ME 8043114 Automatic Control

Lecture 9: Nyquist Stability Criterion

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Lecture Outline

- Nyquist Stability Criterion

Course roadmap

Modeling

- ✓ Laplace transform
- ✓ Transfer function
- ✓ Models for systems
  - ✓ electrical
  - ✓ mechanical
  - ✓ electromechanical
- ✓ Linearization, delay

Analysis

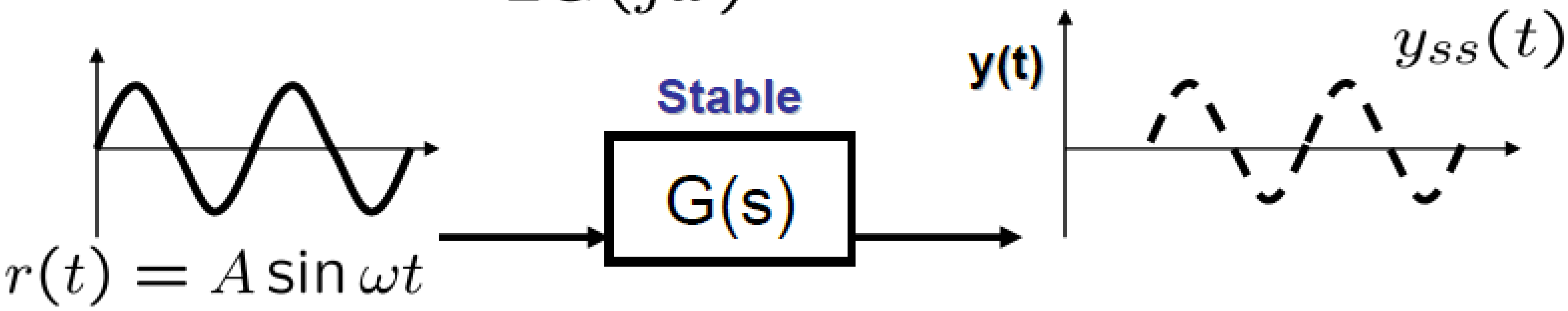
- ✓ Time response
  - ✓ Transient
  - ✓ Steady state
- ✓ Frequency response
  - ✓ Bode plot
- ✓ Stability
  - ✓ Routh-Hurwitz
  - ➤ Nyquist

Design

- Design specs
- ✓ Root locus
- Frequency domain
- ✓ PID & Lead-lag
- Design examples

Frequency response (review)

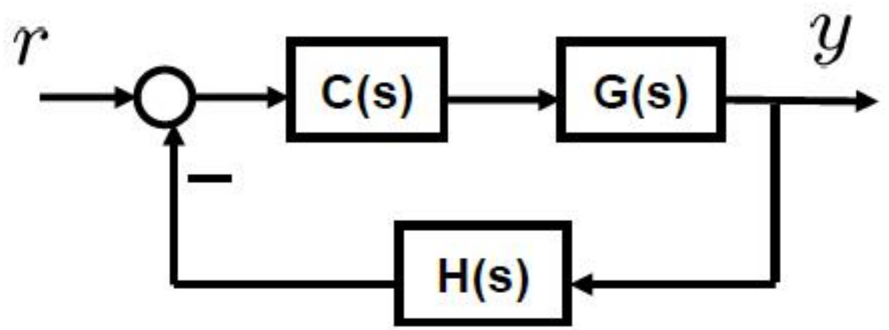
- Steady state output $y_{ss}(t) = A|G(j\omega)|\sin(\omega t + \angle G(j\omega))$
  - Frequency is same as the input frequency ω
  - Amplitude is that of input (A) multiplied by $|G(j\omega)|$ Gain
  - Phase shifts $\angle G(j\omega)$



- Frequency response function (FRF): G(jω)
- *Open-loop FRF contains closed-loop stability info.*

Stability of feedback system

- Consider the feedback system



- Fundamental questions
  - If G and C and H are stable, is closed-loop system *always stable*?
  - If G and C and H are unstable, is closed-loop system *always unstable*?

Closed-loop stability criterion

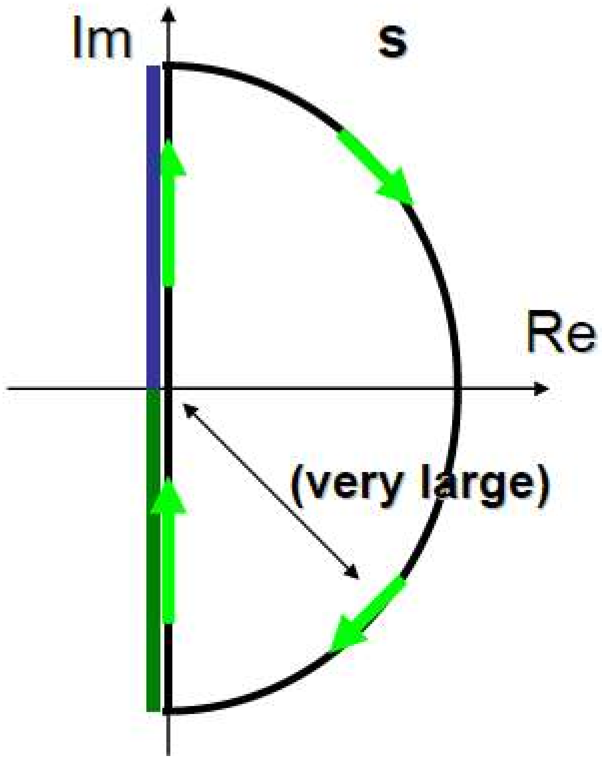
- Closed-loop stability can be determined by the roots of the characteristic equation

$$1 + L(s) = 0, \;\; L(s) := G(s)C(s)H(s)$$

- CL system is stable if the Ch. Eq. has all roots in the open left half plane.
- How to check the stability?
  - Compute all the roots.
  - Routh-Hurwitz stability criterion
  - *Nyquist stability criterion*

