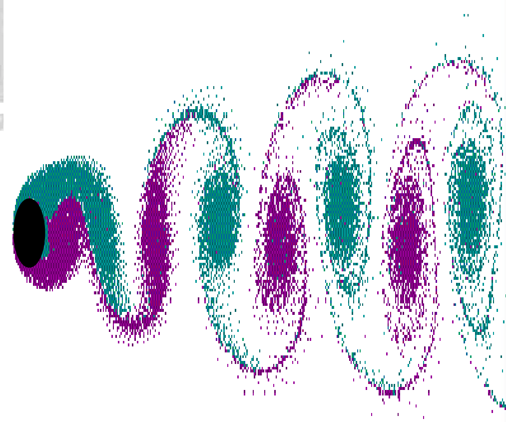
جامعة الإمام عبدالرحمن بن فيصل
IMAM ABDULRAHMAN BIN FAISAL UNIVERSITY

FLUID MECHANICS- ENG 321

Chapter 06_FLOWS IN CONDUITS


2020-2021

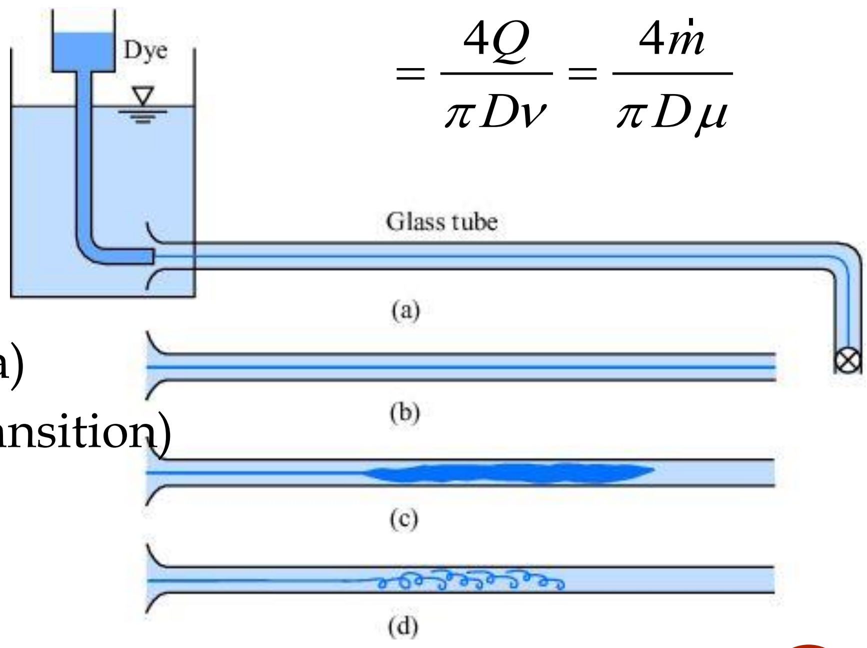
CONDUIT & FLOW CLASSIFICATION

- A conduit is any pipe, tube, or duct that is completely filled with a flowing fluid, e.g., pipeline transporting liquefied natural gas, a micro-channel transporting hydrogen in a fuel cell, a duct transporting air for heating of a building.

- Flow classification:

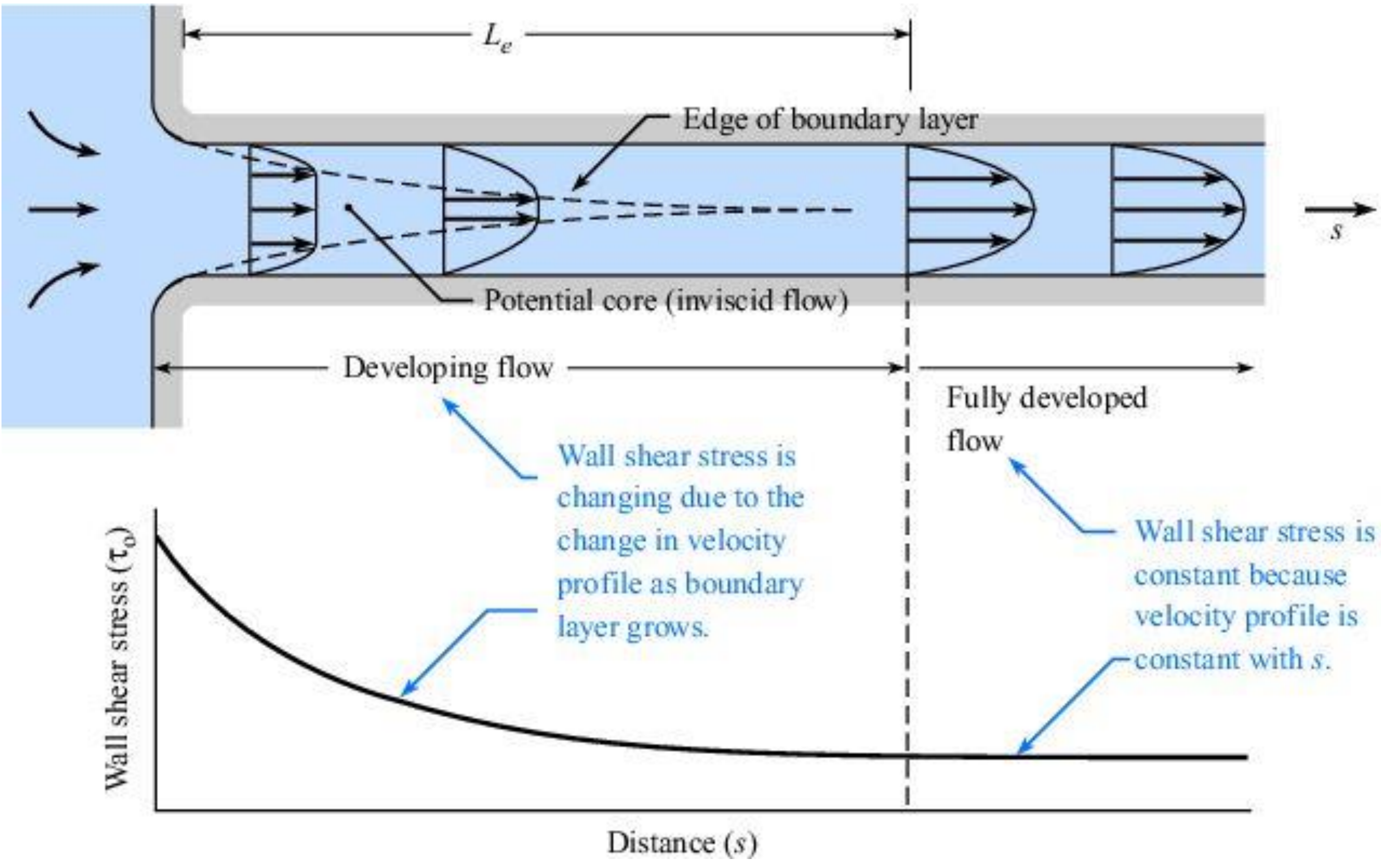
$$\mathrm{Re}_D = \frac{\bar{V}D}{\nu} = \frac{\rho \bar{V} D}{\mu} = \frac{4Q}{\pi D \nu} = \frac{4\dot{m}}{\pi D \mu}$$

- Reynolds' experiment:
  - (a) Apparatus
  - (b) Laminar flow of dye in tube
    $\mathrm{Re}_D \leq 2000$
  - (c) Turbulent flow of dye in tube
    $\mathrm{Re}_D \geq 3000$ (use $\mathrm{Re}_D = 2300$ as criteria)
    $2000 \leq \mathrm{Re}_D \leq 3000$ (unpredictable/transition)
  - (d) Eddies in turbulent flow



