ME 8043114 Automatic Control

Lecture 10: Gain Margin & Phase Margin Examples
Frequency Domain Specifications
Controller Design in Bode Plot (Gain, Lead, Lag)

Dr. Shadi M. Munshi
E-mail: smmunshi@uqu.edu.sa

Dr. Badr AlOufi
E-mail: baawfey@uqu.edu.sa

Lecture Outline

- Gain Margin & Phase Margin Examples
- Frequency Domain Specifications
- Control Design in Bode Plot
  - Gain compensator
  - Lag Compensator
  - Lead Compensator

Course roadmap

Modeling

- ✓ Laplace transform
- ✓ Transfer function
- ✓ Models for systems
  - ✓ electrical
  - ✓ mechanical
  - ✓ electromechanical
- ✓ Linearization, delay

Analysis

- ✓ Time response
  - ✓ Transient
  - ✓ Steady state
- ✓ Frequency response
  - ✓ Bode plot
- ✓ Stability
  - ✓ Routh-Hurwitz
  - ➤ Nyquist

Design

- Design specs
- ✓ Root locus
- Frequency domain
- ✓ PID & Lead-lag
- Design examples

Gain margin (GM) (review)

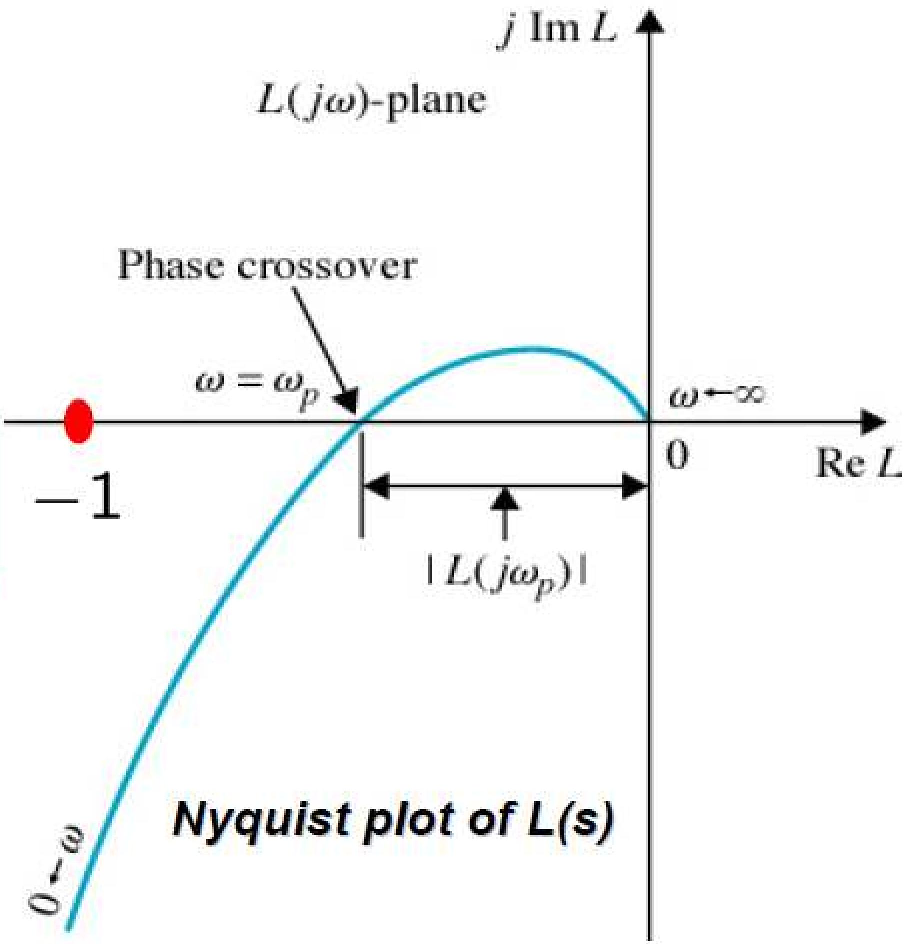
- Phase crossover frequency ω_p:

  $$\angle L(j\omega_p) = -180$$

- Gain margin (in dB)

$$GM = 20\log_{10}\frac{1}{|L(j\omega_p)|}$$

  - Indicates how much OL gain can be multiplied without violating CL stability.



Phase margin (PM) (review)

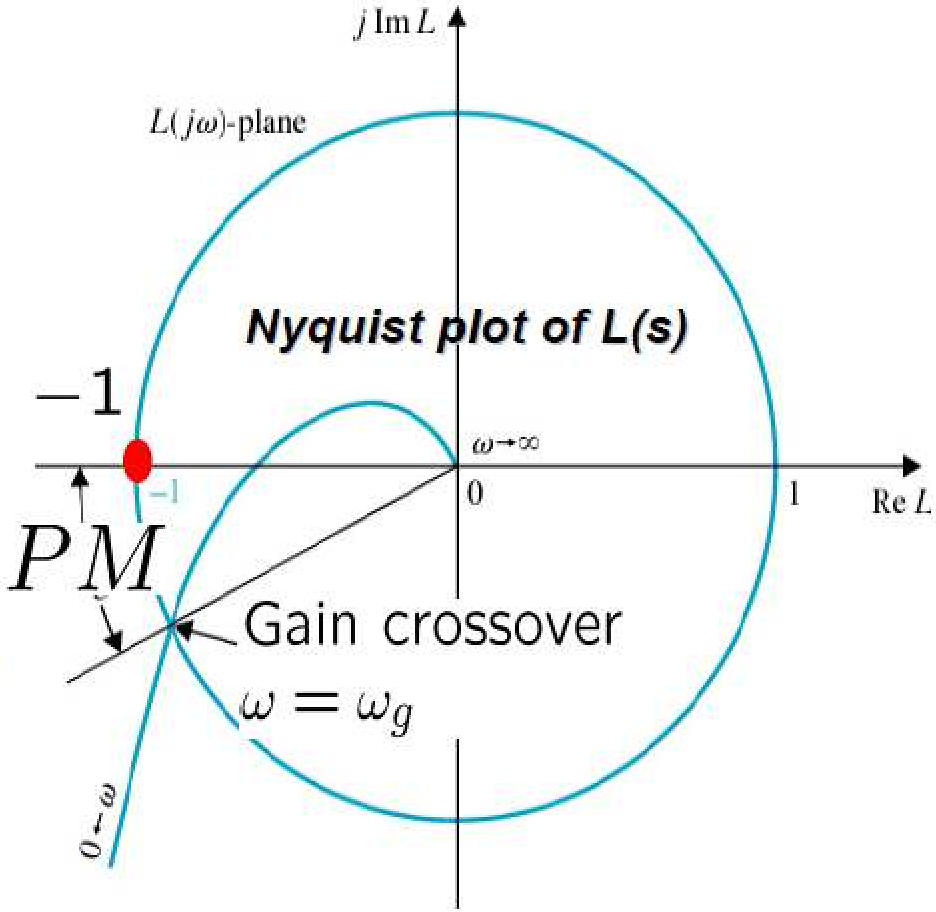
- Gain crossover frequency ωg:

  $$|L(j\omega_g)| = 1$$

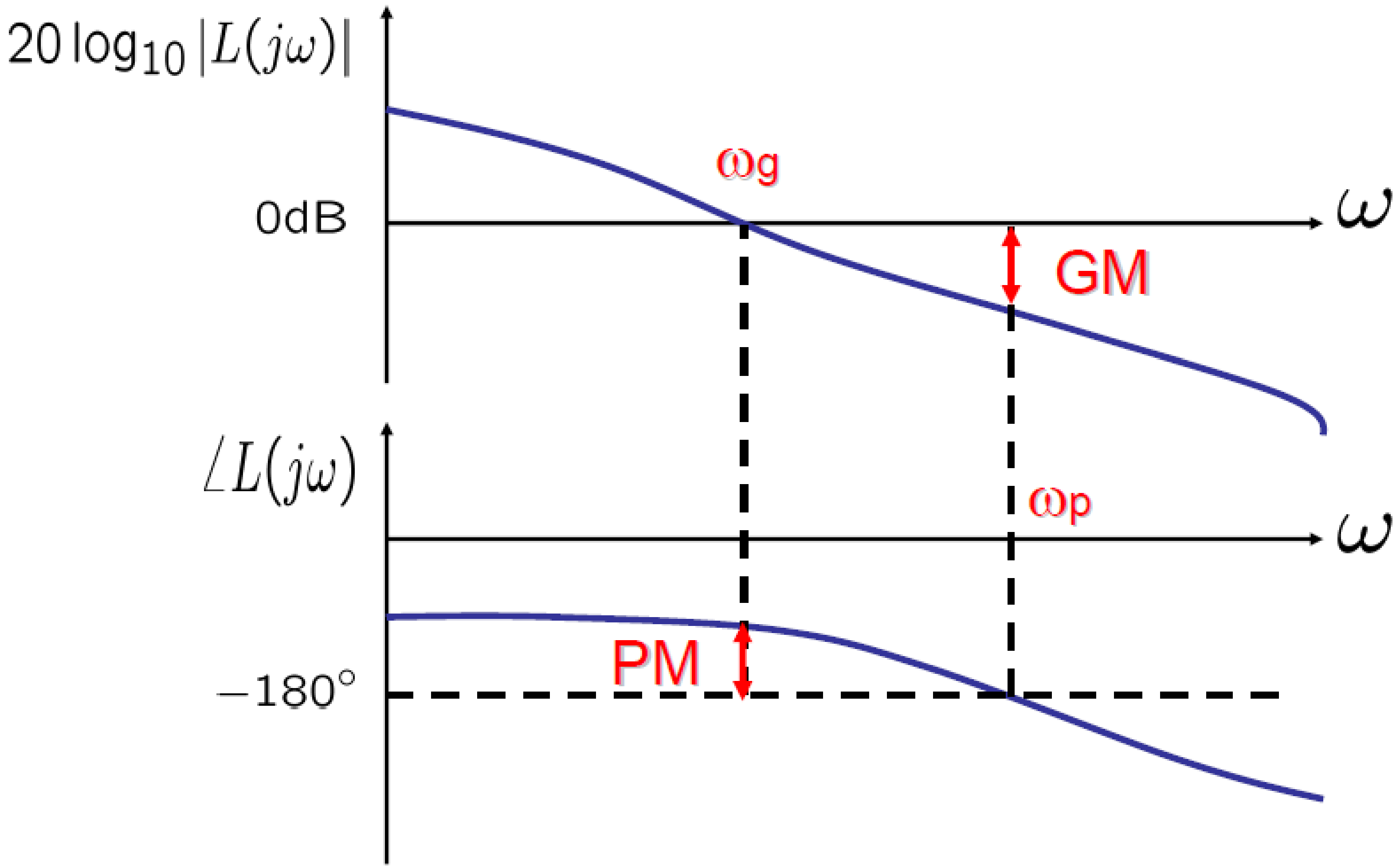
- Phase margin

$$PM = \angle L(j\omega_g) + 180^\circ$$

  - Indicates how much OL phase lag can be added without violating CL stability.



Relative stability on Bode plot (review)