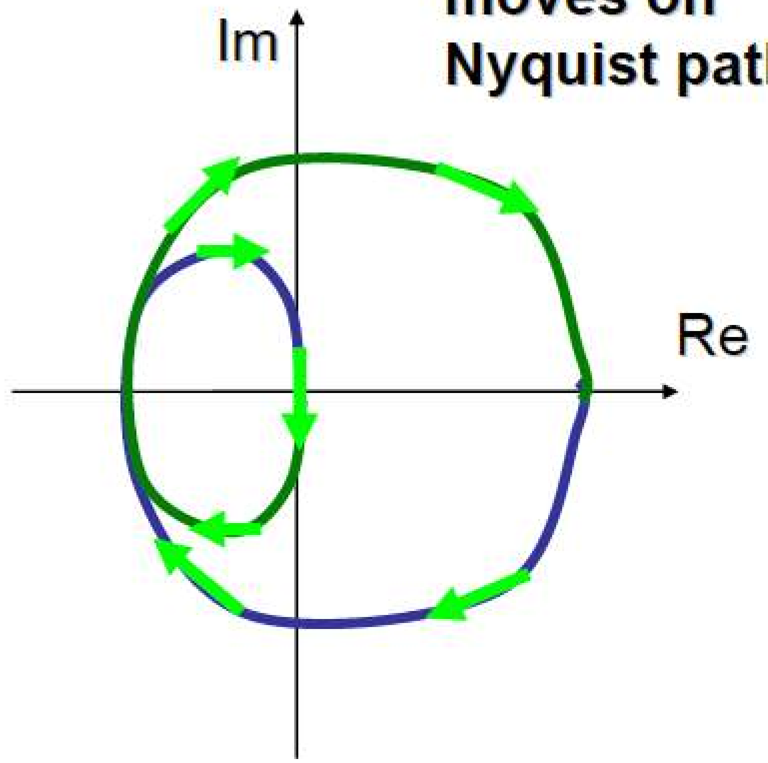
Nyquist plot

- Nyquist path



- Nyquist plot

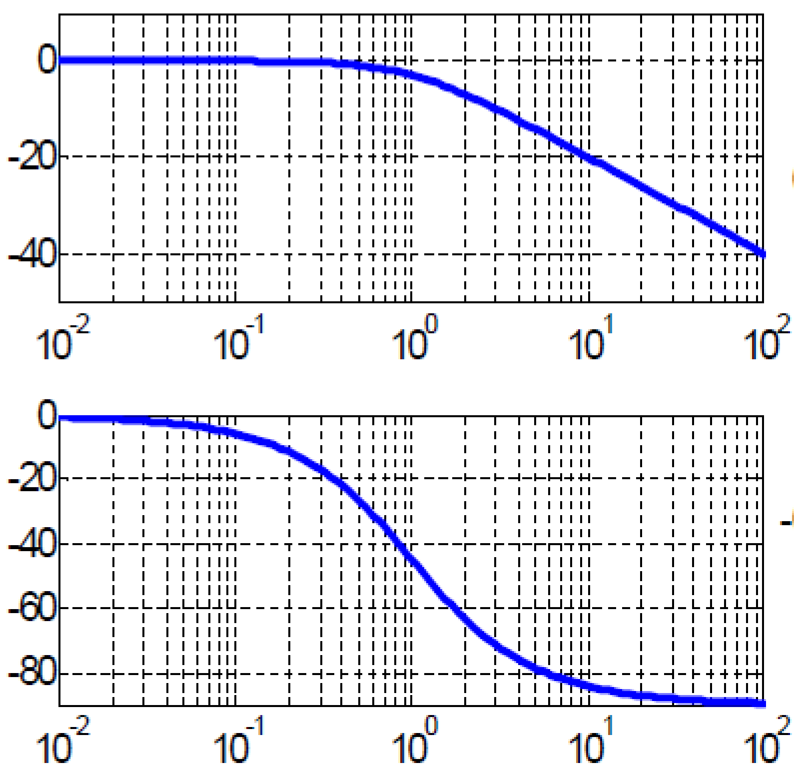
L(s) when s moves on Nyquist path



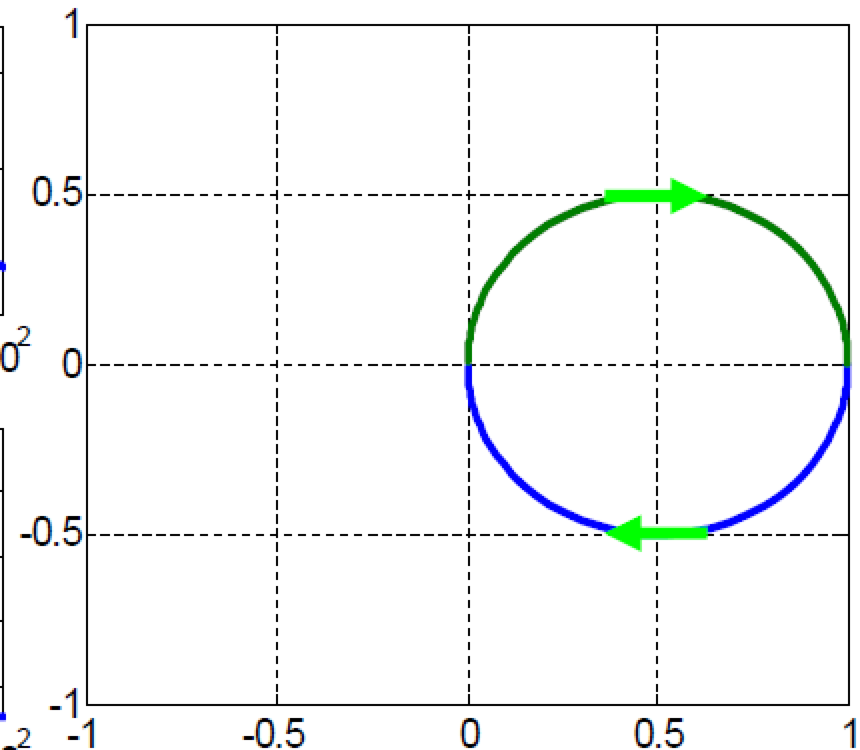
Example of Bode & Nyquist plots

- First order system $L(s) = \dfrac{1}{s+1}$

Bode plot



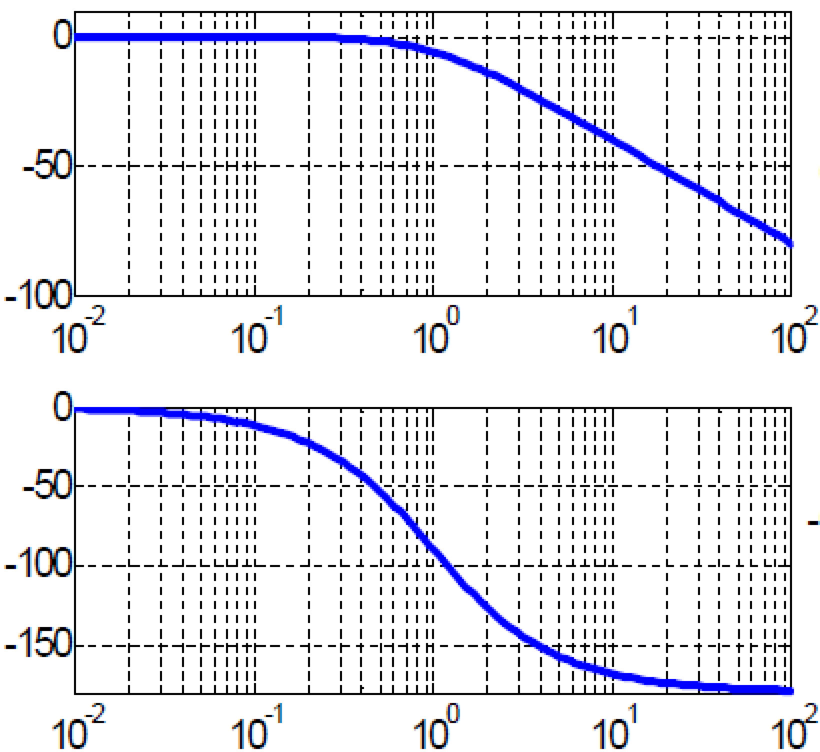
Nyquist plot



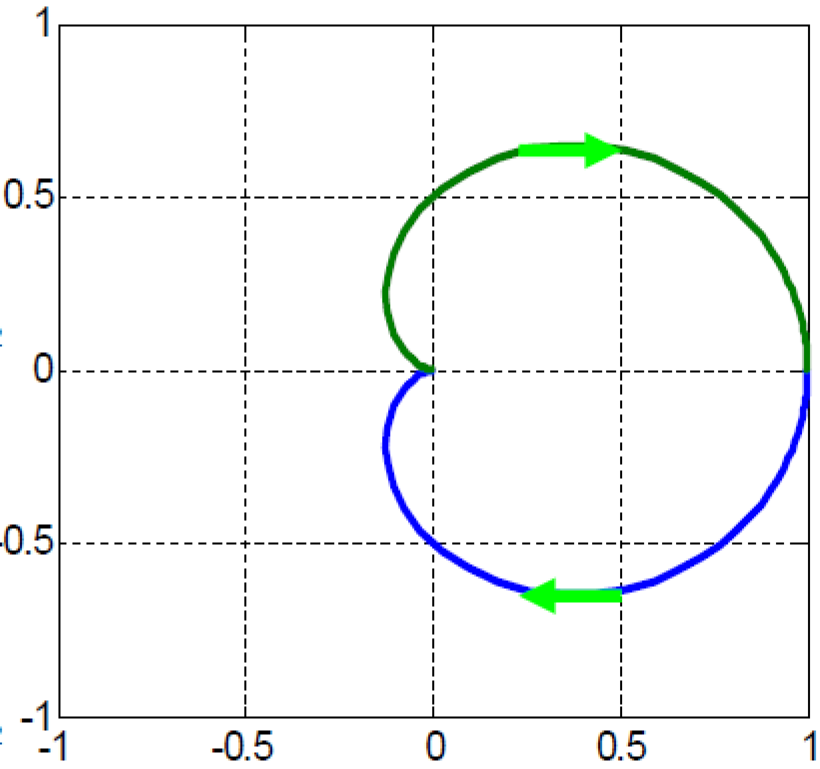
Example of Bode & Nyquist plots

- Second order system $L(s) = \frac{1}{(s+1)^2}$

Bode plot



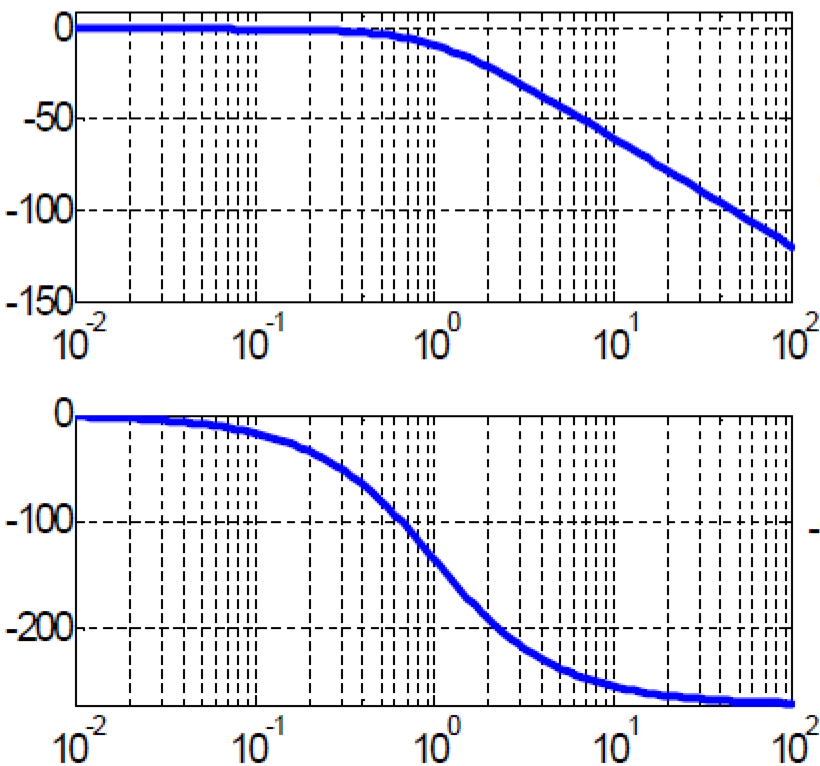
Nyquist plot



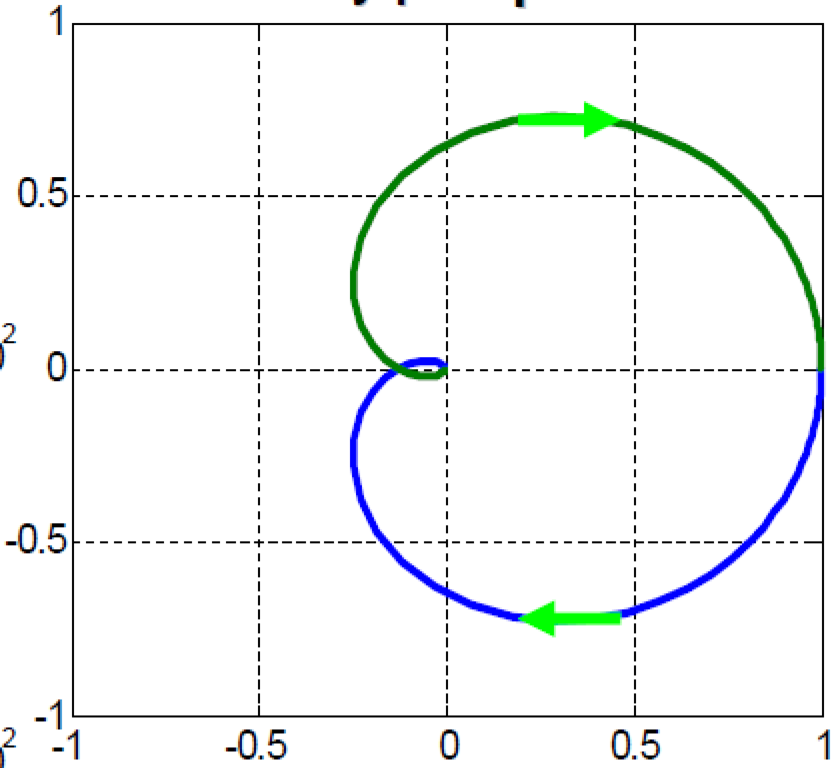
Example of Bode & Nyquist plots

- Third order system $L(s) = \dfrac{1}{(s+1)^3}$

Bode plot



Nyquist plot



Nyquist stability criterion

$$\text{CL system is stable} \Leftrightarrow Z := P + N = 0$$

- Z: # of CL poles in open RHP
- P: # of OL poles in open RHP (given)
- N: # of clockwise encirclement of -1

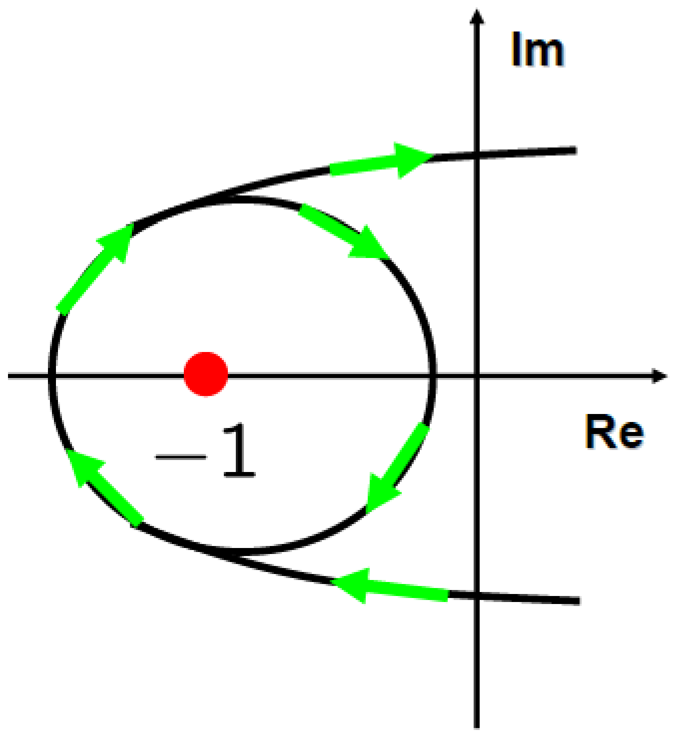
  by Nyquist plot of OL transfer function L(s)

  (counted by using Nyquist plot of L(s))

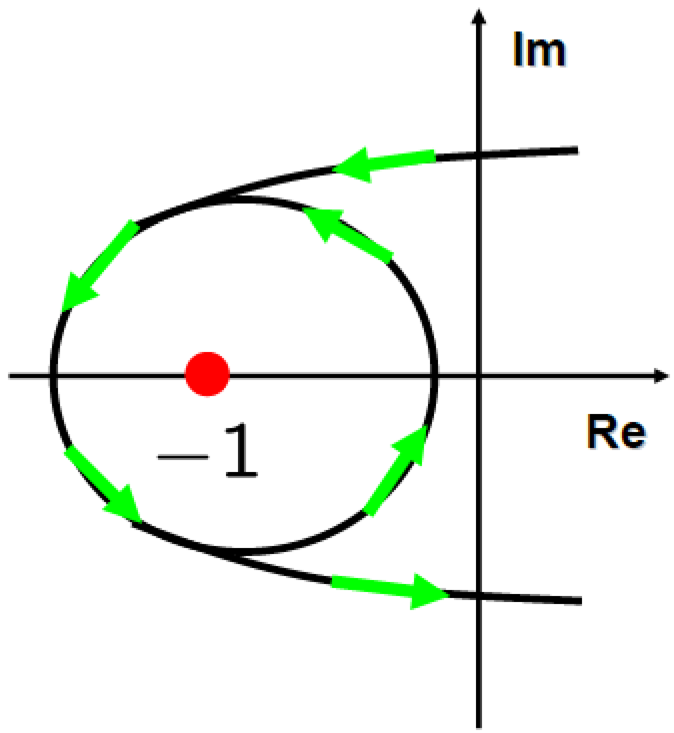
Remark: N=-1: a counter-clockwise encirclement

Encirclements in Nyquist plot

- Clockwise

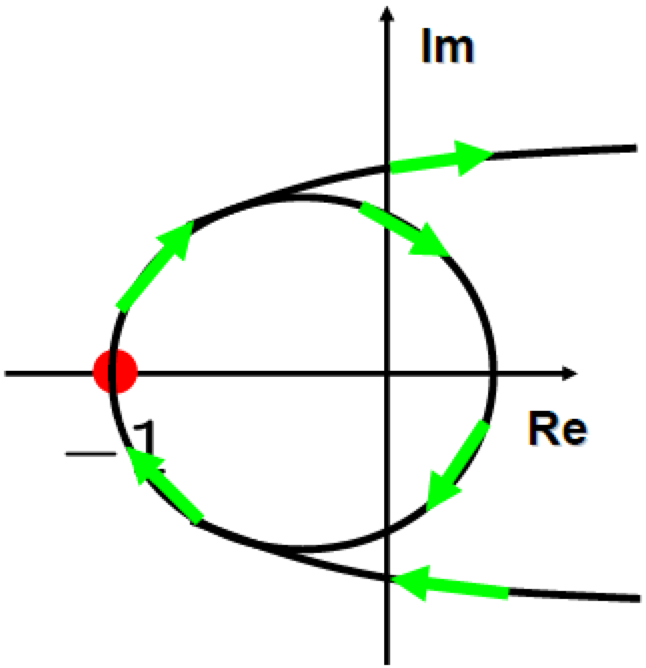


- Counter-clockwise



Remark

- If Nyquist plot passes the point -1, it means that the closed-loop system has a pole on the imaginary axis (and thus, not stable).