DEVELOPING VS. FULLY DEVELOPED FLOW

- **Changing vs. constant velocity profile & wall shear stress**



- **Entry or entrance length:** $L_e / D = 0.05\,\mathrm{Re}_D$ (laminar), $= 50$ (turbulent)

PIPE HEAD LOSS

- The **Combined (Total) Head Loss** is:

  Combined (Total) head loss = Pipe head loss + Component head loss

- **Component head loss** is associated with flow through devices susch as valves, bends and tees. It is sometimes called minor head loss

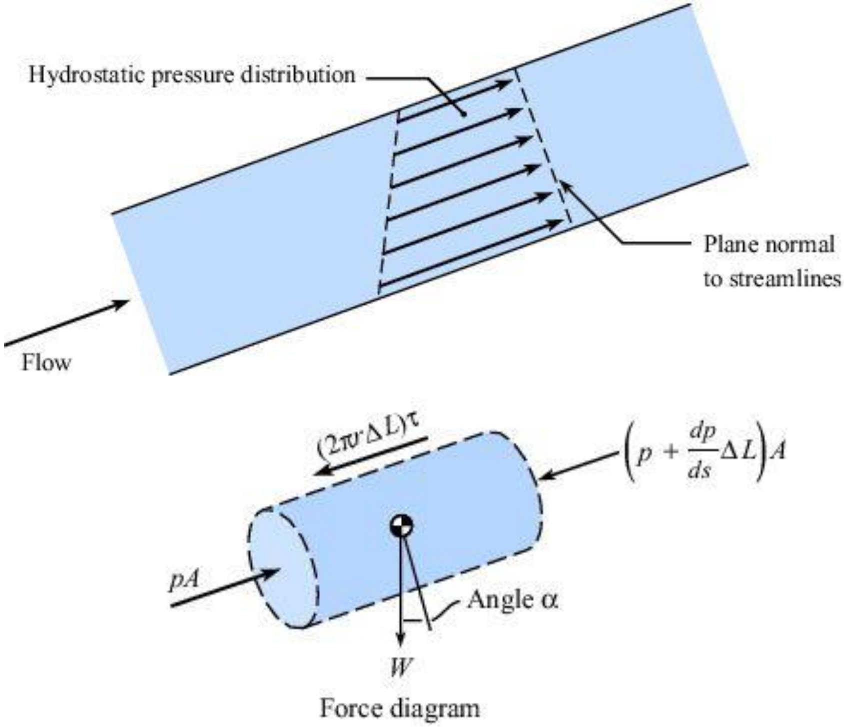
- **Pipe head loss** is associated with fully developed flow in conduits, and is caused by shear stresses that act on the flowing fluid. It is sometimes called major head loss. It is predicted with the **<u>Darcy-Weishbach equation</u>**:

$$h_L = \begin{pmatrix} \text{head loss} \\ \text{in a pipe} \end{pmatrix} \Rightarrow \boxed{h_f = f \frac{L}{D} \frac{V^2}{2g}}$$

$$\text{where :} \quad f = \frac{4\tau_0}{(\rho V^2 / 2)} = \frac{\text{shear stress acting at the wall}}{\text{kinetic or dynamic pressure}}$$

SHEAR STRESS DISTRIBUTION ACROSS A PIPE

- Consider flow in a pipe as shown. It is assumed that the flow is uniform (streamlines are straight and parallel).
- The shear stress distribution can be obtained by applying the momentum equation to the cylindrical control volume shown in figure. (see sec 10.4 for derivation)



- The shear stress distribution is

$$\tau = \frac{r}{2}\left[-\frac{d}{ds}(p+\gamma z)\right] \qquad (1)$$

- Since ***d/ds*(*p+gz*)** is negative and constant across the section for uniform flow, it follows that ***-d/ds*(*p+gz*)** will be positive and constant across the pipe section.

LAMINAR FLOW IN A ROUND PIPE

- The velocity distribution across the pipe can be obtained by substituting for t in equation (1) after a change of variable ($y = r_0 - r$).

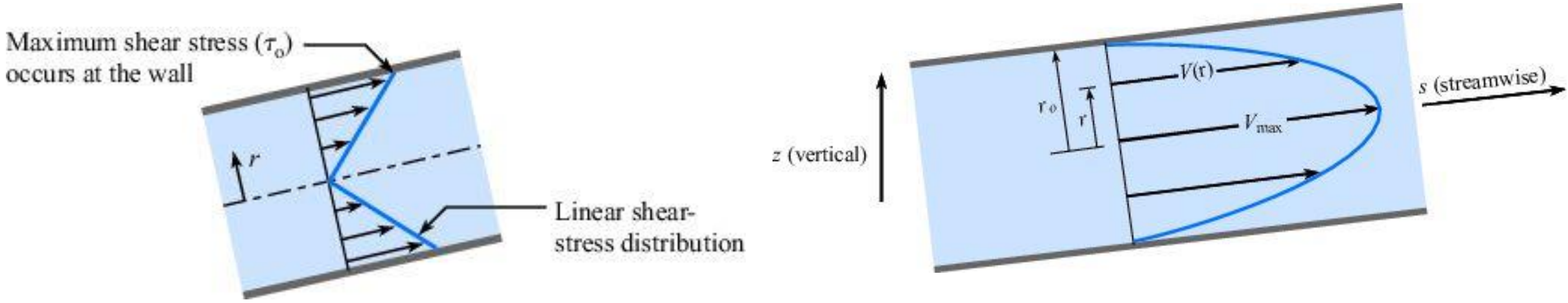
$$\tau = \mu \frac{dV}{dy} = \mu \left(\frac{dV}{dr} \right) \left(\frac{dr}{dy} \right) = -\mu \left(\frac{dV}{dr} \right)$$

- After substituting in equation (1), separating the variables and integrating across the section, we obtain the velocity distribution as:

$$V = \frac{r_o^2 - r^2}{4\mu} \left[-\frac{d}{ds} (p + \gamma z) \right] \qquad (2)$$

LAMINAR FLOW IN A ROUND PIPE…

- Equation (2) indicates that the velocity distribution for laminar flow in a pipe is parabolic across the section with the maximum velocity at the center of the pipe. The shear stress and the velocity variations in the pipe are shown in figs. 10.10 & 10.11.



- **Laminar flow in a round pipe** is known as ***<u>Hagen-Poiseuille</u>*** **<u>flow</u>**.

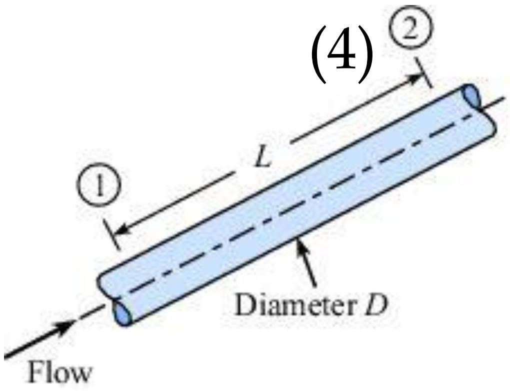
- The rate of flow in the conduit is:

$$Q = \int_A V\, dA = \frac{\pi r_o^4}{8\mu}\left[-\frac{d}{ds}(p + \gamma z)\right] \qquad (3)$$

LAMINAR FLOW IN A ROUND PIPE…

- The mean velocity is obtained as

$$\bar{V} = \frac{Q}{A} = \frac{r_o^2}{8\mu}\left[-\frac{d}{ds}(p+\gamma z)\right] \qquad (4)$$



- Comparing equations (2) and (4) reveals that

$$\bar{V} = V_{max}/2$$

- Integrating equation (4) along the pipe between sections 1 and 2, where $s_2 - s_1 = L$, we obtain