Example of GM & PM

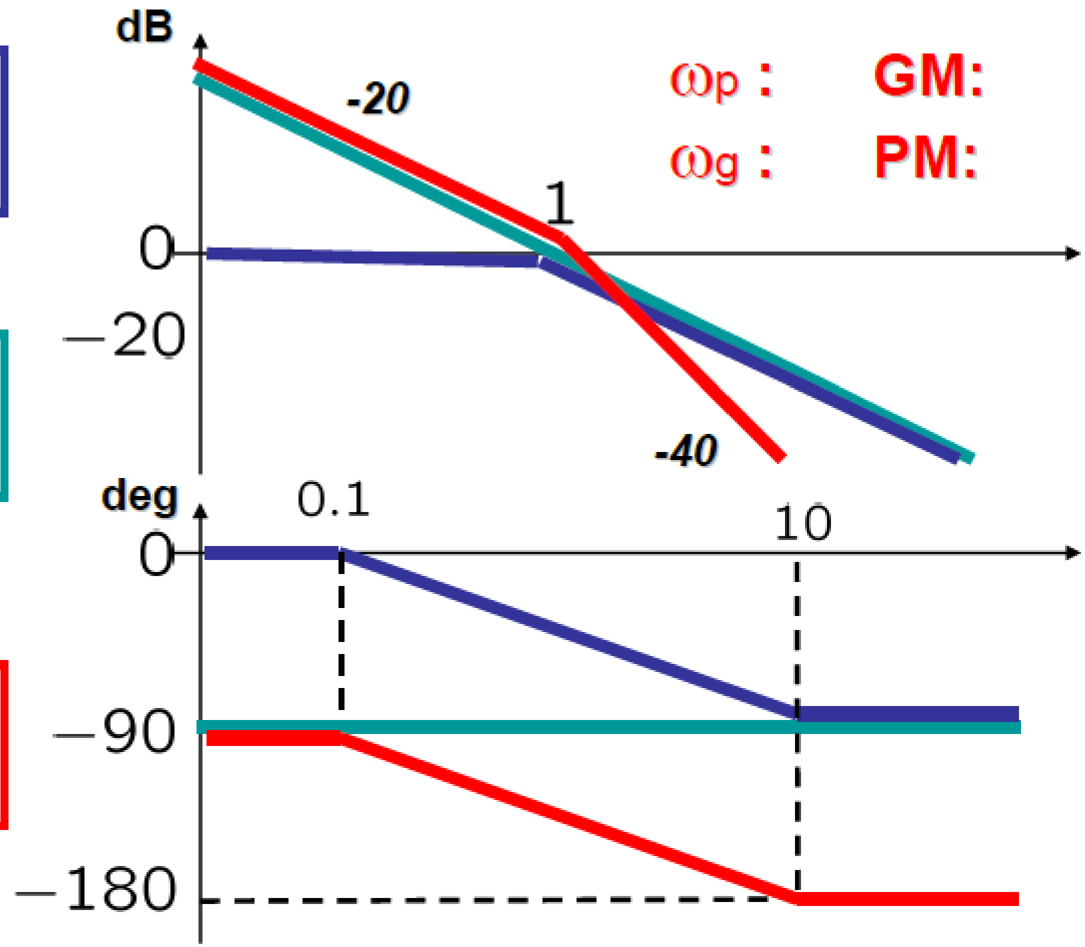
$$G_1(s) = \frac{1}{s+1}$$

$$\times$$

$$G_2(s) = \frac{1}{s}$$

$$\Downarrow$$

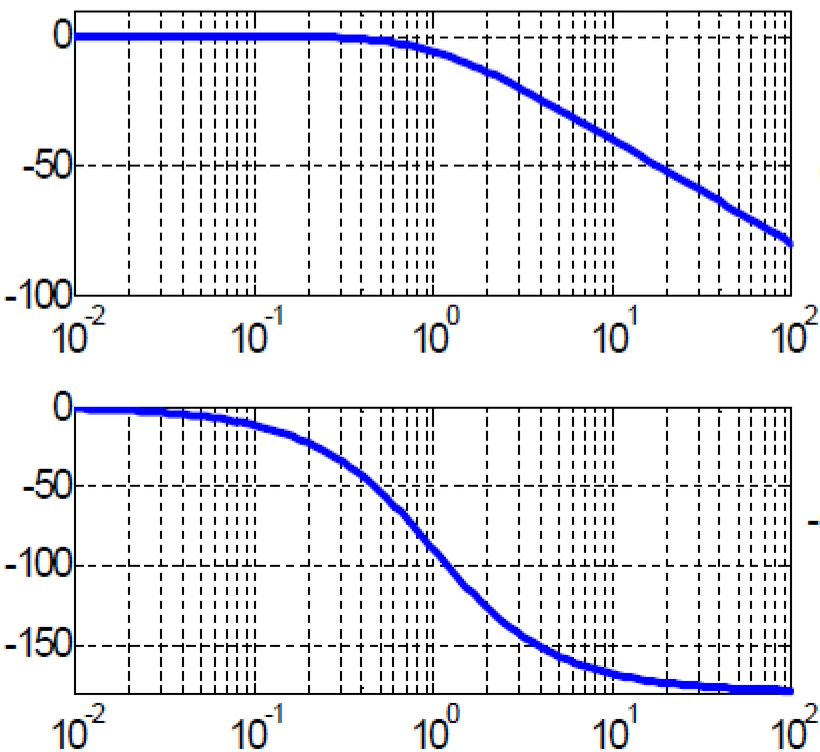
$$L(s) = \frac{1}{s(s+1)}$$



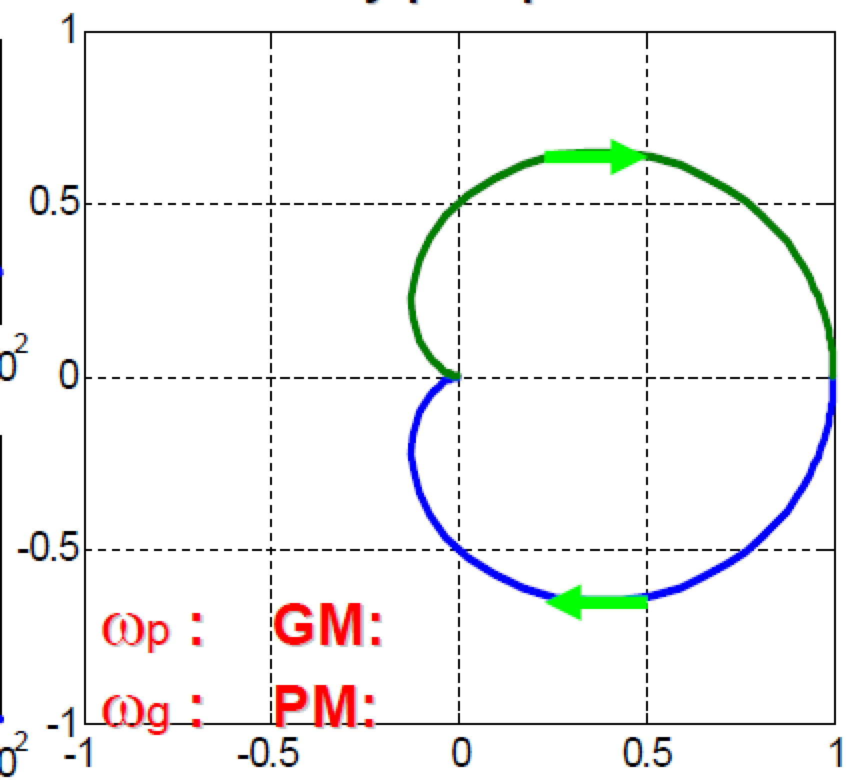
Example of GM & PM

- Second order system $L(s) = \dfrac{1}{(s+1)^2}$

Bode plot



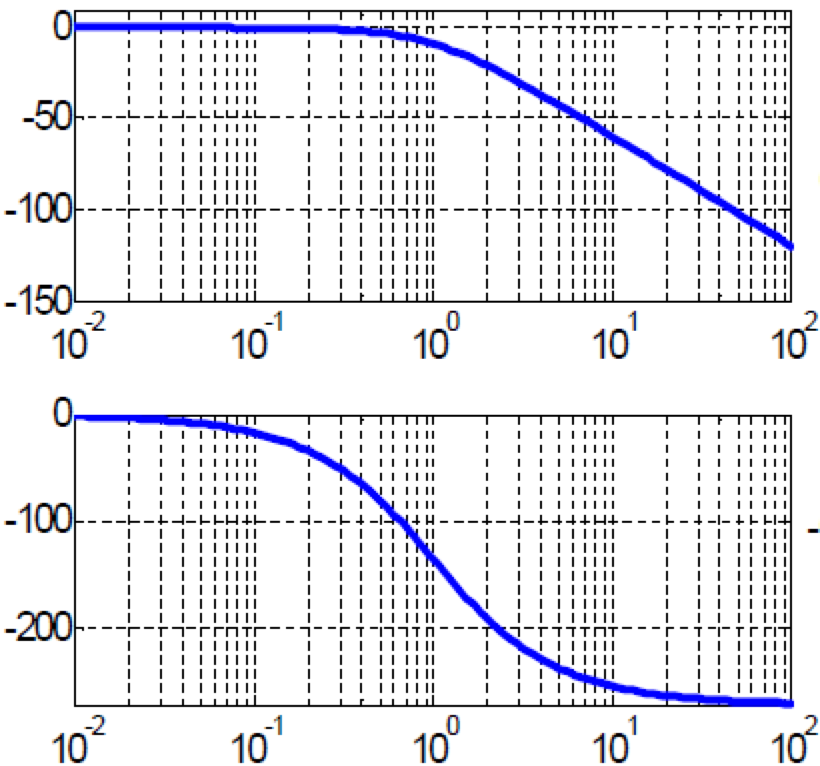
Nyquist plot



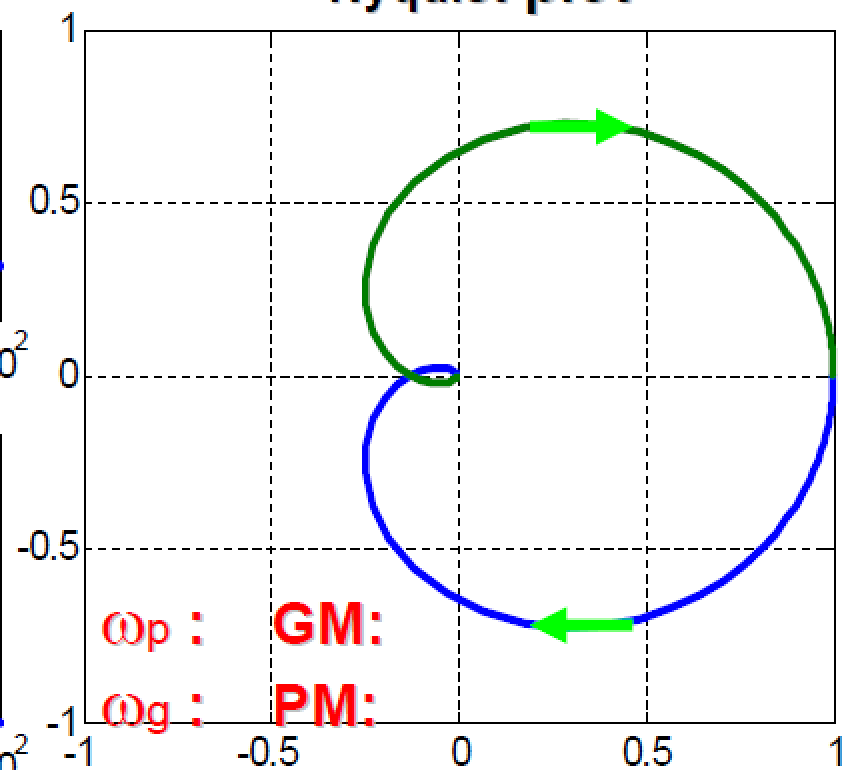
Example of GM & PM

- Third order system $L(s) = \dfrac{1}{(s+1)^3}$

Bode plot



Nyquist plot



How to compute GM?

- Frequency response function

$$\begin{aligned} L(j\omega) &= \frac{1}{(j\omega+1)^3} \\ &= \frac{1}{(j\omega)^3+3(j\omega)^2+3j\omega+1} \\ &= \frac{1}{1-3\omega^2+j\omega(3-\omega^2)} \\ &= \frac{1-3\omega^2-j\omega(3-\omega^2)}{(1-3\omega^2)^2+\omega^2(3-\omega^2)^2} \end{aligned}$$

$$\left[(a+b)^3 = a^3+3a^2b+3ab^2+b^3\right]$$

PM computation often requires computational tools.

$$\mathrm{Im}\{L(j\omega)\}=0 \Rightarrow \omega_p=\sqrt{3} \Rightarrow L(j\sqrt{3})=-\frac{1}{8}$$

Course roadmap

Modeling	*Analysis*	*Design*

✓ Laplace transform
✓ Transfer function
✓ Models for systems
  ✓ electrical
  ✓ mechanical
  ✓ electromechanical
✓ Linearization, delay
	✓ Time response
	  ✓ Transient
	  ✓ Steady state
	✓ Frequency response
	  ✓ Bode plot
	✓ Stability
	  ✓ Routh-Hurwitz
	  ✓ Nyquist
		➤ Design specs
		✓ Root locus
		Frequency domain
		✓ PID & Lead-lag
		Design examples

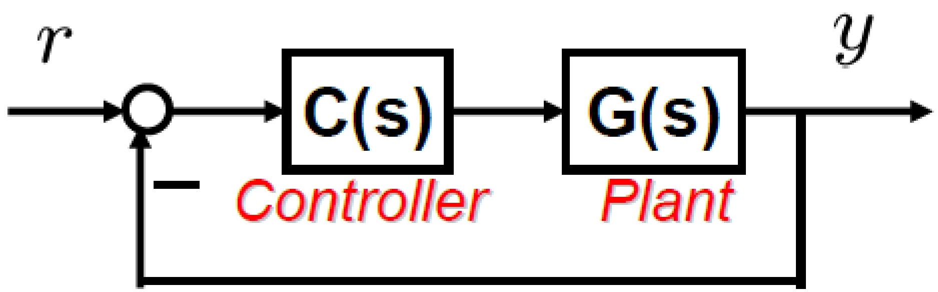
Controller design comparison

Design specifications in time domain
(Rise time, settling time, overshoot, steady state error, etc.)

Approximate translation

Desired closed-loop pole location in s-domain	Desired open-loop frequency response in s-domain
Root locus shaping	***Frequency response shaping (Loop shaping)***

Feedback control system design

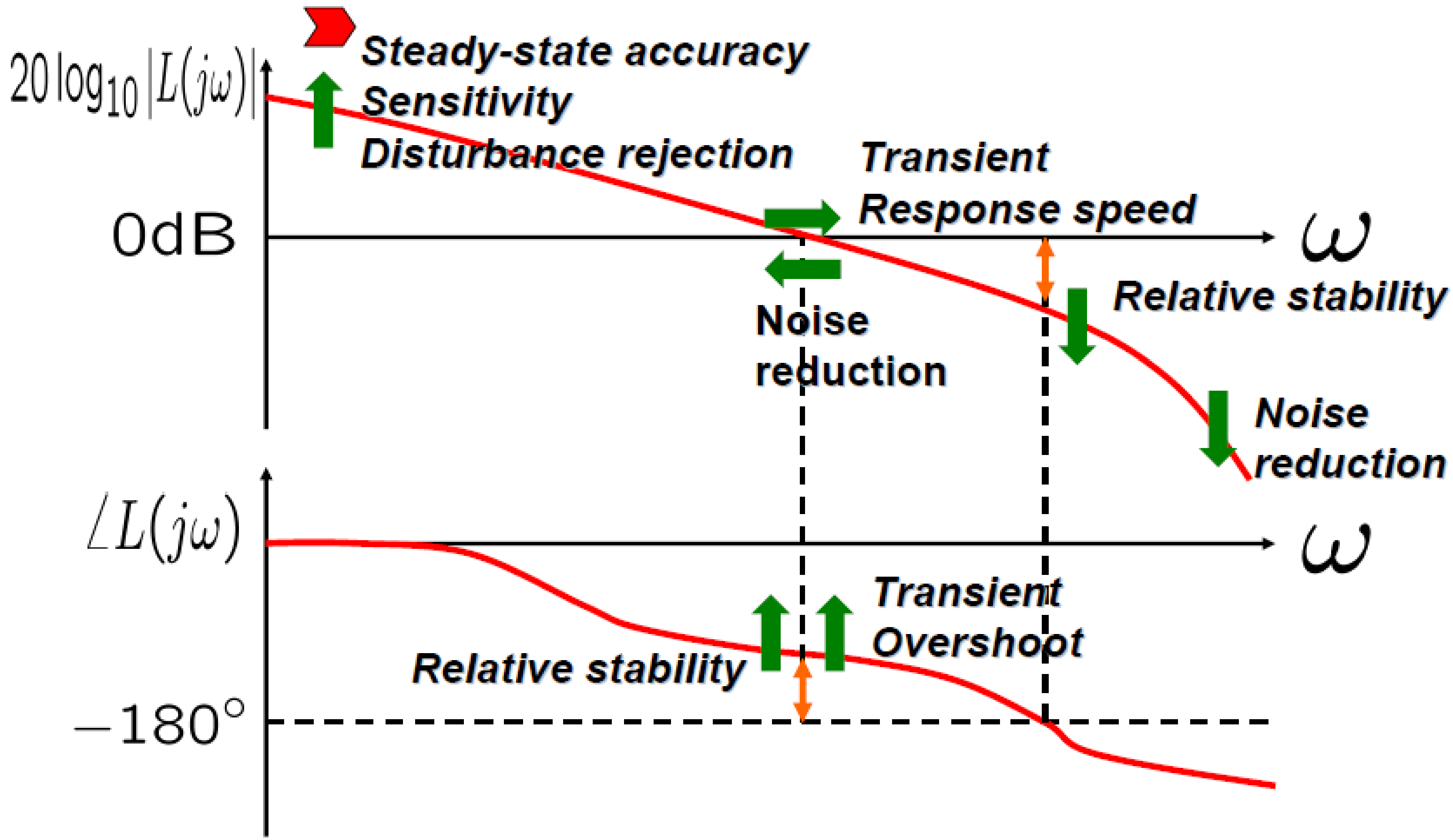


OL: $L(s) := G(s)C(s)$

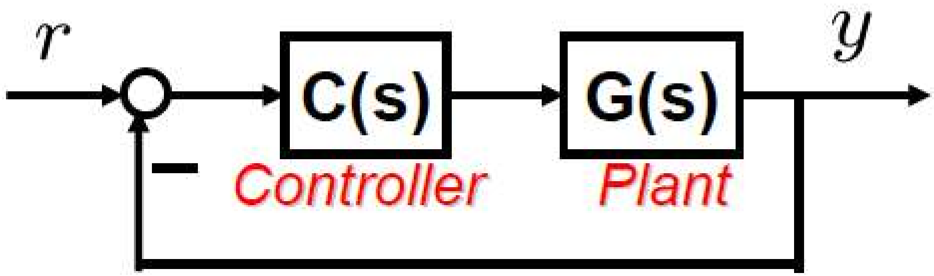
CL: $T(s) := \dfrac{L(s)}{1 + L(s)}$

- Given G(s), design C(s) that satisfies CL stability and time domain specs, i.e., transient and steady-state responses.
- We learn typical qualitative relationships between open-loop Bode plot and time-domain responses.

Typical desired OL Bode plot



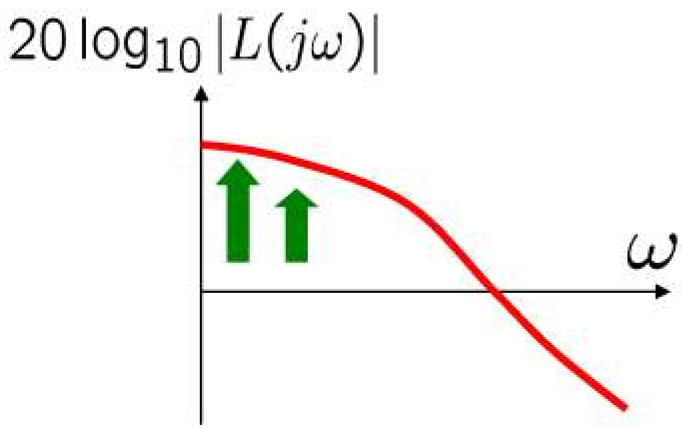
Steady-state accuracy



$$L(s) := G(s)C(s)$$

$$T(s) := \frac{L(s)}{1 + L(s)}$$

For steady-state accuracy,
L should have high gain at low frequencies.



large $|L(j\omega)|$

➡ $T(j\omega) = \dfrac{L(j\omega)}{1 + L(j\omega)} \approx 1$

➡ y(t) tracks r(t) composed of low frequencies very well.