$$L(j\omega_0) = -1 \text{ for some } \omega_0$$

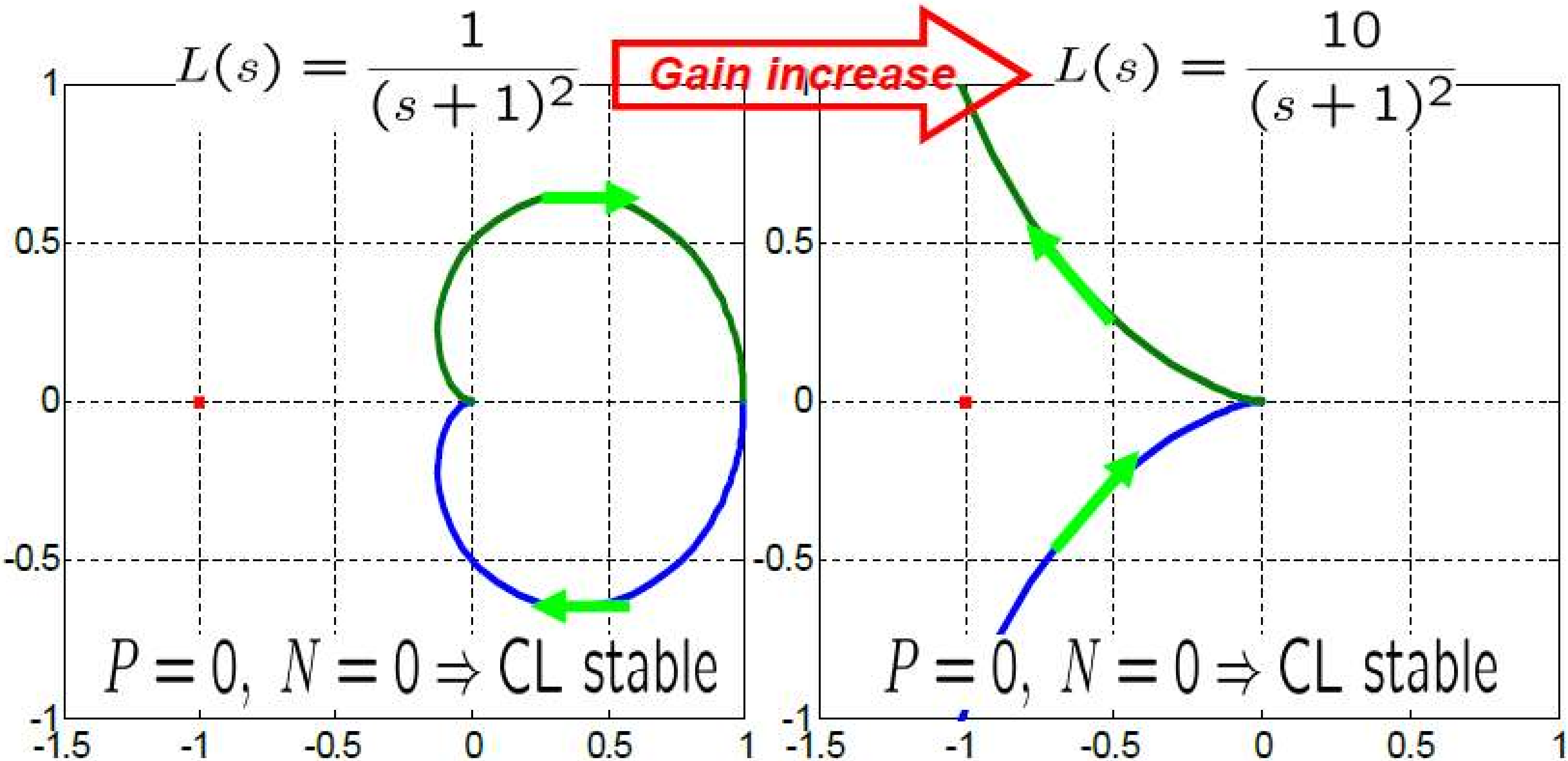
$$\Updownarrow$$

$$1 + L(j\omega_0) = 0 \text{ for some } \omega_0$$

$$\Updownarrow$$

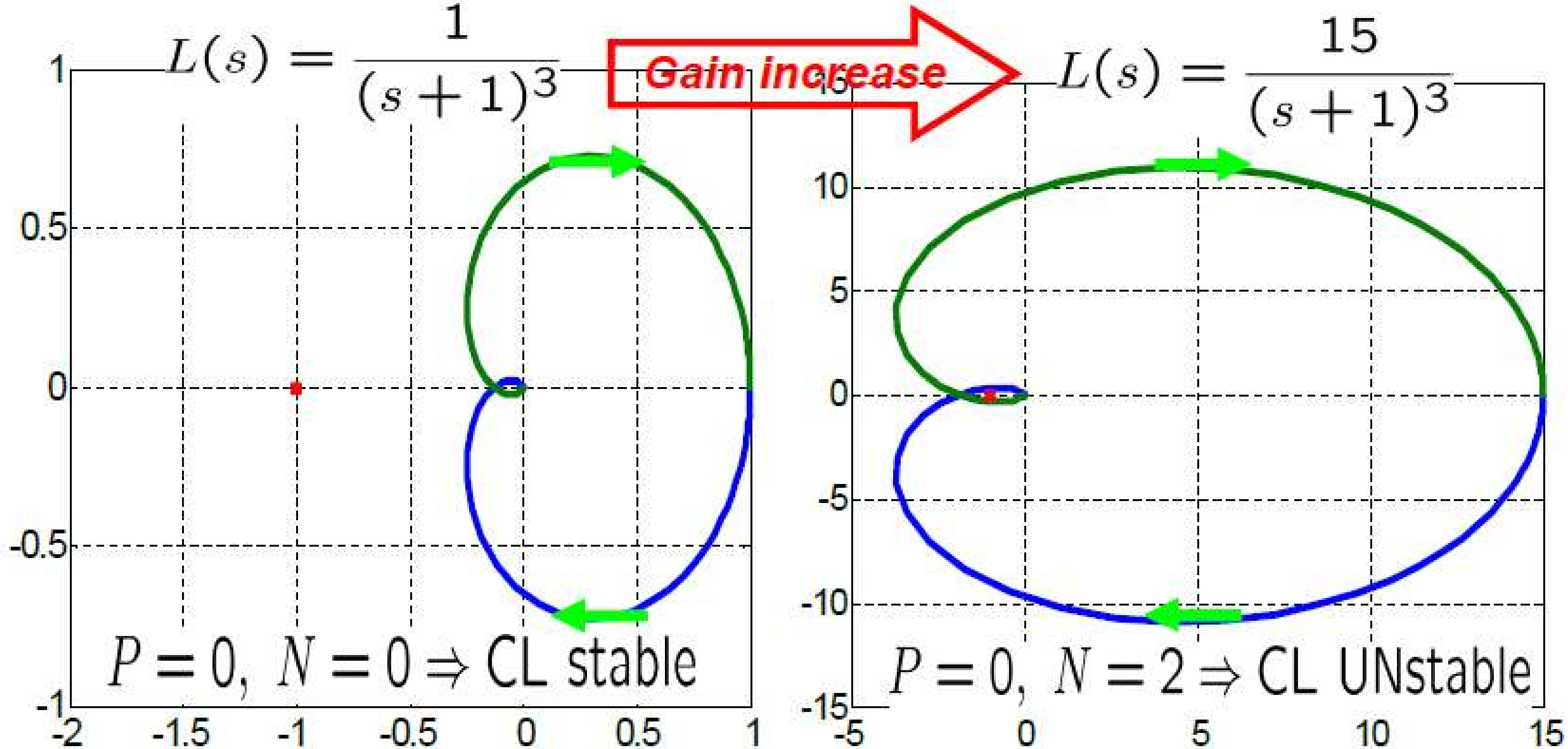
$$\text{CL system has a pole at } j\omega_0.$$

Example for 2nd order L(s)



For L(s) of 2nd order and constant numerator, gain increase never lead to unstable CL system!

Example for 3rd order L(s)

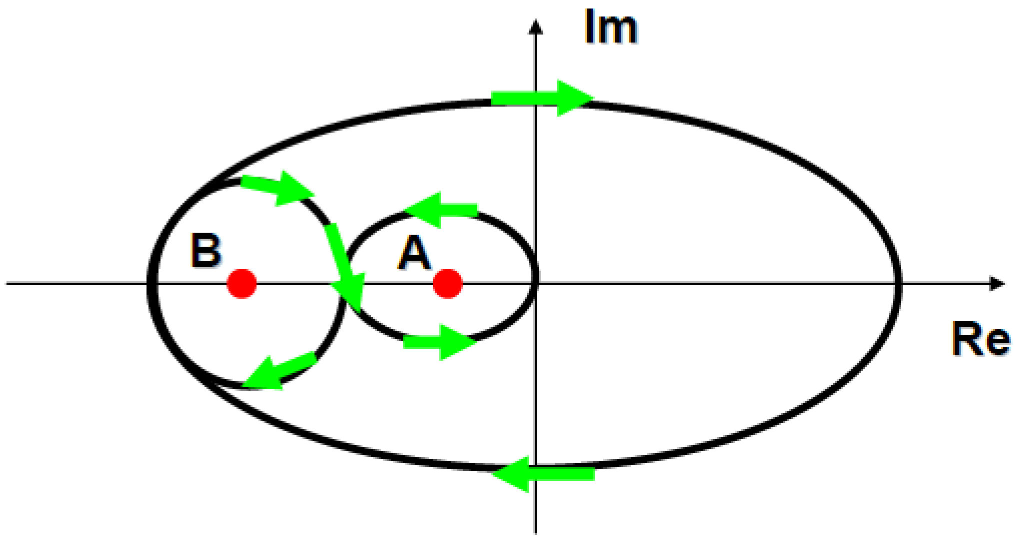


For L(s) with relative degree 3,
gain increase eventually lead to unstable CL system!

How to count # of encirclement

- A ray is drawn from -1 point in any convenient direction. Then,

$$N = (\#\text{ of crossing of ray by Nyquist plot in clockwise direction}) - (\#\text{ of crossing of ray by Nyquist plot in counterclockwise direction})$$



Count N for cases:
- A = -1
- B = -1

Notes on Nyquist stability criterion

- Nyquist stability criterion allows us to determine the stability of CL system from a knowledge of the $G(j\omega)$ of OL system.
- If an OL system is stable, it requires only frequency response data of OL system (TF model L(s) is not necessary).
- It can deal with time delay, which Routh-Hurwitz criterion cannot.
- We often draw only half of Nyquist plot. (The other half is mirror image w.r.t. real axis.)

Nyquist criterion: A special case

$$\text{CL system is stable} \Leftrightarrow Z := P + N = 0$$

- IF P=0 (i.e., if L(s) has no pole in open RHP)

$$\text{CL system is stable} \Leftrightarrow N = 0$$

↑

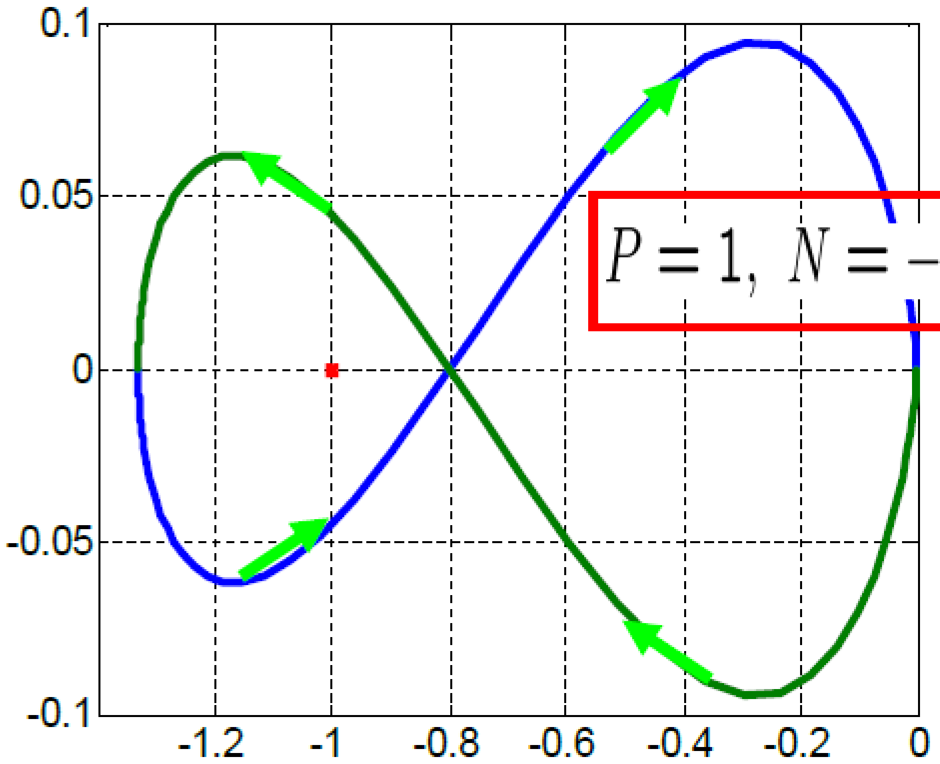
This fact is very important since open-loop systems in many practical problems have no pole in open RHP!

