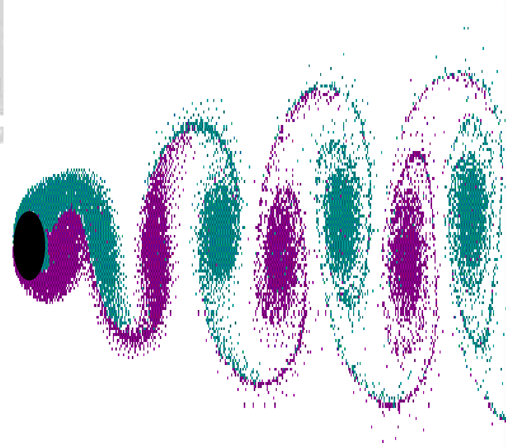
جامعة الإمام عبدالرحمن بن فيصل
IMAM ABDULRAHMAN BIN FAISAL UNIVERSITY

FLUID MECHANICS- ENG 321

Chapter 05_DYNAMICS



2020-2021

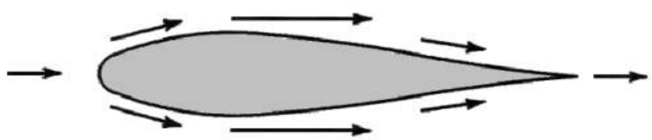
STEADY AND UNSTEADY FLOW

- For a flow that appears the same at all times…

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{0}$$

and likewise for all flow properties.

- We call such a flow **steady**.
- We call time-varying flows **unsteady**.
- Observing flow over a wing from your seat on the plane…

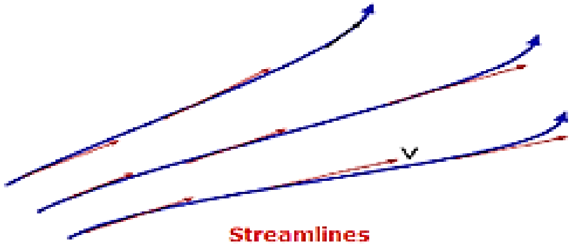


← When the plane is cruising, the flow is steady

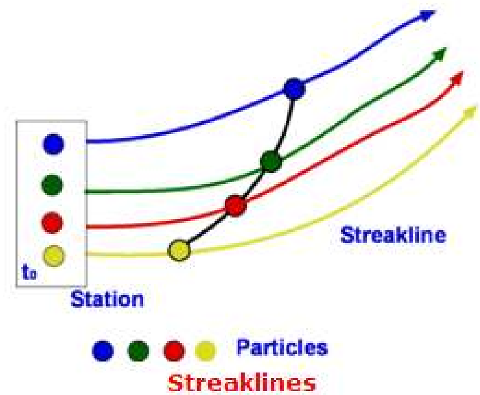
  - Velocity **is the same** at each point over the wing.
  - Velocity **varies** from point to point along the wing.

STREAMLINES, STREAKLINES & PATHLINES

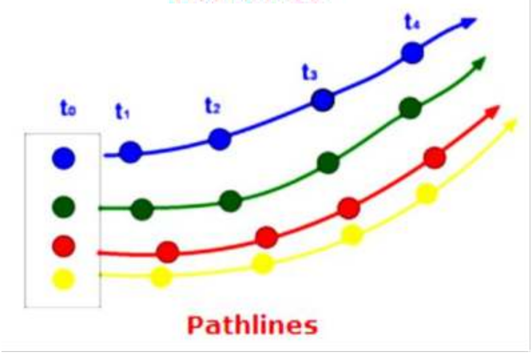
- **Streamline** is one that drawn tangential to the velocity vector at every point in the flow at a given instant and forms a powerful tool in understanding flows.



- **Streakline** concentrates on fluid particles that have gone through a fixed station or point. At some instant of time the position of all these particles are marked and a line is drawn through them. Such a line is called a streakline



- **Pathline** is the line traced by a given particle. This is generated by injecting a dye into the fluid and following its path by photography or other means.



STREAMLINES, STREAKLINES & PATHLINES

- **Streamlines** are contours in a flow field to which the velocity vector is tangent.
- Fluid moves along the streamlines, so they help us visualize a flow field.
- If $d\mathbf{r} = dx\,\mathbf{i} + dy\,\mathbf{j} + dz\,\mathbf{k}$ is a differential vector tangent to the streamline, then its cross product with the velocity vector is zero…

$$\mathbf{u} \times d\mathbf{r} = \mathbf{i}(vdz - wdy) + \mathbf{j}(wdx - udz) + \mathbf{k}(udy - vdx) = \mathbf{0}$$

- Consequently, the differential equation for a streamline is:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

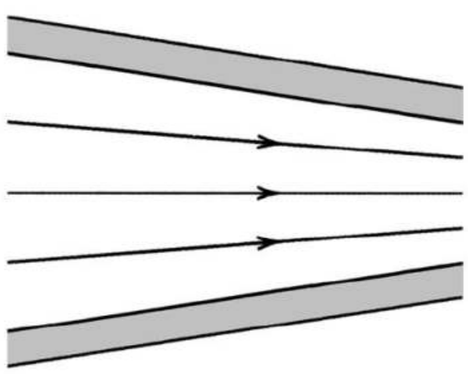
STREAMLINES, STREAKLINES & PATHLINES

- In 2-D, the equation for a streamline is:

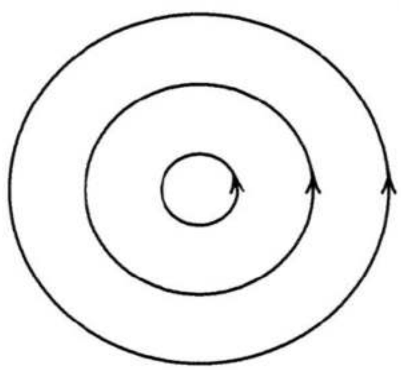
$$\frac{dy}{dx} = \frac{v}{u}$$

which says the **streamline's slope** equals the angle the velocity vector makes to the horizontal.