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + h_f \qquad (5)$$

LAMINAR FLOW IN A ROUND PIPE…

Where:

$$h_f = \frac{32\mu LV}{\gamma D^2} \qquad (6)$$

- It can be seen that equation (5) is the same as energy equation for uniform flow in a constant-diameter pipe.
- Let $t = t_o$ at $r = r_o$. The pressure drop D*p* over a length *L* of pipe is obtained from equation (1) as

$$\Delta(p + \gamma z) = \gamma \Delta h = \frac{2\tau_o L}{r_o} \qquad (7)$$

- Also, from equation (5), pressure drop D*p* over a length *L* of pipe is

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + \frac{32\mu LV}{\gamma D^2} \qquad (8)$$

LAMINAR FLOW IN A ROUND PIPE…

- To derive an equation for the friction factor, combine equation (6) with the **Darcy-Weishbach equation**:

$$h_f = \frac{32\mu LV}{\gamma D^2} = f\frac{L}{D}\frac{V^2}{2g}$$

$$\Rightarrow f = \left(\frac{32\mu LV}{\gamma D^2}\right)\left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right) = \frac{64\mu}{\rho DV} = \frac{64}{\mathrm{Re}_D} \qquad (9)$$

- Equation (9) shows that the friction factor for laminar, steady, fully developed flow of a Newtonian fluid depends only on Reynolds number.

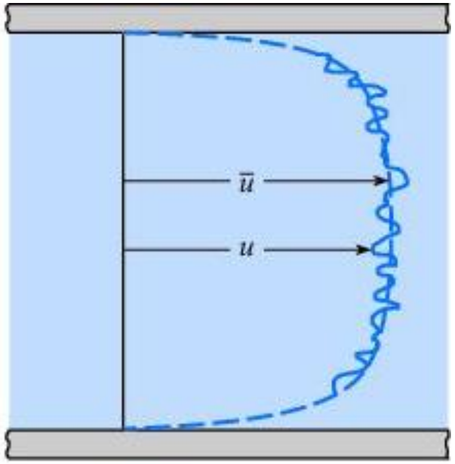
TURBULENT FLOW IN A ROUND PIPE

- Most pipe flows encountered in practical applications are turbulent flows.
- The mixing action of turbulence causes the velocities at a given point in a flow to fluctuate with time (see fig. 4.13 for illustration).
- In turbulent flow, the velocities (and other flow quantities) are considered to be a sum of the mean and the fluctuating components, i.e.

$$u = \overline{u} + u'$$

$$v = \overline{v} + v'$$

$$w = \overline{w} + w'$$



TURBULENT FLOW & MOODY DIAGRAM

- Similar to the laminar pipe flow, the head loss due to viscous effects in the pipe is given by

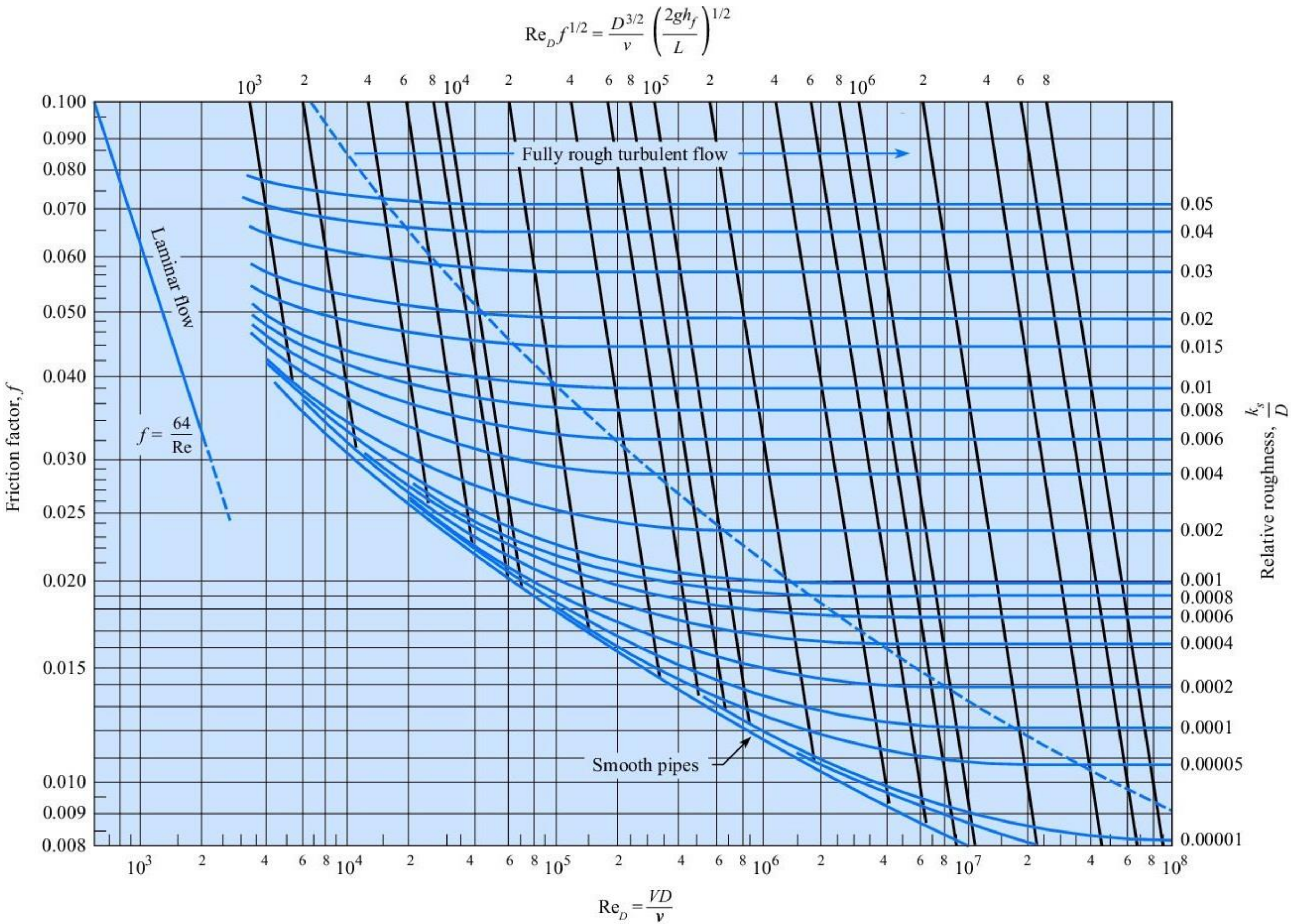
$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (11)$$

where f is the friction factor.

- The **Moody Chart** is used to determine f

- For turbulent pipe flow, f is a function of the **Re** and the pipe relative roughness, k_s/D, where k_s is the average roughness of the pipe and D is the pipe diameter.

- The values of the ***equivalent sand grain roughness*** for various kinds of pipes are presented in **Table 10.4**. These values are used to calculate the relative roughness, which, in turn, is used to find the friction factor.