Some examples

- Unstable L(s)
- Stable L(s)
- L(s) with an integrator
- L(s) with a double integrator
- L(s) with a time-delay

Example for unstable L(s)

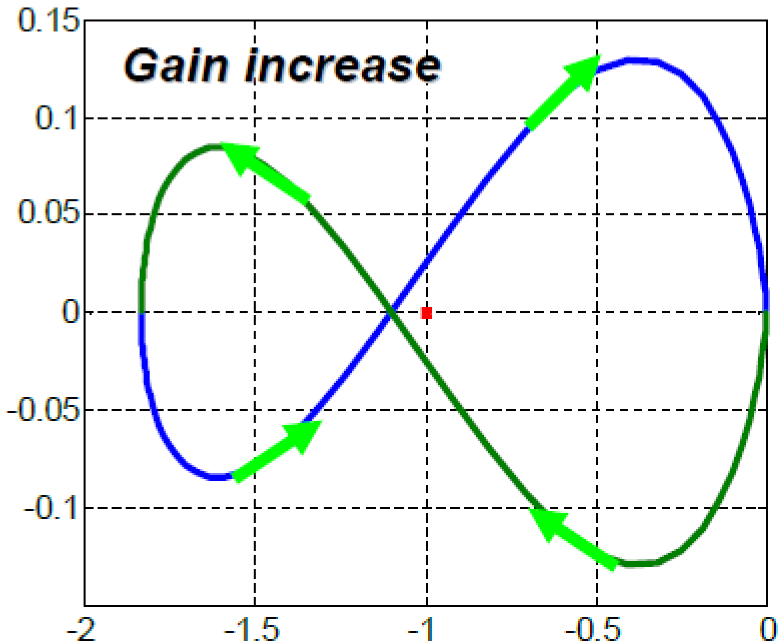
$$L(s) = \frac{8}{(s-1)(s+2)(s+3)}$$



$P = 1,\ N = -1 \Rightarrow \text{CL stable}$

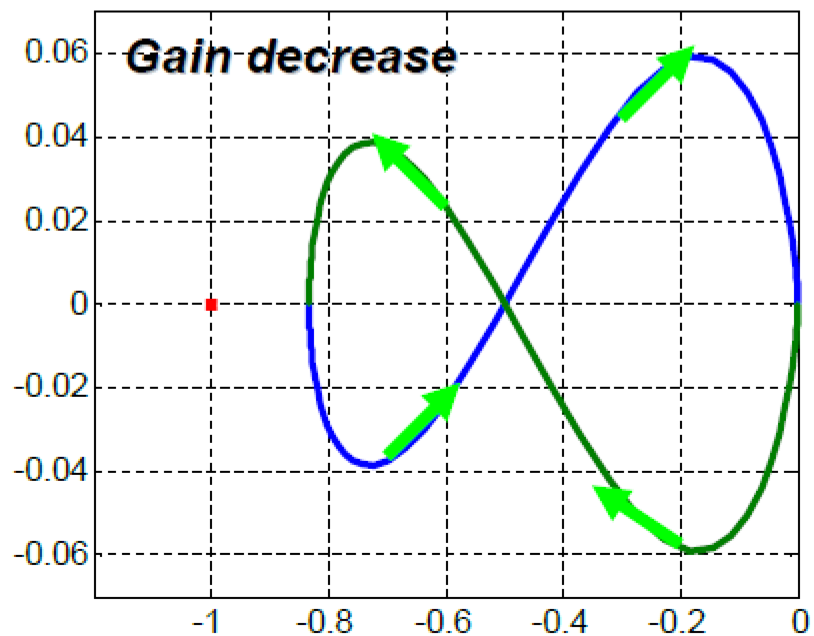
Example of unstable L(s) (cont'd)

$$L(s) = \frac{11}{(s-1)(s+2)(s+3)}$$



$P = 1,\ N = 1 \Rightarrow$ CL unstable

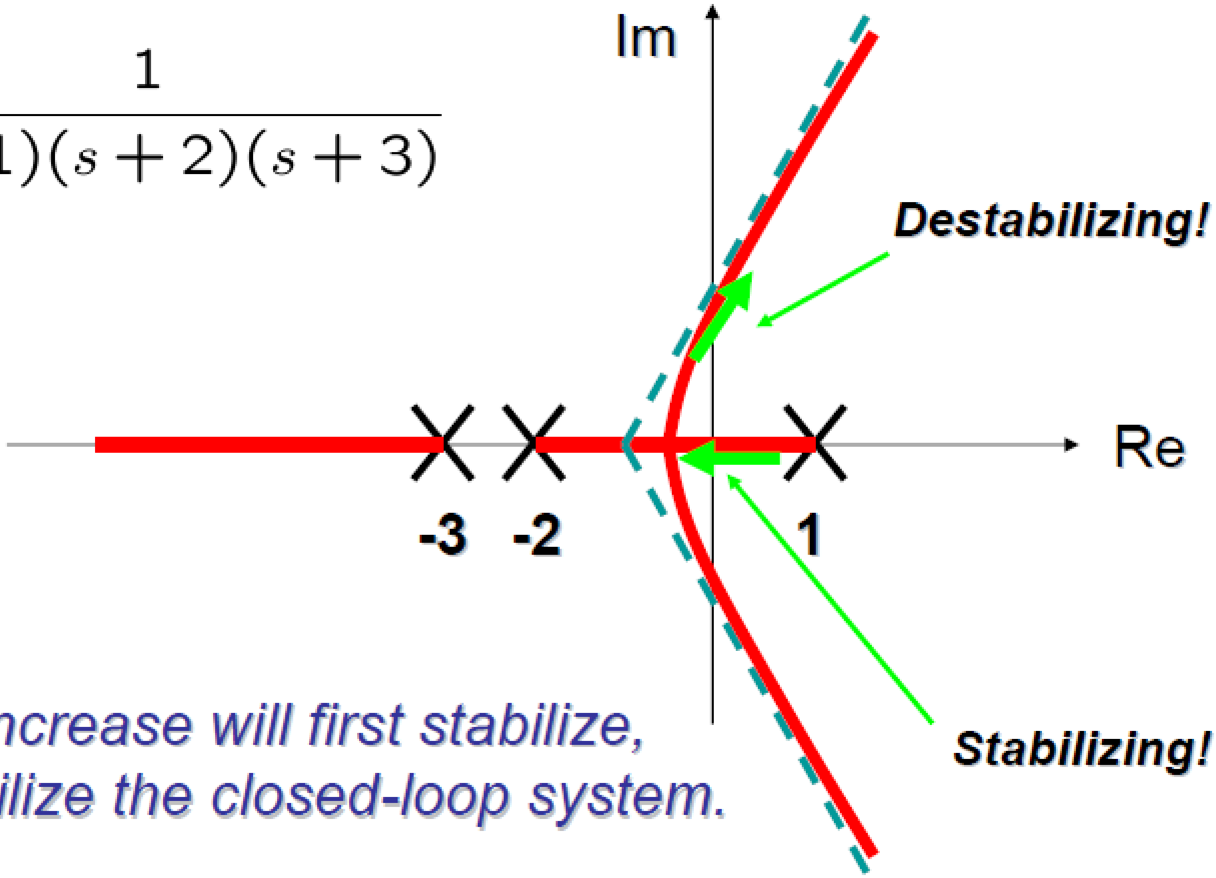
$$L(s) = \frac{5}{(s-1)(s+2)(s+3)}$$



$P = 1,\ N = 0 \Rightarrow$ CL unstable

Interpretation by root locus

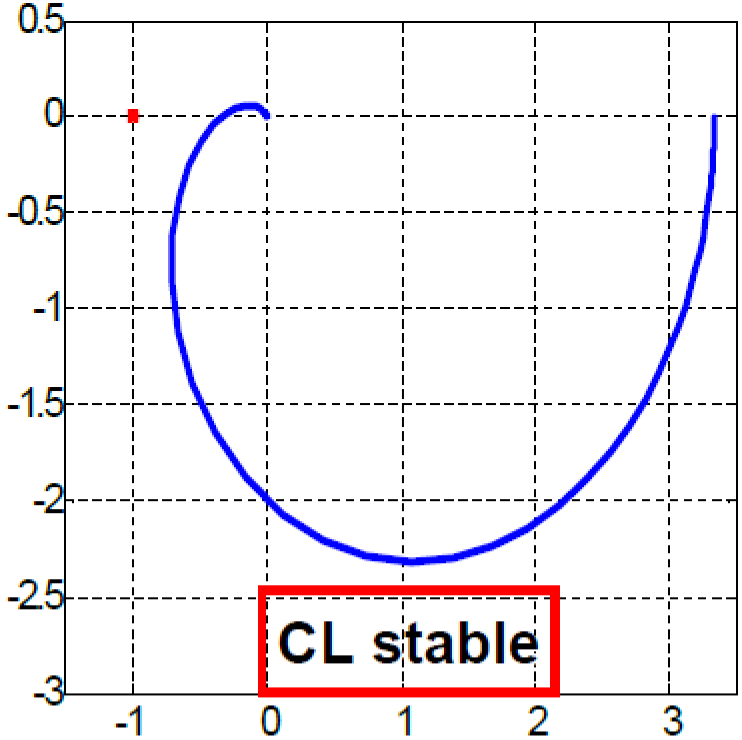
$$L(s) = \frac{1}{(s-1)(s+2)(s+3)}$$



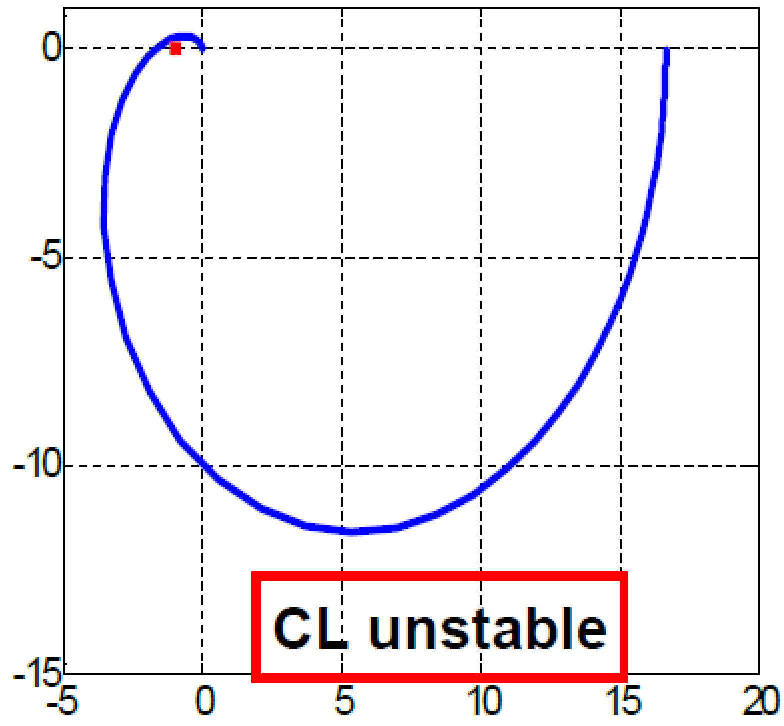
Open loop gain increase will first stabilize, and then, destabilize the closed-loop system.

Example for stable L(s)