- Examples of streamlines…



Converging flow



Potential-vortex flow

STREAMLINES, STREAKLINES & PATHLINES

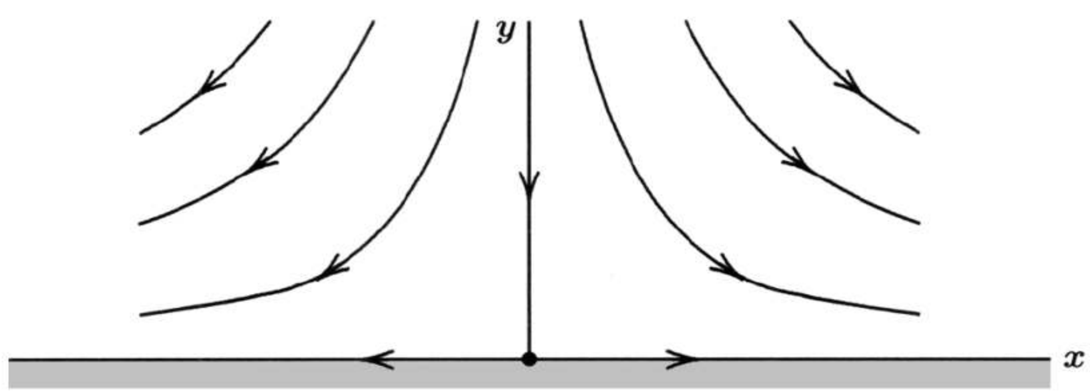
- **For steady flows**, **streamlines** can be observed experimentally by injecting dye, smoke, or other tracer material.

- **For unsteady flows**, injecting tracer material continuously does not display streamlines.
  - Shows the path traveled by all particles that have passed through the injection point…these contours are called **streaklines.**
  - Alternatively, a time-exposure photograph of a marked fluid particle shows the Lagrangian coordinates of the particle…this is called a **pathline.**

- **Streamlines, streaklines and pathlines are identical for steady flows**

STREAMLINES, STREAKLINES & PATHLINES

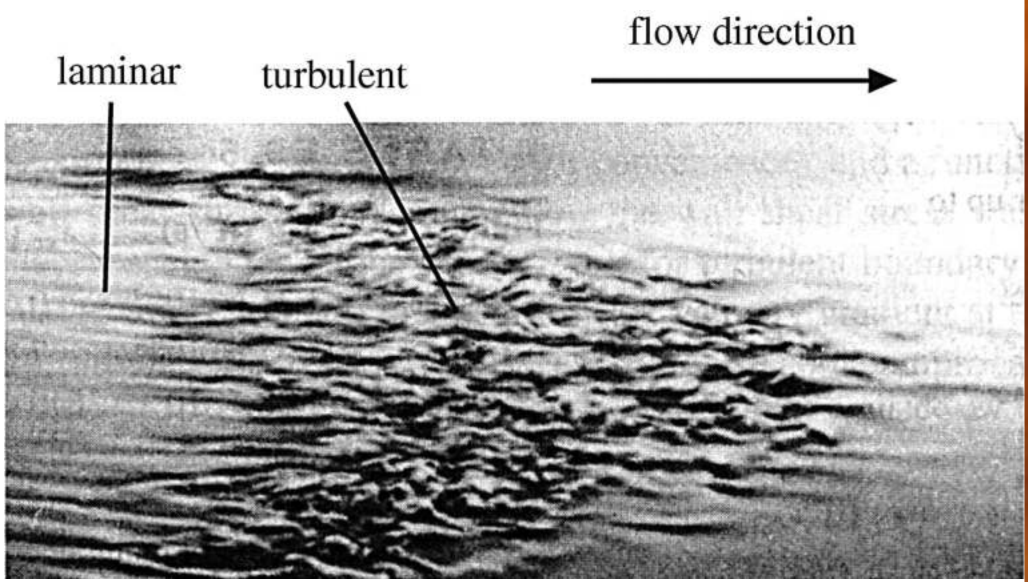
- **Example**: Determine the streamlines for flow near a stagnation point…the velocity is $\mathbf{u} = Ax\,\mathbf{i} - Ay\,\mathbf{j}$, where A is a constant.
  - For 2-D flow, $dy/dx = v/u = (-Ay)/(Ax) = -y/x$
  - Hence, $dx/x + dy/y = 0$
  - Integrating yields $\ln x + \ln y = \ln xy = constant$
  - Therefore, $xy = constant$…a family of hyperbolas.

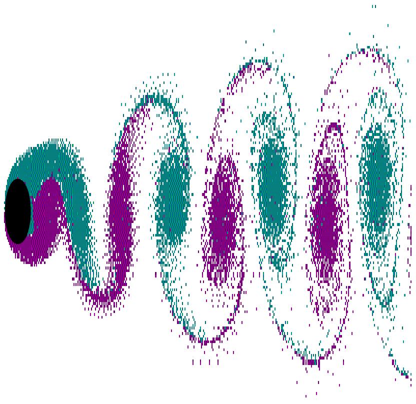


Types of flow

Laminar and Turbulent Flow

- **Laminar flow**: a flow in which the fluid flows with no significant mixing of neighboring fluid particles. The flow may be steady or unsteady.
- **Turbulent flow**: a flow in which the fluid motions vary irregularly so that quantities such as velocity and pressure show a random variation with time and space coordinates.





Reynolds number

- Laminar flows are associated with low velocities and turbulent flows with high velocities. The parameter used to predict the flow regime is the **Reynolds number**.

