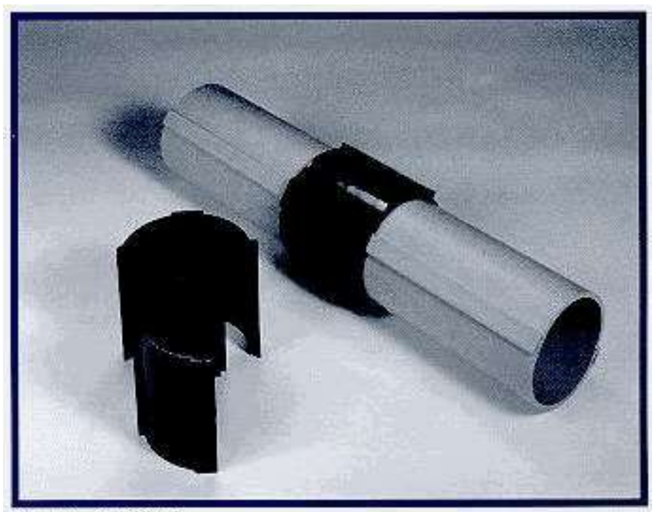


Couplings



Couplings





Patent No. 5,462,312

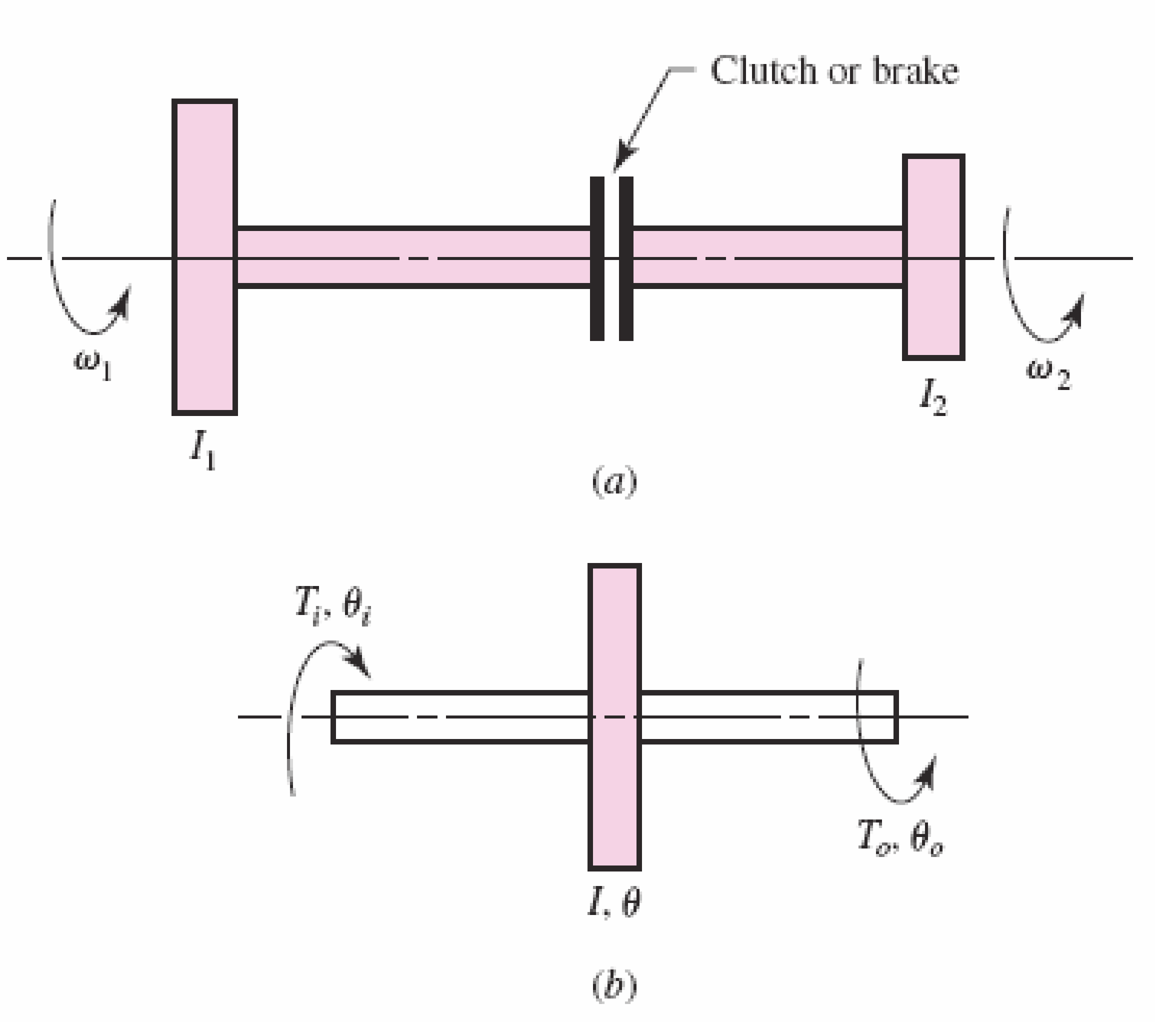
Clutches Brakes, coupling and Fly Wheel

- A group of elements usually associated with rotations that have in common the function of storing and/or transferring rotating energy.
- In analyzing the performance of these devices, following will be of interest
  - The actuating force
  - The torque transmitted
  - The energy loss
  - The temperature rise

Types of devices to be studied

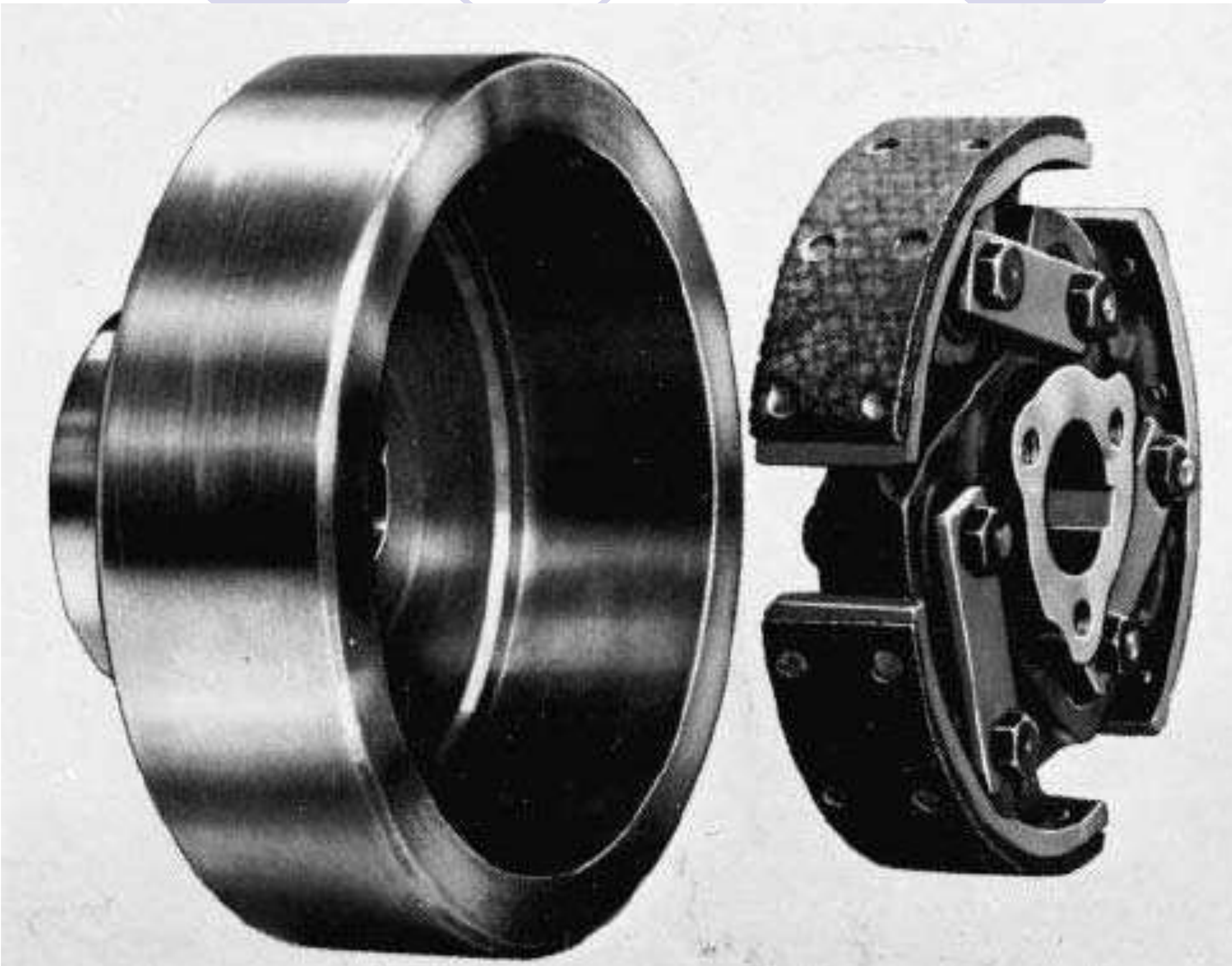
- Rim types with internal expanding shoes
- Rim types with external contracting shoes
- Band types
- Disk or axial types
- Cone types
- Miscellaneous types

Clutch, brake and flywheel



(*a*) Dynamic representation of a clutch or brake; (*b*) mathematical representation of a flywheel.

Internal Expanding Rim Clutches and Brakes



An internal expanding centrifugal-acting rim clutch. (Courtesy of the Hilliard Corporation.)