Steady-state accuracy (cont'd)

- **Step r(t)**

  Increase

  $$K_p := L(0)$$

- **Ramp r(t)**

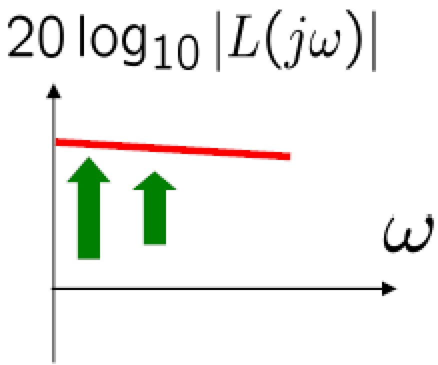
  Increase

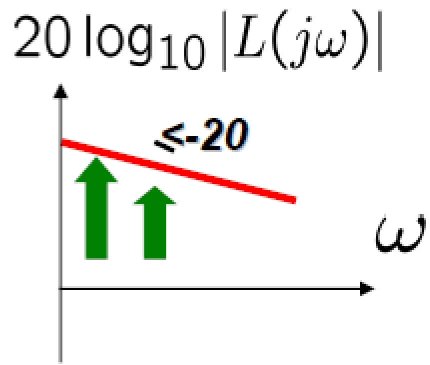
  $$K_v := \lim_{s \to 0} sL(s)$$

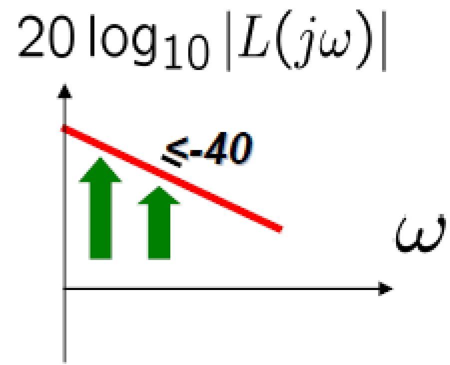
- **Parabolic r(t)**

  Increase

  $$K_a := \lim_{s \to 0} s^2 L(s)$$



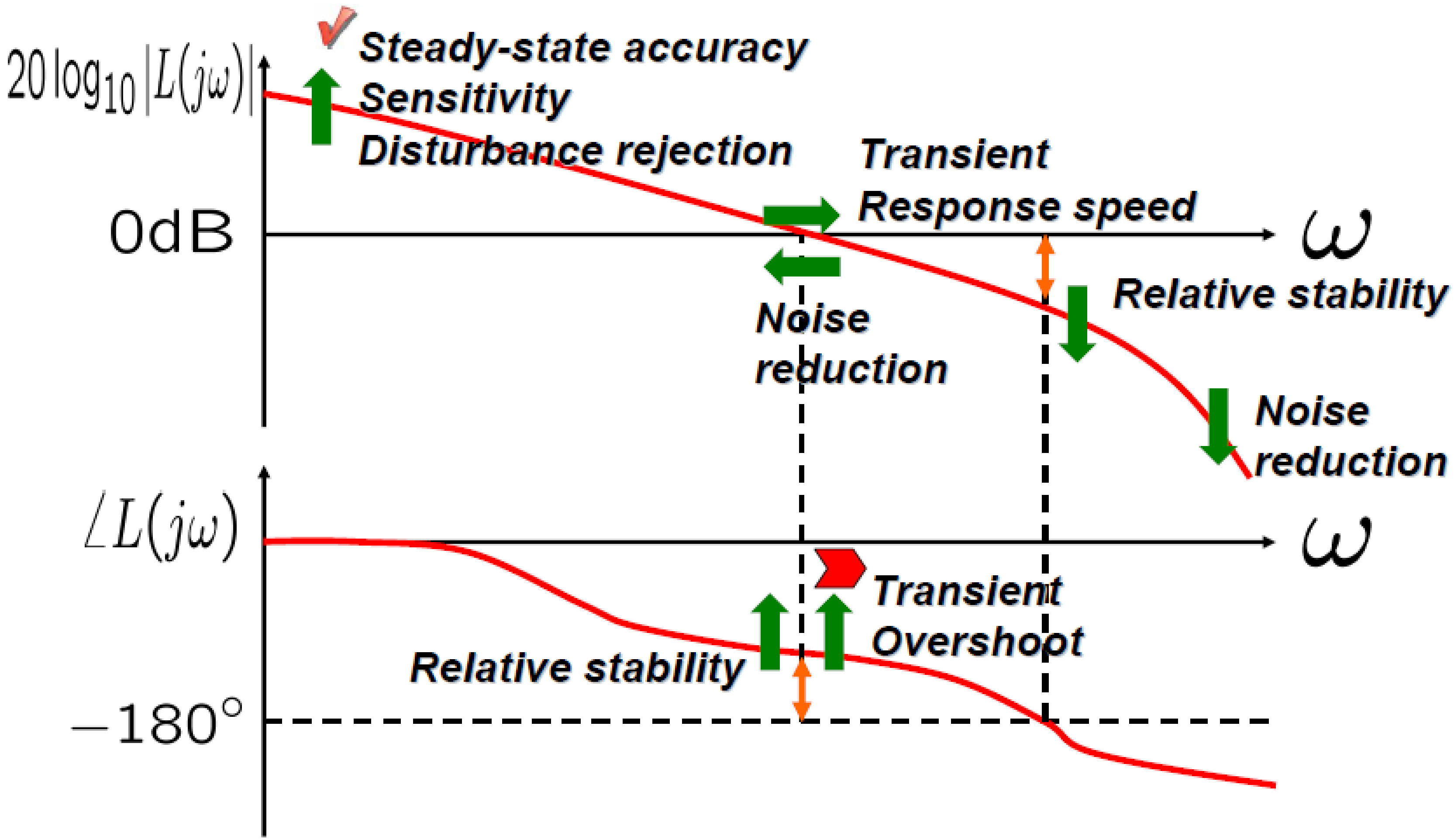




For Kv to be nonzero, L must contain at least one integrator.

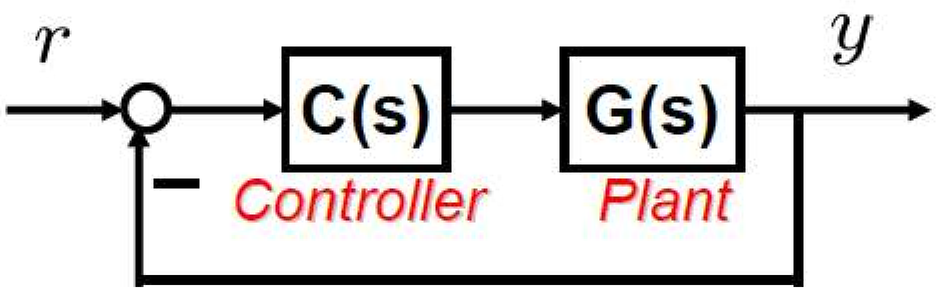
For Ka to be nonzero, L must contain at least two integrators.

Typical desired OL Bode plot



A second order example

- For illustration, we use the feedback system:

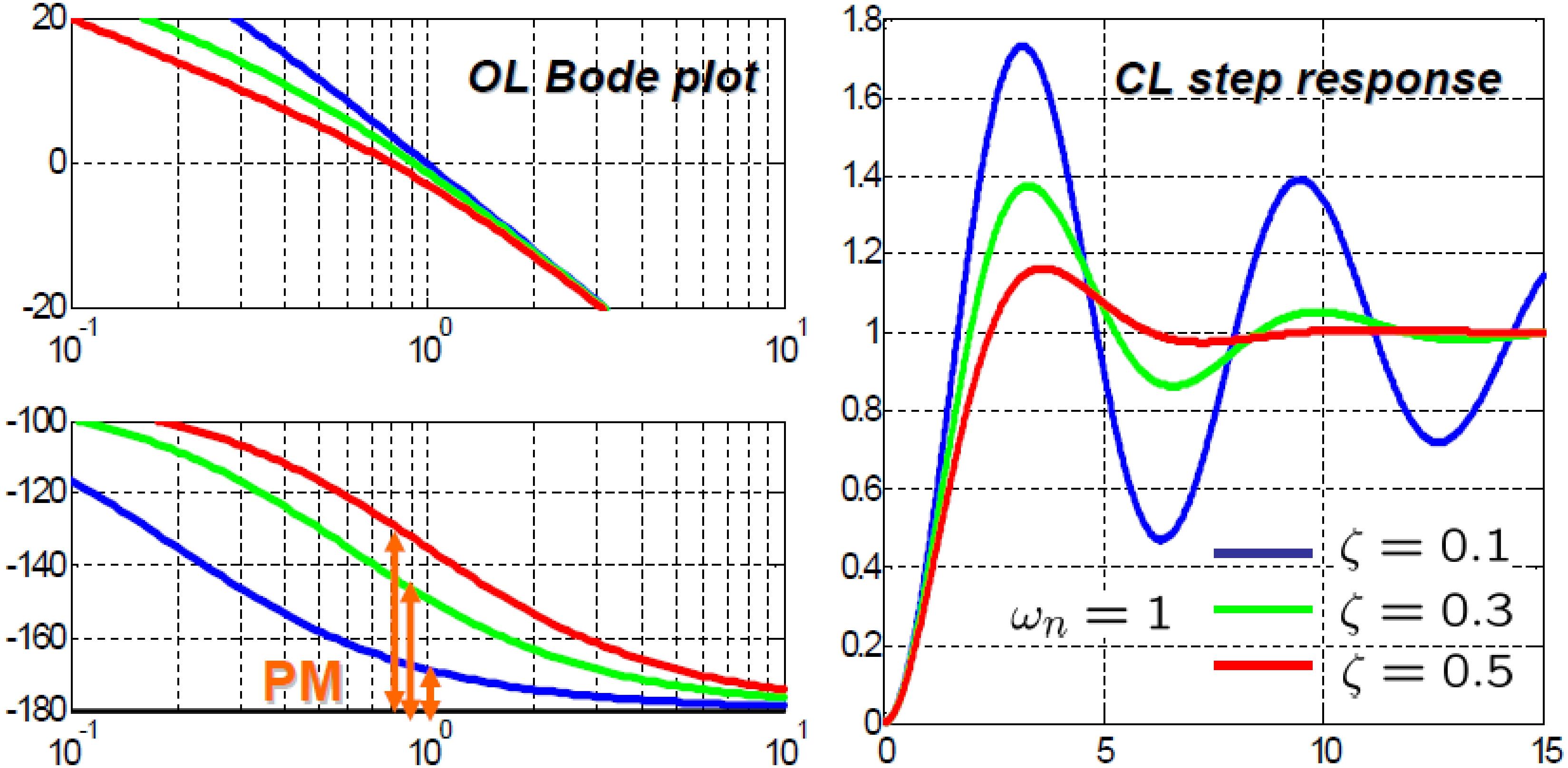


$$L(s) := G(s)C(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

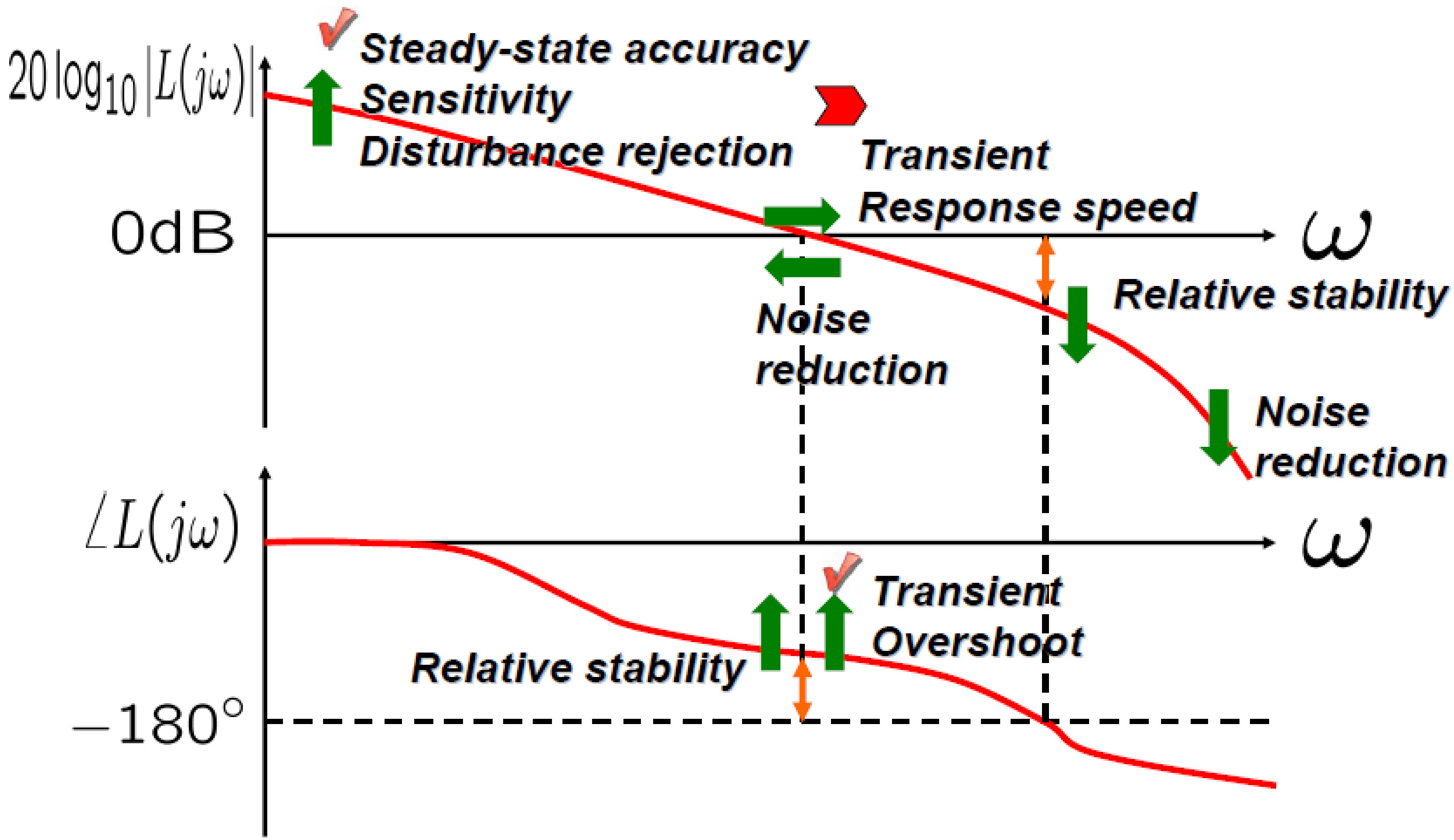
$$T(s) := \frac{L(s)}{1 + L(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Percent overshoot

For small percent overshoot,
L should have larger phase margin.