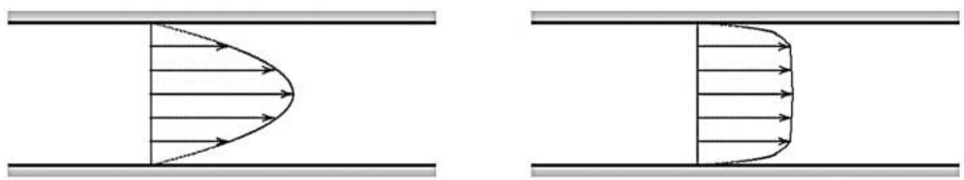
$$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{VL}{\mu / \rho} = \frac{VL}{\nu}$$

where L and V are a characteristic length and velocity, respectively, and ν is the kinematic viscosity. For pipe flow, the characteristic length and velocity are the diameter and the mean velocity, respectively.

- There is a **critical Reynolds number** below which the flow is always laminar. For pipe flow, $\mathbf{Re_{crit} = 2000}$.

Types of flow

- As Reynolds number increases, all flows undergo a transition from laminar to turbulent motion.
- Turbulence is a chaotic phenomenon characterized by rapid fluctuations over a wide range of frequencies in all flow properties.

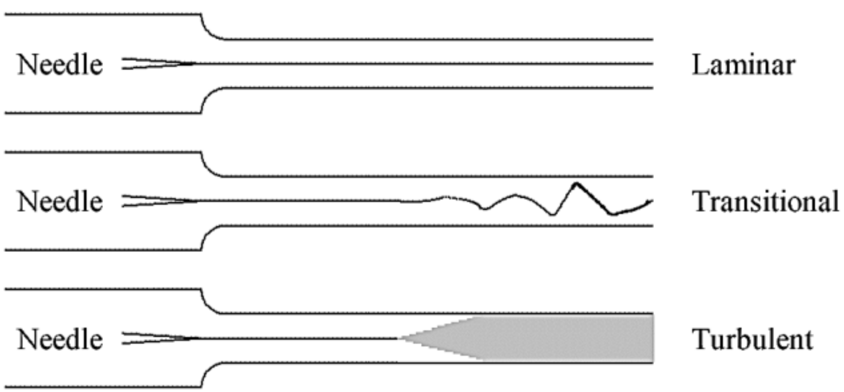


(a) Laminar (b) Turbulent

- Turbulence is present in most fluid flows of engineering interest:
  - Flow past airplanes, automobiles, ships, rockets
  - Flow in pipes, ducts, turbomachines, rivers, atmosphere
  - **Noteworthy example**…blood flow in a human body

Turbulence

- Reynolds (1883) injected dye into fluid flowing in a transparent pipe and observed the following…



- The dye mixes very rapidly when the flow is turbulent…turbulent mixing can be very helpful for:

- Enhancing combustion in a rocket,

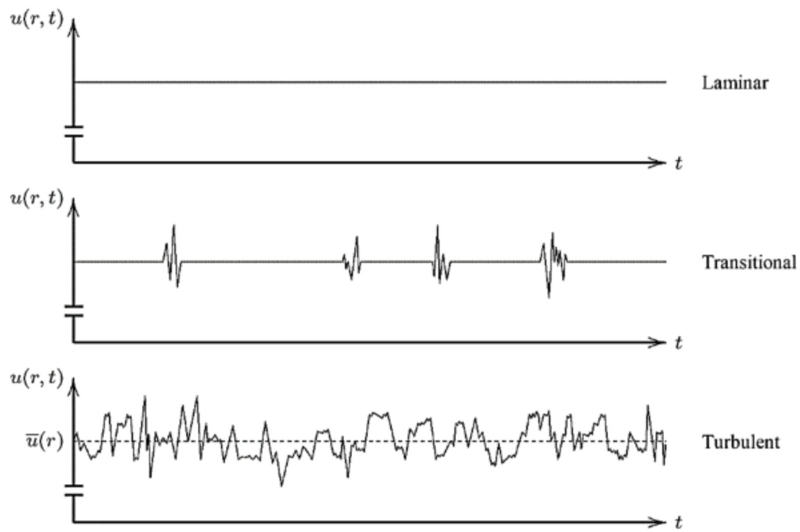
- Improving efficiency of sewage-treatment plants,

- Mixing cream and sugar with coffee…

Turbulence

- Modern devices such as the hot-wire anemometer and the laser-Doppler velocimeter reveal the rapid turbulent fluctuations in flow propert

Intermittent bursts of → turbulence

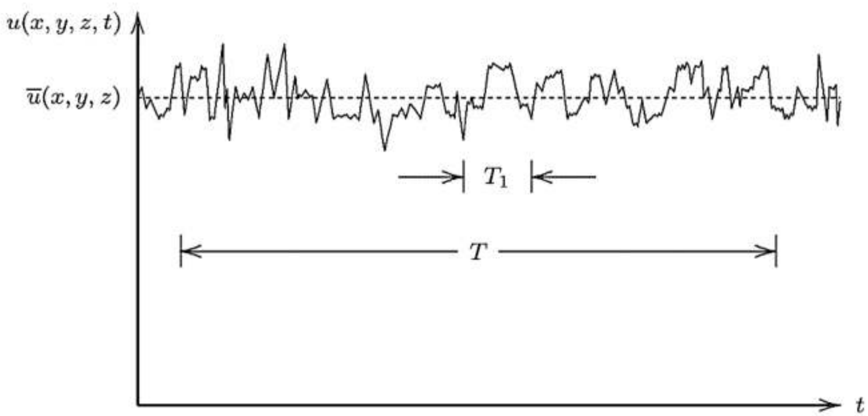


← Mean velocity obtained from time averaging

- Engineering approaches to turbulence focus on statistical averages…design methods depend heavily on empirical formulations.

Reynolds Averaging

- Turbulence consists of random fluctuations in all flow properties…statistical methods are needed.