Internal Expanding Rim Clutches and Brakes

- The internal-shoe rim clutch consists essentially of three elements
  - The mating frictional surface
  - The means of transmitting the torque to and from the surfaces
  - The actuating mechanism
- Depending upon the operating mechanism, such clutches are further classified as
  - Expanding-ring
  - Centrifugal
  - Magnetic
  - Hydraulic
  - pneumatic

Internal Expanding Rim Clutches and Brakes

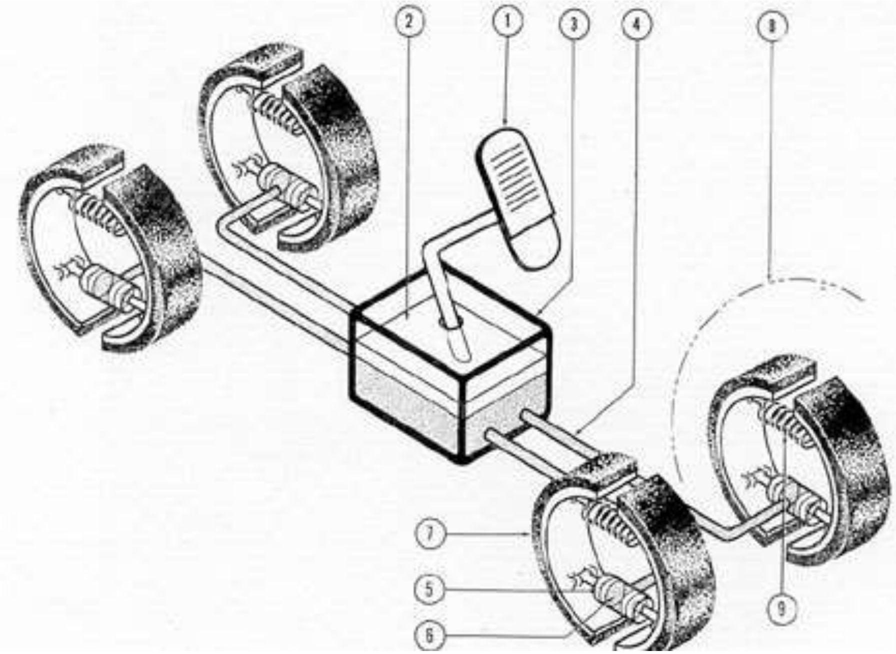
- The expanding-ring clutch is often used in textile machinery, excavators, and machine tools where the clutch may be located within the driving pulley
- Expanding-ring clutches benefit from centrifugal effects; transmit high torque, even at low speeds; and require both positive engagement and ample release force

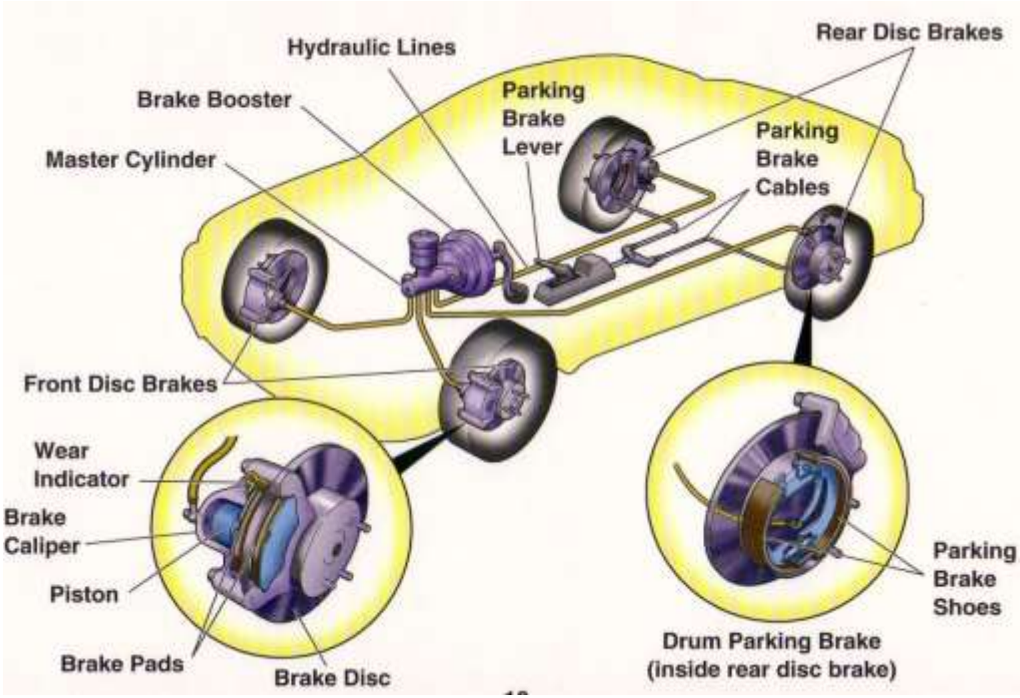
Types of Clutches

- Magnetic clutches are particularly useful for automatic and remote-control systems. Such clutches are also useful in drives subject to complex load cycles
- Hydraulic and pneumatic clutches are also useful in drives having complex loading cycles and in automatic machinery, or in robots
- Here the fluid flow can be controlled remotely using solenoid valves
- These clutches are also available as disk, cone, and multiple-plate clutches

Automated Hydraulic brake system

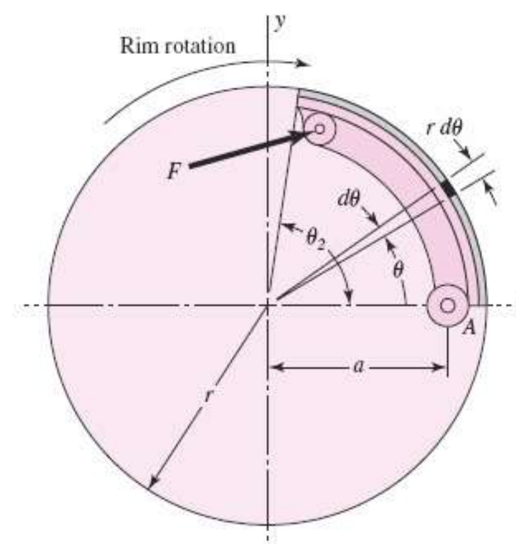
1) Brake pedal; 2) piston; 3) master cylinder; 4) hydraulic line; 5) brake cylinder; 6) brake piston; 7) brake band; 8) wheel; 9) return spring.





Internal Friction Shoe Geometry

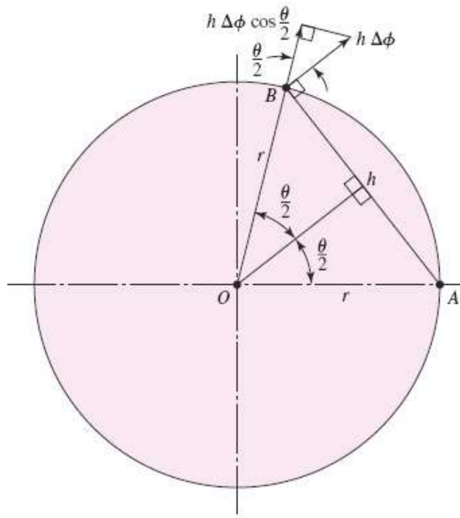
- To analyze an internal-shoe device, refer to figure, which shows a shoe pivoted at point *A*, with the actuating force acting at the other end of the shoe
- Since the shoe is long, the assumption that the distribution of normal forces is uniform cannot be made
- The mechanical arrangement permits no pressure to be applied at the heel, and the pressure will therefore be assumed zero at that point



Internal Friction Shoe Geometry

Let us consider the pressure p acting upon an element of area of the frictional material located at an angle θ from the hinge pin. The maximum pressure p_a is designated as located at an angle θ_a from the hinge pin. To find the pressure distribution on the periphery of the internal shoe, consider point B on the shoe. If the shoe deforms by infinitesimal rotation $\Delta\varphi$ about the pivot A. deformation perpendicular to AB is h $\Delta\varphi$. From the Isosceles triangle AOB, $h=2r\sin(\theta/2)$ so

$$h\,\Delta\varphi=2r\,\Delta\varphi\sin(\theta/2)$$



Deformation perpendicular to the rim is $h\,\Delta\varphi\cos\theta/2$ which is

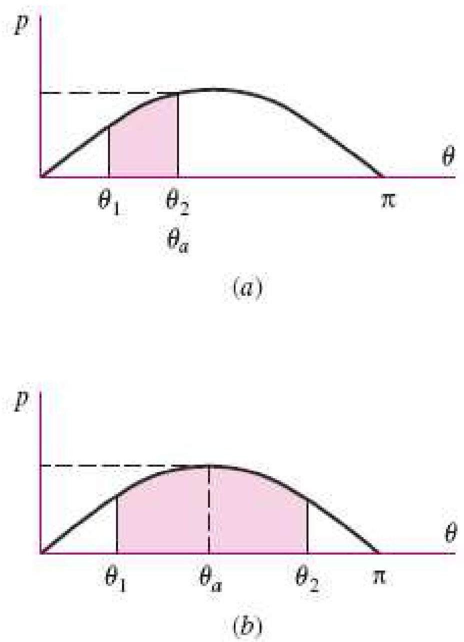
$$h\,\Delta\varphi\cos\theta/2=2r\,\Delta\varphi\sin(\theta/2)\,\cos\theta/2=r\Delta\varphi\sin(\theta/2)$$

Thus deformation, and consequently the pressure is proportional to $\sin\theta$. In terms of the pressure at B and where the pressure is maximum, this means

$$p=\frac{p_a}{\sin\theta_a}\sin\theta$$

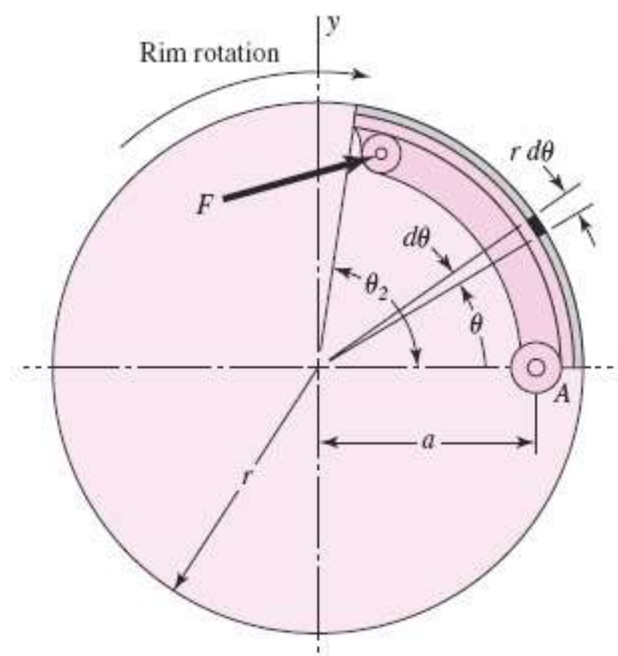
Internal Friction Shoe Geometry

- The pressure distribution is sinusoidal with respect to the central angle θ
- If the shoe is short, as show in (a), the largest pressure on the shoe is p_a occurring at the end of the shoe, θ_2
- If the shoe is long as shown in (b), the largest pressure on the shoe is p_a occurring at $\theta_a = 90^o$.