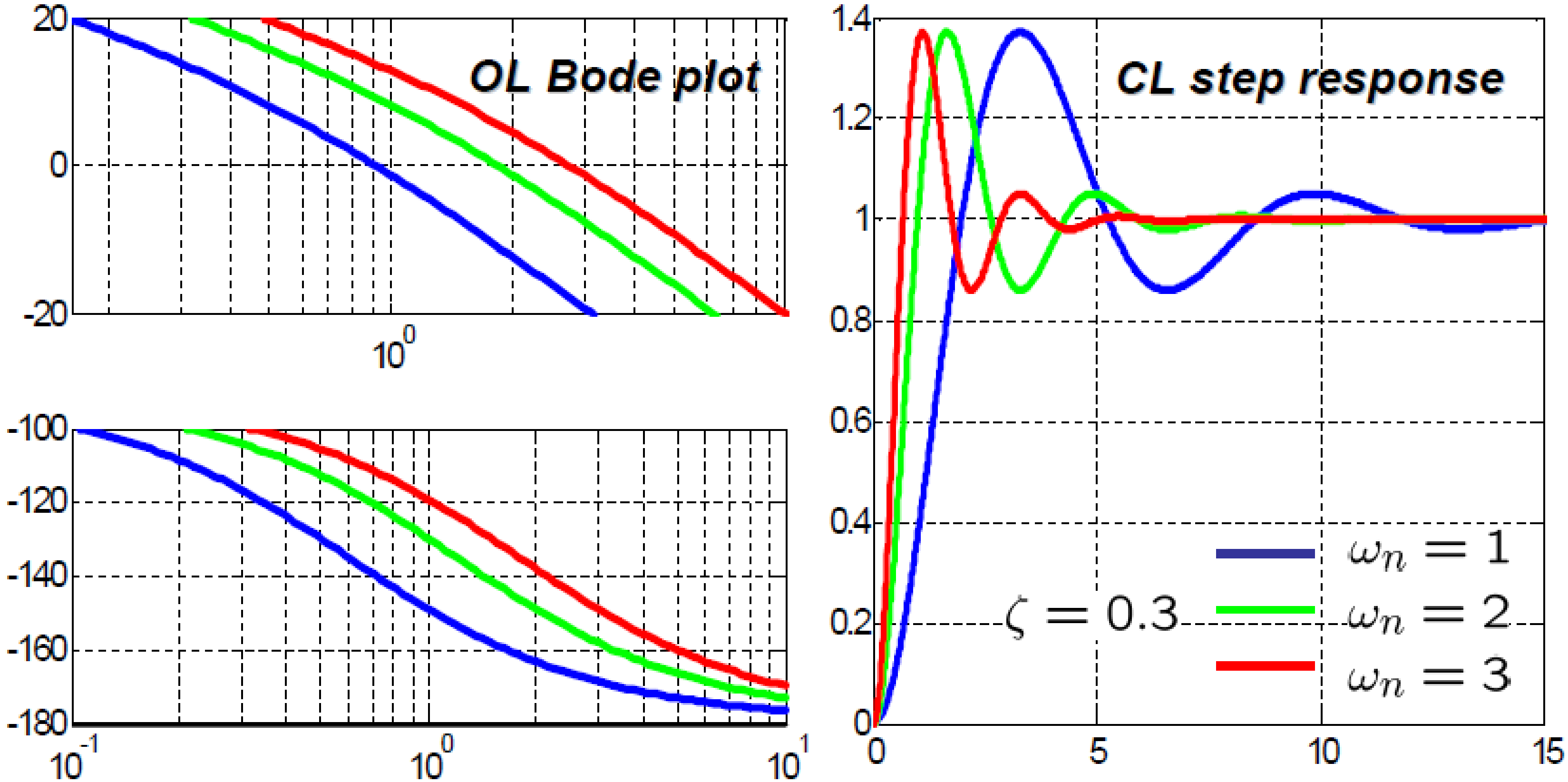


Typical desired OL Bode plot

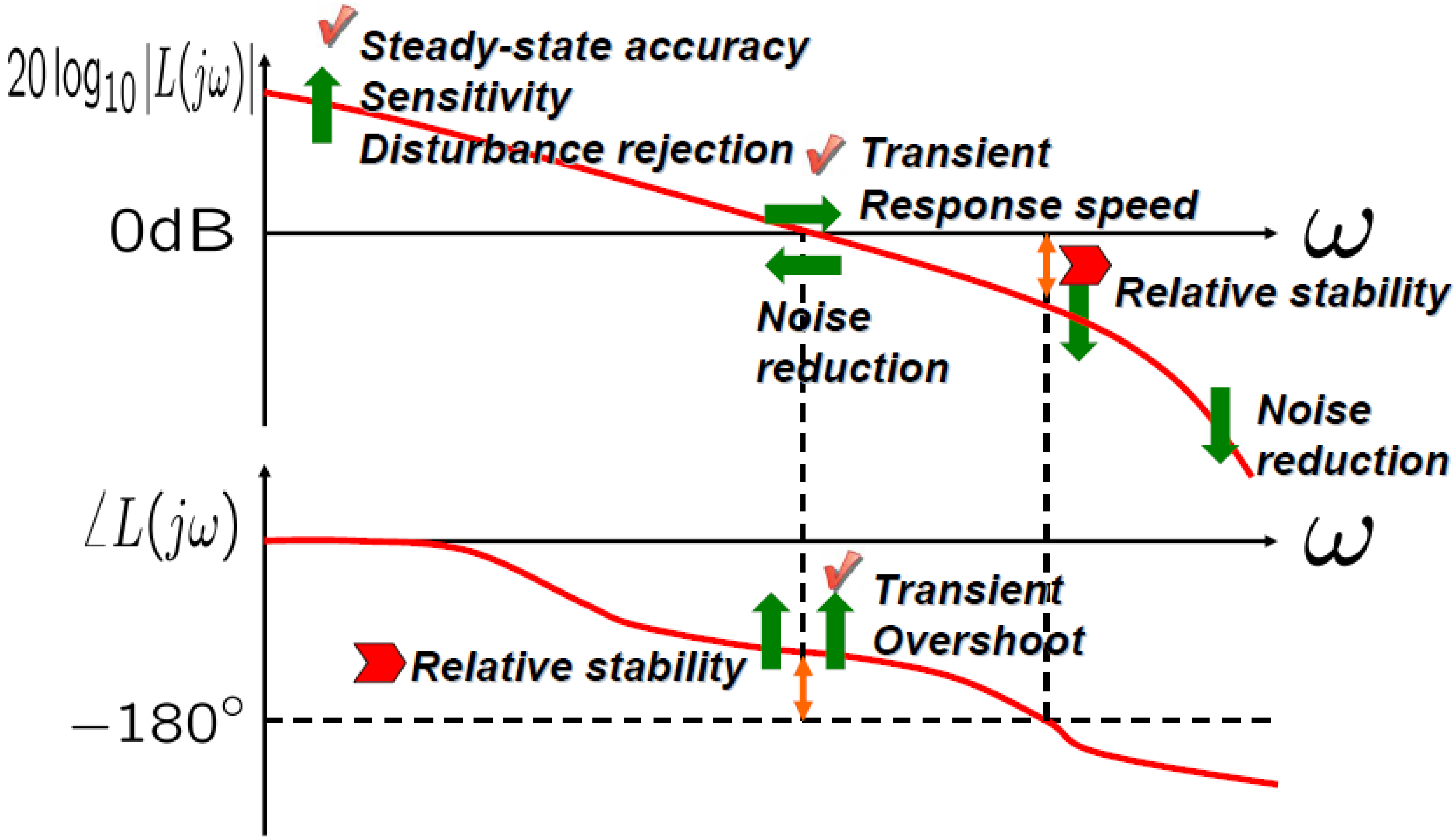


Response speed

For fast response,
L should have larger gain crossover frequency.



Typical desired OL Bode plot

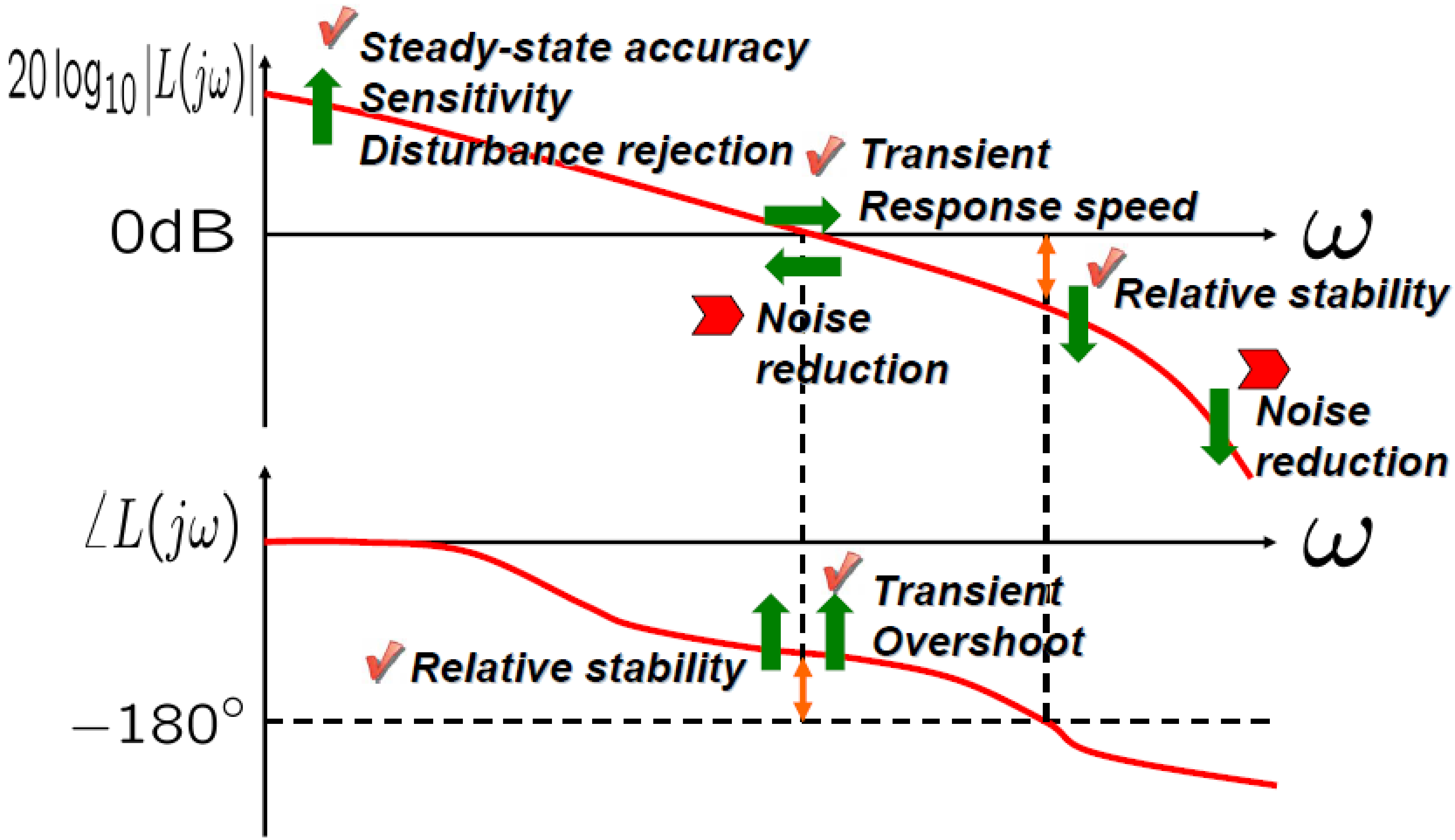


Relative stability

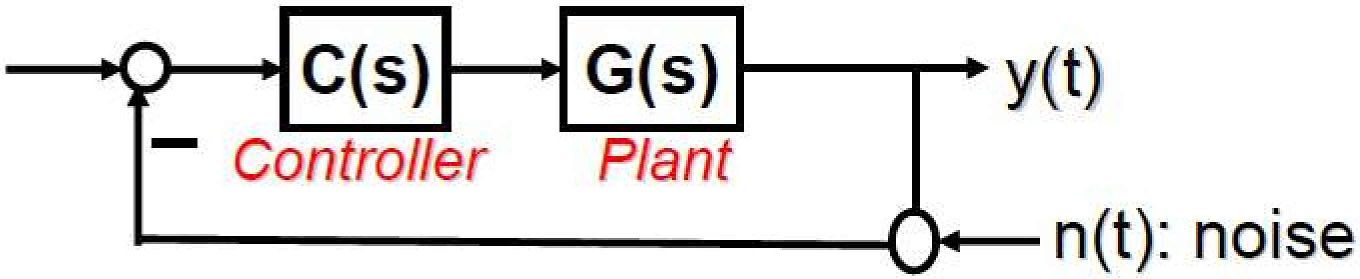
- We require adequate GM and PM for:
  - safety against inaccuracies in modeling
  - reasonable transient response (overshoot)
- It is difficult to give reasonable numbers of GM and PM for general cases, but usually,
  - GM should be at least 6dB
  - PM should be at least 45deg

  (These values are not absolute but approximate!)
- In controller design, we are especially interested in PM (which typically leads to good GM).

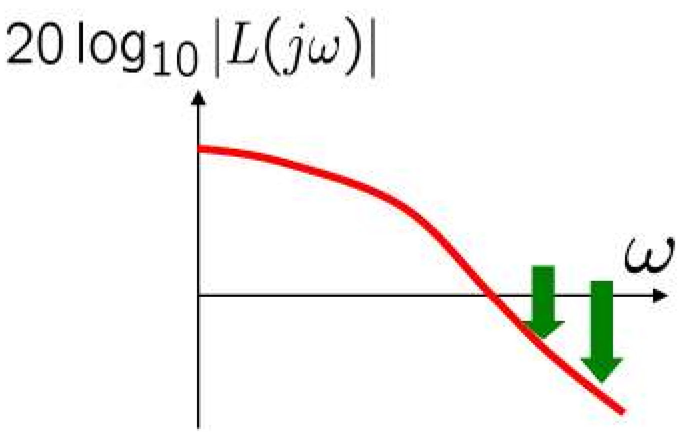
Typical desired OL Bode plot



Noise reduction



For noise reduction,
L should have small gain at high frequencies.

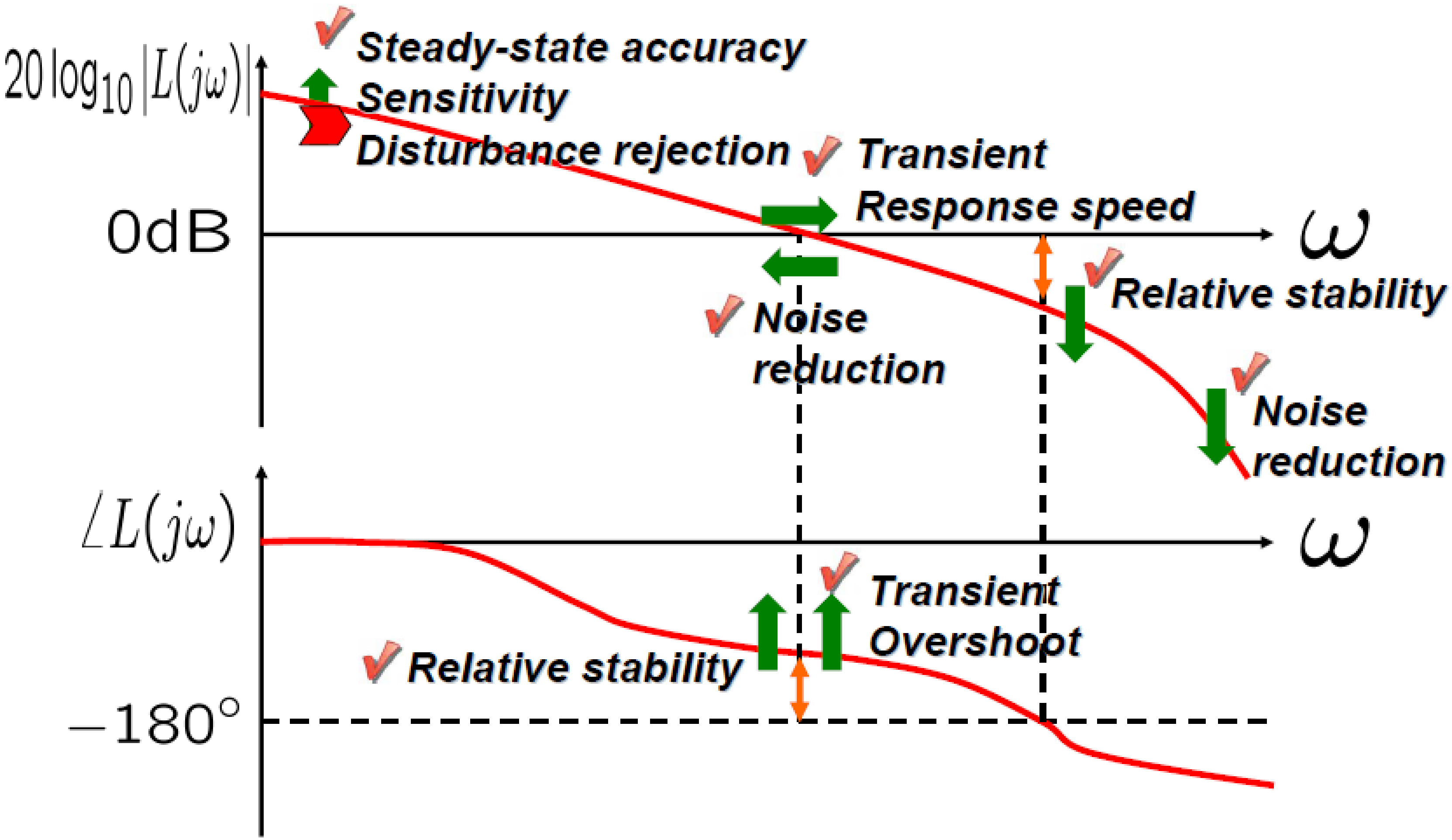


small $|L(j\omega)|$

$$\Rightarrow \frac{Y}{N}(j\omega) = -\frac{L(j\omega)}{1+L(j\omega)} \approx 0$$

$\Rightarrow$ y(t) is not affected by n(t) composed of high frequencies.