$$L(s) = \frac{20}{(s+1)(s+2)(s+3)}$$

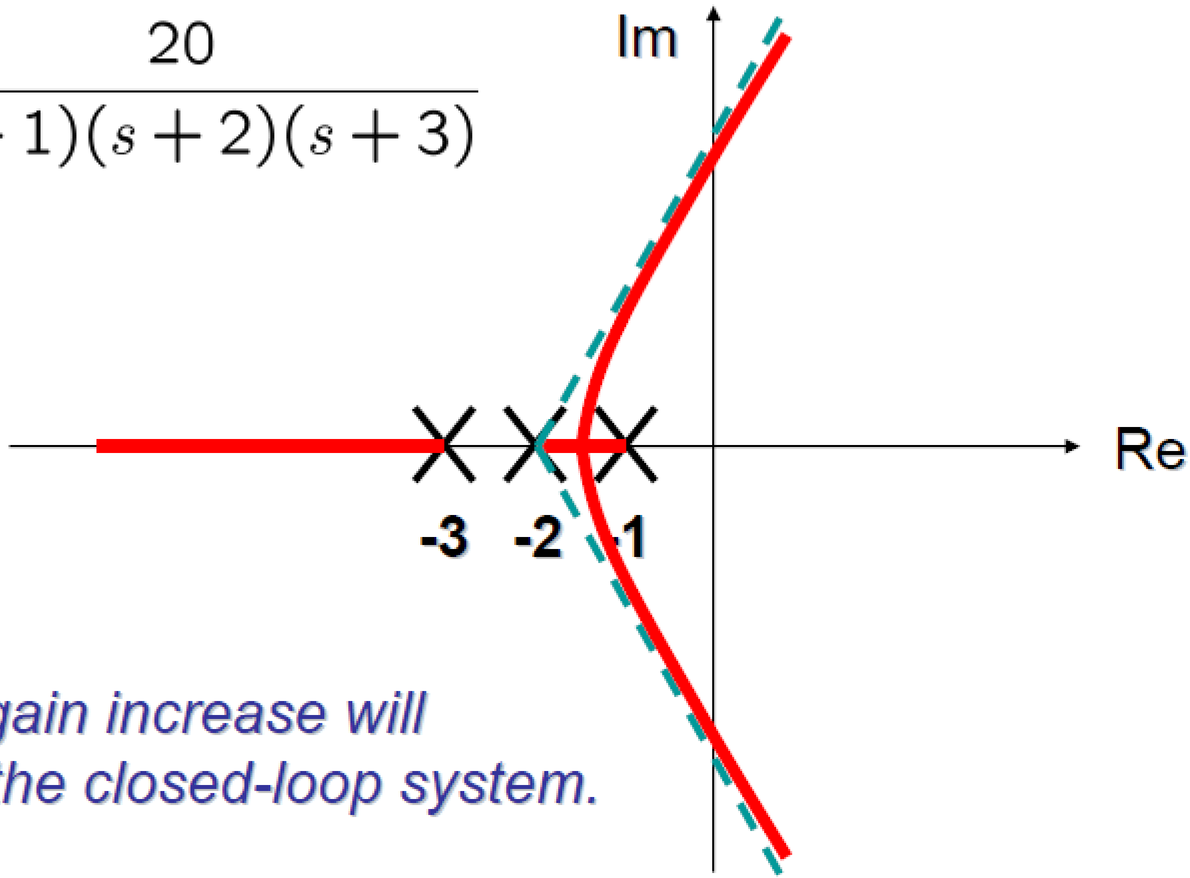


$$L(s) = \frac{100}{(s+1)(s+2)(s+3)}$$



Interpretation by root locus

$$L(s) = \frac{20}{(s+1)(s+2)(s+3)}$$

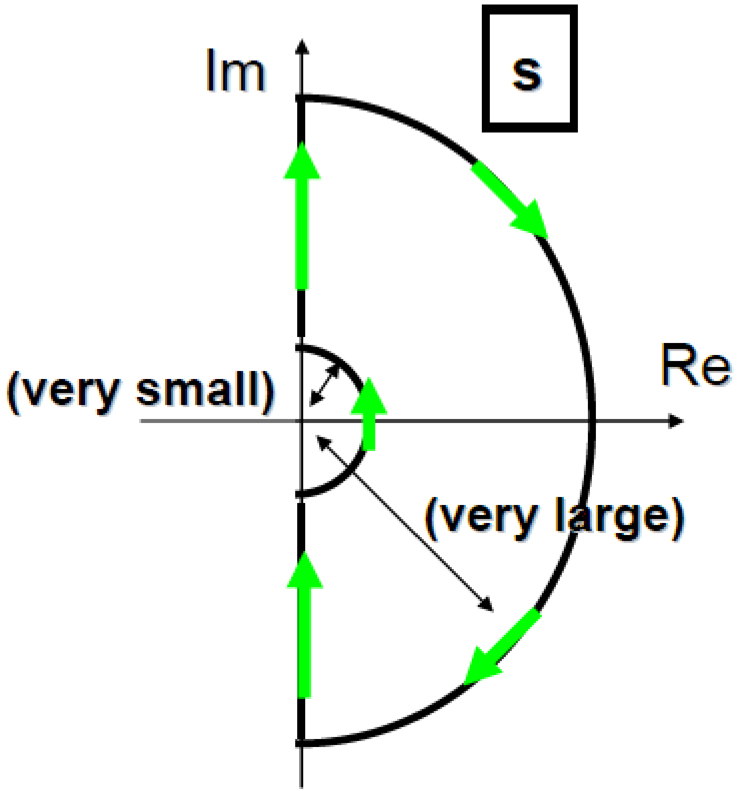


Open loop gain increase will destabilize the closed-loop system.

Example of L(s) with an integrator

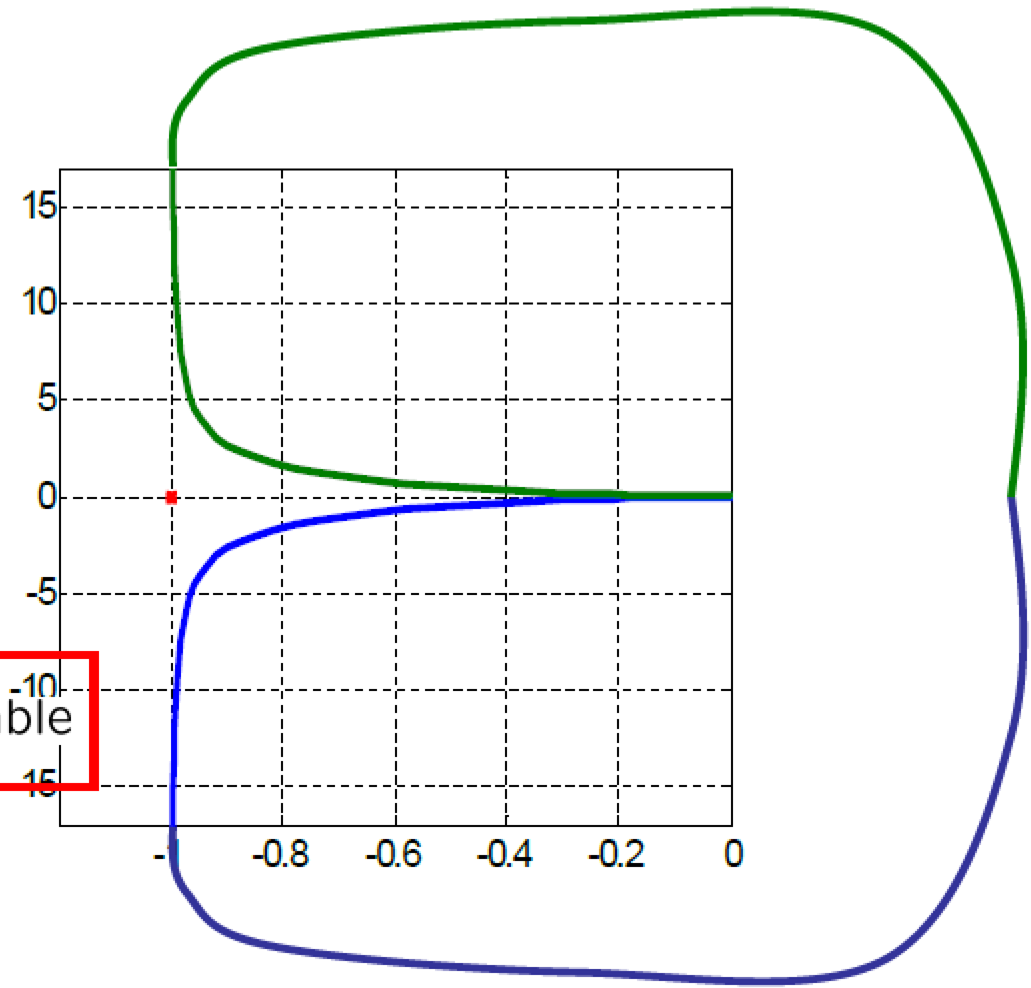
- We modify Nyquist path:

$$L(s) = \frac{1}{s(s+1)}$$



Example of L(s) with an integrator

$$L(s) = \frac{1}{s(s+1)}$$



$P = 0,\ N = 0 \Rightarrow \text{CL stable}$

L(s) for modified Nyquist path

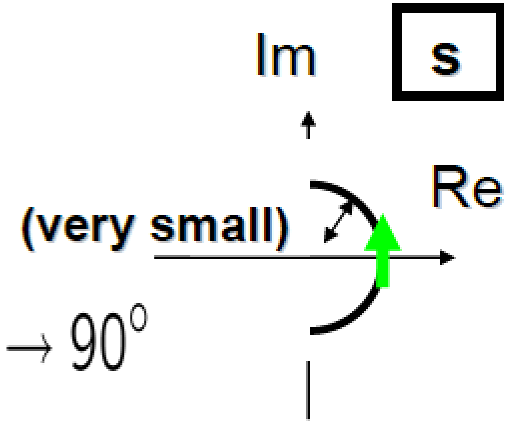
- For small |s|,

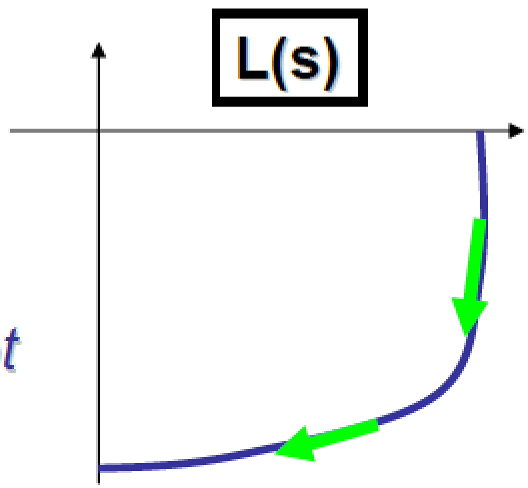
$$L(s) = \frac{1}{s(s+1)} \approx \frac{1}{s}$$

- When s moves as $s = \varepsilon e^{j\theta},\ \theta = 0 \to 90^\circ$

  L(s) moves as

$$L(s) \approx \frac{1}{\varepsilon} e^{-j\theta},\ \theta = 0 \to 90^\circ$$

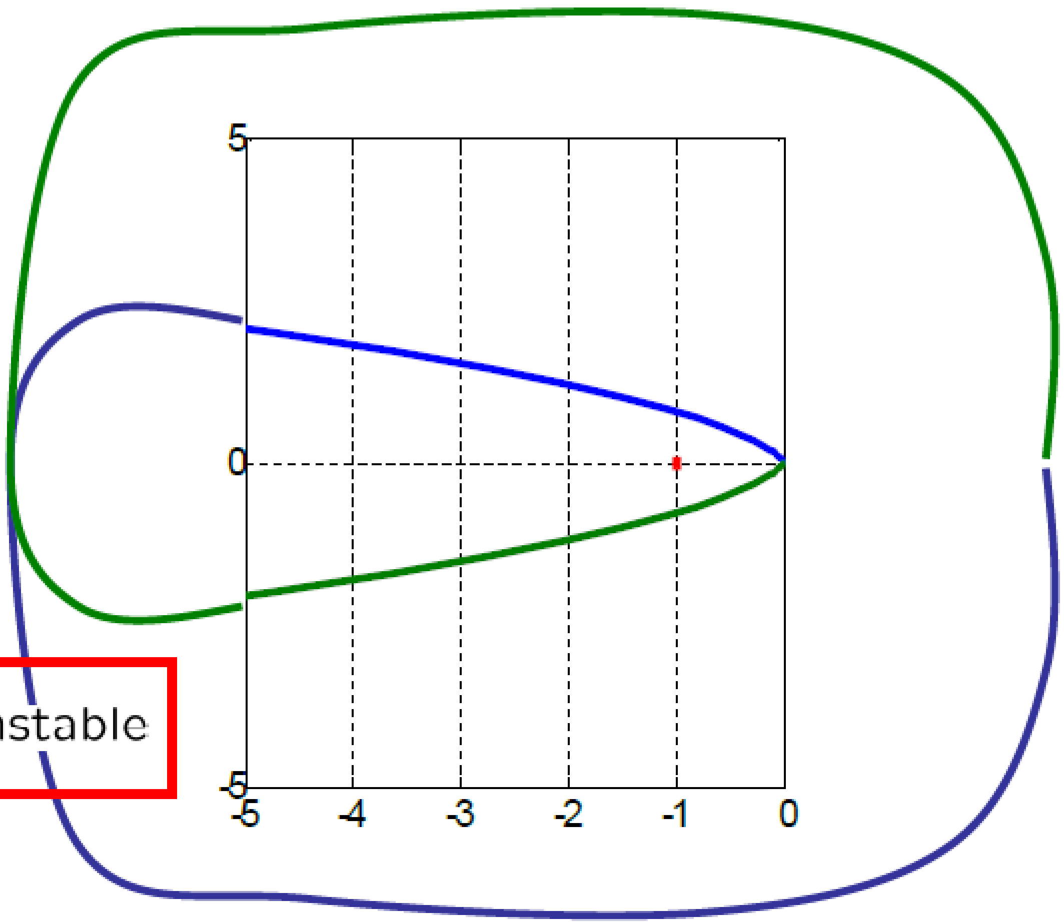




Note: If L(s) has no open RHP pole, we are interested in (and draw) Nyquist plot ONLY around the critical point -1.