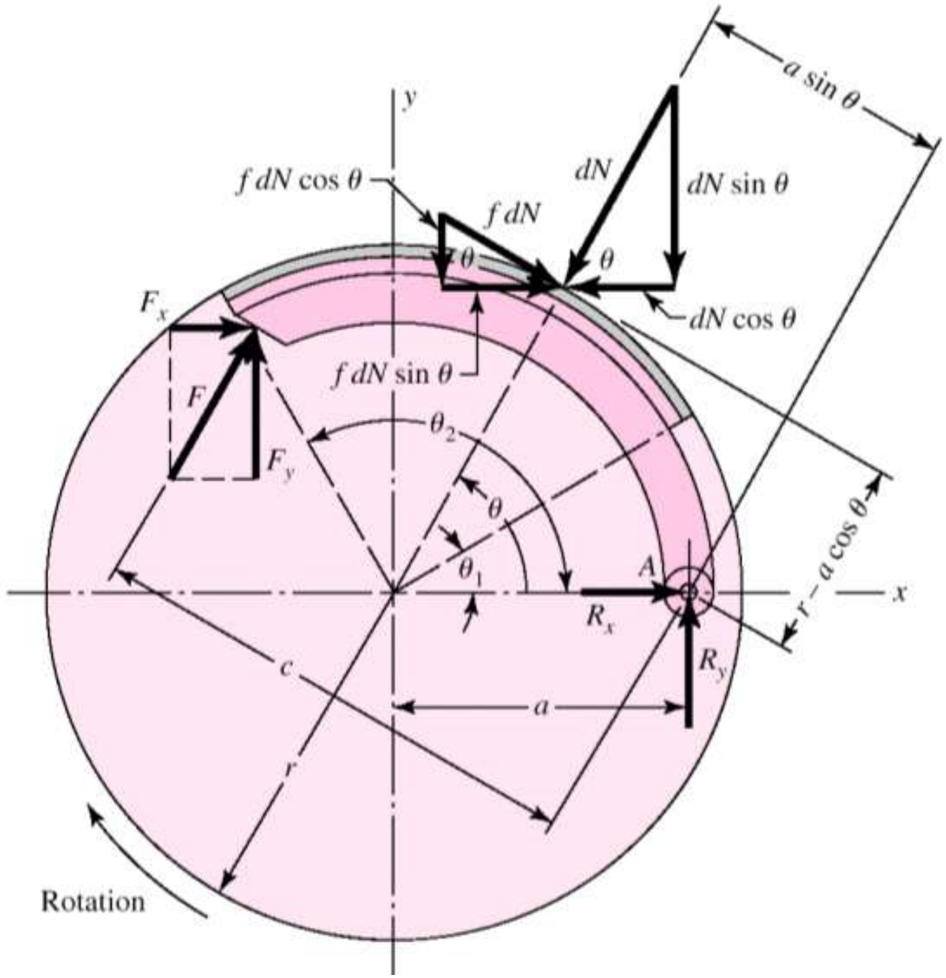


Defining the angle θ_a at which the maximum pressure p_a occurs when (*a*) shoe exists in zone $\theta_1 \le \theta_2 \le \pi/2$ and (*b*) shoe exists in zone $\theta_1 \le \pi/2 \le \theta_2$.

$$p = \frac{p_a}{\sin\theta_a}\sin\theta$$



Forces on the Shoe



Forces on the Shoe

Proceeding now (Fig. 16–7), the hinge-pin reactions are R_x and R_y. The actuating force F has components F_x and F_y and operates at distance c from the hinge pin. At any angle θ from the hinge pin there acts a differential normal force dN whose magnitude is

$$dN = pbr\,d\theta \qquad (b)$$

where b is the face width (perpendicular to the paper) of the friction material. Substituting the value of the pressure from Eq. (16–1), the normal force is

$$dN = \frac{p_a br \sin\theta\, d\theta}{\sin\theta_a} \qquad (c)$$

The normal force dN has horizontal and vertical components $dN\cos\theta$ and $dN\sin\theta$, as shown in the figure. The frictional force $f\,dN$ has horizontal and vertical components whose magnitudes are $f\,dN\sin\theta$ and $f\,dN\cos\theta$, respectively. By applying the conditions of static equilibrium, we may find the actuating force F, the torque T, and the pin reactions R_x and R_y.

Forces on the Shoe

We shall find the actuating force F, using the condition that the summation of the moments about the hinge pin is zero. The frictional forces have a moment arm about the pin of $r - a\cos\theta$. The moment M_f of these frictional forces is

$$M_f = \int f\,dN(r - a\cos\theta) = \frac{f p_a b r}{\sin\theta_a}\int_{\theta_1}^{\theta_2} \sin\theta(r - a\cos\theta)\,d\theta \qquad (16\text{–}2)$$

which is obtained by substituting the value of dN from Eq. (c). It is convenient to integrate Eq. (16–2) for each problem, and we shall therefore retain it in this form. The moment arm of the normal force dN about the pin is $a\sin\theta$. Designating the moment of the normal forces by M_N and summing these about the hinge pin give

$$M_N = \int dN(a\sin\theta) = \frac{p_a b r a}{\sin\theta_a}\int_{\theta_1}^{\theta_2} \sin^2\theta\,d\theta \qquad (16\text{–}3)$$

The actuating force F must balance these moments. Thus

$$F = \frac{M_N - M_f}{c} \qquad (16\text{–}4)$$

We see here that a condition for zero actuating force exists. In other words, if we make $M_N = M_f$, self-locking is obtained, and no actuating force is required. This furnishes us with a method for obtaining the dimensions for some self-energizing action. Thus the dimension a in Fig. 16–7 must be such that

$$M_N > M_f \qquad (16\text{–}5)$$

A self-energizing brake is one that uses the actual braking drag force to increase the force with which the brake is applied. The principle is well known in automotive engineering. The action of a rotating brake drum that increases the application pressure of the brake shoe(s).

Forces on the Shoe

The torque T applied to the drum by the brake shoe is the sum of the frictional forces $f\,dN$ times the radius of the drum:

$$T = \int f r\,dN = \frac{f p_a b r^2}{\sin\theta_a}\int_{\theta_1}^{\theta_2} \sin\theta\,d\theta$$

$$= \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} \qquad (16\text{–}6)$$

The hinge-pin reactions are found by taking a summation of the horizontal and vertical forces. Thus, for R_x, we have

$$R_x = \int dN\cos\theta - \int f\,dN\sin\theta - F_x$$

$$= \frac{p_a b r}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\,d\theta - f\int_{\theta_1}^{\theta_2} \sin^2\theta\,d\theta\right) - F_x \qquad (d)$$

Forces on the Shoe

The vertical reaction is found in the same way:

$$R_y = \int dN \sin\theta + \int f\, dN \cos\theta - F_y$$

$$= \frac{p_a br}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta + f\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\, d\theta\right) - F_y \qquad (e)$$

The direction of the frictional forces is reversed if the rotation is reversed. Thus, for counterclockwise rotation the actuating force is

$$F = \frac{M_N + M_f}{c} \qquad (16\text{–}7)$$

and since both moments have the same sense, the self-energizing effect is lost. Also, for counterclockwise rotation the signs of the frictional terms in the equations for the pin reactions change, and Eqs. (*d*) and (*e*) become

$$R_x = \frac{p_a br}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\, d\theta + f\int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta\right) - F_x \qquad (f)$$

$$R_y = \frac{p_a br}{\sin\theta_a}\left(\int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta - f\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\, d\theta\right) - F_y \qquad (g)$$

Forces on the Shoe

Equations (d), (e), (f), and (g) can be simplified to ease computations. Thus, let

$$A = \int_{\theta_1}^{\theta_2} \sin\theta \cos\theta \, d\theta = \left(\frac{1}{2}\sin^2\theta\right)_{\theta_1}^{\theta_2}$$

$$B = \int_{\theta_1}^{\theta_2} \sin^2\theta \, d\theta = \left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_{\theta_1}^{\theta_2} \tag{16–8}$$

Then, for clockwise rotation as shown in Fig. 16–7, the hinge-pin reactions are

$$R_x = \frac{p_a br}{\sin\theta_a}(A - fB) - F_x$$

$$R_y = \frac{p_a br}{\sin\theta_a}(B + fA) - F_y \tag{16–9}$$

For counterclockwise rotation, Eqs. (f) and (g) become

$$R_x = \frac{p_a br}{\sin\theta_a}(A + fB) - F_x$$

$$R_y = \frac{p_a br}{\sin\theta_a}(B - fA) - F_y \tag{16–10}$$

In using these equations, the reference system always has its origin at the center of the drum. The positive x axis is taken through the hinge pin. The positive y axis is always in the direction of the shoe, even if this should result in a left-handed system.

To Remember

The following assumption are implied by the preceding analysis

- The pressure at any point on the shoe is assumed to be proportional to the distance from the hinge pin, being zero at the heel.
- The effect of centrifugal force has been neglected. In this case the brakes, the shoes are not rotating, and no centrifugal forces exists. In clutch design, the effect of this force must be considered in writing the equations of static equilibrium
- The shoe is assumed to be rigid. Since this cannot be true, some deflection will occur, depending upon the load, pressure, and stiffness of the shoe. The resulting pressure distribution may be different from that which has been assumed
- The entire analysis has been based upon a coefficient of friction which does not vary with pressure. Actually, the coefficient may vary with a number of conditions, including temperature, wear and environment

Example 16-2

The brake shown in Fig. 16–8 is 300 mm in diameter and is actuated by a mechanism that exerts the same force F on each shoe. The shoes are identical and have a face width of 32 mm. The lining is a molded asbestos having a coefficient of friction of 0.32 and a pressure limitation of 1000 kPa. Estimate the maximum

(a) Actuating force F.

(b) Braking capacity.

(c) Hinge-pin reactions.

(a) The right-hand shoe is self-energizing, and so the force F is found on the basis that the maximum pressure will occur on this shoe. Here $\theta_1 = 0°$, $\theta_2 = 126°$, $\theta_a = 90°$, and $\sin\theta_a = 1$. Also,

$$a = \sqrt{(112)^2 + (50)^2} = 122.7 \text{ mm}$$

Integrating Eq. (16–2) from 0 to θ_2 yields

$$M_f = \frac{f p_a b r}{\sin\theta_a}\left[\left(-r\cos\theta\right)_0^{\theta_2} - a\left(\frac{1}{2}\sin^2\theta\right)_0^{\theta_2}\right]$$

$$= \frac{f p_a b r}{\sin\theta_a}\left(r - r\cos\theta_2 - \frac{a}{2}\sin^2\theta_2\right)$$

θ_1 is the start of friction material on the brake, θ_2 is the end of frictional material and θ_a is the point of maximum pressure: See Figure 16-6 for further explanation

Example 16-2

Changing all lengths to meters, we have

$$M_f = (0.32)[1000(10)^3](0.032)(0.150)$$
$$\times \left[0.150 - 0.150\cos 126^\circ - \left(\frac{0.1227}{2}\right)\sin^2 126^\circ\right]$$
$$= 304\ \text{N}\cdot\text{m}$$

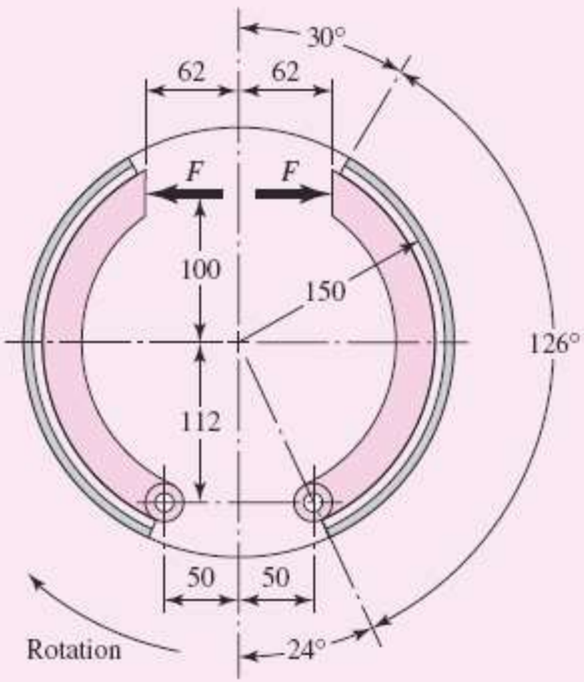
The moment of the normal forces is obtained from Eq. (16–3). Integrating from 0 to θ_2 gives

$$M_N = \frac{p_a bra}{\sin\theta_a}\left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_0^{\theta_2}$$
$$= \frac{p_a bra}{\sin\theta_a}\left(\frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2\right)$$
$$= [1000(10)^3](0.032)(0.150)(0.1227)\left\{\frac{\pi}{2}\frac{126}{180} - \frac{1}{4}\sin[(2)(126^\circ)]\right\}$$