Typical desired OL Bode plot

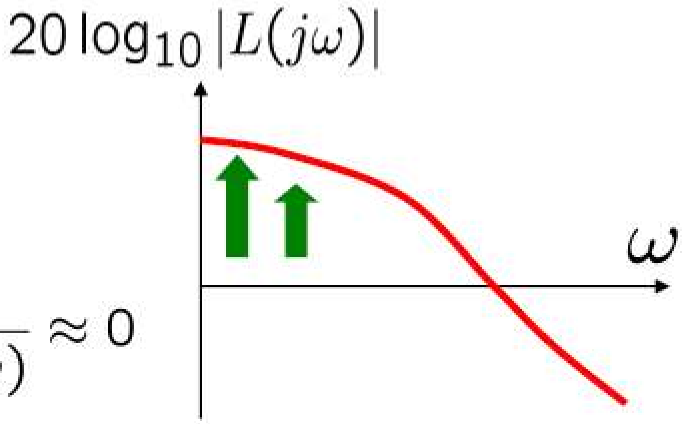


Sensitivity reduction

- Sensitivity indicates the influence of plant variations (due to temperature, humidity, age.) on closed-loop performance.
- Sensitivity function $$S(s) := \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)} = \frac{1}{1+G(s)C(s)} = \frac{1}{1+L(s)}$$

For sensitivity reduction, L should have large gain at low frequencies.

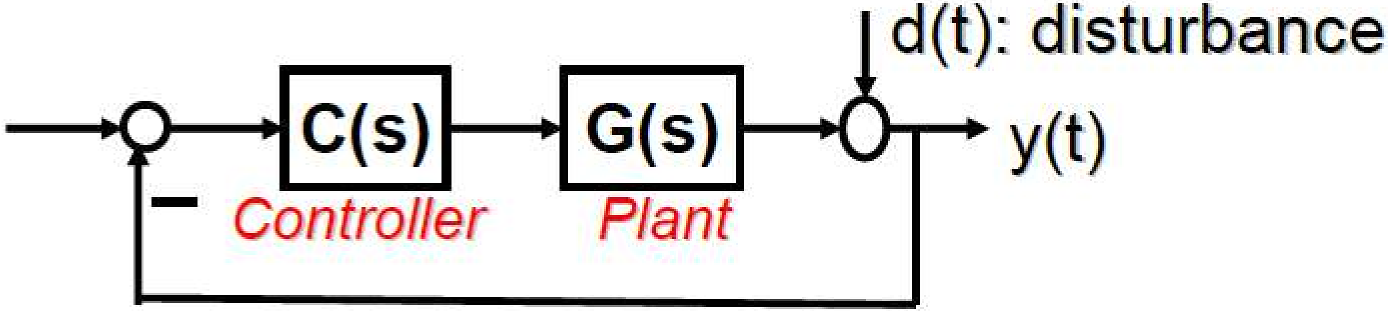
large $|L(j\omega)|$ ⟹ $S(j\omega) = \frac{1}{1+L(j\omega)} \approx 0$



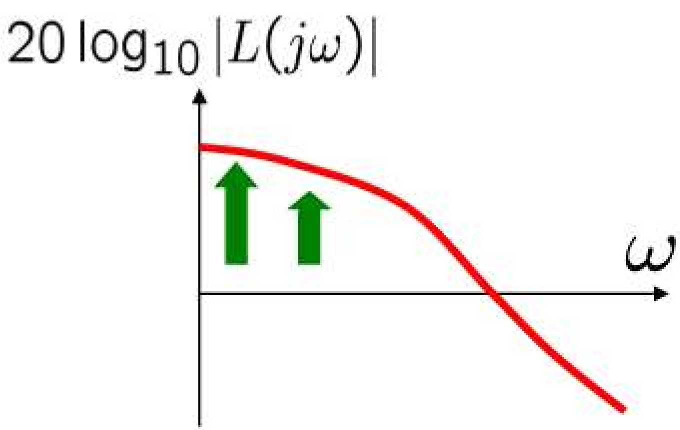
Disturbance

- Unwanted signal
- Examples
  - Wind turbulence in airplane altitude control
  - Wave in ship direction control
  - Sudden temperature change outside the temperature-controlled room
  - Air pressure brake to DC motor
  - Bumpy road in cruise control
- Often, disturbance is neither measurable nor predictable. (Use feedback to compensate it!)

Disturbance rejection



For disturbance rejection,
L should have large gain at low frequencies.

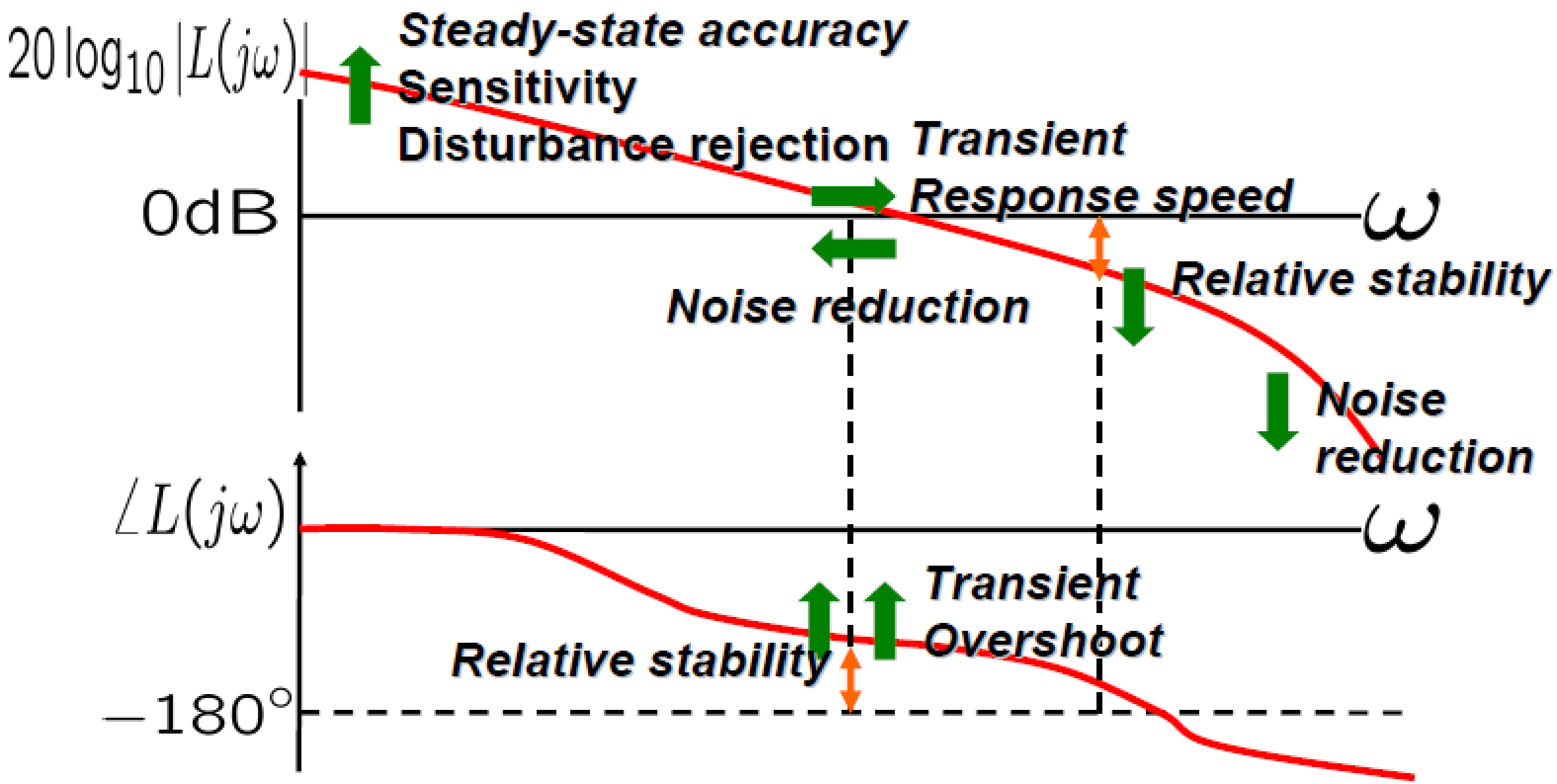


large $|L(j\omega)|$

→ $$\frac{Y}{D}(j\omega) = \frac{1}{1+L(j\omega)} \approx 0$$

→ y(t) is not affected by d(t) composed of low frequencies.

Typical shaping goal (Summary)



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  - Routh-Hurwitz
  - Nyquist

Design

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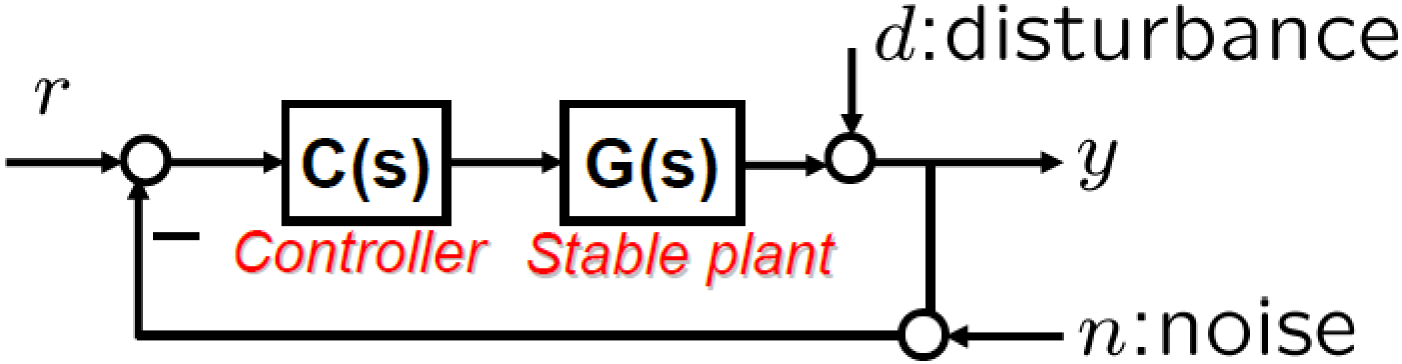
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

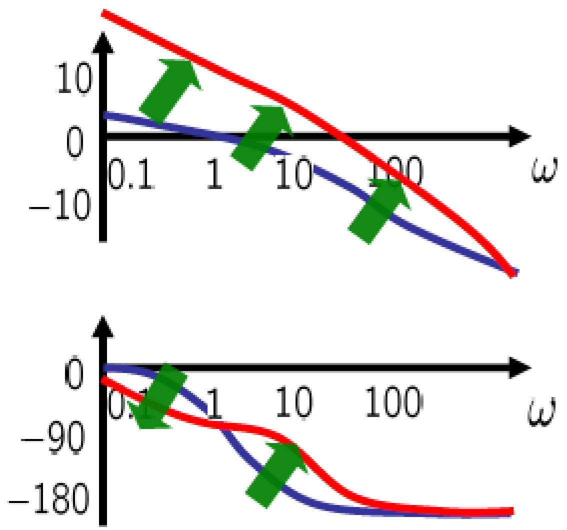
Frequency response shaping (Loop shaping)



- Reshape Bode plot of G(jω) into a “desired” shape of

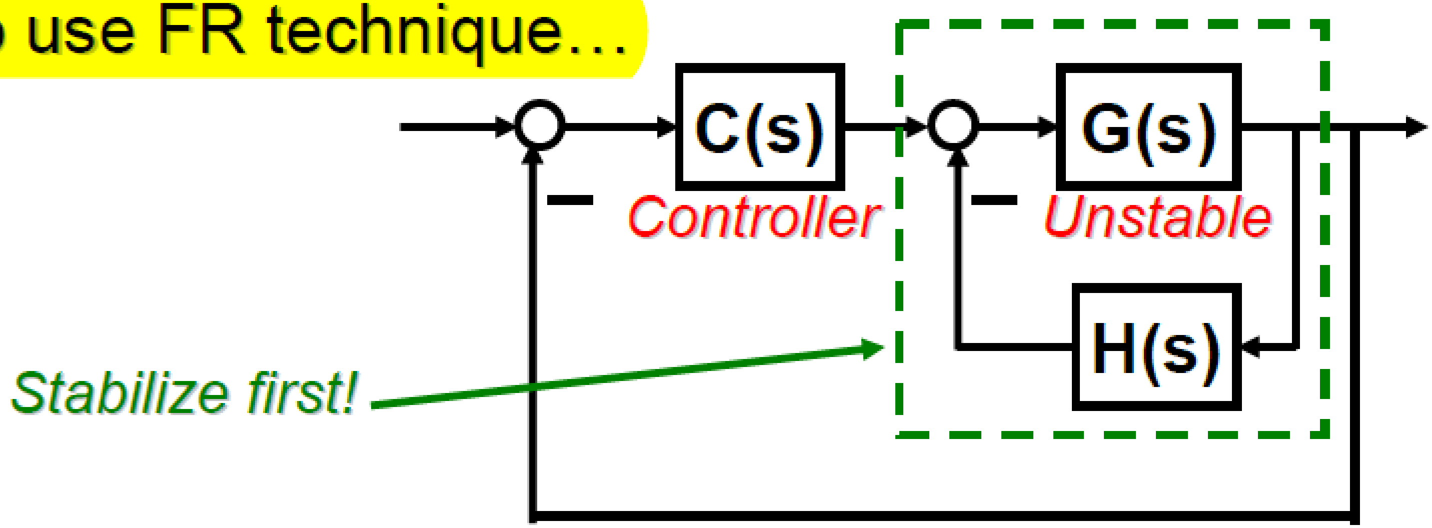
$$L(j\omega) := G(j\omega)C(j\omega)$$

by a series connection of appropriate C(s).



If G(s) has open RHP poles…

- What is problematic?
  - Nyquist stability criterion says that, for closed-loop stability, Nyquist plot of open-loop system must encircle -1 point.
  - It is hard to translate this condition into Bode plot.
- To use FR technique…



How to stabilize? An example

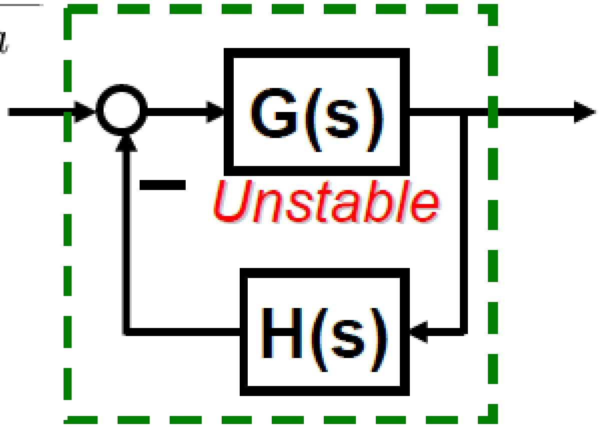
- An unstable plant $G(s) = \frac{1}{s(s-1)}$
- Characteristic eq. $1 + G(s)H(s) = 0$
  - Case 1: Constant H

    $s^2 - s + H = 0$ *Never be stable!*
  - Case 2: 1st order H $H(s) = \frac{b_1 s + b_2}{s + a}$

$$s(s-1)(s+a) + b_1 s + b_2 = 0$$

$$\Rightarrow s^3 + (a-1)s^2 + (b_1 - a)s + b_2 = 0$$

Select controller parameters!



An advantage of Bode plot (review)

- Bode plot of a series connection $G_1(s)G_2(s)$ is the addition of each Bode plot of G_1 and G_2.
  - Gain

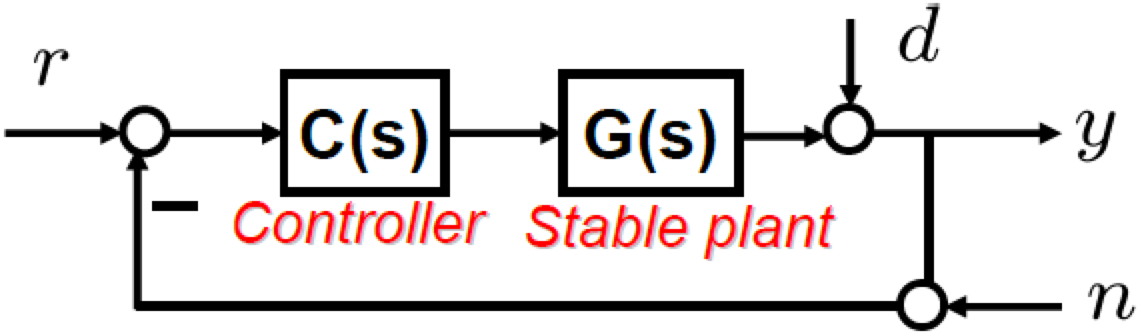
$$20\log_{10}|G_1(j\omega)G_2(j\omega)| = 20\log_{10}|G_1(j\omega)| + 20\log_{10}|G_2(j\omega)|$$

  - Phase

$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

- We use this property to design C(s) so that G(s)C(s) has a "desired" shape of Bode plot.