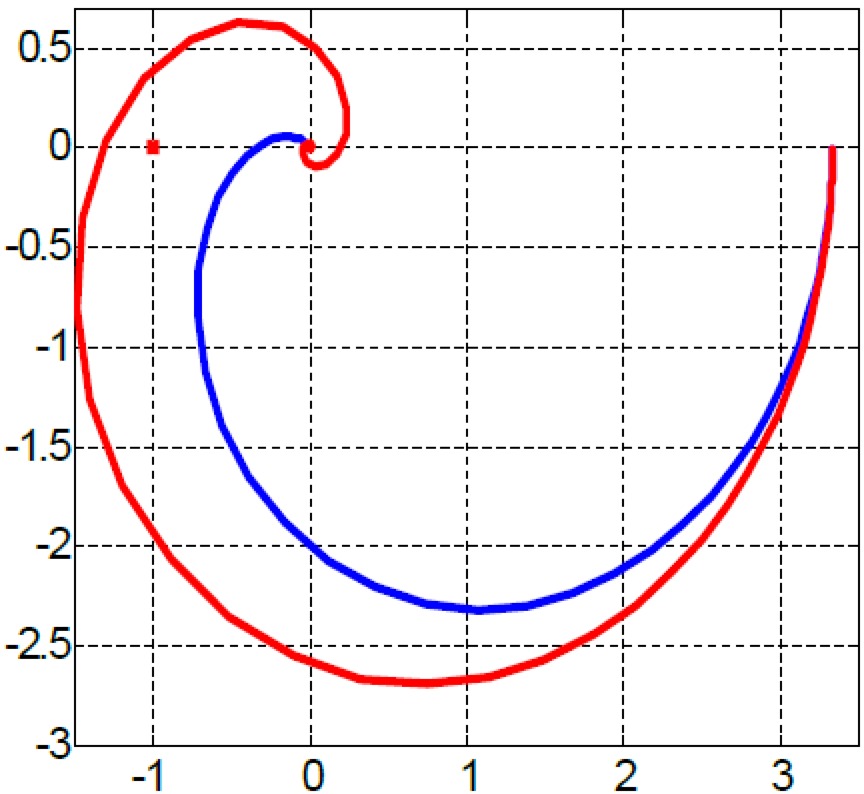
Example of L(s) with double integrator

$$L(s) = \frac{1}{s^2(s+1)}$$



$P = 0,\ N = 2 \Rightarrow \text{CL unstable}$

L(s) with a time-delay



$$L(s) = \frac{20}{(s+1)(s+2)(s+3)}$$

$$P = 0,\ N = 0 \Rightarrow \text{CL stable}$$

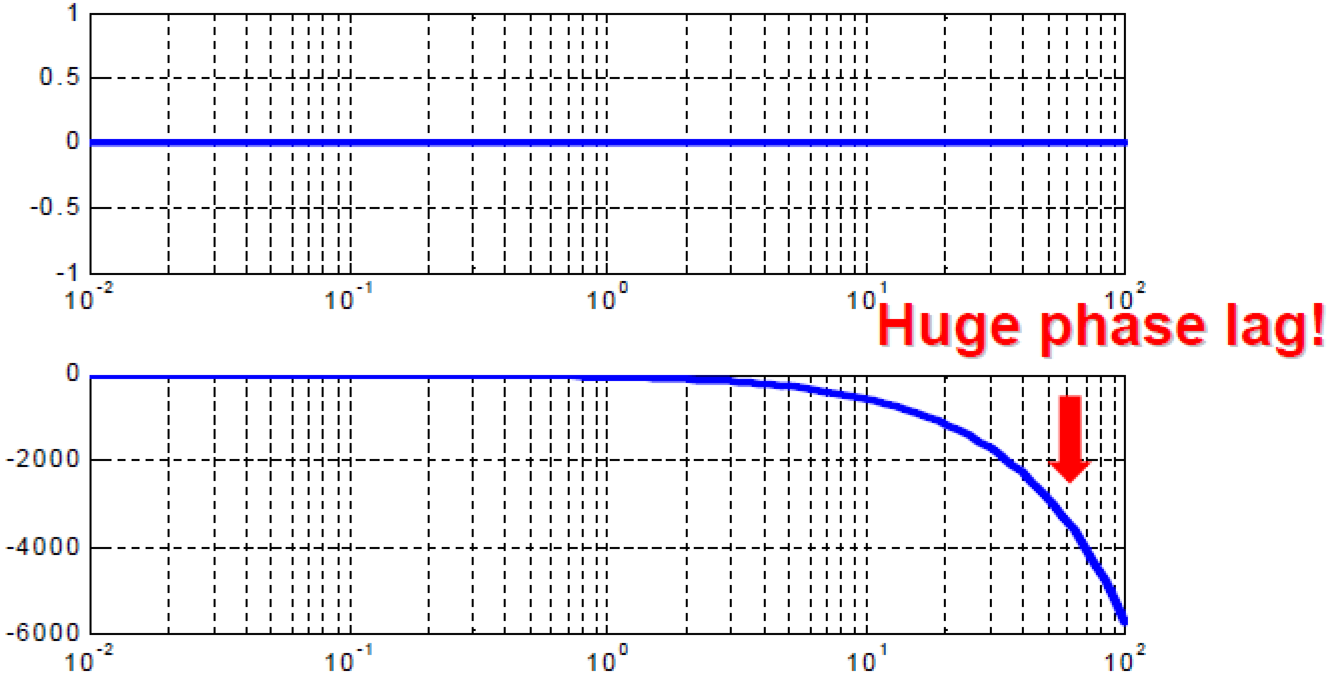
$$L(s) = \frac{20e^{-0.7s}}{(s+1)(s+2)(s+3)}$$

$$P = 0,\ N = 1 \Rightarrow \text{CL unstable}$$

Routh-Hurwitz is NOT applicable!

Bode plot of a time delay (review)

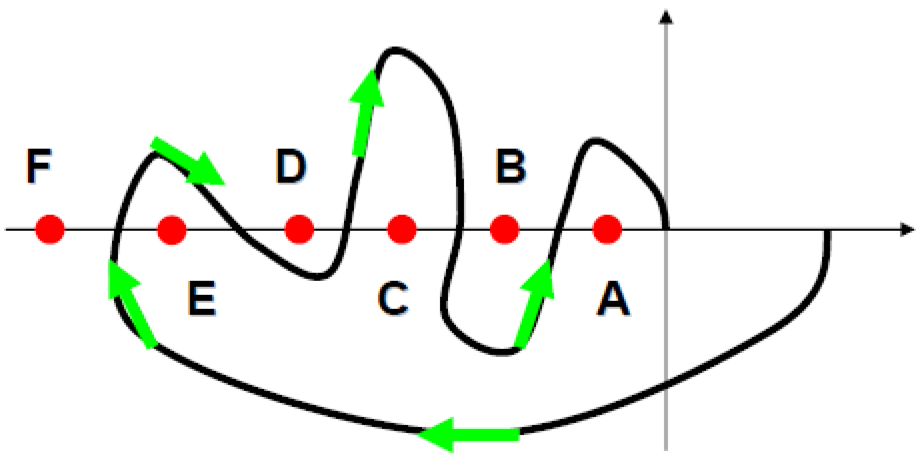
$$G(s) = e^{-Ts} \Rightarrow |G(j\omega)| = 1, \forall\omega,\ \angle G(j\omega) = -\omega T(\text{rad})$$



The phase lag causes instability of the closed-loop system, and thus, the difficulty in control.

Summary and exercises

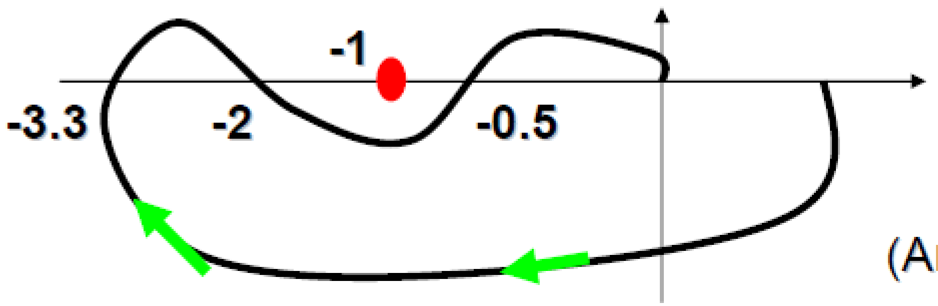
- Nyquist plot (Matlab command: "*nyquist.m*")
- Nyquist stability criterion for feedback stability
- Examples for Nyquist stability criterion
- Exercises: For (half of) Nyquist plot below, count N for each case.



- **A=-1** (Ans. N=2)
- **B=-1** (Ans. N=0)
- **C=-1** (Ans. N=2)
- **D=-1** (Ans. N=0)
- **E=-1** (Ans. N=2)
- **F=-1** (Ans. N=0)

Exercise

- Exercise: Suppose L(s) is stable and has Nyquist plot below. Find the range of OL gain K>0 for which CL system is stable.



(Ans. 0<K<1/3.3, 1/2<K<2)

Some remarks

- Nyquist stability criterion gives not only *absolute* but also *relative stability*.
  - Absolute stability: Is the closed-loop system stable or not? (Answer is yes or no.)
  - Relative stability: How "much" is the closed-loop system stable? (Margin of safety)
- Relative stability is important because a mathematical model is never accurate.
- How to measure relative stability?
  - Use a "distance" from the critical point -1.
  - Gain margin (GM) & Phase margin (PM)

Gain margin (GM)

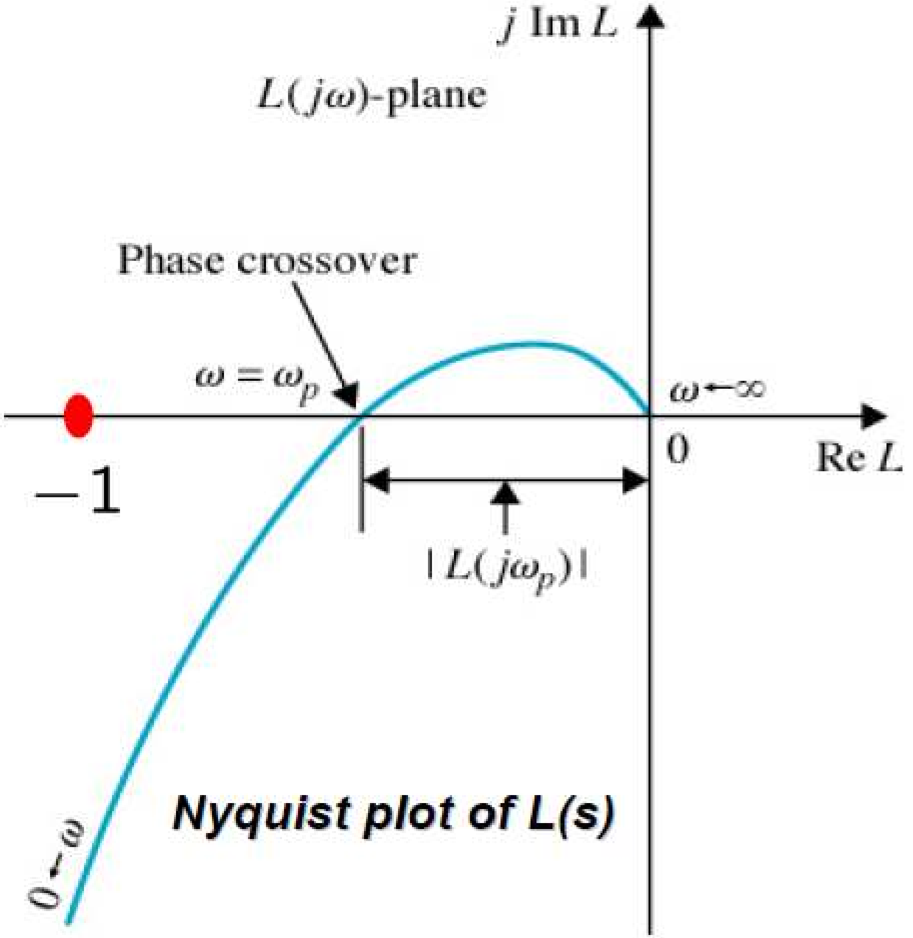
- Phase crossover frequency ωp:

$$\angle L(j\omega_p) = -180$$

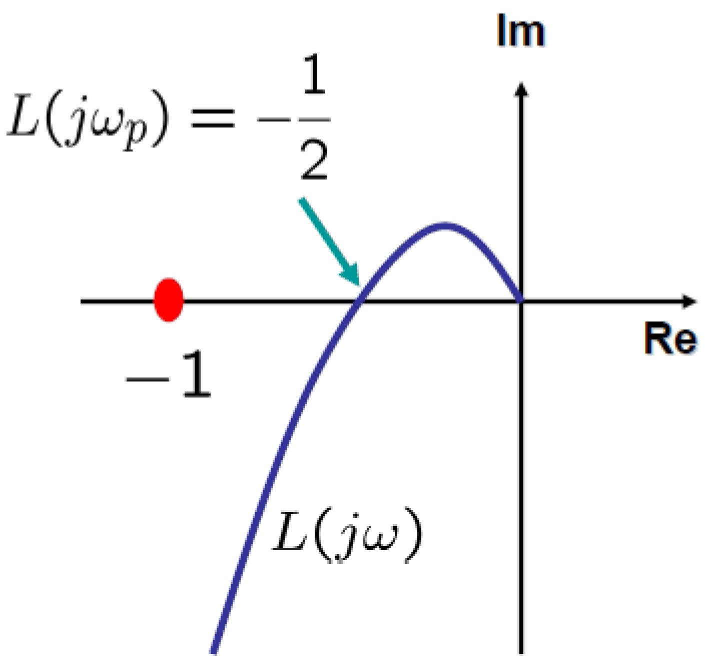
- Gain margin (in dB)

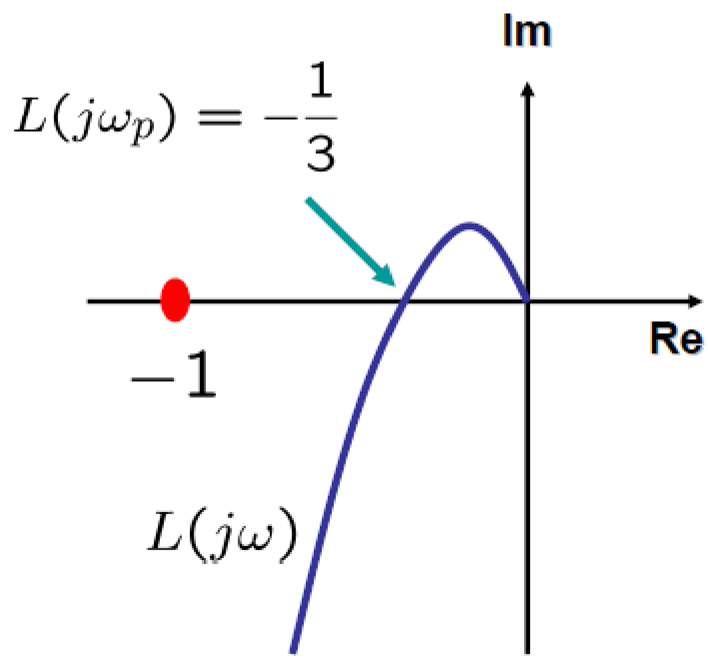
$$GM = 20\log_{10}\frac{1}{|L(j\omega_p)|}$$

  - Indicates how much OL gain can be multiplied without violating CL stability.