- Decompose into mean and fluctuating parts:

$$\mathbf{u}(x,y,z,t) = \overline{\mathbf{u}}(x,y,z) + \mathbf{u}'(x,y,z,t)$$

- The mean velocity is calculated as:

$$\overline{\mathbf{u}}(x,y,z) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \mathbf{u}(x,y,z,t)\, dt$$

Acceleration and Euler's Equation

- Basing on Eulerian description:

$$a_x = \frac{Du}{Dt} = \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{convective acceleration}} + \underbrace{\frac{\partial u}{\partial t}}_{\text{local acceleration}}$$

- Euler's equation concerns pressure variation in an accelerating fluid in which there is no velocity gradient within the fluid.

$$\underset{-\left(\substack{\text{gradient of}\\ \text{piezometric}\\ \text{pressure}\\ \text{field}}\right)}{-\nabla p_z} = \underset{\left(\frac{\text{mass}}{\text{volume}}\right)\left(\substack{\text{acceleration}\\ \text{of particle}}\right)}{\rho \mathbf{a}}$$

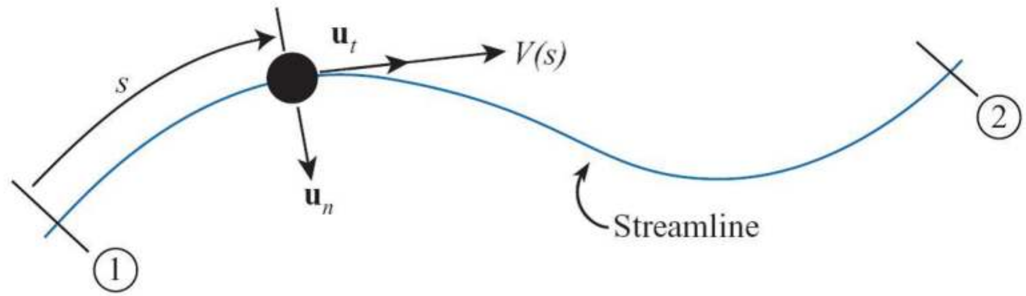
- Euler's equation can be applied in any direction in the fluid.

- Note that when there is no acceleration, the Euler's equation reduces to the hydrostatic pressure variation in a fluid ($p + g\ z =$ const.). In other words, along a direction in which there is no acceleration the pressure distribution is hydrostatic. For example, pressure variation normal to streamlines is hydrostatic.

Bernoulli Equation

The Bernoulli Equation along a Pathline/Streamline

- The Bernoulli equation is obtained from the integration of Euler's equation **along a streamline**. Assuming **steady incompressible inviscid flow**, the velocity of a particle depends on position only, i.e., $V = V(s)$.



Euler's Equation: $-\dfrac{\partial}{\partial s}(p+\gamma z) = \rho a_t$

$$\Rightarrow a_t = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} = V\frac{\partial V}{\partial s}$$

$$\Rightarrow -\frac{d}{ds}(p+\gamma z) = \rho V\frac{dV}{ds} = \rho\frac{d}{ds}\left(\frac{V^2}{2}\right) = \frac{d}{ds}\left(\rho\frac{V^2}{2}\right)$$

$$\Rightarrow \frac{d}{ds}\left(p+\gamma z+\rho\frac{V^2}{2}\right) = 0 \Rightarrow \boxed{p+\gamma z+\rho\frac{V^2}{2} = \text{const}}$$

Bernoulli Equation

The Bernoulli Equation along a Streamline

- The Bernoulli equation is stated in any of the following forms:

Pressure form: $$\underbrace{p+\gamma z}_{\text{piezometric pressure}} + \underbrace{\frac{1}{2}\rho V^2}_{\text{kinetic pressure}} = \text{constant along a streamline}$$

Head form: $$\frac{p}{\gamma}+z + \frac{V^2}{2g} = \text{constant along a streamline}$$