Simple controllers



- We use simple controllers for shaping.
  - Gain

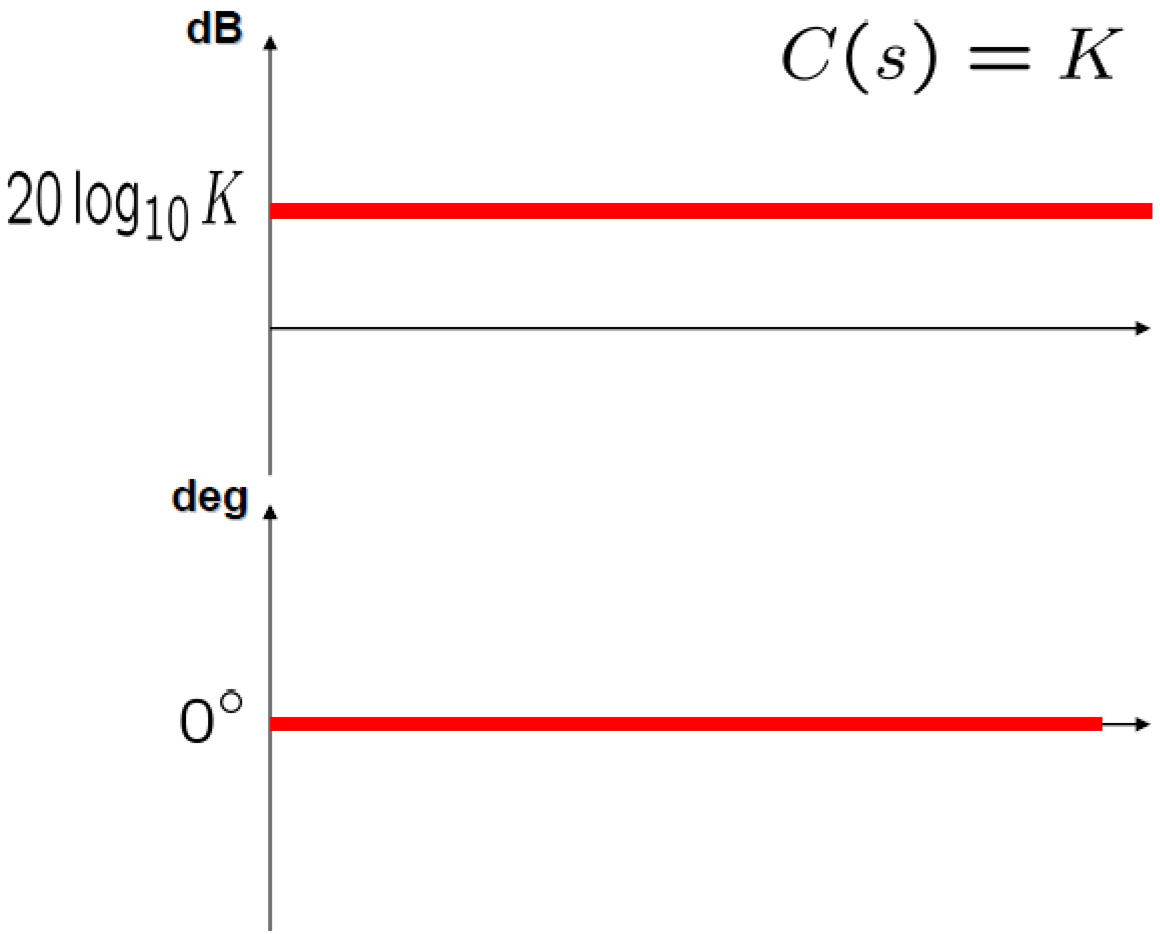
$$C(s) = K$$

  - Lead and lag compensators

$$C(s) = \frac{\text{(1st-order poly.)}}{\text{(1st-order poly.)}} = \frac{\frac{s}{z}+1}{\frac{s}{p}+1}$$

Bode plot of a gain (review)

$$C(s) = K$$

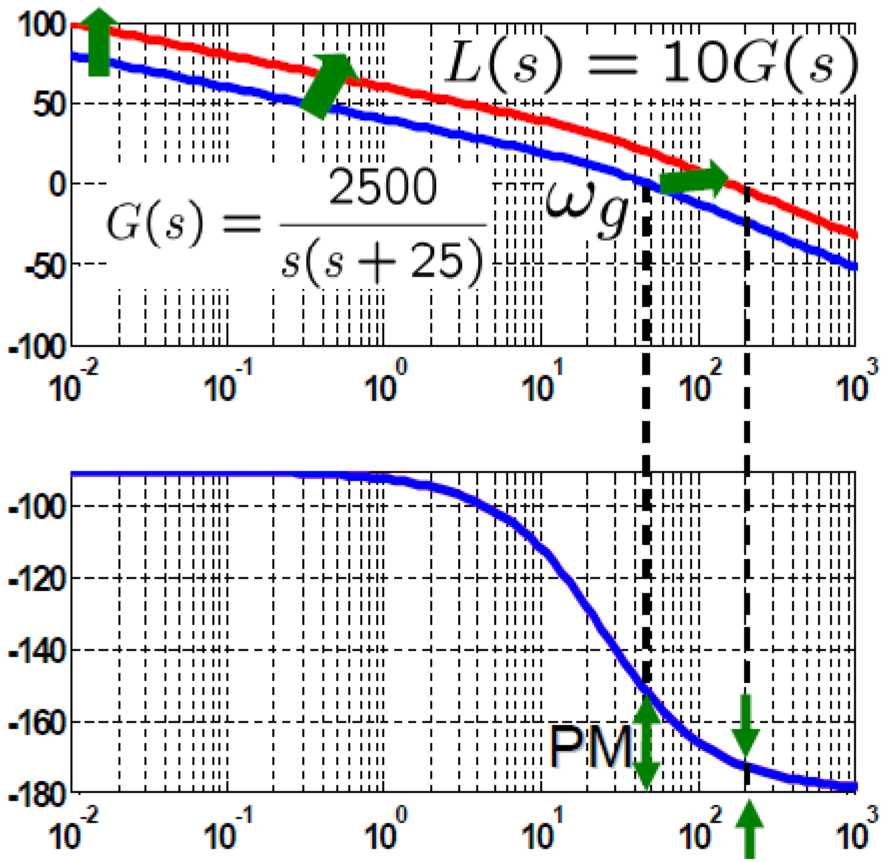


Effect of gain C(s) on L(jω)

$$C(s) = K(>0)$$

In case of K>1,

- Gain increases uniformly, but phase does not change.
- Typically,
  - (Steady state) L(0) ↑
  - (Speed) ω_g ↑
  - (Stability & overshoot) PM ↓

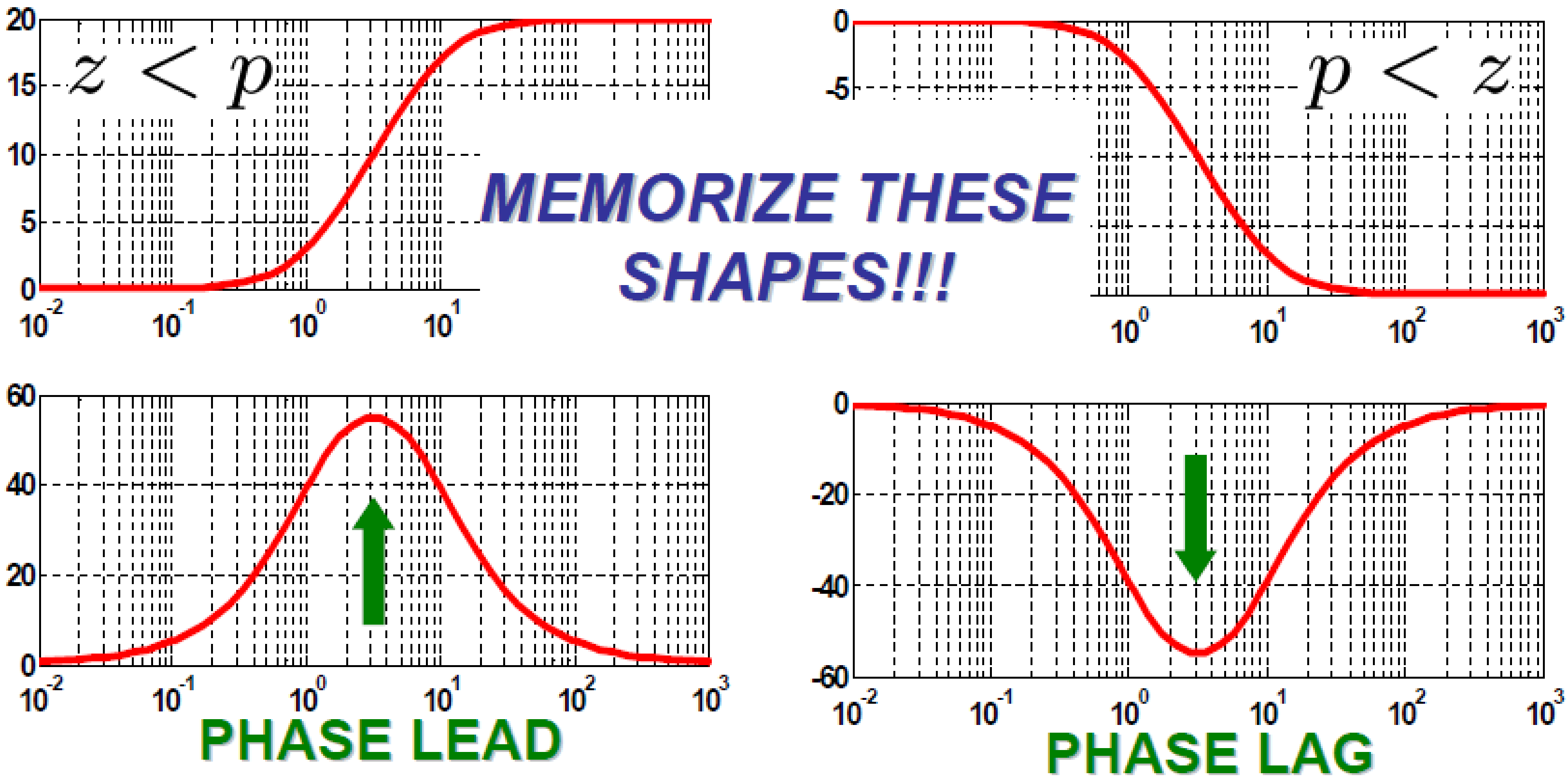


Bode plots of lead and lag C(s)

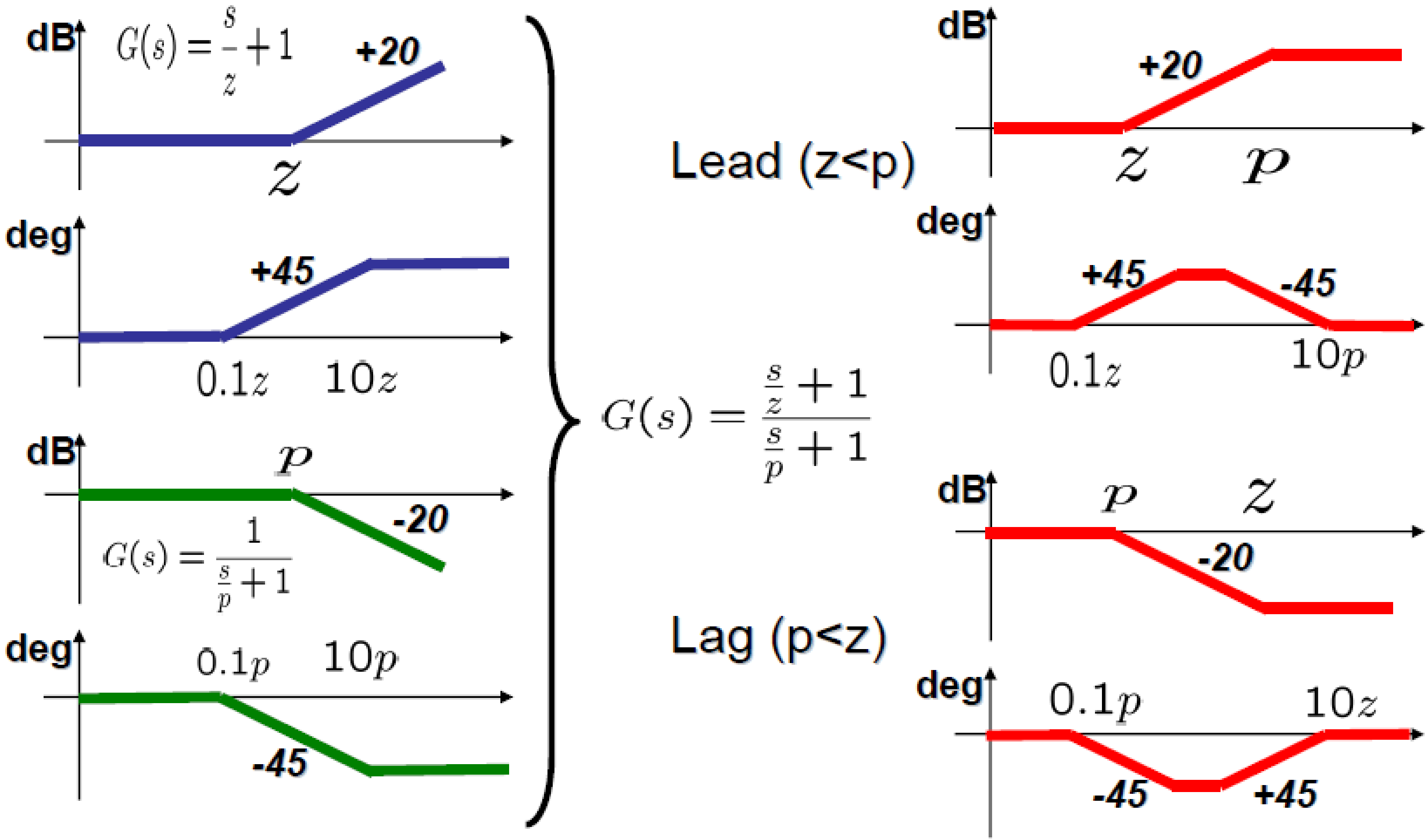
$$C(s) = \frac{\frac{s}{z} + 1}{\frac{s}{p} + 1}$$