Examples of GM

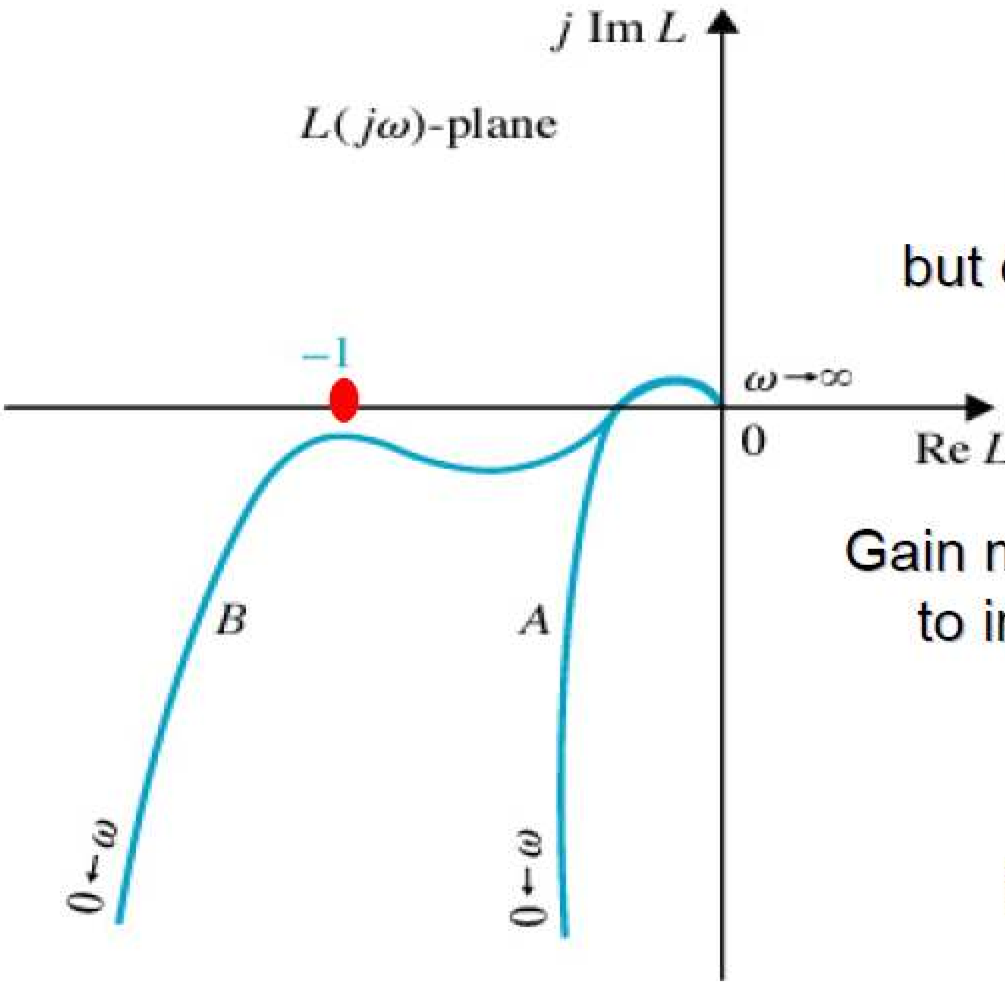




$$GM = 20\log_{10}\underbrace{\frac{1}{|L(j\omega_p)|}}_{2} \approx 6\text{dB}$$

$$GM = 20\log_{10}\underbrace{\frac{1}{|L(j\omega_p)|}}_{3} \approx 9.5\text{dB}$$

Reason why GM is inadequate



Same gain margin,
but different relative stability

Gain margin is often inadequate to indicate relative stability

Phase margin!

Phase margin (PM)

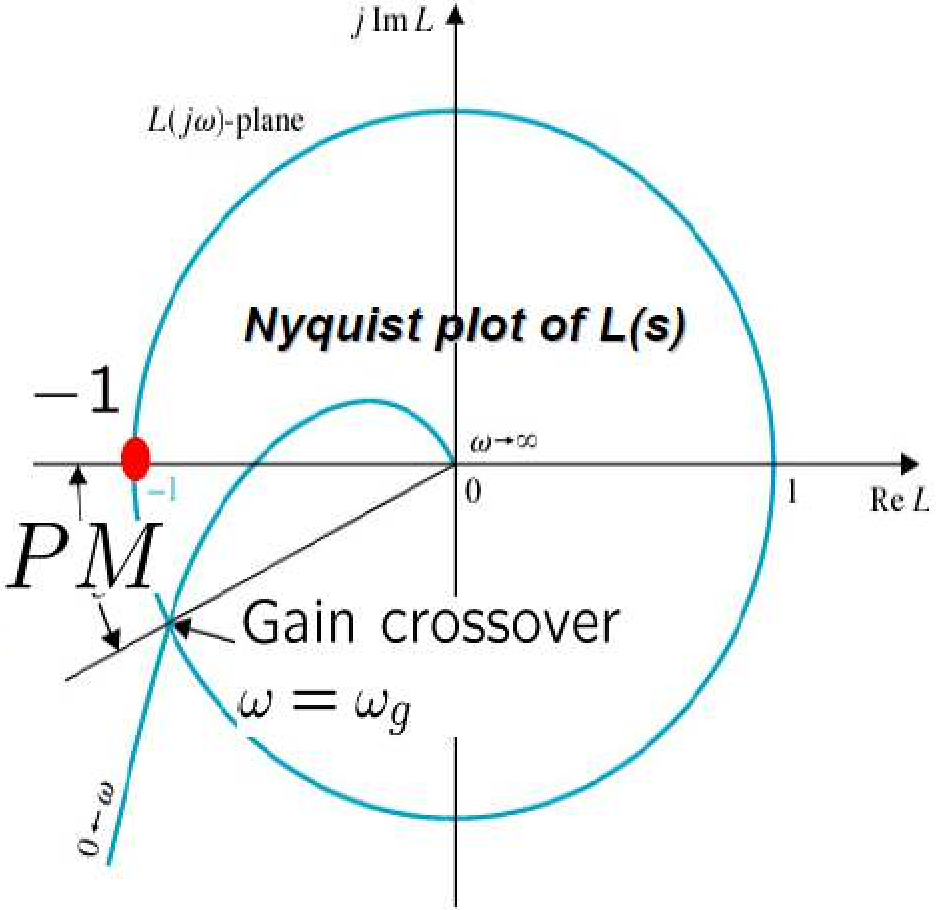
- Gain crossover frequency ω_g:

$$|L(j\omega_g)| = 1$$

- Phase margin

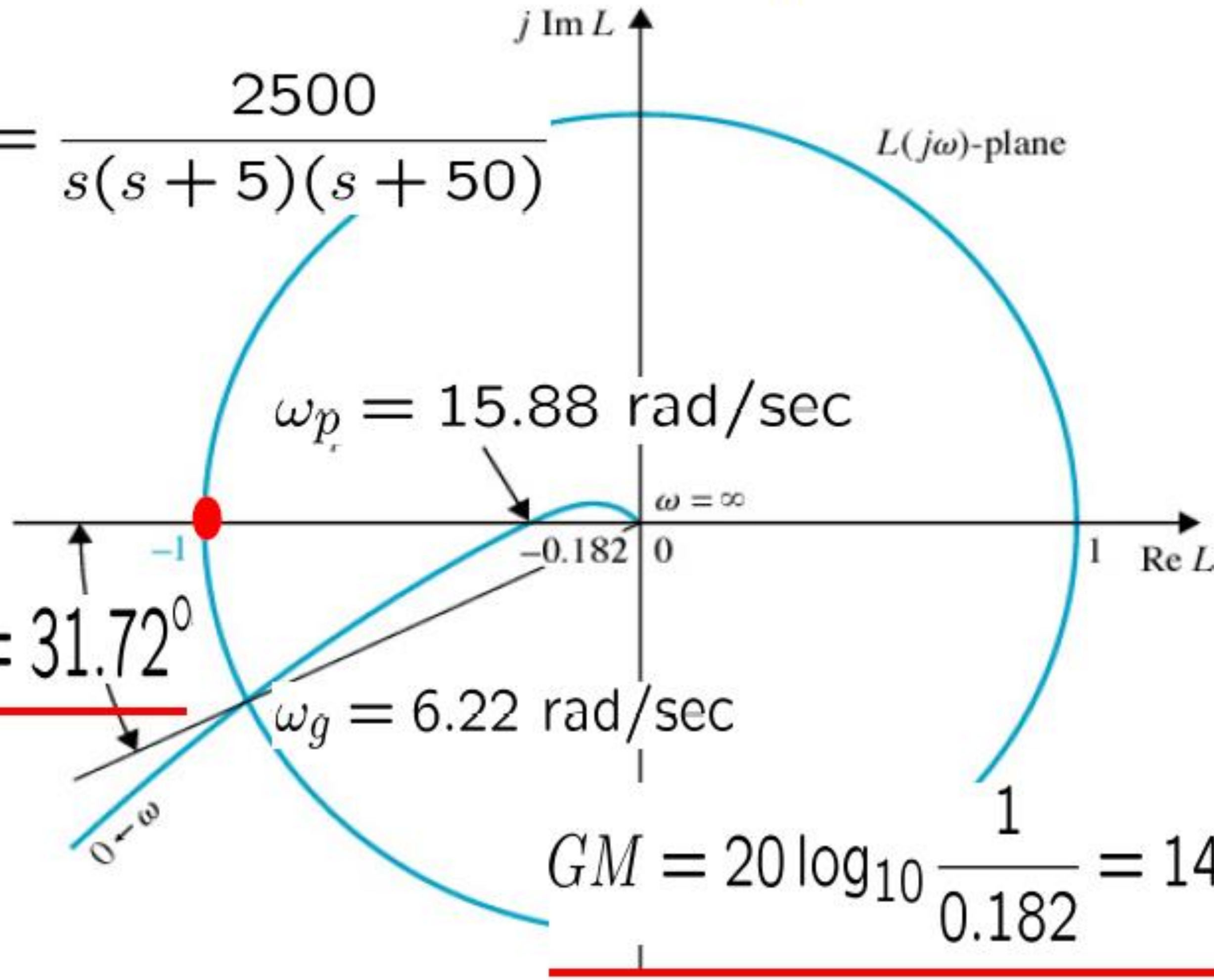
$$PM = \angle L(j\omega_g) + 180^\circ$$

  - Indicates how much OL phase lag can be added without violating CL stability.



An example

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$



$$\omega_p = 15.88 \ \text{rad/sec}$$

$$\omega_g = 6.22 \ \text{rad/sec}$$

$$PM = 31.72^0$$

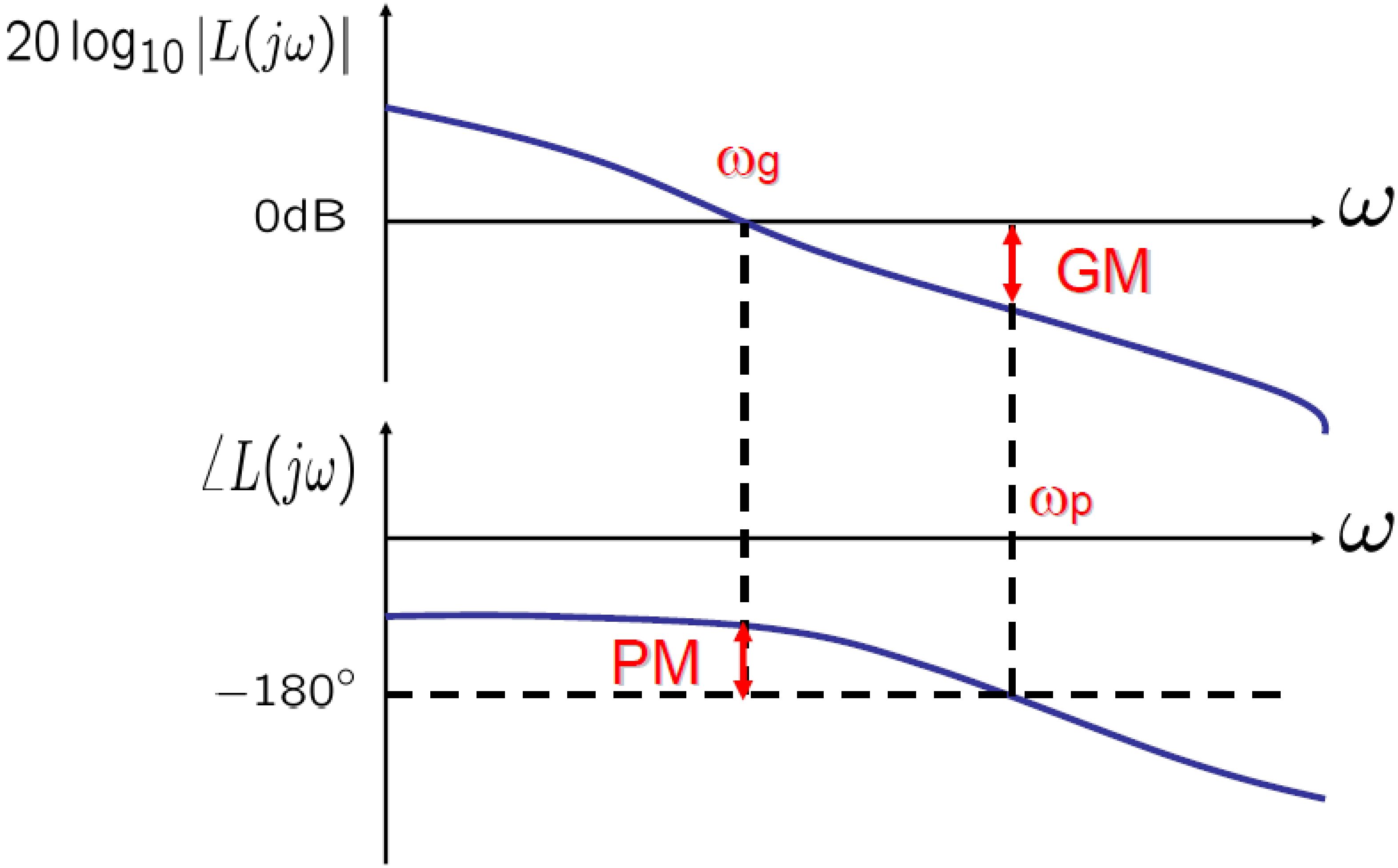
$$GM = 20\log_{10}\frac{1}{0.182} = 14.80\text{dB}$$

Notes on Nyquist plot

- Advantages
  - Nyquist plot can be used for study of closed-loop stability, for open loop systems which is unstable and includes time-delay.
- Disadvantage
  - Controller design on Nyquist plot is difficult. (Controller design on Bode plot is much simpler.)