Answer $$= 788\ \text{N}\cdot\text{m}$$

From Eq. (16–4), the actuating force is

Answer $$F = \frac{M_N - M_f}{c} = \frac{788 - 304}{100 + 112} = 2.28\ \text{kN}$$



(*b*) From Eq. (16–6), the torque applied by the right-hand shoe is

$$T_R = \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

$$= \frac{0.32[1000(10)^3](0.032)(0.150)^2(\cos 0° - \cos 126°)}{\sin 90°} = 366 \text{ N} \cdot \text{m} \qquad \text{Answer}$$

The torque contributed by the left-hand shoe cannot be obtained until we learn its maximum operating pressure. Equations (16–2) and (16–3) indicate that the frictional and normal moments are proportional to this pressure. Thus, for the left-hand shoe,

$$M_N = \frac{788 p_a}{1000} \qquad M_f = \frac{304 p_a}{1000}$$

Then, from Eq. (16–7),

$$F = \frac{M_N + M_f}{c}$$

or

$$2.28 = \frac{(788/1000)p_a + (304/1000)p_a}{100 + 112}$$

Solving gives $p_a = 443$ kPa. Then, from Eq. (16–6), the torque on the left-hand shoe is

$$T_L = \frac{f p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

Since $\sin\theta_a = \sin 90° = 1$, we have

Answer $$T_L = 0.32[443(10)^3](0.032)(0.150)^2(\cos 0° - \cos 126°) = 162 \text{ N} \cdot \text{m}$$

The braking capacity is the total torque:

Answer $$T = T_R + T_L = 366 + 162 = 528\ \text{N} \cdot \text{m}$$

(c) In order to find the hinge-pin reactions, we note that $\sin \theta_a = 1$ and $\theta_1 = 0$. Then Eq. (16–8) gives

$$A = \frac{1}{2}\sin^2\theta_2 = \frac{1}{2}\sin^2 126° = 0.3273$$

$$B = \frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2 = \frac{\pi(126)}{2(180)} - \frac{1}{4}\sin[(2)(126°)] = 1.3373$$

Also, let

$$D = \frac{p_a b r}{\sin\theta_a} = \frac{1000(0.032)(0.150)}{1} = 4.8\ \text{kN}$$

where $p_a = 1000$ kPa for the right-hand shoe. Then, using Eq. (16–9), we have

Answer $$R_x = D(A - fB) - F_x = 4.8[0.3273 - 0.32(1.3373)] - 2.28\sin 24° = -1.410\ \text{kN}$$

Answer $$R_y = D(B + fA) - F_y = 4.8[1.3373 + 0.32(0.3273)] - 2.28\cos 24° = 4.839\ \text{kN}$$

The resultant on this hinge pin is

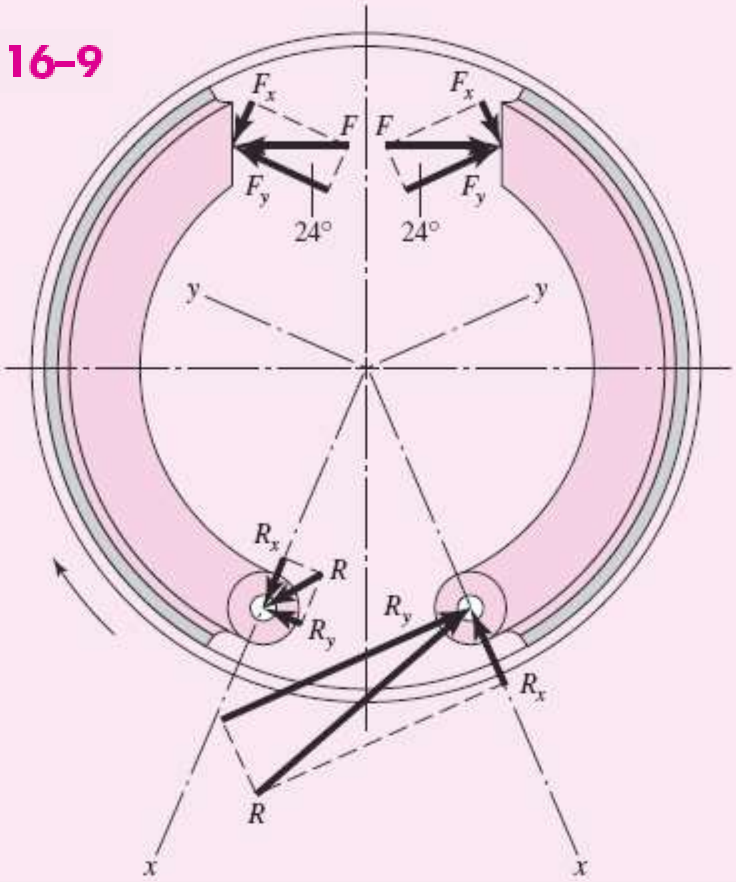
Answer $$R = \sqrt{(-1.410)^2 + (4.839)^2} = 5.04\ \text{kN}$$

The reactions at the hinge pin of the left-hand shoe are found using Eqs. (16–10) for a pressure of 443 kPa. They are found to be $R_x = 0.678$ kN and $R_y = 0.538$ kN. The resultant is

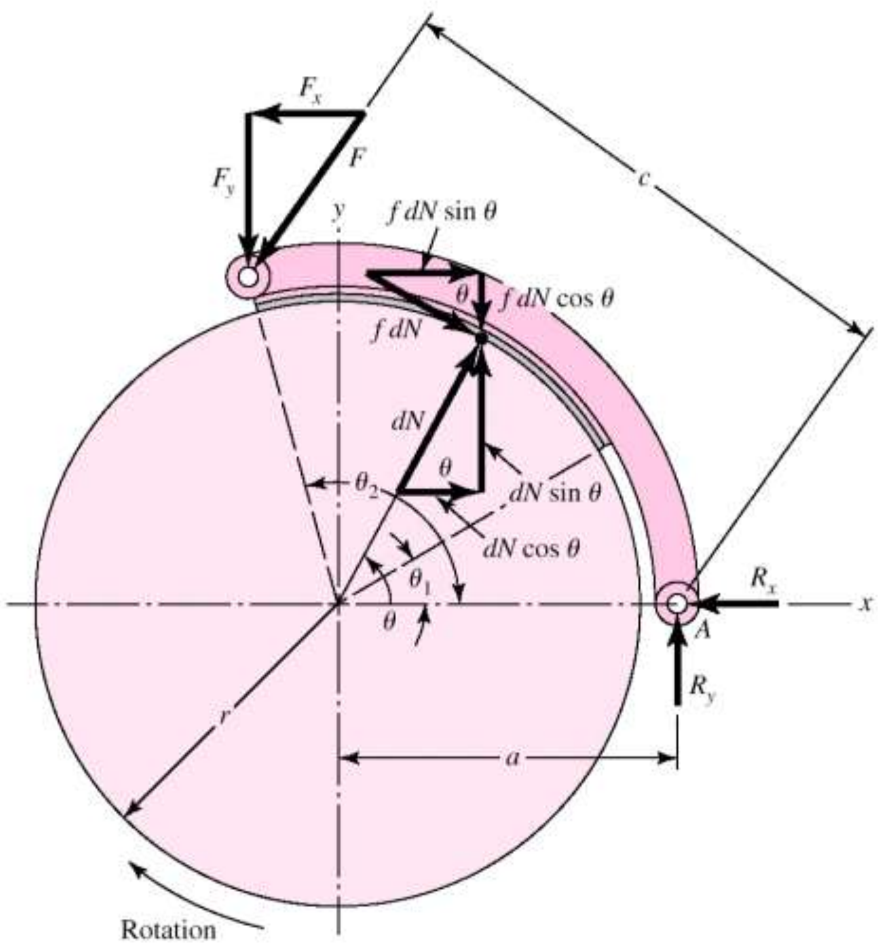
Answer $$R = \sqrt{(0.678)^2 + (0.538)^2} = 0.866 \text{ kN}$$

The reactions for both hinge pins, together with their directions, are shown in Fig. 16–9.

Figure 16–9



External Contracting Rim Clurches and Brakes



External Contracting Rim Clurches and Brakes

The notation for external contracting shoes is shown in Fig. 16–11. The moments of the frictional and normal forces about the hinge pin are the same as for the internal expanding shoes. Equations (16–2) and (16–3) apply and are repeated here for convenience:

$$M_f = \frac{f p_a b r}{\sin \theta_a} \int_{\theta_1}^{\theta_2} \sin \theta (r - a \cos \theta)\, d\theta \tag{16–2}$$

$$M_N = \frac{p_a b r a}{\sin \theta_a} \int_{\theta_1}^{\theta_2} \sin^2 \theta \, d\theta \tag{16–3}$$

Both these equations give positive values for clockwise moments (Fig. 16–11) when used for external contracting shoes. The actuating force must be large enough to balance both moments:

$$F = \frac{M_N + M_f}{c} \tag{16–11}$$

External Contracting Rim Clurches and Brakes

The horizontal and vertical reactions at the hinge pin are found in the same manner as for internal expanding shoes. They are

$$R_x = \int dN \cos\theta + \int f\, dN \sin\theta - F_x \tag{a}$$

$$R_y = \int f\, dN \cos\theta - \int dN \sin\theta + F_y \tag{b}$$

By using Eq. (16–8), we have

$$\begin{aligned} R_x &= \frac{p_a b r}{\sin\theta_a}(A + f B) - F_x \\ R_y &= \frac{p_a b r}{\sin\theta_a}(f A - B) + F_y \end{aligned} \tag{16–12}$$

External Contracting Rim Clurches and Brakes

If the rotation is counterclockwise, the sign of the frictional term in each equation is reversed. Thus Eq. (16–11) for the actuating force becomes

$$F = \frac{M_N - M_f}{c} \tag{16–13}$$

and self-energization exists for counterclockwise rotation. The horizontal and vertical reactions are found, in the same manner as before, to be

$$R_x = \frac{p_a br}{\sin\theta_a}(A - fB) - F_x$$
$$R_y = \frac{p_a br}{\sin\theta_a}(-fA - B) + F_y \tag{16–14}$$

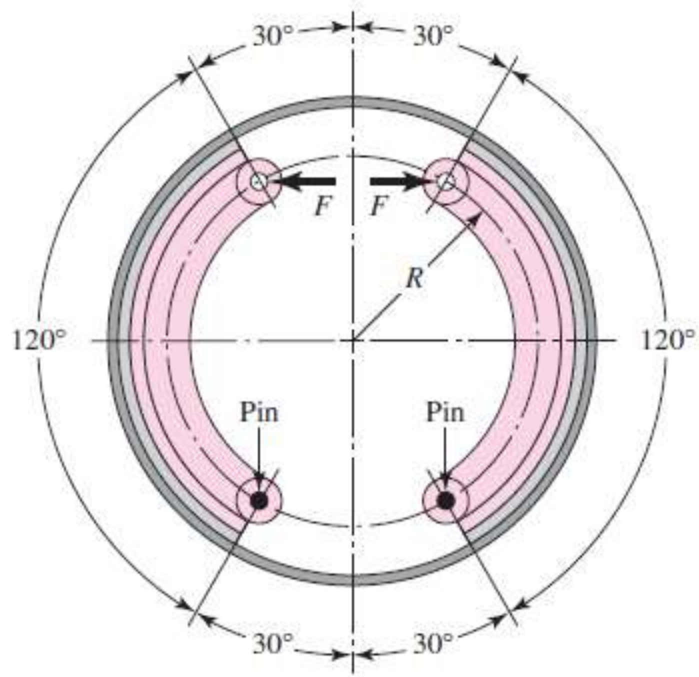
It should be noted that, when external contracting designs are used as clutches, the effect of centrifugal force is to decrease the normal force. Thus, as the speed increases, a larger value of the actuating force F is required.