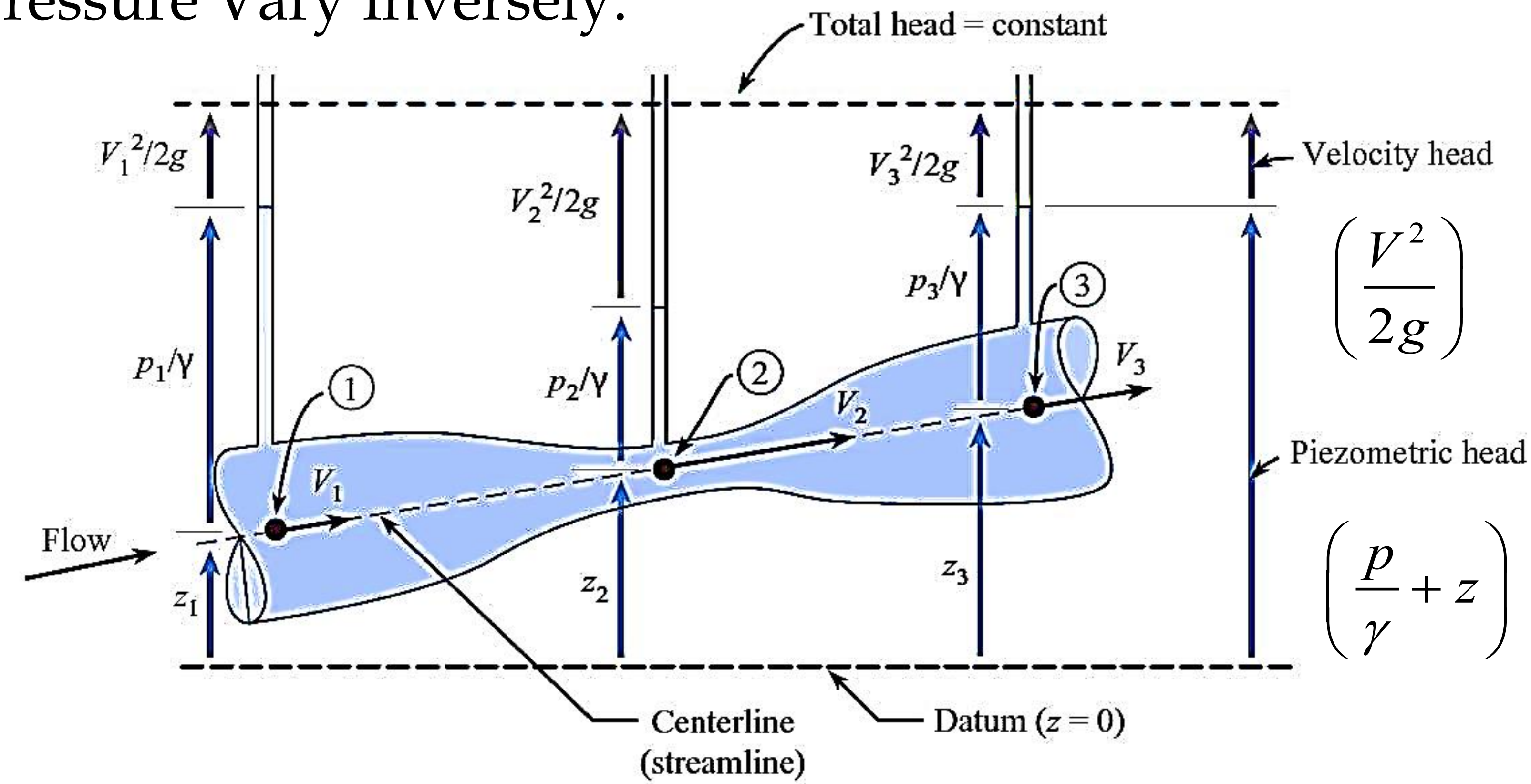
piezometric head velocity head

Note: The kinetic pressure ($rV^2/2$) is also called **dynamic pressure**.

- **Head:** a term used to characterize the balance of work and energy in a flowing fluid.
- **Velocity Head**: characterizes the kinetic energy in a flowing fluid
- **Elevation Head**: characterizes the gravitational potential energy of a fluid
- **Pressure Head**: relates work done by pressure forces.

Bernoulli Equation

- Physical Interpretation 01: Energy is Conserved
- Physical Interpretation 02: Velocity & Piezometric Pressure Vary Inversely.



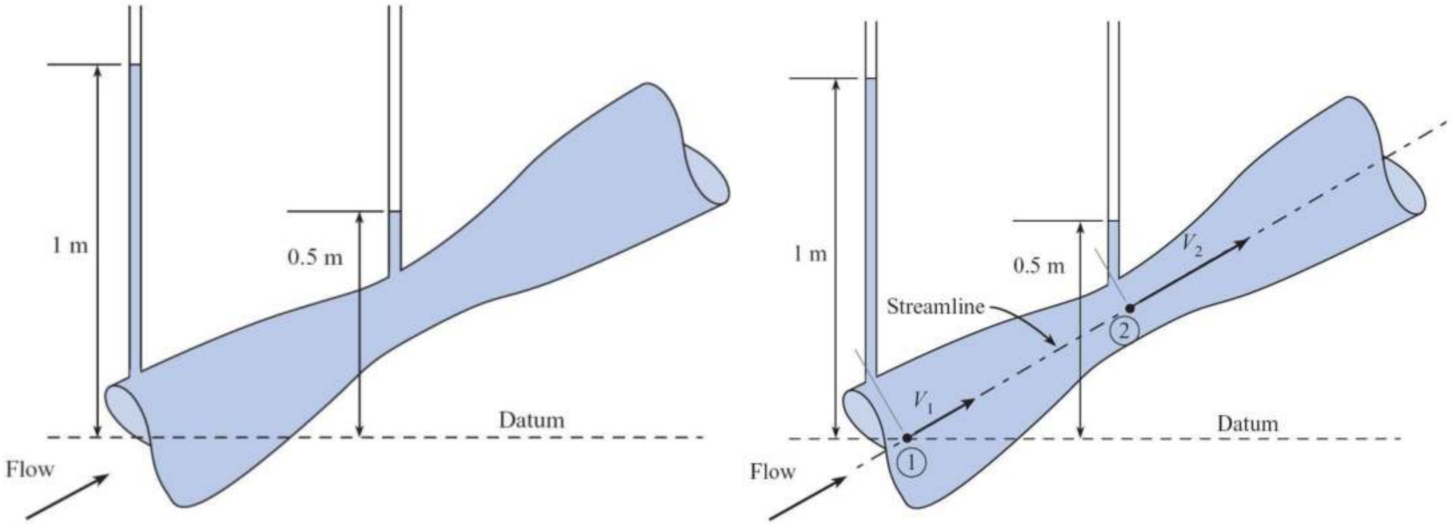
$$\left(\frac{V^2}{2g}\right)$$

$$\left(\frac{p}{\gamma} + z\right)$$

Bernoulli Equation

- **Example:**

We assume that: $V_1 = \frac{V_2}{2}$



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} \Rightarrow \boxed{h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g}}$$

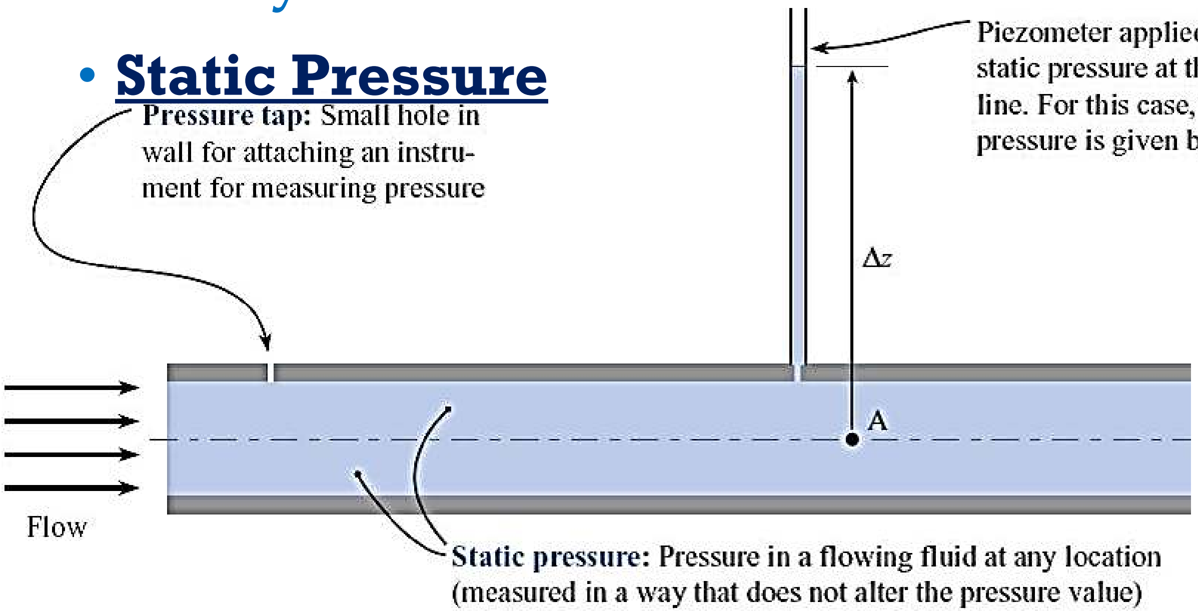
$$\therefore (1.0 \text{ m}) + \frac{V_1^2}{2g} = (0.5 \text{ m}) + \frac{V_2^2}{2g}, \quad V_1 = 0.5V_2$$