$Re_D f^{1/2} = \frac{D^{3/2}}{\nu}\left(\frac{2gh_f}{L}\right)^{1/2}$
Fully rough turbulent flow
Laminar flow
$f = \frac{64}{Re}$
Smooth pipes
Friction factor, f
Relative roughness, $\frac{k_s}{D}$
$Re_D = \frac{VD}{\nu}$

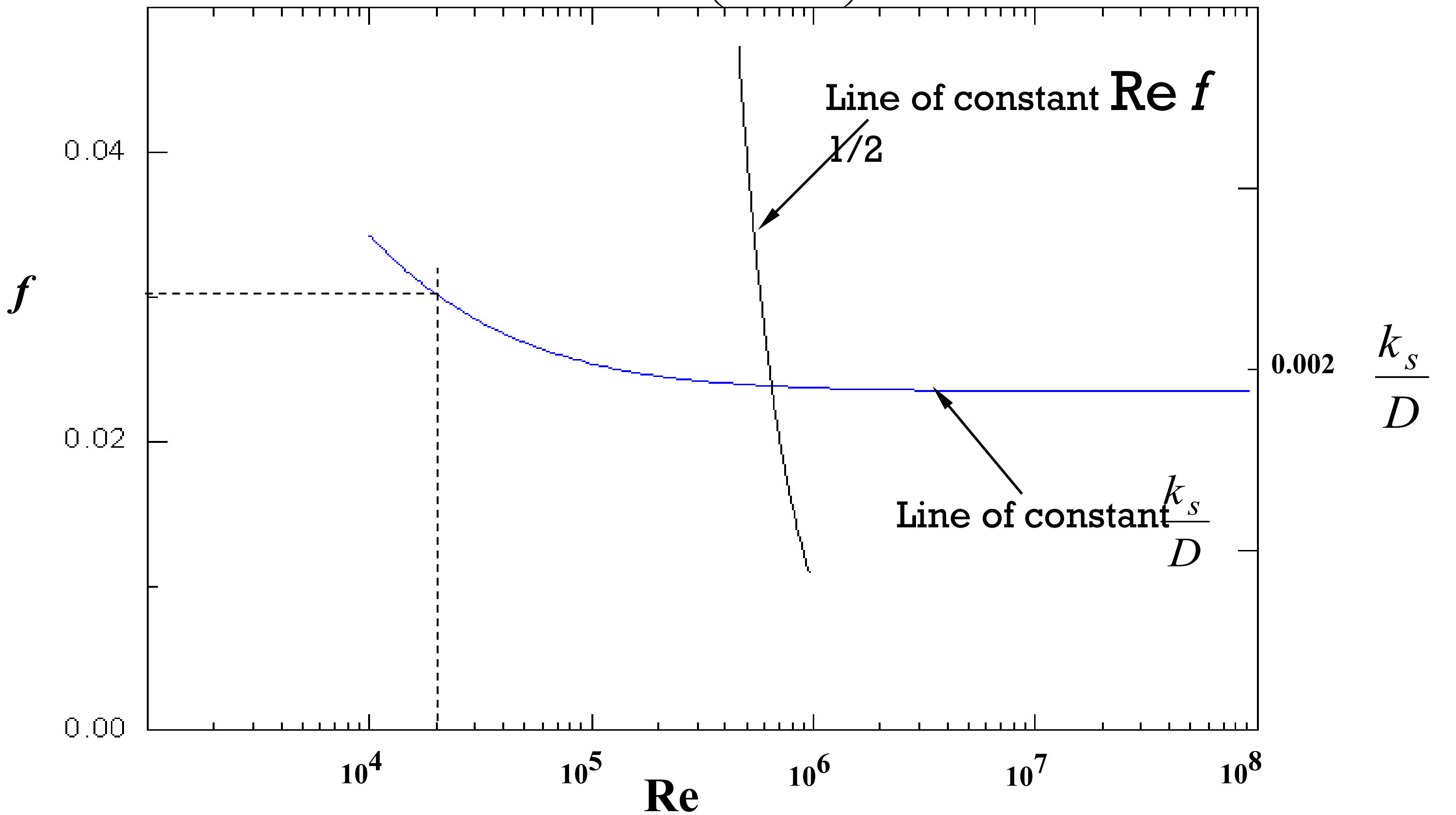
TURBULENT FLOW & MOODY DIAGRAM…

- The friction factor can be obtained with the use of the **Moody diagram** shown in fig. 10.14.
- The Moody diagram is used as follows:
  - **When Re and k_s/D are known**, pick the correct curve of constant k_s/D from the right vertical axis, follow the curve until the correct value of Re is reached, then move horizontally to the left vertical scale to read the value of f.
  - **When Re and h_f are known**, pick the correct curve of constant $\mathbf{Re}f^{1/2}$ from the top horizontal axis, follow the curve until the correct value of k_s/D is reached, then move horizontally to the left vertical scale to read the value of f. This is illustrated in the next slide.

TURBULENT FLOW & MOODY DIAGRAM…

- Example illustration for k_s/D = 0.002 and Re = 20000, giving f = 0.03

$$\mathrm{Re}\, f^{1/2} = \frac{D^{3/2}}{\nu}\left(\frac{2 g h_f}{L}\right)^{1/2}$$



TURBULENT FLOW & MOODY DIAGRAM…

- The Moody diagram is used as follows:

  When h_f and k_s/D are known (but V is not known), calculate **Re$f^{1/2}$**, using

$$\mathrm{Re}\, f^{1/2} = \frac{D^{3/2}}{\nu}\left(\frac{2 g h_f}{L}\right)^{1/2} \qquad (12)$$

  pick the correct curve of constant **Re$f^{1/2}$** from the top horizontal axis, follow the curve until the correct value of k_s/D is reached, then move horizontally to the left vertical scale to read the value of f.

- See examples 10.3 to 10.5 for both approaches.

TURBULENT FLOW & MOODY DIAGRAM…

- When using computer to carry out pipe flow calculations, the following formula can be used to compute the friction factor:

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7\,D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \qquad (13)$$

- The above equation is known as the Colebrook-White correlation and was developed by Swamee and Jain.
- In some problems, the friction factor *f* cannot be determined directly. Solution involves using a guessed value of *f* initially and obtaining the actual value through iteration.
- See examples 10.6 and 10.7 for illustrations.

MINOR HEAD LOSS

- In addition to frictional losses, additional pressure losses occur in pipe systems. These additional pressure losses, referred to as minor losses, are due to valves, elbows, enlargements, contractions, inlets, outlets, bends, branches, and other fittings.

- A minor loss is expressed in terms of a **loss coefficient** ***K***, defined by:

$$h_L = K \frac{V^2}{2g}$$

where

$$K = \frac{(\Delta h)}{(V^2/2g)} = \frac{(\Delta p_z)}{(\rho V^2/2)}$$

$$K = \frac{\text{drop in piezometric head across component}}{\text{velocity head}} = \frac{\text{pressure drop due to component}}{\text{kinetic or dynamic pressure}}$$

- Values of ***K*** have been determined experimentally for various fittings and geometry changes and are presented in Table 10.5 of the textbook

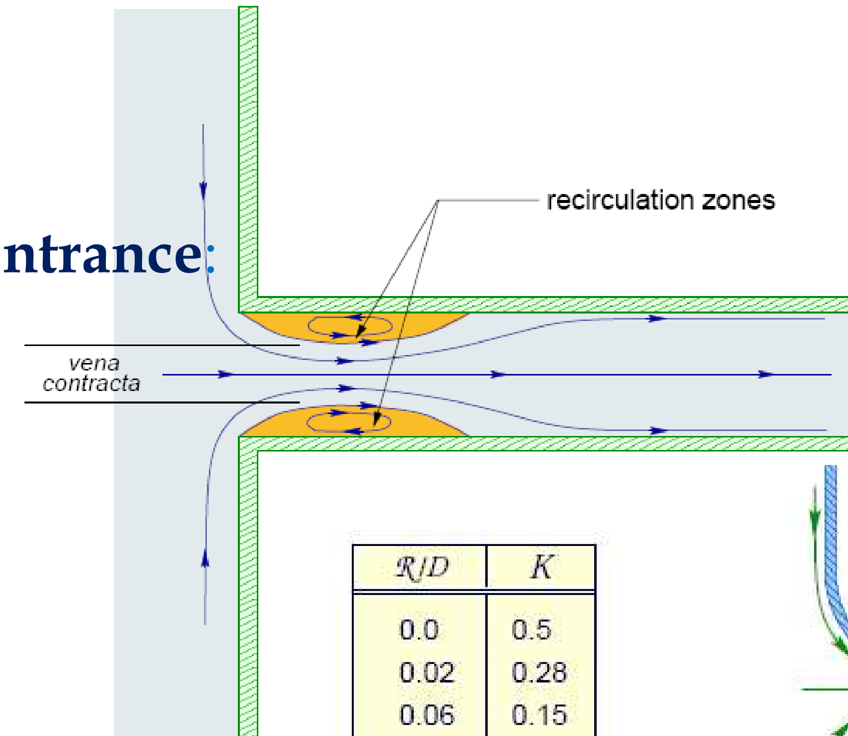
MINOR HEAD LOSS…

- In the above equation, *V* represents the average velocity usually just upstream of the region whose minor loss is to be estimated.