Lead compensator

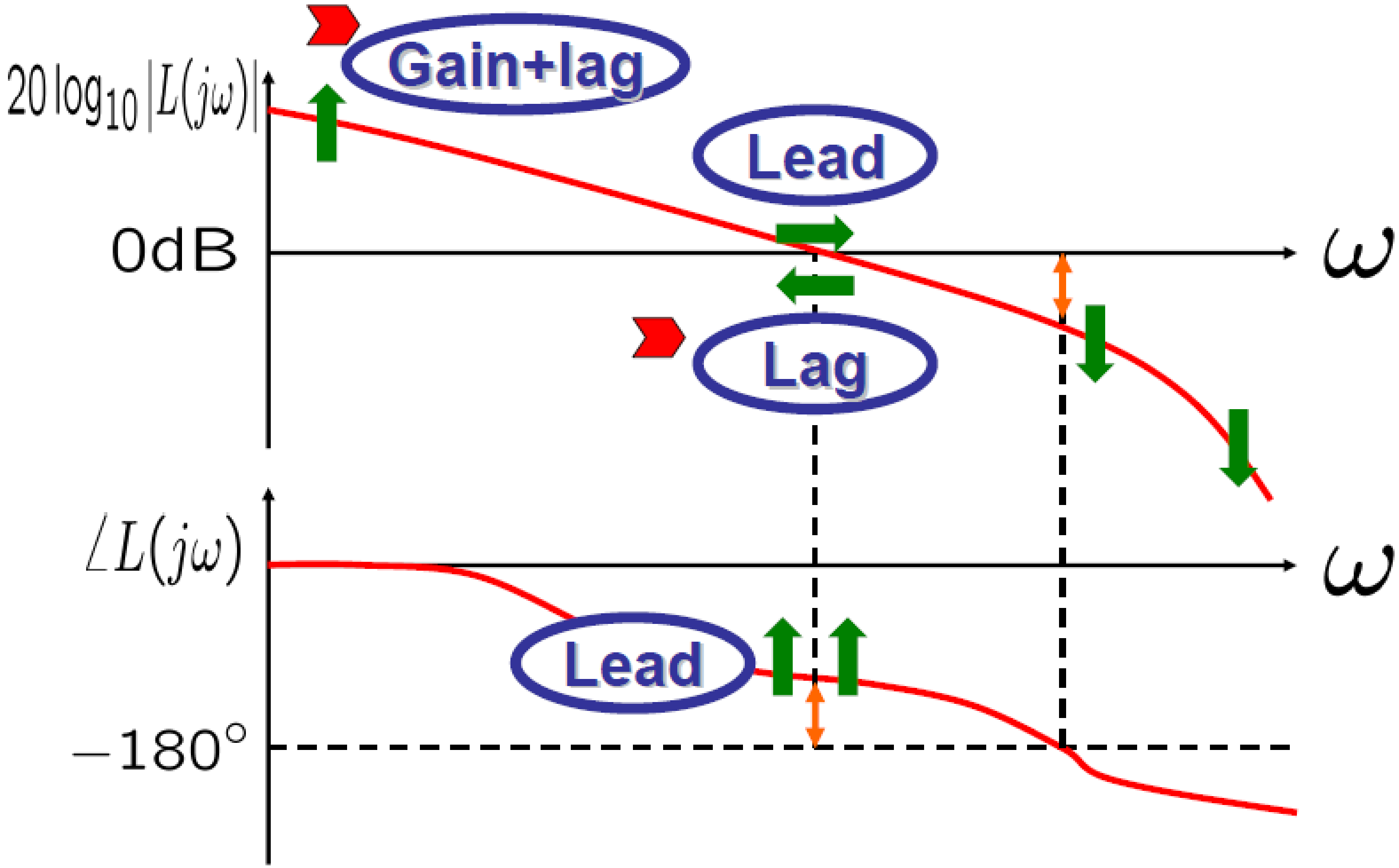
Lag compensator



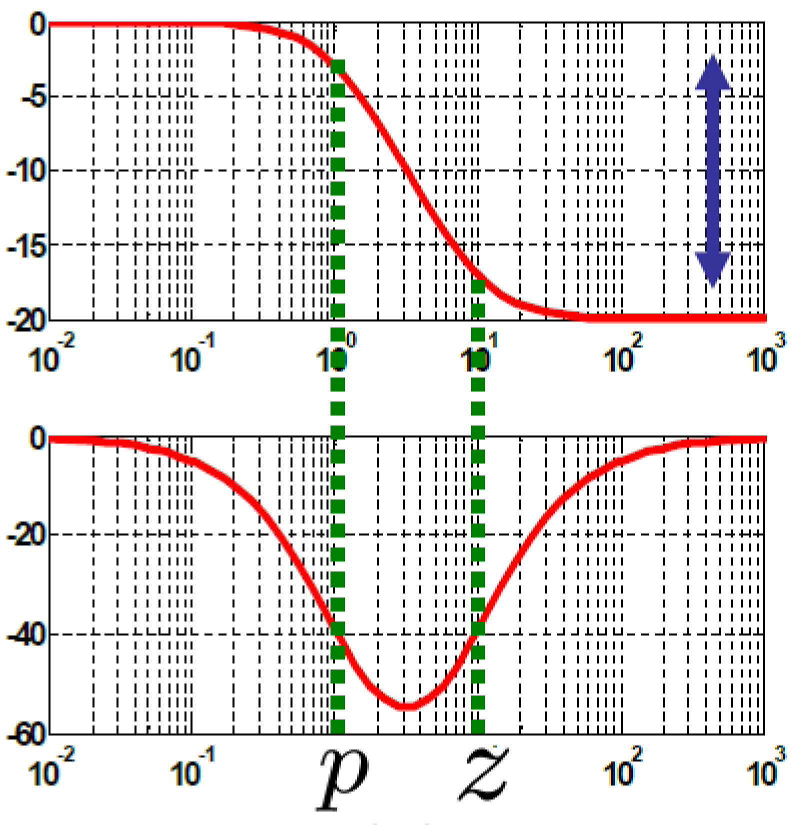
Straight-line approximations



Guideline of lead/lag design



Effect of a lag C(s) on L(jω)



$20 \log_{10} \frac{z}{p}$

- ***Decreasing*** ω_g

Destabilizing effect

Select z much (at least 1 decade) less than ω_g

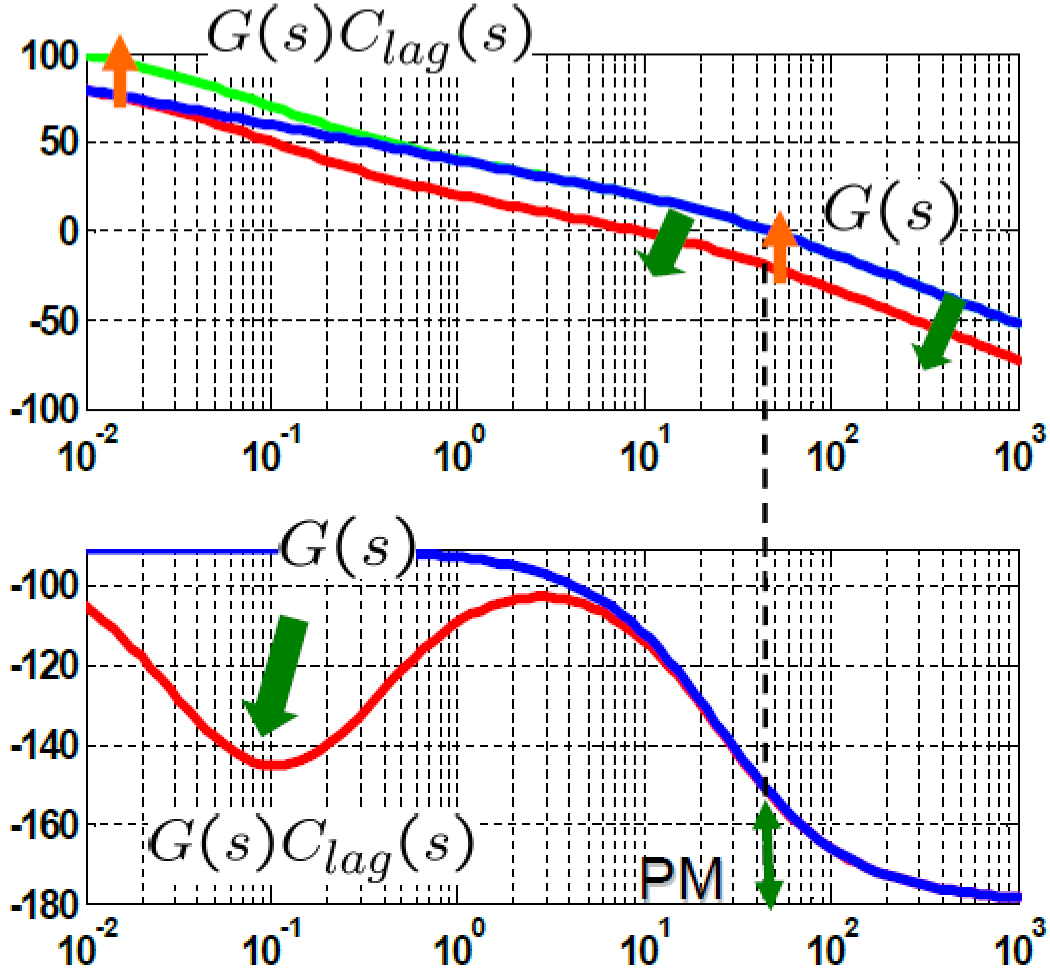
Lag+gain C(s) design

$$G(s) = \frac{2500}{s(s+25)}$$

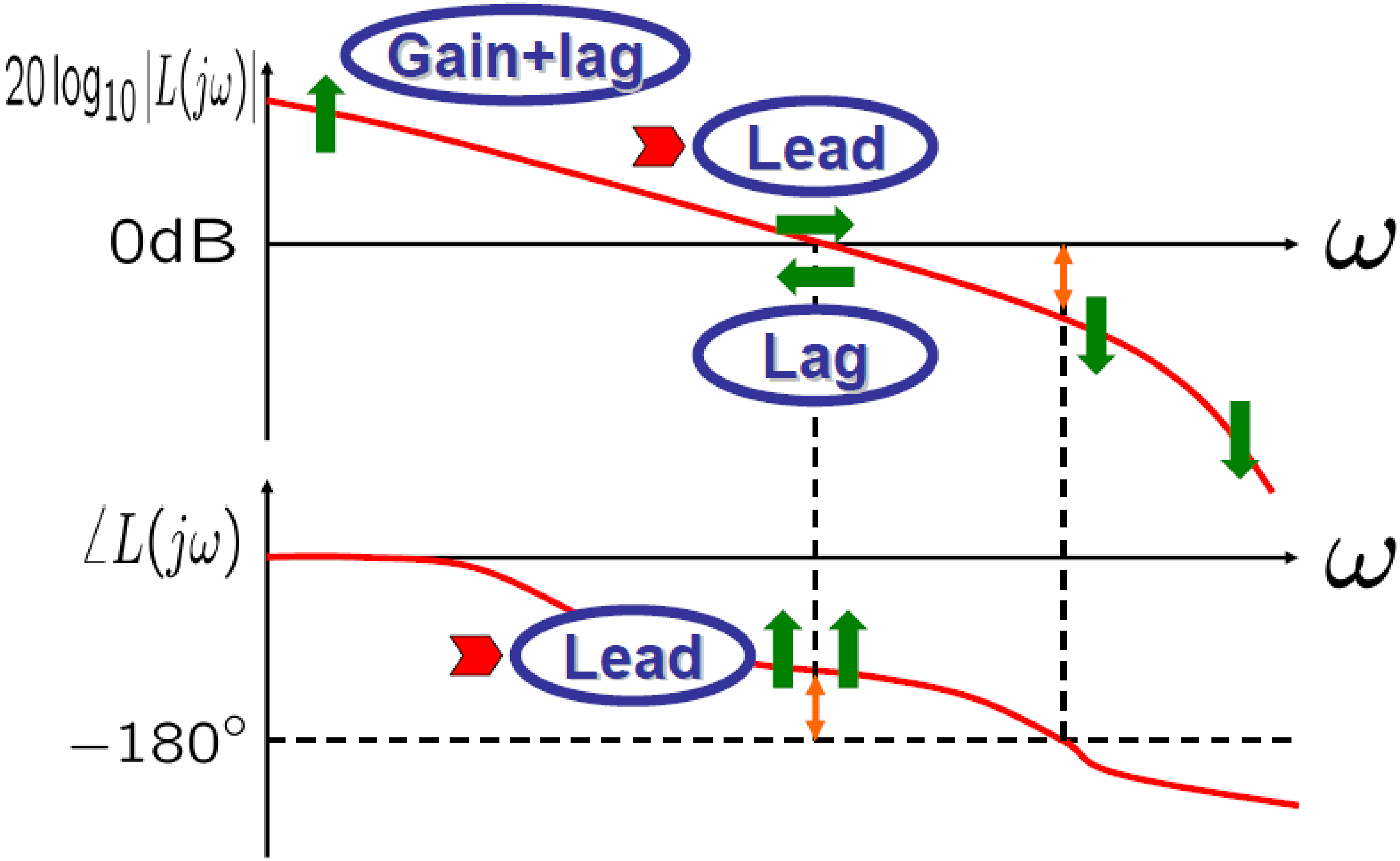
PM: 28 deg at
ωg=47 rad/s

$$C_{lag}(s) = 10\,\frac{\frac{s}{1/3}+1}{\frac{s}{1/30}+1}$$

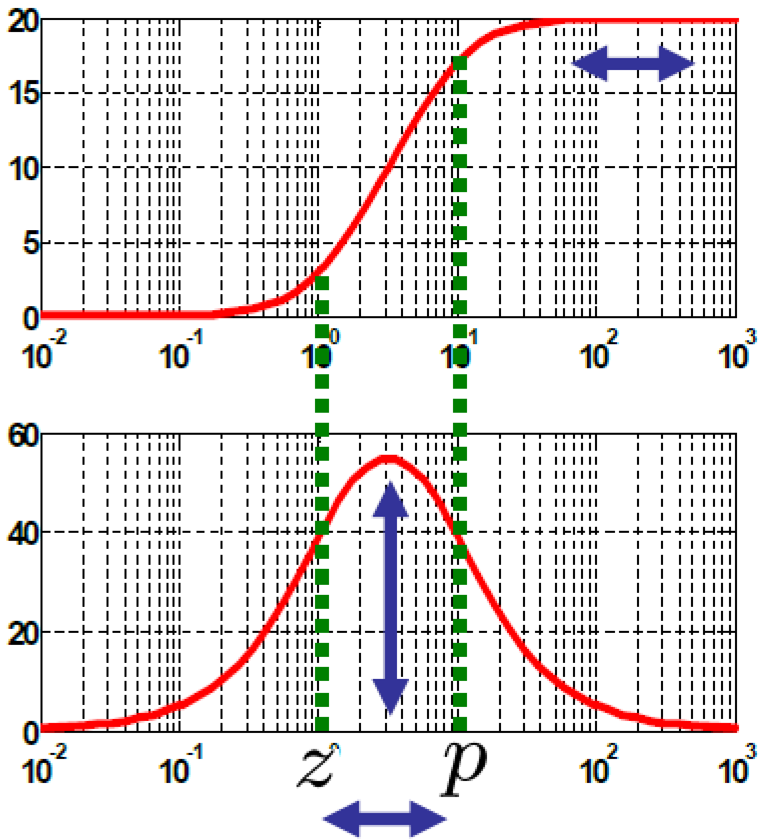
PM: 27 deg at
ωg=47 rad/s



Guideline of lead/lag design



Effect of a lead C(s) on L(jω)