$$\Rightarrow (1.0 \text{ m}) + \frac{(0.5V_2)^2}{2g} = (0.5 \text{ m}) + \frac{V_2^2}{2g} \Rightarrow \frac{0.75V_2^2}{2g} = (0.5 \text{ m})$$

Application of Bernoulli Equation

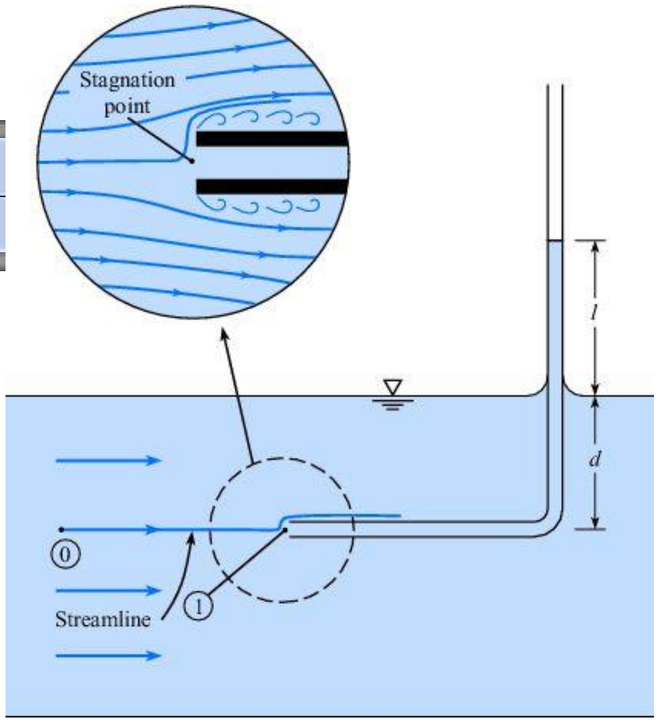
Velocity and Pressure Measurement

- **Static Pressure**



- **Stagnation Tube**

  Also known as **total head tube** is an open-ended tube directed upstream in a flow as shown.



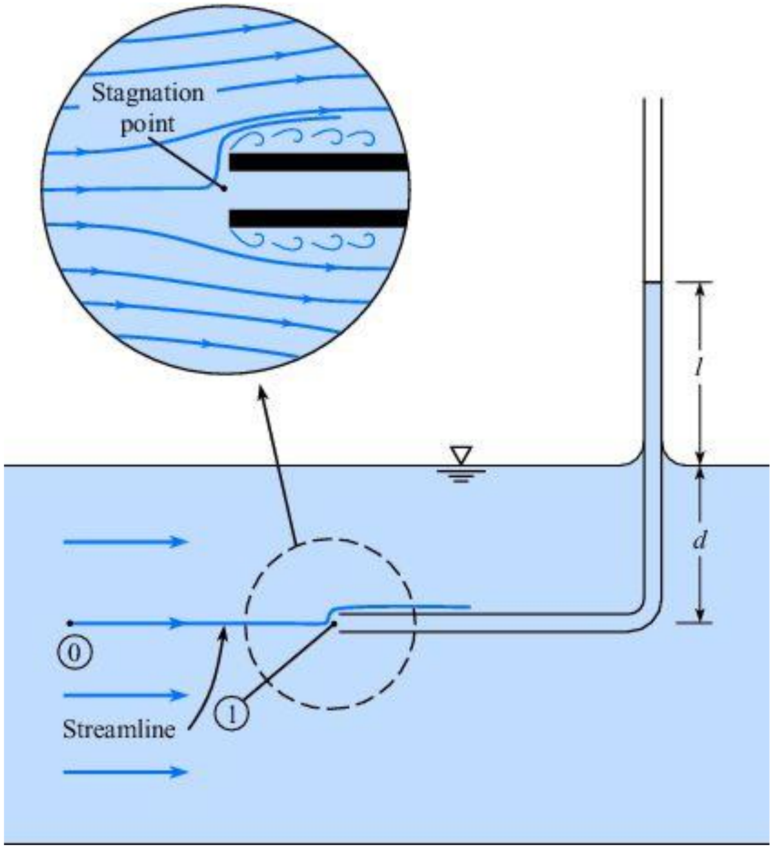
Application of Bernoulli Equation

- **Stagnation Tube**

Applying Bernoulli equation:

$$p_1 + \rho \frac{V_1^2}{2} = p_0 + \rho \frac{V_0^2}{2}; \quad V_1 = 0$$

$$\Rightarrow \boxed{V_0 = \sqrt{\frac{2(p_1 - p_0)}{\rho}}}$$



Applying the hydrostatic equation:

$$p_0 = \gamma d; \quad p_1 = \gamma(l + d)$$

$$\therefore V_0 = \sqrt{\frac{2(\gamma l)}{\rho}} \Rightarrow \boxed{V_0 = \sqrt{2gl}}$$