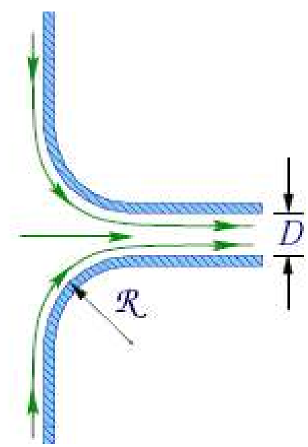
Examples:

- **Sharp-edged entrance:**

$$h_m = 0.5\ (V^2/2g)$$

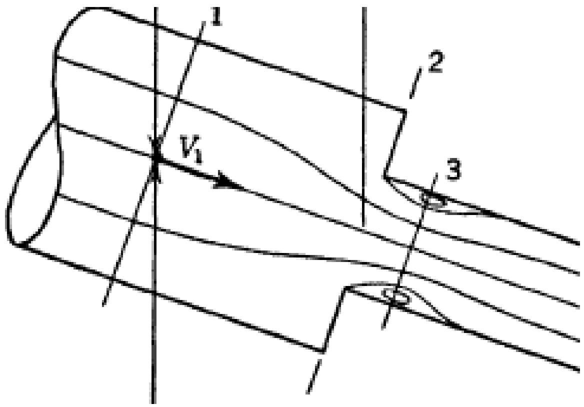


$\mathcal{R}/D$	K
0.0	0.5
0.02	0.28
0.06	0.15
≥ 0.15	0.04



MINOR HEAD LOSS…

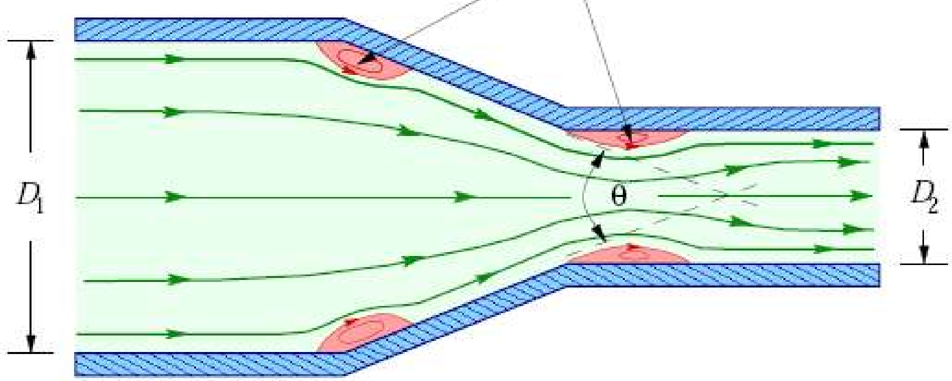
- **Sudden Contraction:**



$$h_m = K\,(V_2^{\,2}/2g)$$

D_2/D_1	K	
	$\theta = 60°$	$\theta = 180°$
0.2	0.08	0.49
0.4	0.07	0.42
0.6	0.06	0.32
0.8	0.05	0.18

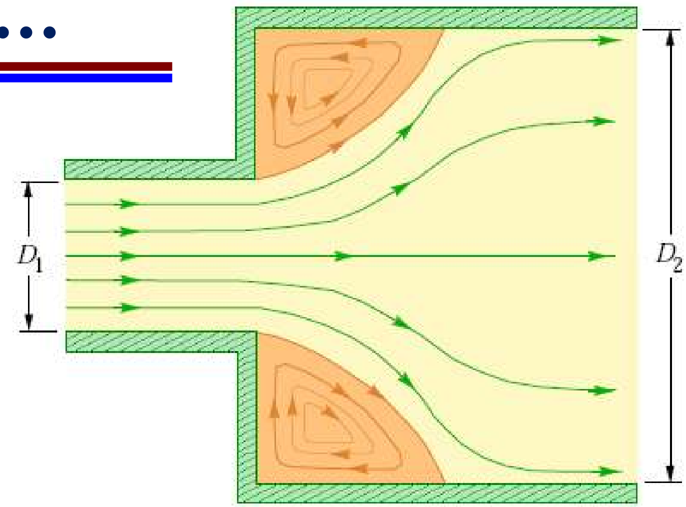
- **Gradual Contraction:**



D_2/D_1	K	
	$\theta = 60°$	$\theta = 180°$
0.2	0.08	0.49
0.4	0.07	0.42
0.6	0.06	0.32
0.8	0.05	0.18

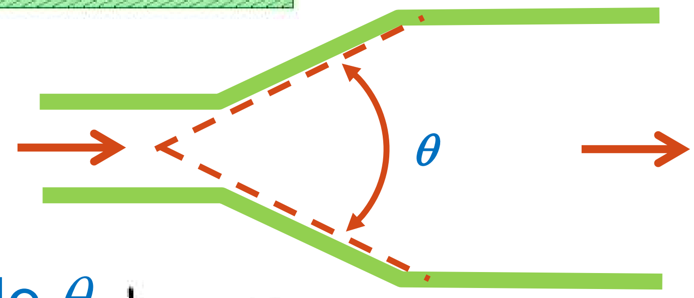
MINOR HEAD LOSS…

- **Sudden Expansion**:



$$h_m = (V_1^2 - V_2^2)/2g)$$

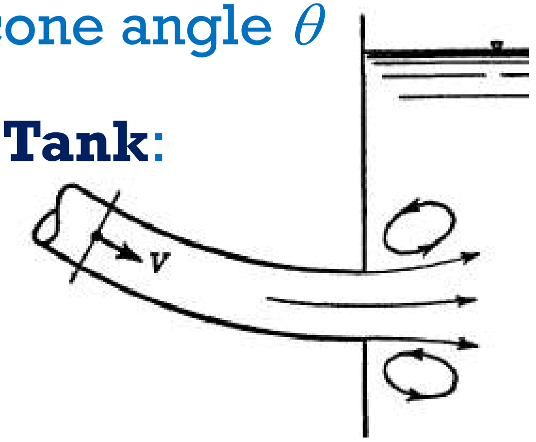
- **Gradual Expansion**:



$$h_m = K(V_1^2 - V_2^2)/2g)$$

In this case, ***K*** is a function of cone angle θ

- **Pipe Discharge into a Large Tank**:



$$h_m = V^2/2g$$

MINOR HEAD LOSS…

- **Pipe Fittings**:

$$h_m = K_L\,(V_2^2/2g)$$

Loss coefficient K_L for commercial pipe fittings

Globe valve, wide open	10.0
Angle valve, wide open	5.0
Gate valve, wide open	.19
¾ open	1.15
½ open	5.6
¼ open	24.0
Return bend	2.2
Standard tee	1.8
90° elbow	.90
45° elbow	.42

COMBINED HEAD LOSS

- As mentioned earlier, the **Combined (Total) Head Loss** is:

Combined (Total) head loss = Pipe head loss + Component head loss

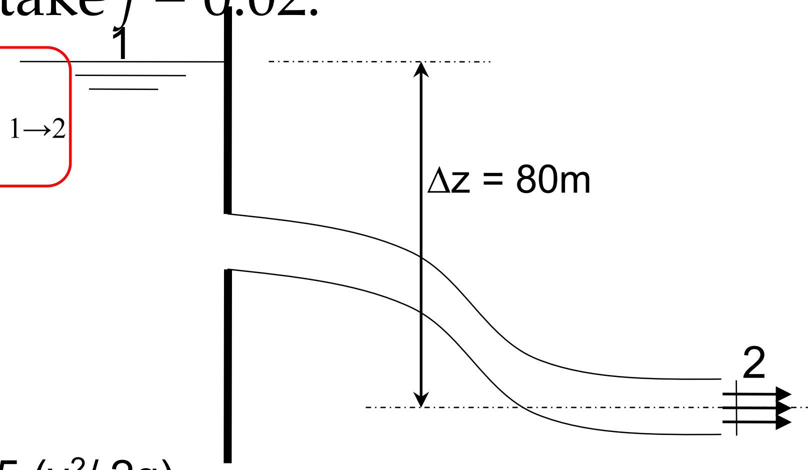
which can be expressed as:

$$h_L = \sum_{\text{pipes}} f \frac{L}{D} \frac{V^2}{2g} + \sum_{\text{components}} K \frac{V^2}{2g}$$

- **See example 10.7 for illustrations.**

COMBINED HEAD LOSS…

Example: Referring to the shown figure, find the flow rate through a 25 cm diameter pipe of length 1500m with Δz = 80m. Consider the losses in the system and take f = 0.02.



Solution:

$$\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + \sum \text{Losses}_{1\to 2}$$

$$\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + \sum \text{Losses}_{1\to 2}$$

$\sum \text{Losses}_{1\to 2} = f\ (L/D)\ (v^2/2g) + 0.5\ (v^2/2g)$

$\sum \text{Losses}_{1\to 2} = 0.02\ (1500/0.25)\ (v^2/2g) + 0.5\ (v^2/2g)$

$\sum \text{Losses}_{1\to 2} = 120.5\ (v^2/2g)$

$80 = (v^2/2g) + 120.5(v^2/2g)$

$(v^2/2g) = 80/121.5 = 0.658$

$V^2 = 0.658 \times 2g$

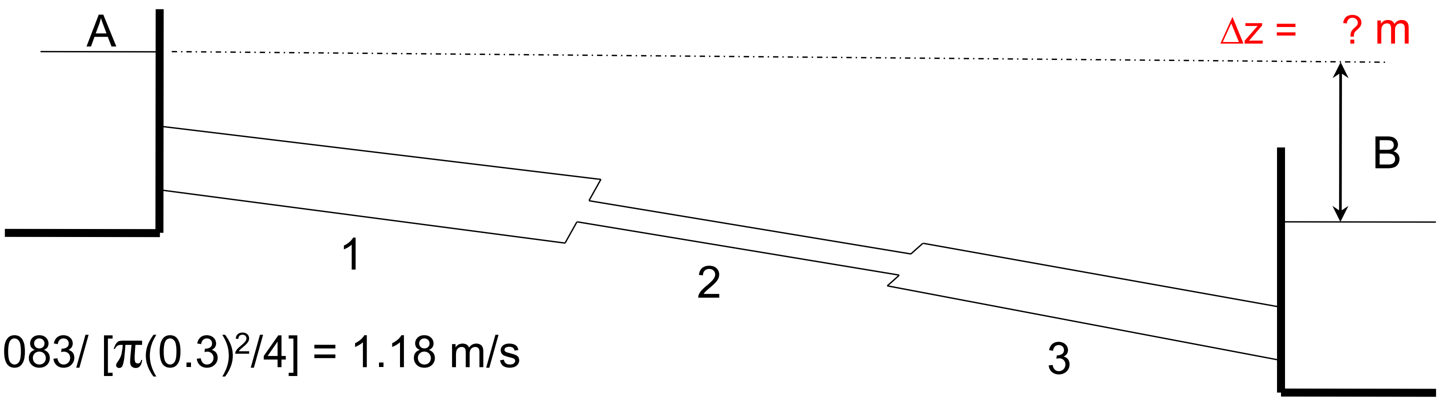
$v = 3.6$ m/s

$Q = A\,v = [\pi(0.25)^2/4] \times 3.6 = 0.18\ m^3/s$

COMBINED HEAD LOSS…

Example: The pipeline shown 1, 2 and 3 are 300m of 30cm diameter, 150m of 20 cm diameter and 250m of 25cm diameter, respectively. If the flow rate of water=0.083 m^3/s ,find the losses in the pipeline. Take, f = 0.02 for all pipeline, $K_c=0.03$.

Solution:



$Q = A_1 v_1$

$v_1 = Q/A_1 = 0.083/ [\pi(0.3)^2/4] = 1.18$ m/s

$v_2 = Q/A_2 = 0.083/ [\pi(0.2)^2/4] = 2.64$ m/s

$v_3 = Q/A_3 = 0.083/ [\pi(0.25)^2/4] = 1.69$ m/s

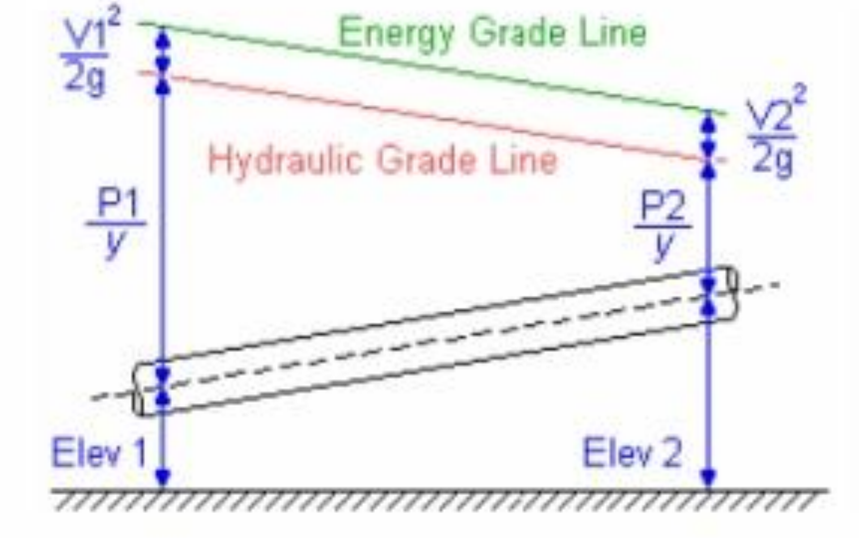
$$\sum \text{Losses}_{A\rightarrow B} = f\ (L_1/D_1)\ (v_1^2/ 2g) + f\ (L_2/D_2)\ (v_2^2/ 2g) + f\ (L_3/D_3)\ (v_3^2/ 2g) + 0.5\ (V_1^2/ 2g) + K_c\ (V_2^2/2g) + (V_2 - V_3)^2/2g + V_3^2/2g$$

$$\sum \text{Losses}_{A\rightarrow B} = 9.89 \text{ m}$$

TRANSITION LOSSES & GRADE LINES…

- Recall that in Kinematics Chapter, it was stated that the EGL and HGL drop whenever useful energy is extracted from the flow, e.g. with turbine.