We translate GM and PM defined in Nyquist plot into those in Bode plot!

Relative stability on Bode plot

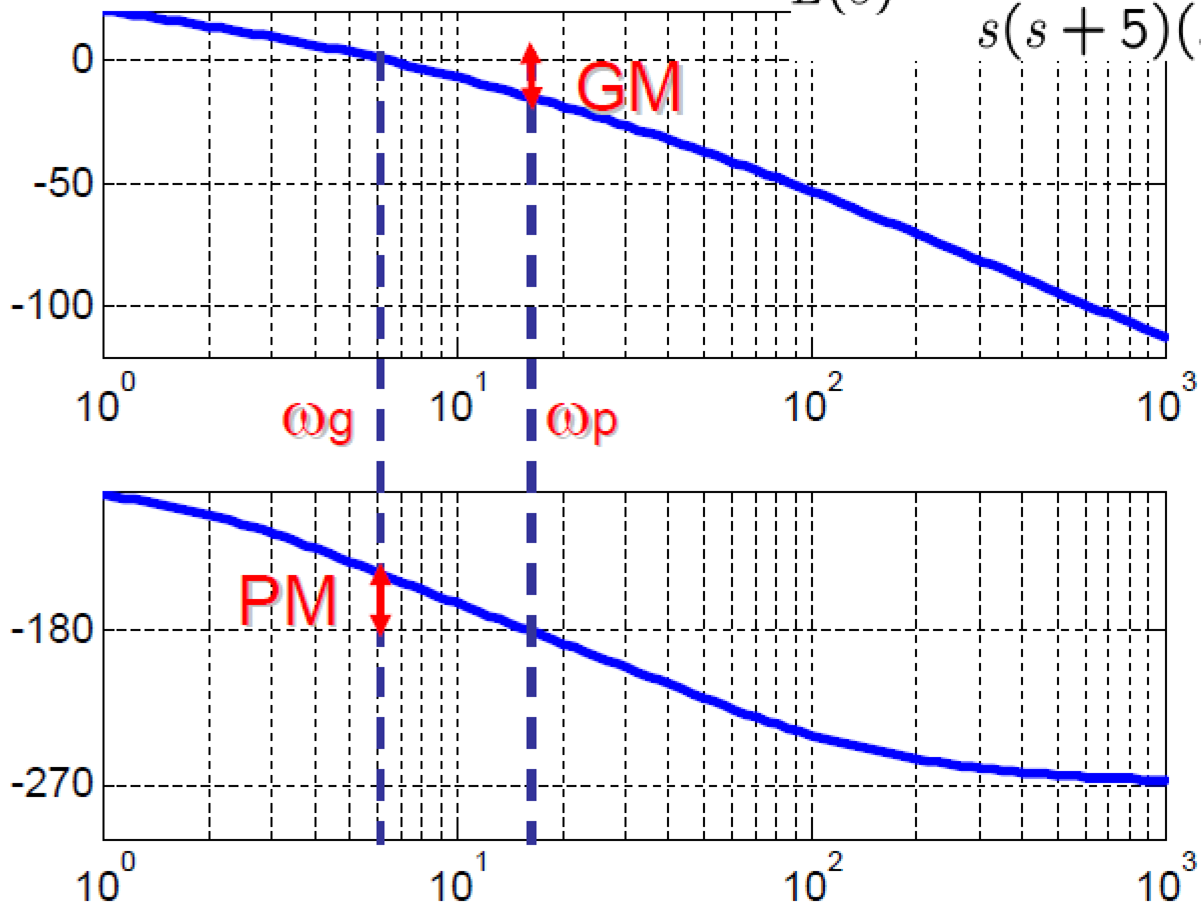


Notes on Bode plot

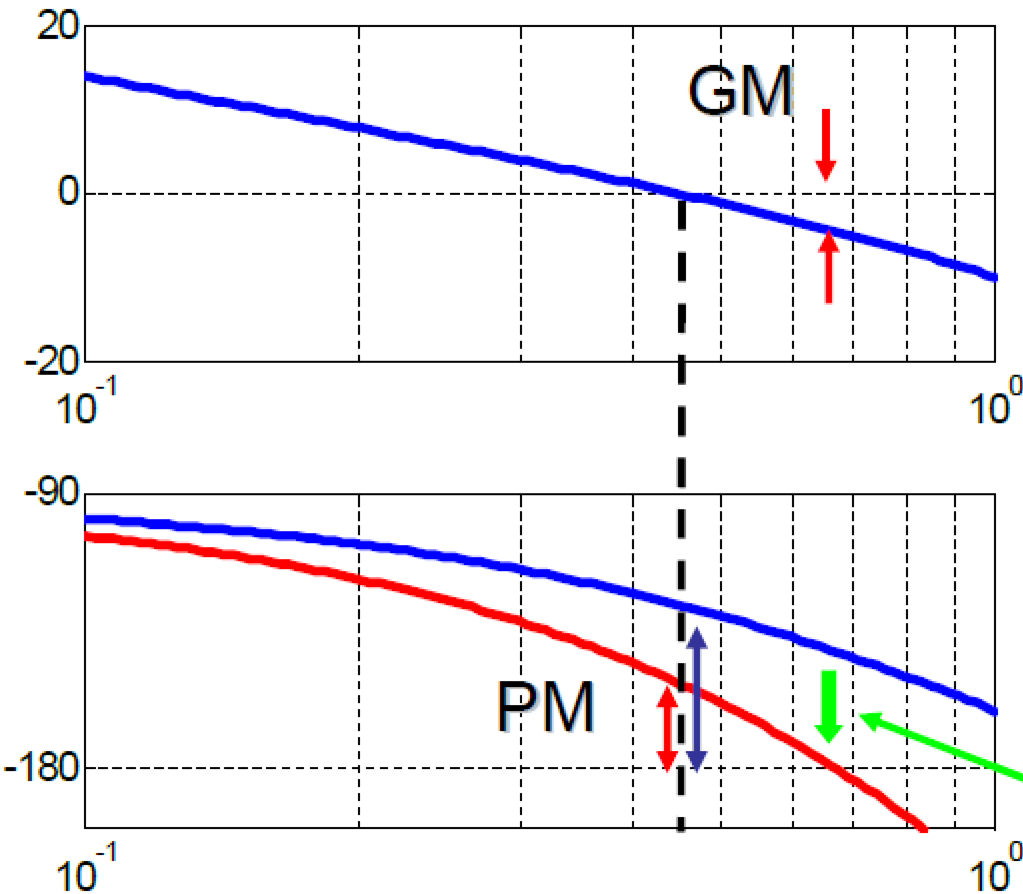
- Advantages
  - Without computer, Bode plot can be sketched easily.
  - GM, PM, crossover frequencies are easily determined on Bode plot.
  - Controller design on Bode plot is simple. (Next week)
- Disadvantage
  - If OL system is unstable, we cannot use Bode plot for closed-loop stability analysis.

An example

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$



Relative stability with time delay



$$L(s) = \frac{1}{s(s+1)(s+2)}$$

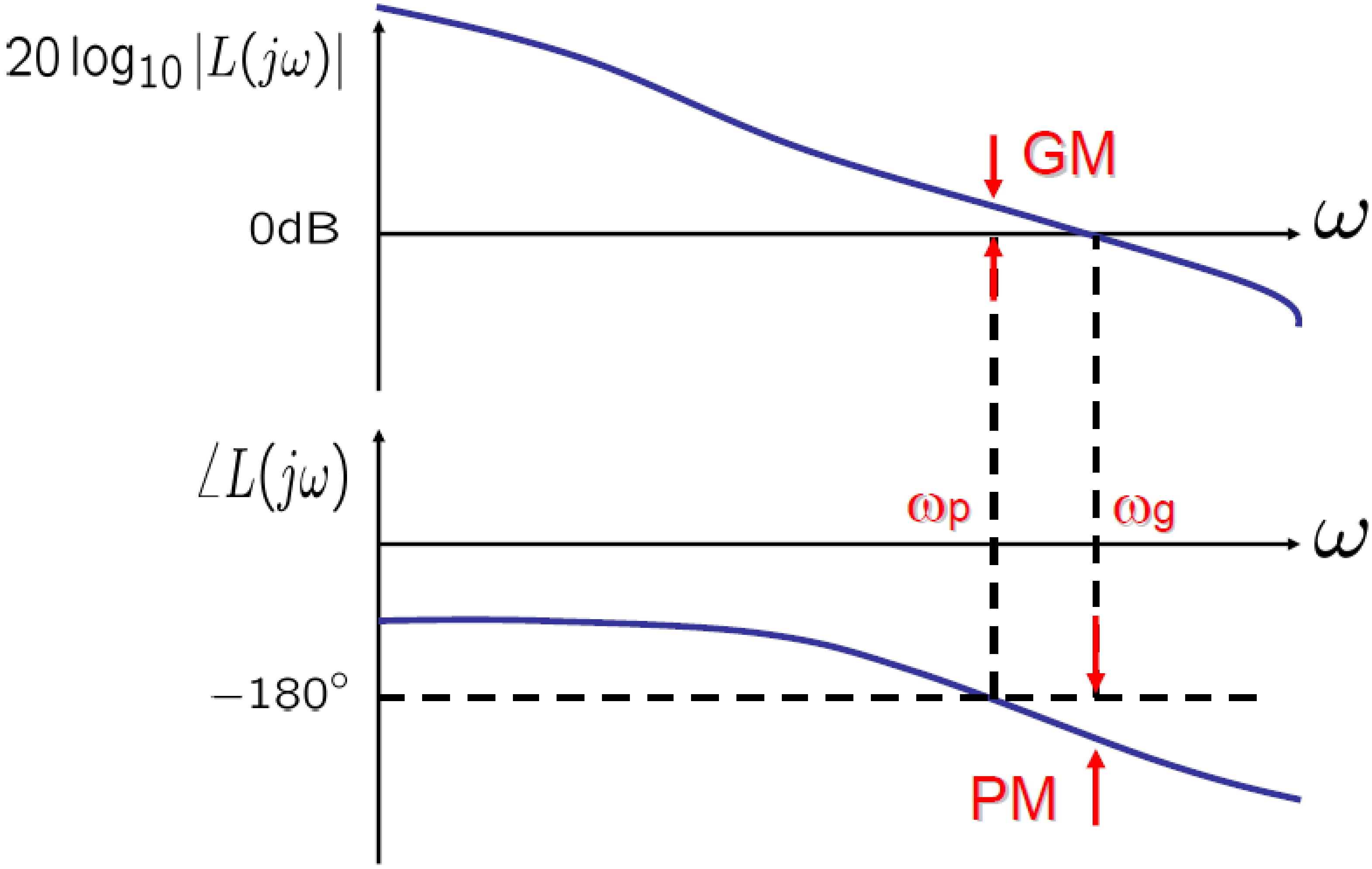
$$L(s) = \frac{e^{-s}}{s(s+1)(s+2)}$$

Time delay reduces relative stability!

$\frac{180}{\pi}\omega T_d$ deg phase lag

Delay time

Unstable closed-loop case



Summary

- Relative stability: Closeness of Nyquist plot to the critical point -1
  - Gain margin, phase crossover frequency
  - Phase margin, gain crossover frequency
- Relative stability on Bode plot
- We normally emphasize PM in controller design

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- Next
  - Examples of gain margin and phase margin frequency domain specifications.
  - Frequency response shaping

End of Lecture 9