16–1 The figure shows an internal rim-type brake having an inside rim diameter of 12 in and a dimension $R = 5$ in. The shoes have a face width of $1\frac{1}{2}$ in and are both actuated by a force of 500 lbf. The mean coefficient of friction is 0.28.

(*a*) Find the maximum pressure and indicate the shoe on which it occurs.

(*b*) Estimate the braking torque effected by each shoe, and find the total braking torque.

(*c*) Estimate the resulting hinge-pin reactions.



Problem 16–1

16–2 For the brake in Prob. 16–1, consider the pin and actuator locations to be the same. However, instead of 120°, let the friction surface of the brake shoes be 90° and centrally located. Find the maximum pressure and the total braking torque.

(a) $\theta_1 = 0°, \quad \theta_2 = 120°, \quad \theta_a = 90°, \quad \sin\theta_a = 1, \quad a = 5 \text{ in}$

$$\text{Eq. (16-2):} \quad M_f = \frac{0.28 p_a (1.5)(6)}{1} \int_{0°}^{120°} \sin\theta(6 - 5\cos\theta)\, d\theta$$

$$= 17.96 p_a \text{ lbf} \cdot \text{in}$$

$$\text{Eq. (16-3):} \quad M_N = \frac{p_a(1.5)(6)(5)}{1} \int_{0°}^{120°} \sin^2\theta\, d\theta = 56.87 p_a \text{ lbf} \cdot \text{in}$$

$$c = 2(5\cos 30°) = 8.66 \text{ in}$$

$$\text{Eq. (16-4):} \quad F = \frac{56.87 p_a - 17.96 p_a}{8.66} = 4.49 p_a$$

$$p_a = F/4.49 = 500/4.49 = 111.4 \text{ psi} \quad \text{for cw rotation}$$

$$\text{Eq. (16-7):} \quad 500 = \frac{56.87 p_a + 17.96 p_a}{8.66}$$

$$p_a = 57.9 \text{ psi for ccw rotation}$$

A maximum pressure of 111.4 psioccurs on the RH shoe for cw rotation. *Ans.*

(b) *RH shoe*:

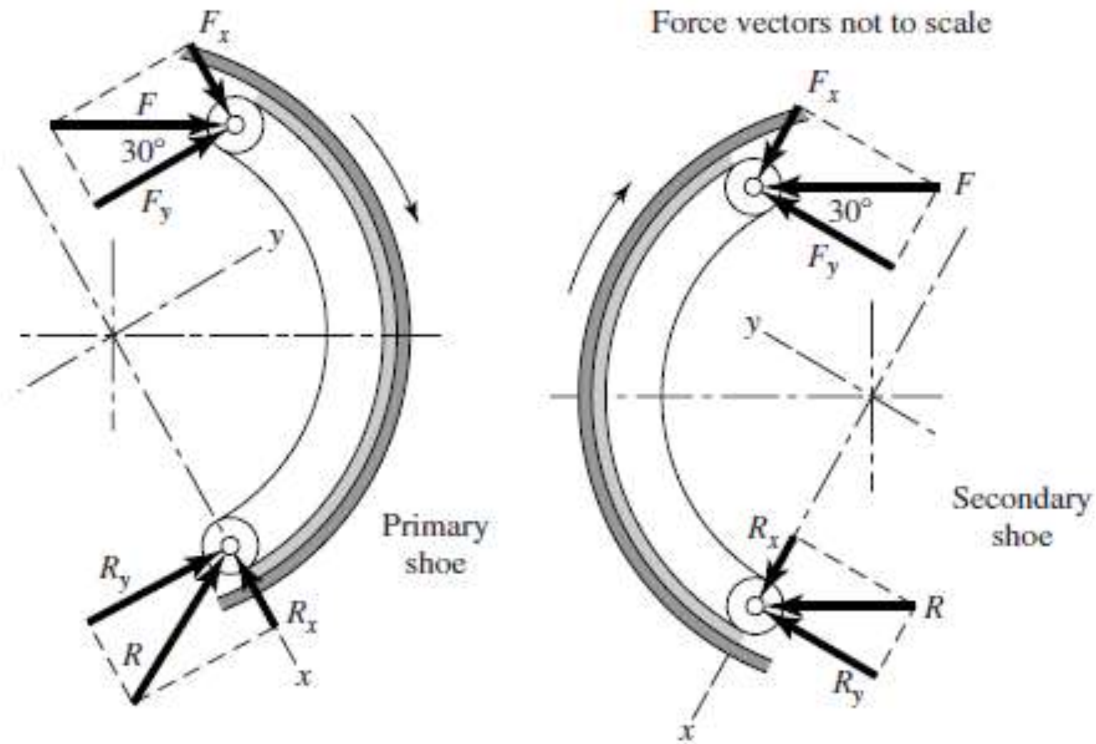
Eq. (16-6): $$T_R = \frac{0.28(111.4)(1.5)(6)^2(\cos 0^\circ - \cos 120^\circ)}{1} = 2530\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

LH shoe:

Eq. (16-6): $$T_L = \frac{0.28(57.9)(1.5)(6)^2(\cos 0^\circ - \cos 120^\circ)}{1} = 1310\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$T_{\text{total}} = 2530 + 1310 = 3840\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

(c)


Force vectors not to scale
F_x
F
30°
F_y
y
R_y
R
R_x
x
Primary
shoe
F_x
F
30°
F_y
y
R_x
R
R_y
x
Secondary
shoe

RH shoe: $F_x = 500\sin 30^\circ = 250\text{ lbf}, \quad F_y = 500\cos 30^\circ = 433\text{ lbf}$

Eqs. (16-8): $A = \left(\frac{1}{2}\sin^2\theta\right)_{0^\circ}^{120^\circ} = 0.375, \quad B = \left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_{0}^{2\pi/3\text{ rad}} = 1.26$

Eqs. (16-9): $R_x = \frac{111.4(1.5)(6)}{1}[0.375 - 0.28(1.264)] - 250 = -229\text{ lbf}$

$$R_y = \frac{111.4(1.5)(6)}{1}[1.264 + 0.28(0.375)] - 433 = 940\text{ lbf}$$

$$R = [(-229)^2 + (940)^2]^{1/2} = 967\text{ lbf} \quad \textit{Ans.}$$

LH shoe: $F_x = 250\text{ lbf}, \quad F_y = 433\text{ lbf}$

Eqs. (16-10): $R_x = \frac{57.9(1.5)(6)}{1}[0.375 + 0.28(1.264)] - 250 = 130\text{ lbf}$

$$R_y = \frac{57.9(1.5)(6)}{1}[1.264 - 0.28(0.375)] - 433 = 171\text{ lbf}$$

$$R = [(130)^2 + (171)^2]^{1/2} = 215\text{ lbf} \quad \textit{Ans.}$$

Home Assignment

16-2 For the brake in Prob. 16–1, consider the pin and actuator locations to be the same. However, instead of 120°, let the friction surface of the brake shoes be 90° and centrally located. Find the maximum pressure and the total braking torque.

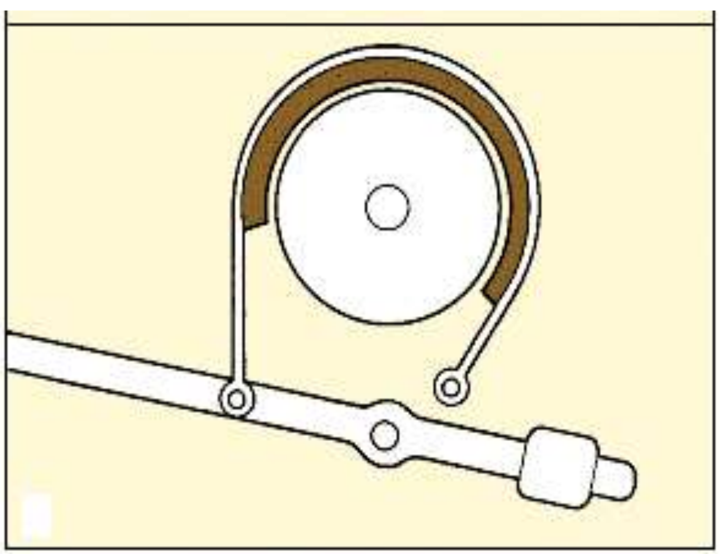
16-3 In the figure for Prob. 16–1, the inside rim diameter is 280 mm and the dimension R is 90 mm. The shoes have a face width of 30 mm. Find the braking torque and the maximum pressure for each shoe if the actuating force is 1000 N, the drum rotation is counterclockwise, and $f = 0.30$.

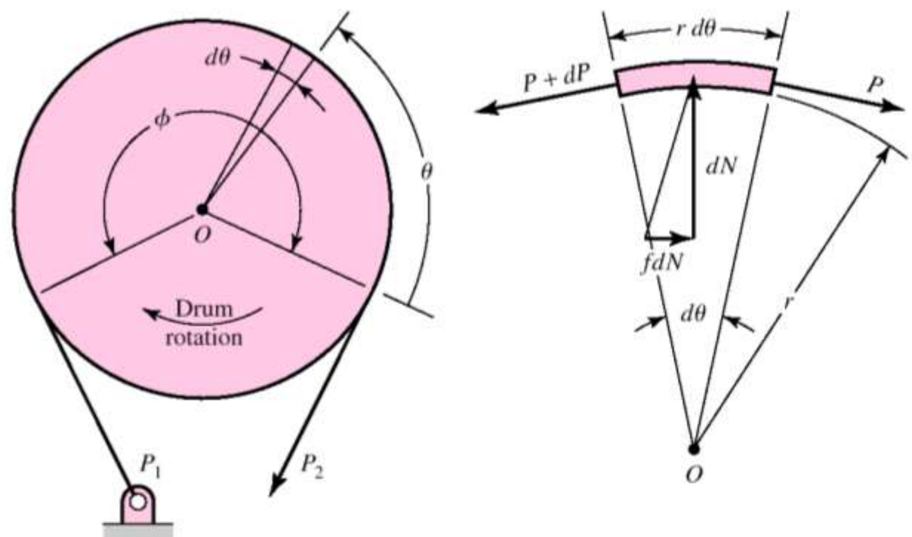
Band type clutches and brakes

Flexible clutch and brake bands are used in power excavators and in hoisting and other machinery. The analysis follows the notation of Fig. 16–13.