- Whenever there are valves, elbows, enlargements, contractions, inlets, outlets, bends, and other fittings, the **EGL and HGL drop by an amount equal to the head loss produced by that transition.**



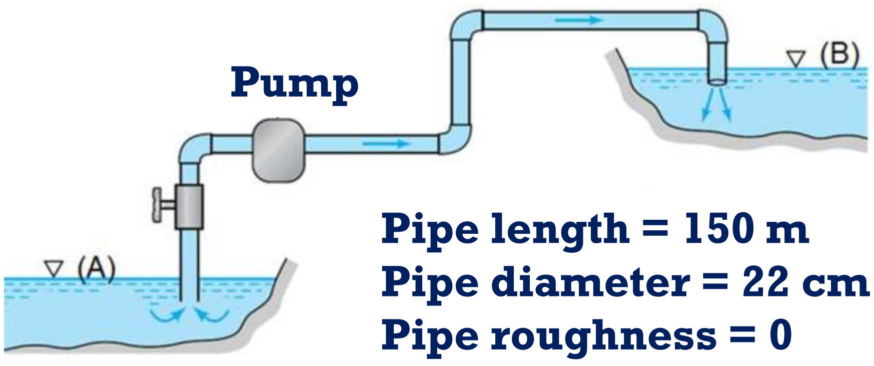
TRANSITION LOSSES & GRADE LINES…

Example: The pump shown in the figure delivers a head of 75 m to the water (water kinematic viscosity $\nu = 1.0 \times 10^{-6}$ m²/s). The difference in elevation of the two lakes (A and B) is 60 m. Assume that the total minor head loss from all fittings is 7 m.

Determine:

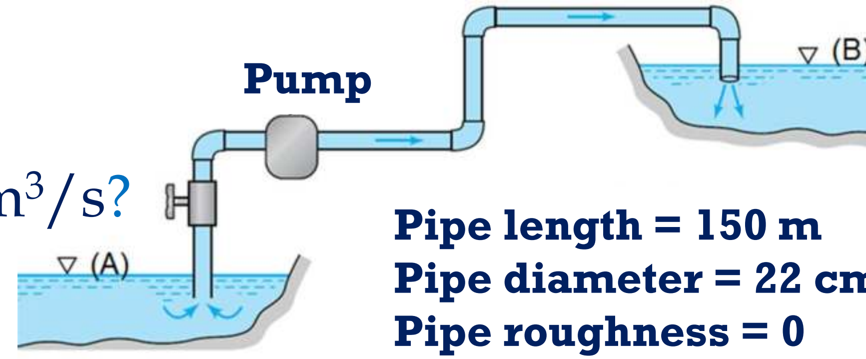
1) The water volume flow rate in m³/s

2) The power that the pump adds to the water in Watt.



Pipe length = 150 m
Pipe diameter = 22 cm
Pipe roughness = 0

TRANSITION LOSSES & GRADE LINES…



Pipe length = 150 m
Pipe diameter = 22 cm
Pipe roughness = 0

Solution:

1) The water volume flow rate in m^3/s?

- From Energy equation:

$$\frac{p_A}{\gamma} + z_A + \alpha_1 \frac{\bar{V}_A^2}{2g} + h_p = \frac{p_B}{\gamma} + z_B + \alpha_1 \frac{\bar{V}_B^2}{2g} + \sum h_L$$

$$\Rightarrow z_A + h_p = z_B + h_f + h_{fitting} \Rightarrow h_f = h_p + (z_A - z_B) - h_{fitting} = 75 - 60 - 7 \Rightarrow \boxed{h_f = 8 \text{ m}}$$

- Moody chart using **Re $f^{1/2}$** and **smooth pipe**

$$\therefore \text{Re}\, f^{1/2} = \frac{D^{3/2}}{\upsilon}\left(\frac{2gh_f}{L}\right)^{1/2} = \frac{0.22^{3/2}}{1\times10^{-6}}\left(\frac{2\times9.81.\times8}{150}\right)^{1/2} = 1\times10^5$$

$f = 0.012$

2) The power that the pump adds to the water in Watt?

$$h_f = f\frac{L}{D}\left(\frac{V^2}{2g}\right) \Rightarrow 8 = 0.012\frac{150}{0.22}\left(\frac{V^2}{2\times9.81}\right) \Rightarrow V = 4.4 \text{ m/s}$$

$$Q = VA = 4.4\left(\frac{\pi}{4}0.22^2\right) = 0.166 \text{ m}^3/\text{s}, \;\; \text{Power} = \gamma Q h_p = 9810(0.166)75 = 122 \text{ kW}$$

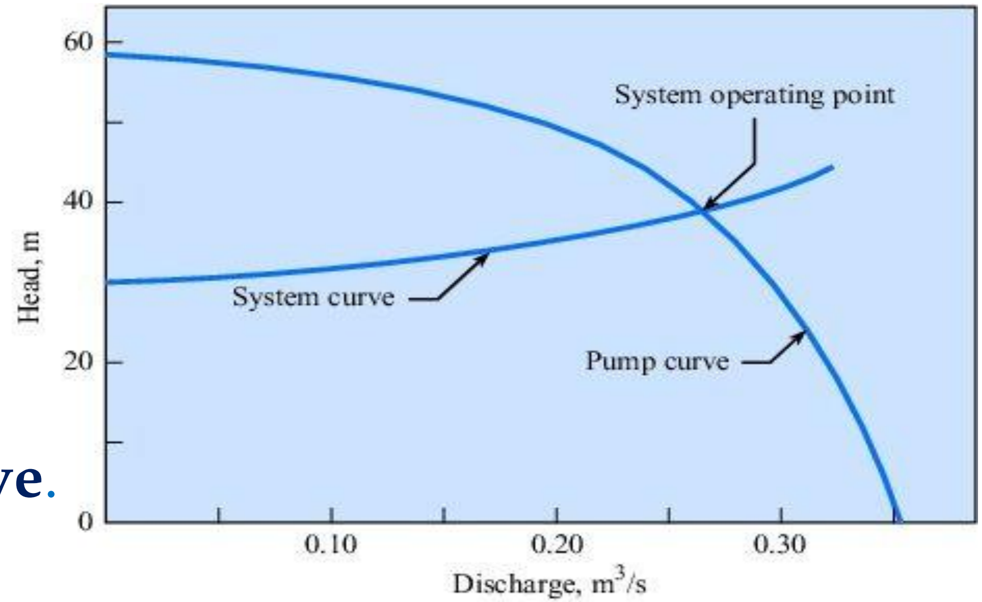
PUMP AND PIPE SYSTEMS

Simple Pump in a Pipeline

- Consider the flow of liquid in a pump and pipe combination as shown in figure 10.15. The energy equation for this system simplifies to:

$$h_p = (z_2 - z_1) + \frac{V^2}{2g}\left(1 + f\frac{L}{D} + \sum K_L\right) = C_1 + C_2 Q^2$$

- The above equation shows that the head required for maintaining the flow increases as the discharge increases. The **head vs. discharge curve system curve**.