Increasing ω_g

Stabilizing effect

Select z&p around ω_g

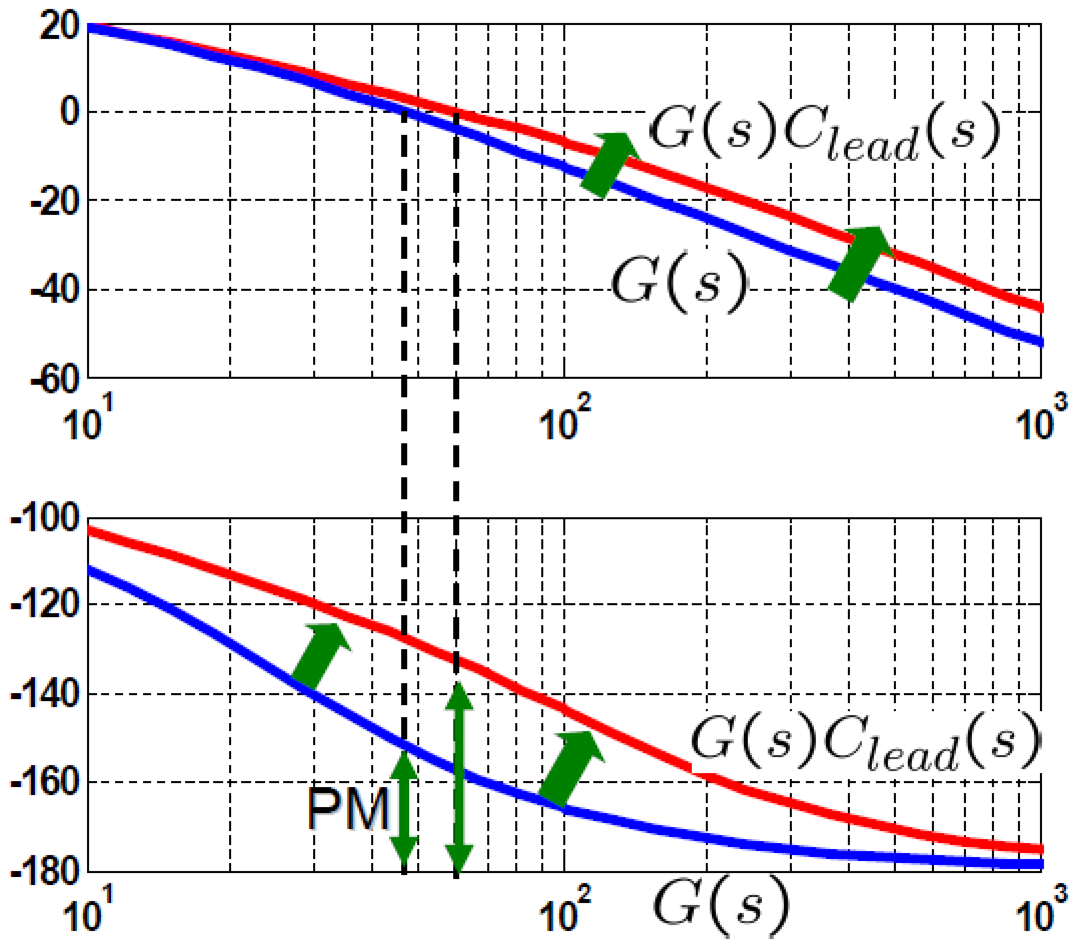
Example of a lead C(s) design

$$G(s) = \frac{2500}{s(s+25)}$$

PM: 28 deg at
ωg=47 rad/s

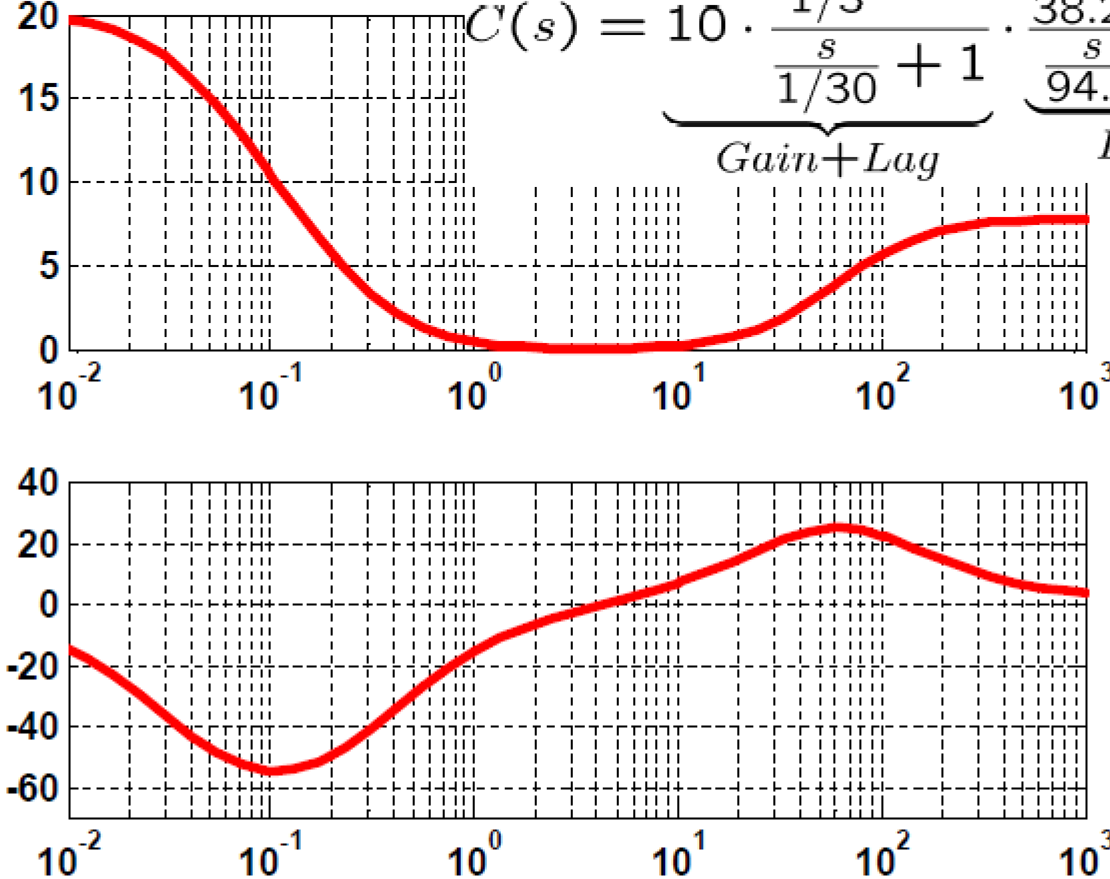
$$C_{lead}(s) = \frac{\frac{s}{38.21}+1}{\frac{s}{94.1}+1}$$

PM: 47 deg at
ωg=60 rad/s



Lead-lag compensator

$$C(s) = \underbrace{10 \cdot \frac{\frac{s}{1/3}+1}{\frac{s}{1/30}+1}}_{Gain+Lag} \cdot \underbrace{\frac{\frac{s}{38.21}+1}{\frac{s}{94.1}+1}}_{Lead}$$



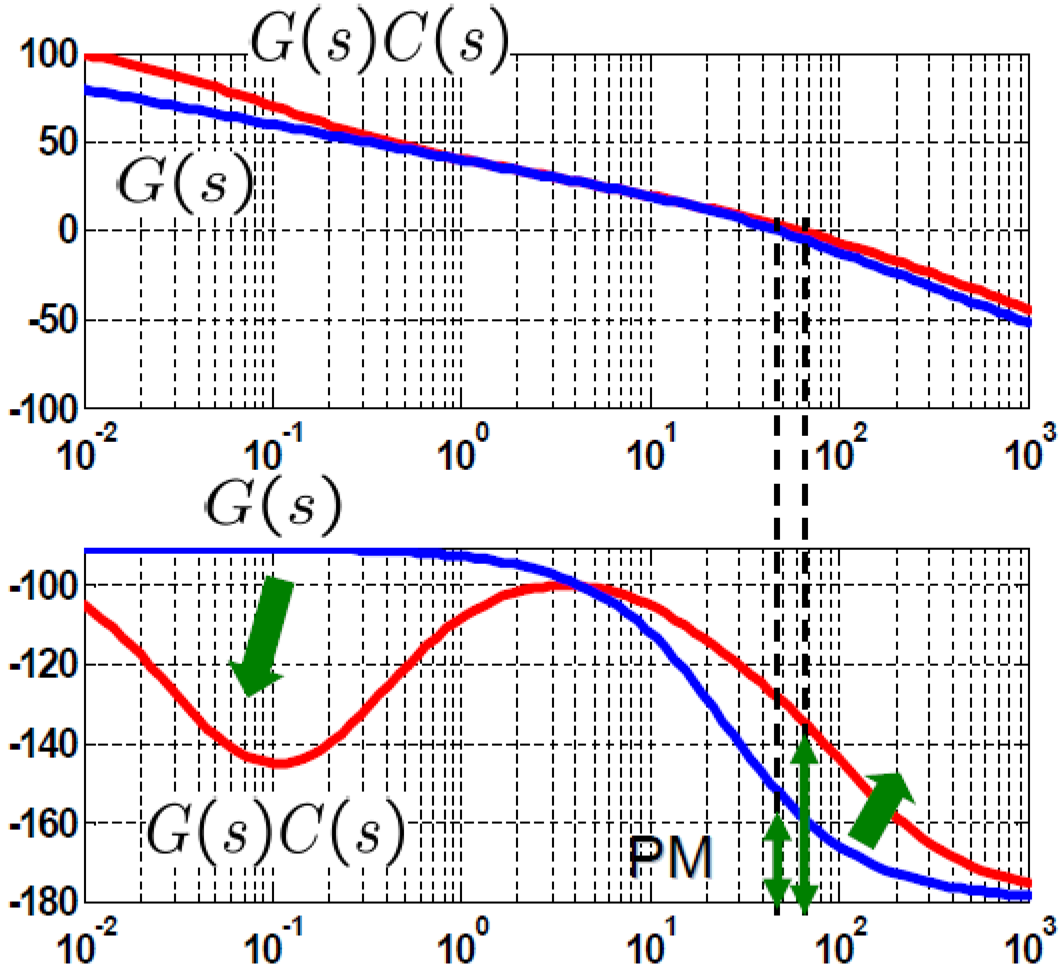
Example of a lead-lag C(s) design

$$G(s) = \frac{2500}{s(s+25)}$$

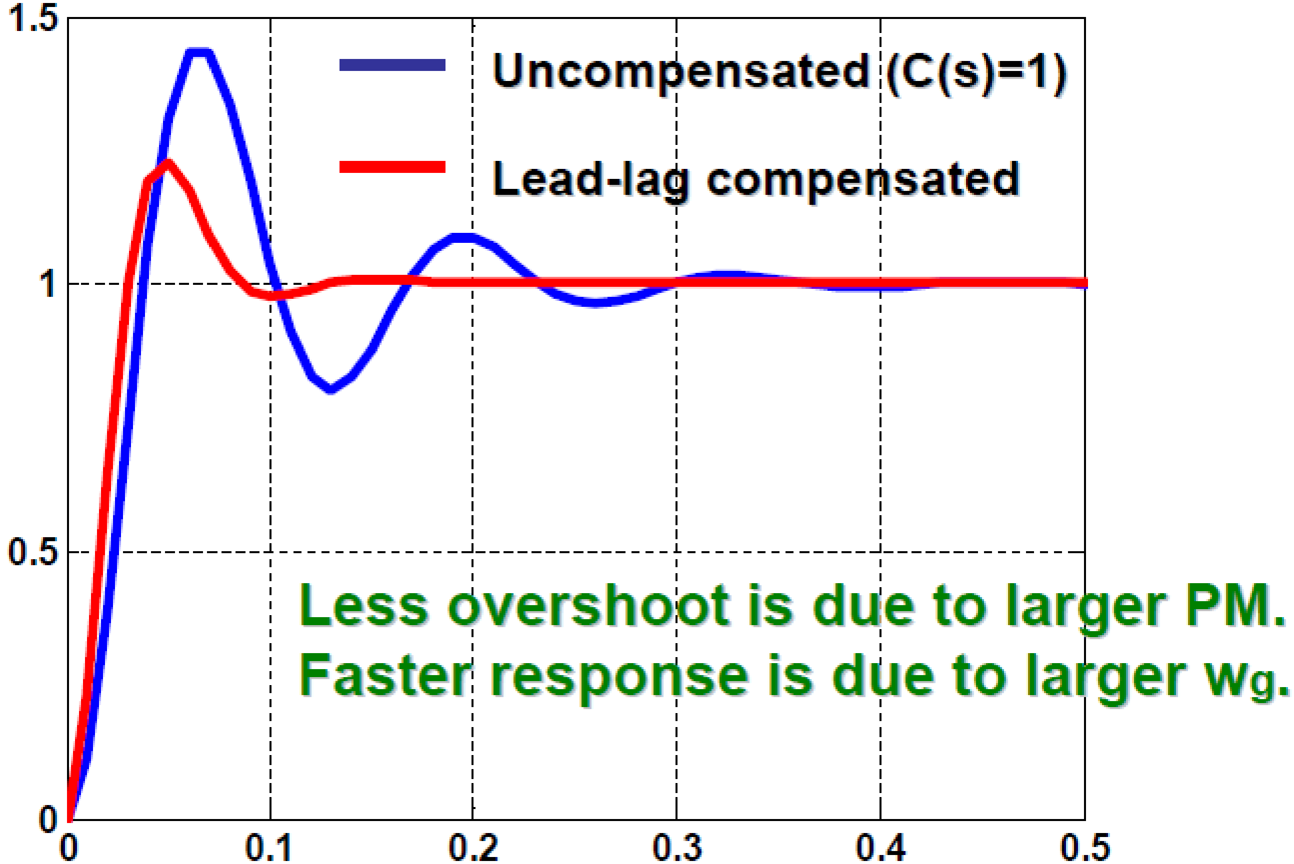
PM: 28 deg at
ωg=47 rad/s

$$C(s) = C_{lead}(s)C_{lag}(s)$$

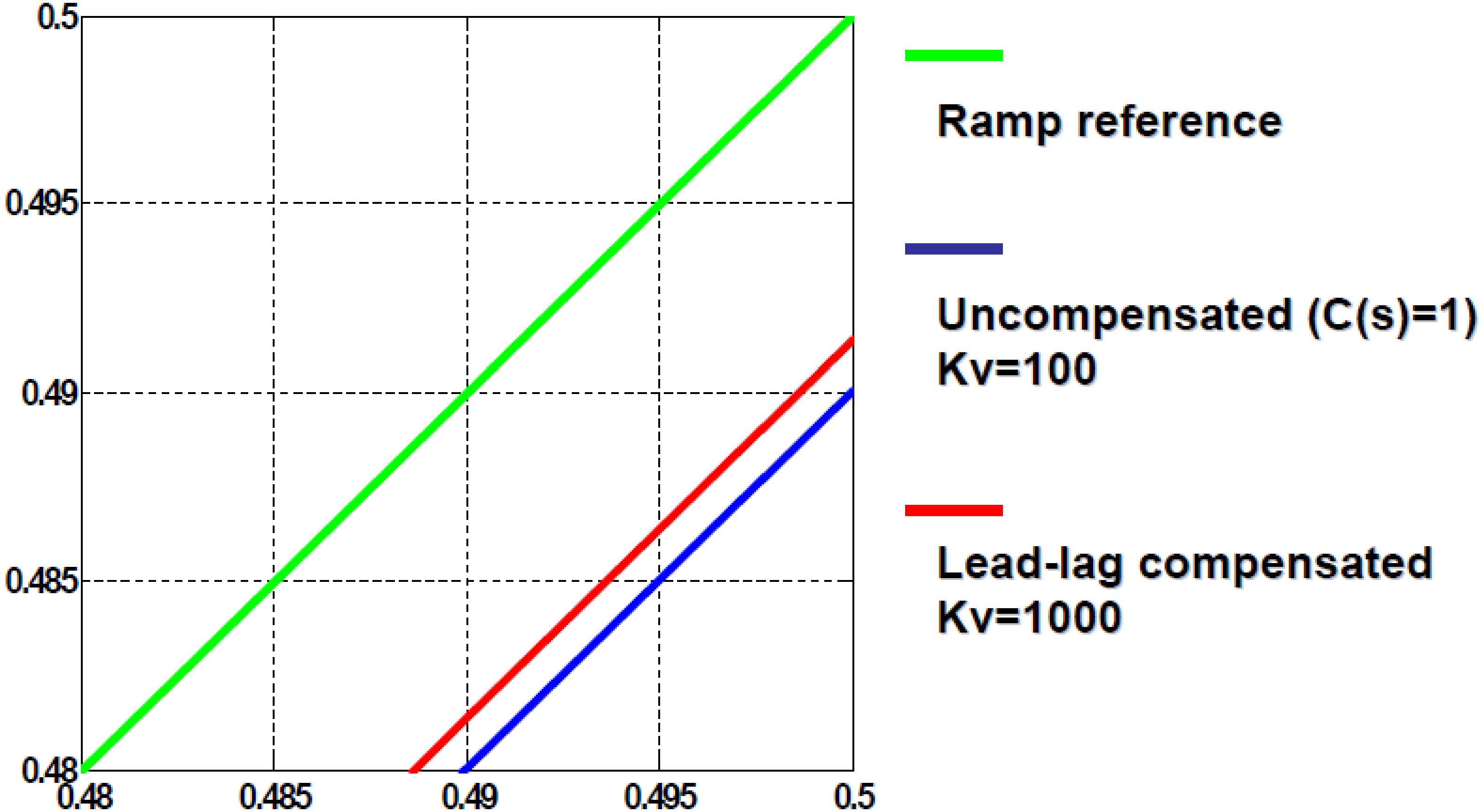
PM: 47 deg at
ωg=60 rad/s



Step responses



Ramp responses



Smaller steady-state error is due to larger Kv.

Summary

- Examples of gain margin and phase margin
  - Gain margin, phase crossover frequency
  - Phase margin, gain crossover frequency
- Frequency domain specifications
- Controller design in Bode plot
  - Gain controller changes gain uniformly over frequencies, but phase does not change
  - Lag compensator can be used for improving
    - PM by maintaining low freq. gain or
    - Low freq. gain by maintaining PM
  - Lead compensator can be used for improving
    - Gain crossover frequency ω_g and PM, by maintaining low frequency gain.

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- Next
  - Frequency domain lead-lag compensator design
  - Frequency response shaping in Matlab

End of Lecture 10