Because of friction and the rotation of the drum, the actuating force P_2 is less than the pin reaction P_1. Any element of the band, of angular length $d\theta$, will be in equilibrium under the action of the forces shown in the figure. Summing these forces in the vertical direction, we have

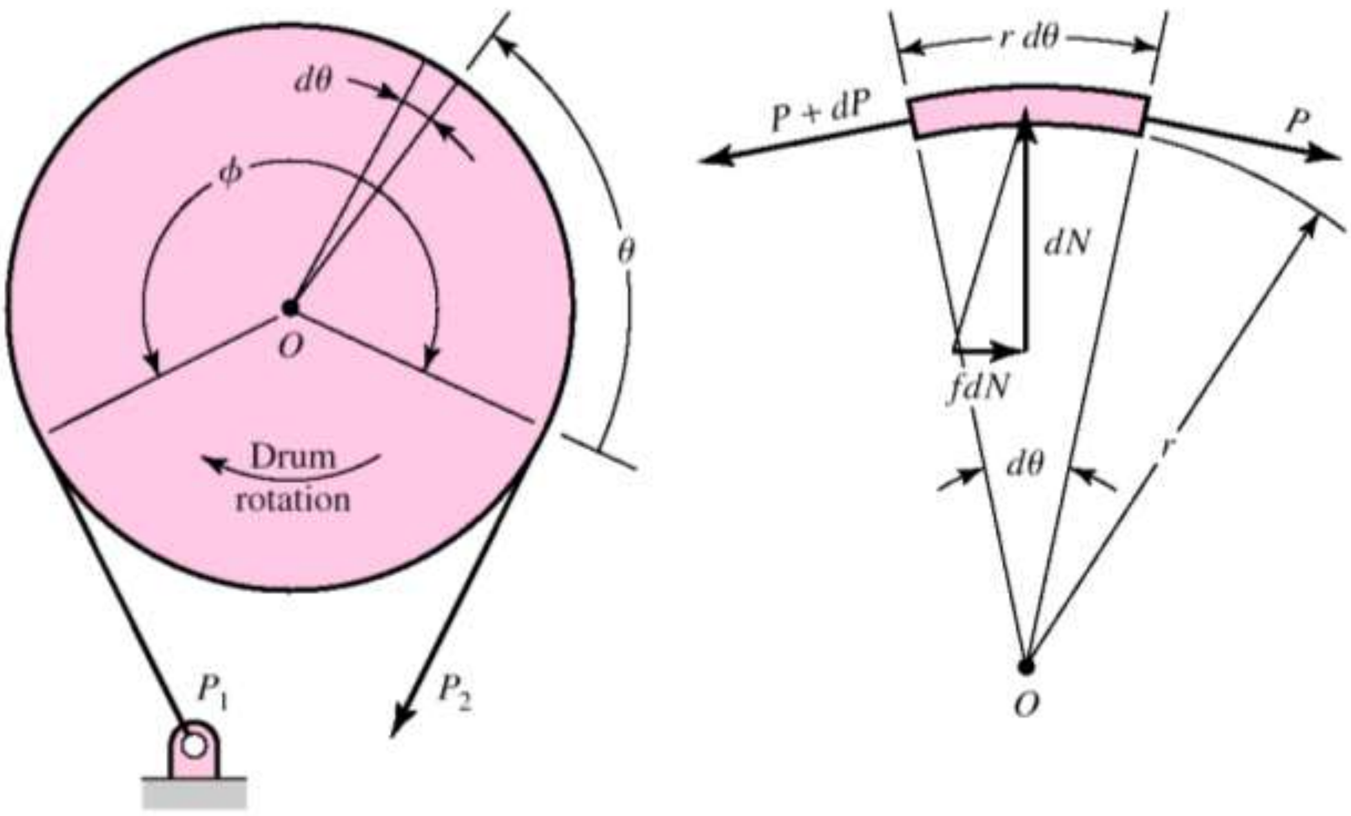
$$(P + dP)\sin\frac{d\theta}{2} + P\,\sin\frac{d\theta}{2} - dN = 0 \qquad (a)$$





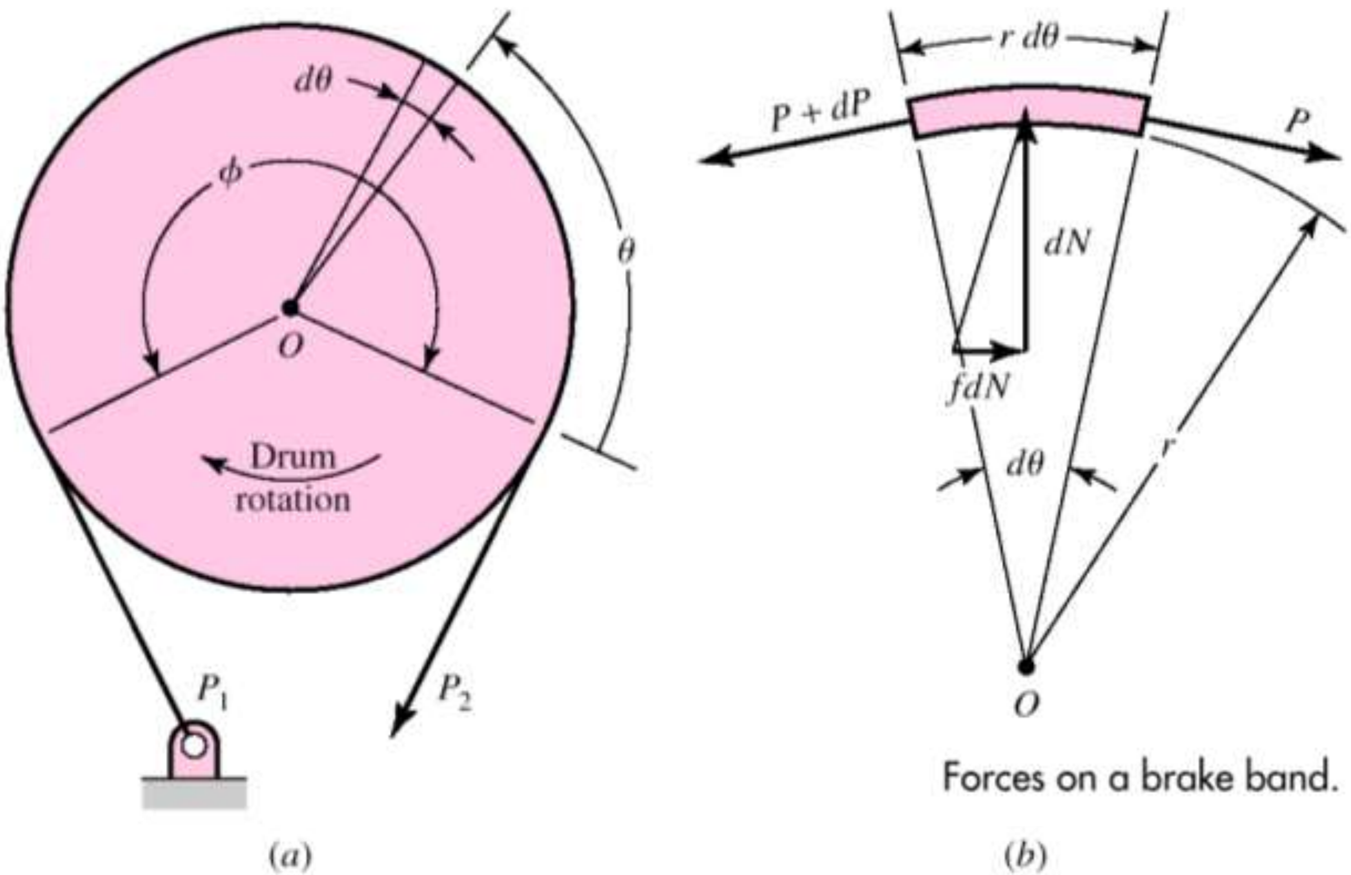
(a) (b)

Band type clutches and brakes



(a) (b)

Band type clutches and brakes



Forces on a brake band.

Band type clutches and brakes

$$dN = P\,d\theta \qquad (b)$$

since for small angles $\sin d\theta/2 = d\theta/2$. Summing the forces in the horizontal direction gives

$$(P + dP)\cos\frac{d\theta}{2} - P\cos\frac{d\theta}{2} - f\,dN = 0 \qquad (c)$$

$$dP - f\,dN = 0 \qquad (d)$$

Band type clutches and brakes

Substituting the value of dN from Eq. (b) in (d) and integrating,

$$\int_{P_2}^{P_1} \frac{dP}{P} = f \int_0^{\phi} d\theta \qquad \ln \frac{P_1}{P_2} = f\phi$$

and

$$\frac{P_1}{P_2} = e^{f\phi} \tag{16–19}$$

The torque may be obtained from the equation

$$T = (P_1 - P_2)\frac{D}{2} \tag{16–20}$$

Band type clutches and brakes

The normal force dN acting on an element of area of width b and length $r\,d\theta$ is

$$dN = pbr\,d\theta \tag{e}$$

where p is the pressure. Substitution of the value of dN from Eq. (b) gives

$$P\,d\theta = pbr\,d\theta$$

Therefore

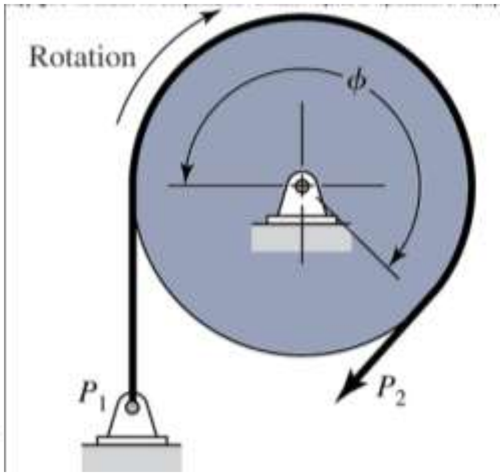
$$p = \frac{P}{br} = \frac{2P}{bD} \tag{16–21}$$

The pressure is therefore proportional to the tension in the band. The maximum pressure p_a will occur at the toe and has the value

$$p_a = \frac{2P_1}{bD} \tag{16–22}$$

Problem 16-11

The maximum band interface pressure on the brake shown in the figure is 90 psi. Use a 14-in-diameter drum, and band width of 4 in, a coefficient of friction of 0.25, and an angle-of-wrap of 270°. Find the band tensions and the torque capacity.