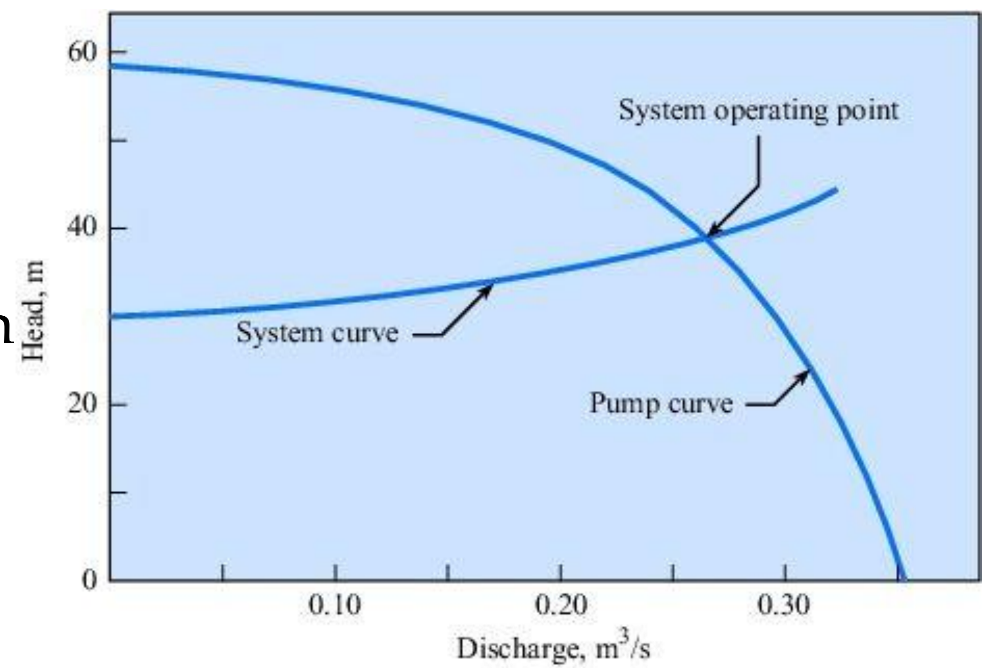


PUMP AND PIPE SYSTEMS

Simple Pump in a Pipeline…

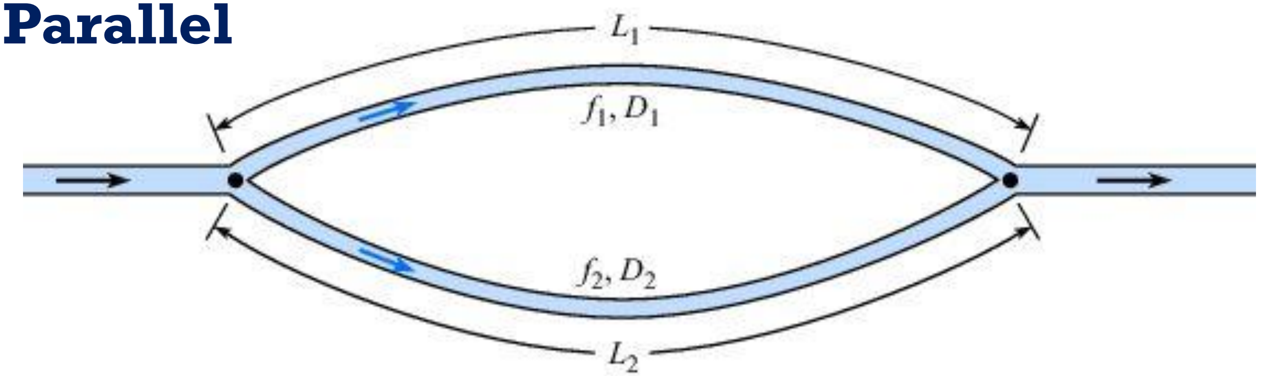
- The head produced by a centrifugal pump decreases as the discharge increases. A typical **head vs. discharge curve**, called the **pump curve**, is as shown in figure. It is usually supplied by the pump manufacturer.

- The point of intersection of the two curves, called the **system operating point**, gives the operating condition of the system.



PIPE LINES

Pipes in Parallel



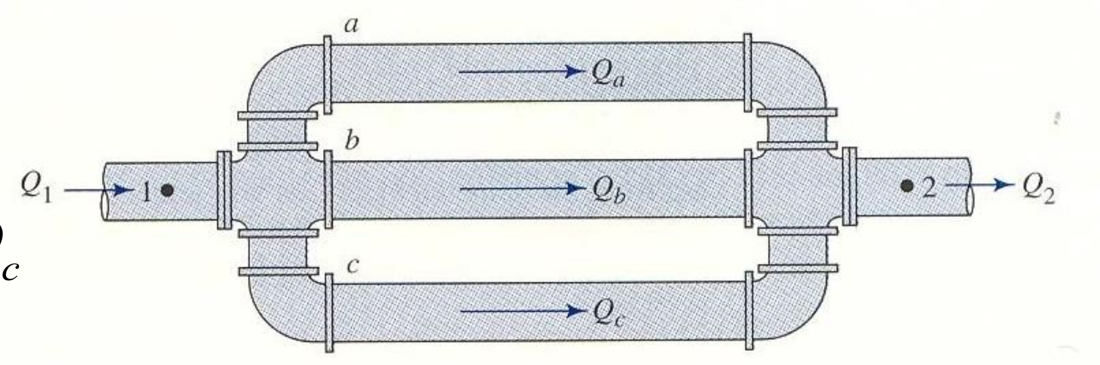
- A typical problem might be to determine the flow in each pipe, given the total flow rate. No matter which pipe is involved, the D*p* and elevation z between the two junctions is the same. Now since,

$$h_L = \left(p_1 + \gamma z_1\right) - \left(p_1 + \gamma z_2\right) \Rightarrow h_{L_1} = h_{L_2}$$

$$\therefore f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} \Rightarrow \boxed{\frac{V_1}{V_2} = \sqrt{\frac{f_2 L_2 D_1}{f_1 L_1 D_2}}}$$

PIPE LINES

Pipes in Parallel



- Continuity:

$$Q_1 = Q_2 = Q_a + Q_b + Q_c$$

- Energy equation:

$$\frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{\bar{V}_1^{\,2}}{2g} = \frac{p_2}{\gamma} + z_2 + \alpha_1 \frac{\bar{V}_2^{\,2}}{2g} + h_L$$

- All elements of the flow converging at 2 will have the same head loss. The flow will adjust automatically so that the **head loss h_L** in each branch pipe will be the same

$$h_L = h_{L_a} = h_{L_b} = h_{L_c} \Rightarrow h_L = f_a \frac{L_a}{D_a}\frac{V_a^{\,2}}{2g} = f_b \frac{L_b}{D_b}\frac{V_b^{\,2}}{2g} = f_c \frac{L_c}{D_c}\frac{V_c^{\,2}}{2g}$$

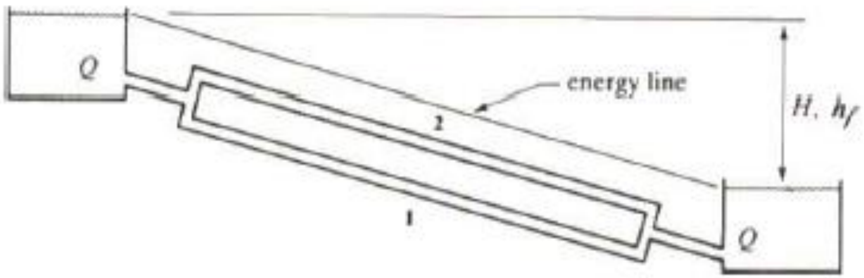
- If the entrance and exit loss coefficients (K_e & K_E) are given, then

$$h_{L_a} = f_a \frac{L_a}{D_a}\frac{V_a^{\,2}}{2g} + K_e \frac{V_a^{\,2}}{2g} + K_E \frac{V_a^{\,2}}{2g} = \left(f_a \frac{L_a}{D_a} + K_e + K_E \right)\frac{V_a^{\,2}}{2g}$$

PIPE LINES

Example: Two pipes connect two reservoirs (A and B) which have a height difference of 10 m. Pipe 1 has diameter 50 mm and length 100 m. Pipe 2 has diameter 100 mm and length 100 m. Both have entry loss k_L = 0.5 and exit loss k_L = 1.0 and Darcy-friction factor f of 0.008. Calculate the rate of flow for each pipe.

Solution:



- Continuity: $Q_1 = Q_2 = Q_a + Q_b$
- Energy equation:

$$\frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{\bar{V}_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \alpha_1 \frac{\bar{V}_2^2}{2g} + h_L \Rightarrow \boxed{h_L = z_1 - z_2}$$

- Head loss in each pipe will yield velocity and then flow rate

$$h_{L_a} = \left(f_a \frac{L_a}{D_a} + K_e + K_E \right) \frac{V_a^2}{2g} = z_1 - z_2 \Rightarrow V_a \Rightarrow Q_a = V_a A_a$$