Note: The stagnation tube is not suitable for velocity measurement in a pressurized pipe

Application of Bernoulli Equation

❑ Pitot Tube

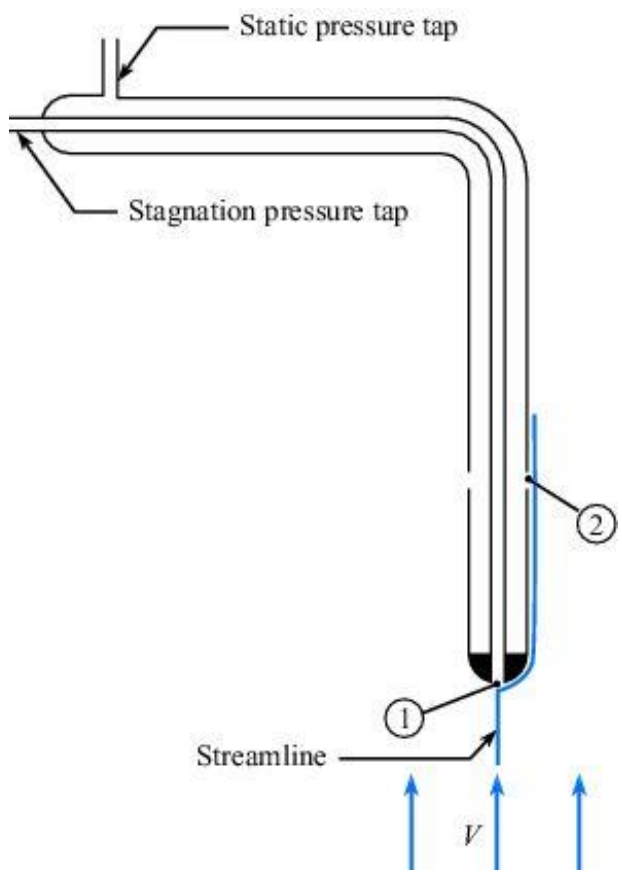
The pitot tube is used to measure the local flow velocity at a given point in the flow stream and not the average flow velocity in the pipe.

The Pitot tube has a pressure tap at the upstream end of the tube for sensing the stagnation pressure. It also has ports for sensing static pressure in the fluid.

Applying the Bernoulli equation gives:

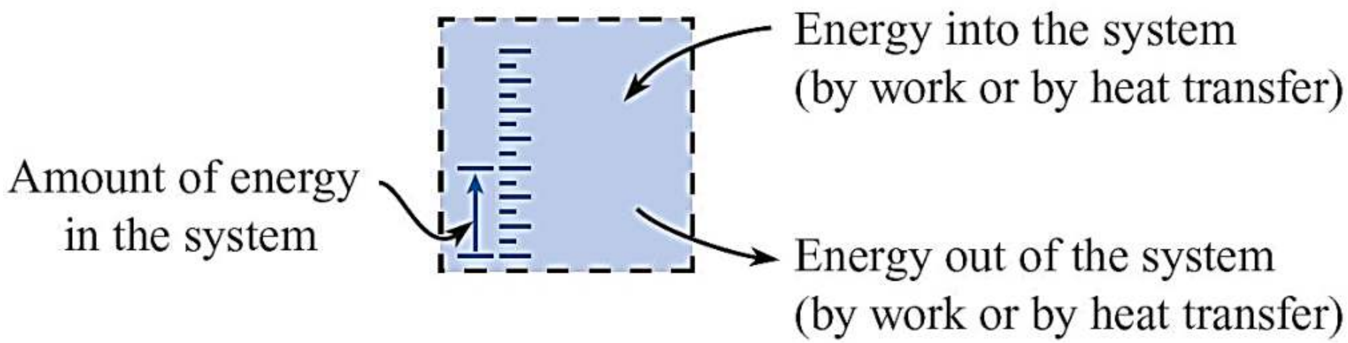
$$p_1 + \gamma z_1 + \rho \frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho \frac{V_2^2}{2}; \quad V_1 = 0, V_2 = V$$

$$\Rightarrow \boxed{V = \sqrt{\frac{2}{\rho}(p_{z,1} - p_{z,2})}}$$



Modified-Bernoulli Equation

- **Joule's Theory of Energy Conservation:** The energy (property) of a **closed system** can be changed in only two ways:
  - **Work**: by work interactions at the boundary.
  - **Heat transfer**: by heat transfer (interaction) across the boundary.



- **First Law of Thermodynamics:** The work & energy balance proposed by Joule:

$$\Delta \mathbf{E} = \mathbf{Q} - \mathbf{W}$$

- To modify the above equation so that it applies at an instant in time, take the derivative to give:

$$\frac{dE}{dt} = \dot{Q} - \dot{W}$$

Modified-Bernoulli Equation

Simplified forms of the energy principle

- For **non-uniform velocity distribution** at the inlet and outlet ports, **the kinetic energy correction factor** ***α*** is introduced to account for the velocity distribution. Energy equation is then written (after dividing through by $\dot{m}$) as:

$$\frac{1}{\dot{m}}\left(\dot{Q}-\dot{W}_{\text{shaft}}\right)+\left(\frac{p_1}{\rho}+gz_1+u_1+\alpha_1\frac{\bar{V}_1^{\,2}}{2}\right)=\left(\frac{p_2}{\rho}+gz_2+u_2+\alpha_2\frac{\bar{V}_2^{\,2}}{2}\right)$$

where $\bar{V}_1$ and $\bar{V}_2$ are the mean velocities at the inlet and outlet ports.

The kinetic energy correction factor, α, is defined by

$$\alpha=\frac{1}{A}\int_A\left(\frac{V}{\bar{V}}\right)^3 dA$$

- Thus, $\alpha = 1$ when the velocity is uniform across the section. For most cases of turbulent flow, $\boldsymbol{\alpha} \approx 1.05$ and therefore it is common practice in engineering applications to let $\boldsymbol{\alpha} = 1$.