From Eq. (16-22),

$$P_1 = \frac{p_a b D}{2} = \frac{90(4)(14)}{2} = 2520 \text{ lbf} \quad Ans.$$

$$f\phi = 0.25(\pi)(270°/180°) = 1.178$$

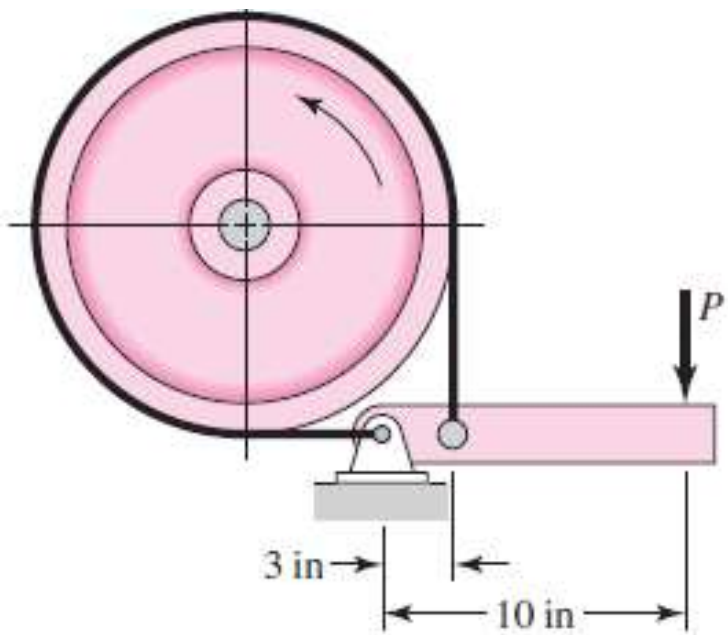
Eq. (16-19): $P_2 = P_1 \exp(-f\phi) = 2520 \exp(-1.178) = 776 \text{ lbf} \quad Ans.$

$$T = \frac{(P_1 - P_2)D}{2} = \frac{(2520 - 776)14}{2} = 12\,200 \text{ lbf} \cdot \text{in} \quad Ans.$$

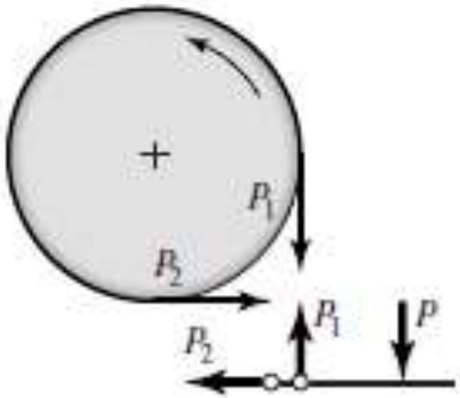
16-14 The figure depicts a band brake whose drum rotates counterclockwise at 200 rev/min. The drum diameter is 16 in and the band lining 3 in wide. The coefficient of friction is 0.20. The maximum lining interface pressure is 70 psi.

(*a*) Find the brake torque, necessary force *P*, and steady-state power.

Problem 16–14



(a)



$D = 16"$, $b = 3"$

$n = 200$ rev/min

$f = 0.20$, $p_a = 70$ psi

Eq. (16-22): $$P_1 = \frac{p_a b D}{2} = \frac{70(3)(16)}{2} = 1680 \text{ lbf}$$

$$f\phi = 0.20(3\pi/2) = 0.942$$

Eq. (16-14): $P_2 = P_1 \exp(-f\phi) = 1680 \exp(-0.942) = 655 \text{ lbf}$

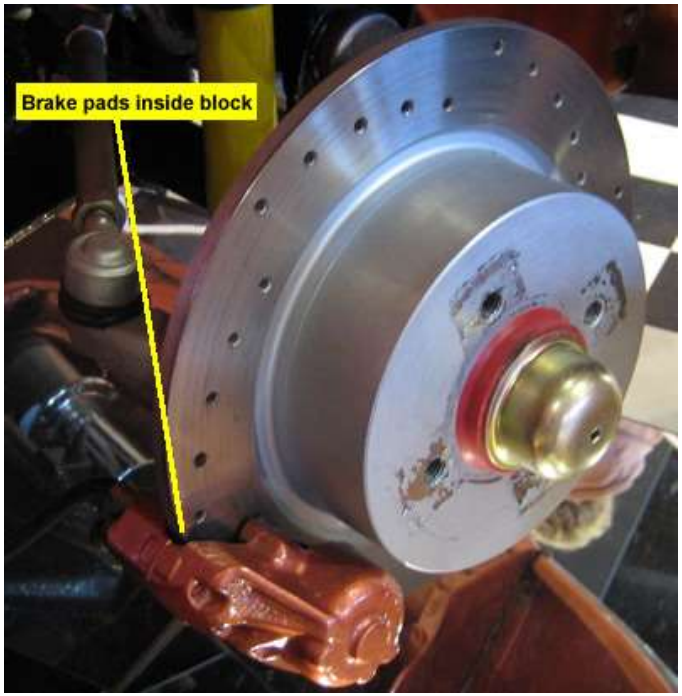
$$T = (P_1 - P_2)\frac{D}{2} = (1680 - 655)\frac{16}{2}$$

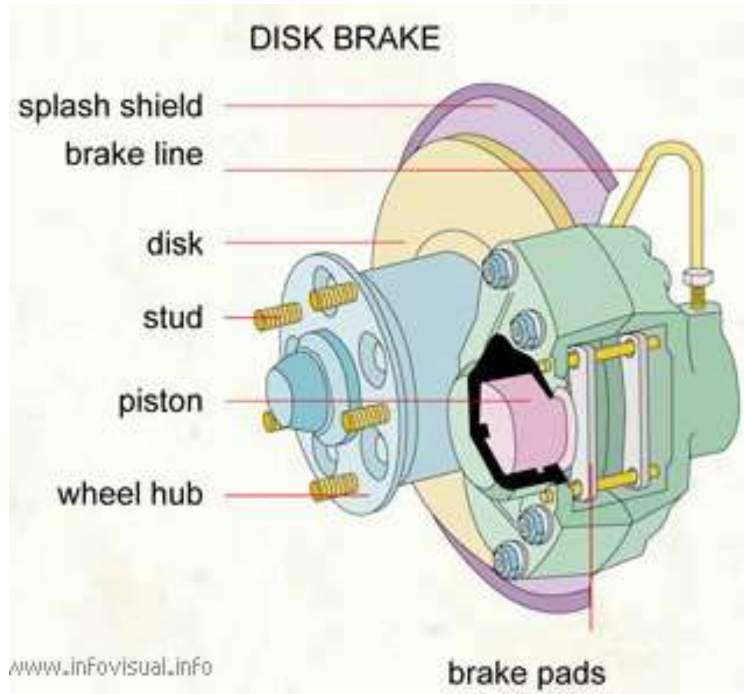
$$= 8200 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$H = \frac{Tn}{63\,025} = \frac{8200(200)}{63\,025} = 26.0 \text{ hp} \quad \textit{Ans.}$$

$$P = \frac{3P_1}{10} = \frac{3(1680)}{10} = 504 \text{ lbf} \quad \textit{Ans.}$$

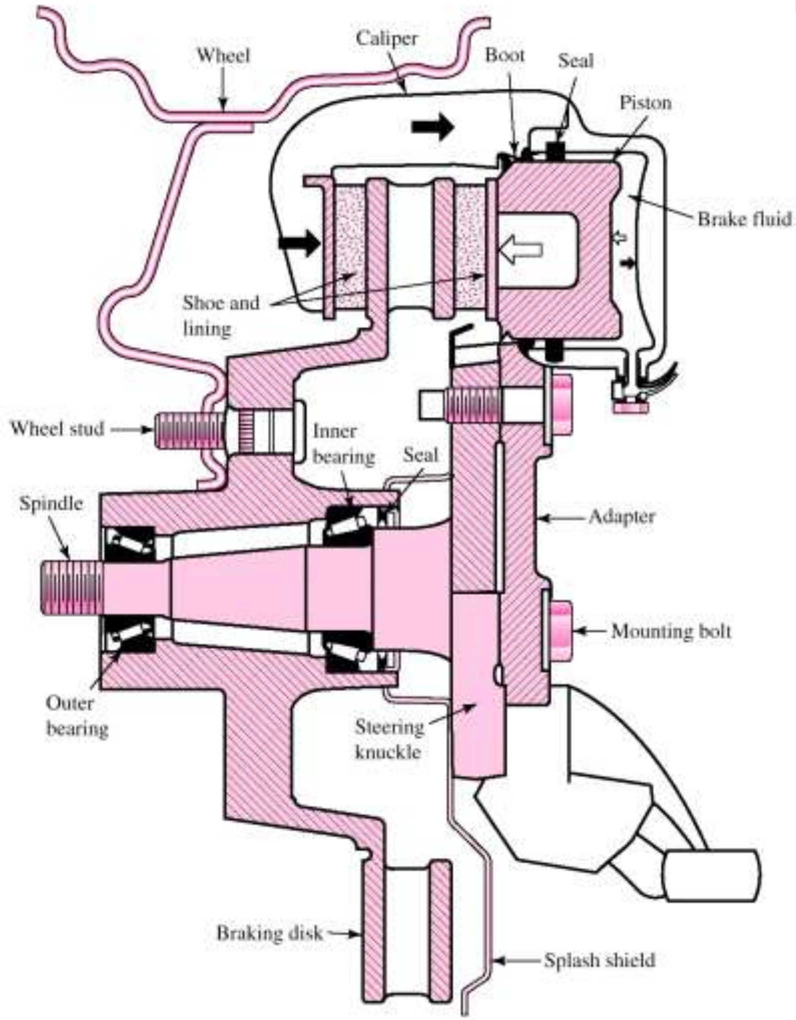
Disk Brakes





We have seen that rim or drum brakes can be designed for self-energization. While this feature is important in reducing the braking effort required, it also has a disadvantage. When drum brakes are used as vehicle brakes, only a slight change in the coefficient of friction will cause a large change in the pedal force required for braking. A not unusual 30 percent reduction in the coefficient of friction due to a temperature change or moisture, for example, can result in a 50 percent change in the pedal force required to obtain the same braking torque obtainable prior to the change. The disk brake has no self-energization, and hence is not so susceptible to changes in the coefficient of friction.