Modified-Bernoulli Equation

Simplified forms of the energy principle

- The **shaft-work** term in energy equation is usually the result of a **turbine** or **pump** in the flow system. It is convenient to represent the shaft-work term as

$$\dot{W}_s = \dot{W}_t - \dot{W}_p$$

where $\dot{W}_t$ and $\dot{W}_p$ are the magnitudes of power delivered by a turbine or supplied to a pump, respectively.

- It is often conventional to express the term associated with a pump and a turbine as the **pump head** (h_p) and the **turbine head** (h_t), respectively, defined as

$$h_p = \frac{\text{work/time done by pump on flow}}{\text{weight/time of flowing fluid}} = \frac{\dot{W}_p}{\dot{m}\,g} \mapsto \text{head added by pump}$$

$$h_t = \frac{\text{work/time done by flow on turbine}}{\text{weight/time of flowing fluid}} = \frac{\dot{W}_t}{\dot{m}\,g} \mapsto \text{head removed by turbine}$$

Modified-Bernoulli Equation

Simplified forms of the energy principle

- Introducing the **pump head** (h_p) and the **turbine head** (h_t) terms in the energy equation yields after some re-arrangement:

$$\frac{p_1}{\gamma}+\alpha_1\frac{\bar{V}_1^{\,2}}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\alpha_2\frac{\bar{V}_2^{\,2}}{2g}+z_2+h_t+\left[\frac{1}{g}\left(u_2-u_1\right)-\frac{\dot{Q}}{\dot{m}g}\right]$$

- The last term enclosed in square brackets in the above equation represents energy loss due to viscous action between fluid particles and due to heat transfer. It is simply referred to as the **head loss** and is represented by (h_L). It is **always positive** because of second law of thermodynamics. Thus, the **final form of the energy equation** is:

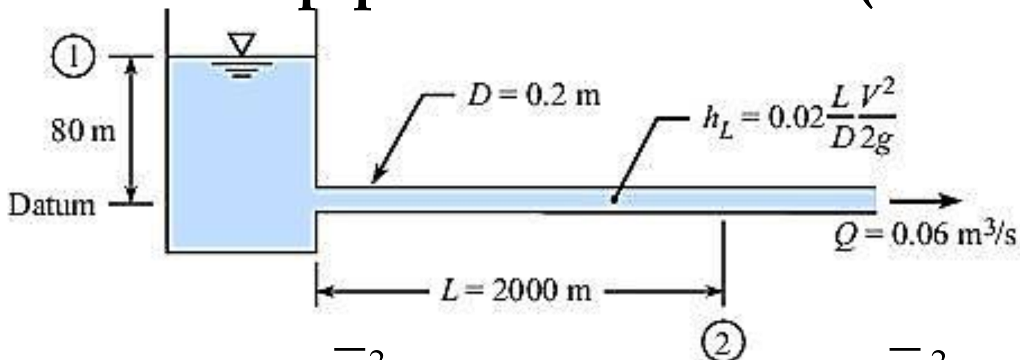
$$\boxed{\frac{p_1}{\gamma}+\alpha_1\frac{\bar{V}_1^{\,2}}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\alpha_2\frac{\bar{V}_2^{\,2}}{2g}+z_2+h_t+h_L}$$

Modified-Bernoulli Equation

Example :

- To find the pressure in pipe at location 2 (2000 m from reservoir)

Solution:



The energy equation: $\frac{p_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + h_t + h_L$

$$z_1 = 100 \text{ m},\ z_2 = 20 \text{ m},\ \alpha_2 = 1,\ Q = 0.06 \text{ m}^3/\text{s}$$

Data:

$$p_1 = 0 \text{ gage},\ V_1 \approx 0 \text{ reservoir level nearly constant},\ h_p = h_t = 0$$

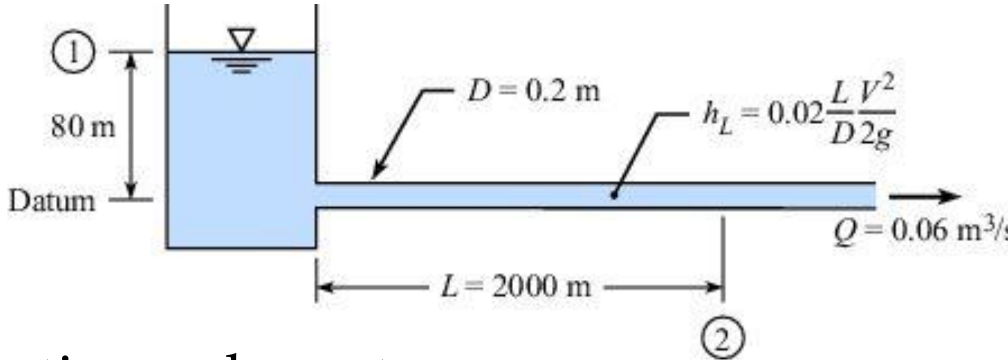
Term-by-term analysis:

Modified-Bernoulli Equation

Example :

- To find the pressure in pipe at location 2 (2000 m from reservoir)

Solution:



The energy equation reduces to:

$$z_1 - z_2 = \frac{p_2}{\gamma} + \frac{V_2^{\,2}}{2g} + h_L \Rightarrow \frac{p_2}{\gamma} = \left(z_1 - z_2 - \frac{V_2^{\,2}}{2g} - h_L \right)$$

where: $\bar{V}_2 = V_2 = \dfrac{Q}{A} = 1.910 \text{ m/s}, \; h_L = \dfrac{0.02(L/D)V_2^{\,2}}{2g} = 37.2 \text{ m}$

Thus: $$p_2 = \gamma\left(z_1 - z_2 - \frac{V_2^{\,2}}{2g} - h_L \right) = 418 \text{ kPa}$$