An Automotive disk brake



Friction Materials

- A brake or friction clutch should have the following lining material characteristics to a degree that is dependent on the severity of service
  - High and reproducible coefficient of friction
  - Imperviousness to environmental conditions, such as moisture
  - The ability to withstand high temperatures, together with good thermal conductivity, as well as high specific heat capacity
  - Good resiliency
  - High resistance to wear, scoring, and galling
  - Compatible with environment
  - Flexibility

Friction Materials

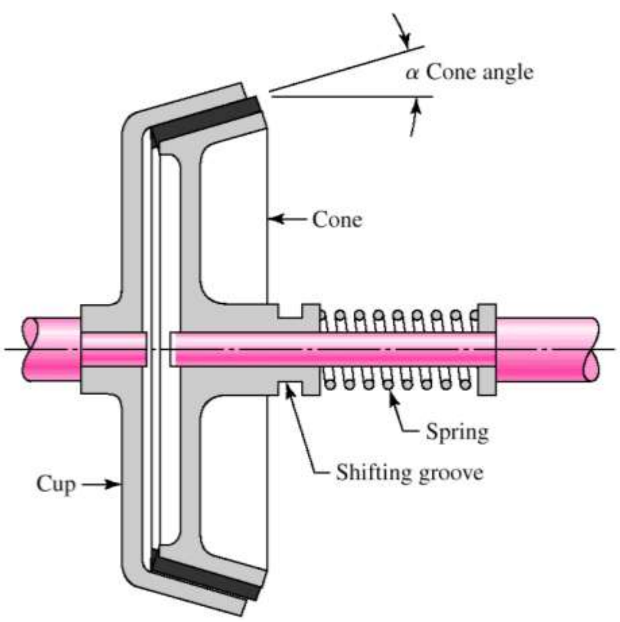
Table 16–3

Area of Friction Material Required for a Given Average Braking Power *Sources:* M. J. Neale, *The Tribology Handbook*, Butterworth, London, 1973; *Friction Materials for Engineers*, Ferodo Ltd., Chapel-en-le-frith, England, 1968.

Duty Cycle	Typical Applications	Ratio of Area to Average Braking Power, $in^2/(Btu/s)$		
		Band and Drum Brakes	Plate Disk Brakes	Caliper Disk Brakes
Infrequent	Emergency brakes	0.85	2.8	0.28
Intermittent	Elevators, cranes, and winches	2.8	7.1	0.70
Heavy-duty	Excavators, presses	5.6–6.9	13.6	1.41

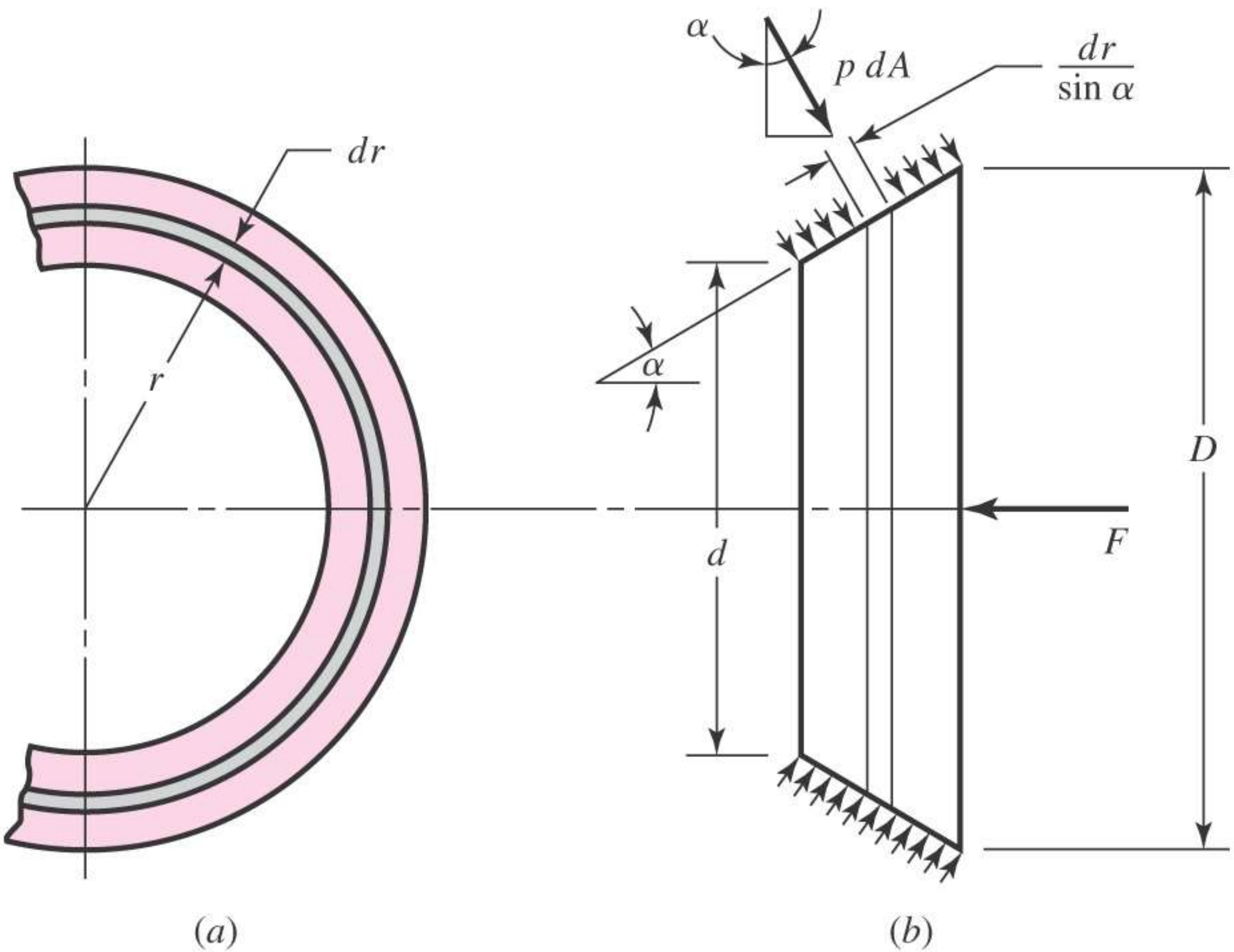
Cone clutches and brakes

The drawing of a *cone clutch* in Fig. 16–21 shows that it consists of a *cup* keyed or splined to one of the shafts, a *cone* which must slide axially on splines or keys on the mating shaft, and a helical *spring* to hold the clutch in engagement. The clutch is disengaged by means of a fork which fits into the shifting groove on the friction cone. The *cone angle* α and the diameter and face width of the cone are the important geometric design parameters. If the cone angle is too small, say, less than about 8°, then the force required to disengage the clutch may be quite large. And the wedging effect lessens rapidly when larger cone angles are used. Depending upon the characteristics of the friction materials, a good compromise can usually be found using cone angles between 10 and 15°.

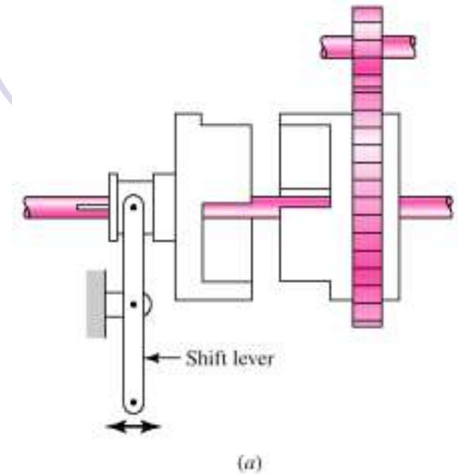


Cross section of a cone clutch.

Cone clutches and brakes

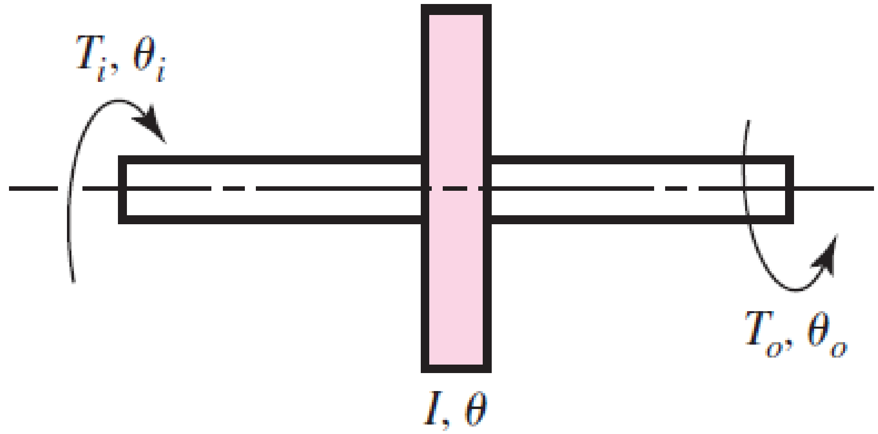


Miscellaneous Clutches and Couplings



- Square-jaw Clutch
  - The do not slip
  - No heat is generated
  - They cannot be engaged at high speeds
  - Sometimes they cannot be engaged when both shafts are at rest
  - Engagement at any speed is accompanied by shock

Flywheels



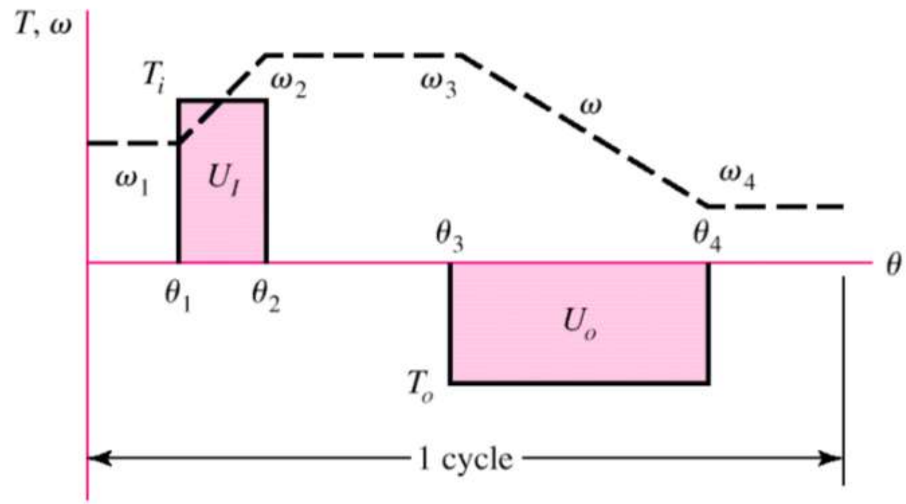
The equation of motion for the flywheel represented in Fig.

$$\sum M = T_i(\theta_i, \dot{\theta}_i) - T_o(\theta_o, \dot{\theta}_o) - I\ddot{\theta} = 0$$

or

$$I\ddot{\theta} = T_i(\theta_i, \omega_i) - T_o(\theta_o, \omega_o)$$

where T_i is considered positive and T_o negative, and where $\dot{\theta}$ and $\ddot{\theta}$ are the first and second time derivatives of θ, respectively. Note that both T_i and T_o may depend for their values on the angular displacements θ_i and θ_o as well as their angular velocities ω_i and ω_o. In many cases the torque characteristic depends upon only one of these. Thus, the torque delivered by an induction motor depends upon the speed of the motor. In fact, motor manufacturers publish charts detailing the torque-speed characteristics of their various motors.



We can also write these relations in terms of kinetic energy. At $\theta = \theta_1$ the flywheel has a velocity of ω_1 rad/s, and so its kinetic energy is

$$E_1 = \frac{1}{2} I \omega_1^2 \tag{e}$$

At $\theta = \theta_2$ the velocity is ω_2, and so

$$E_2 = \frac{1}{2} I \omega_2^2 \tag{f}$$

Thus the change in kinetic energy is

$$E_2 - E_1 = \frac{1}{2} I (\omega_2^2 - \omega_1^2) \tag{16–63}$$