CEN 534 Review of Data Transmission (1)
Data Transmission Basics

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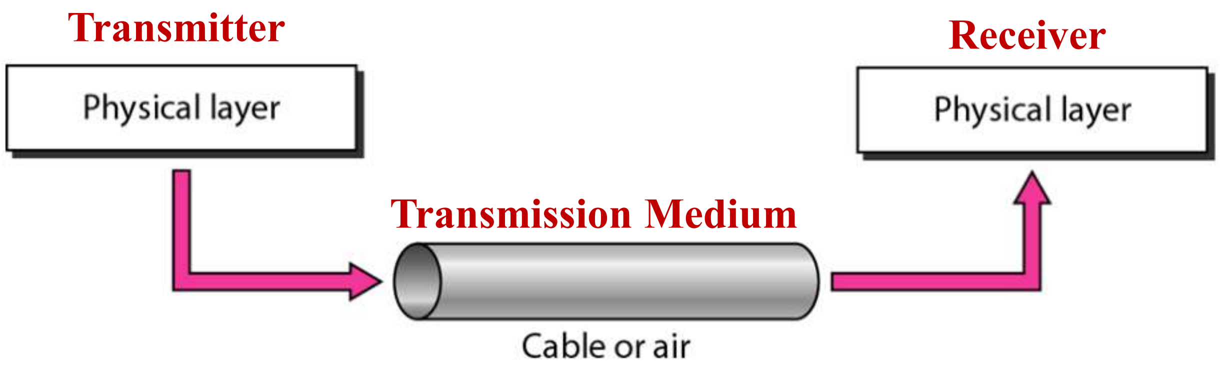
Chapter 3

Data Transmission

Terminology (1)

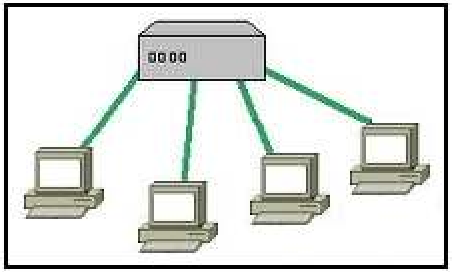
- **Transmitter**
- **Receiver**
- **Transmission Medium**
  - **Guided medium**
    - e.g. twisted pair, optical fiber
  - **Unguided medium**
    - e.g. air, water, vacuum



Terminology (2)

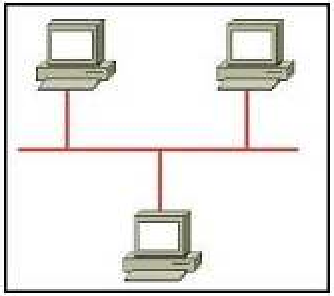
Network Topology:

- **Point-to-point**
  - **Direct link:** No intermediate devices other than amplifiers or repeaters used to increase signal strength
  - Only 2 devices share link



Point-to-Point Topology

- **Multi-point**
  - More than two devices share the link

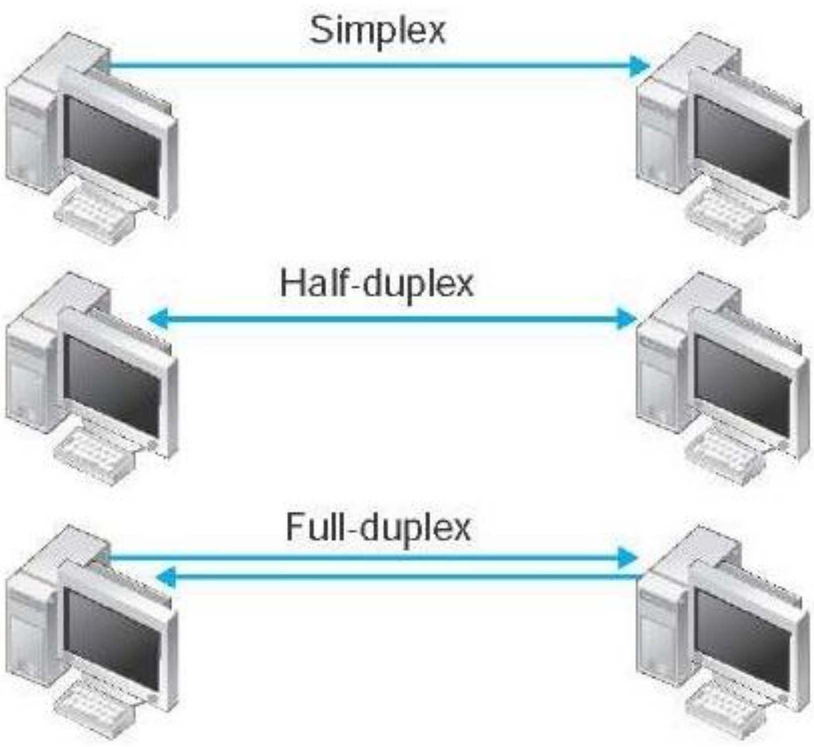


Multi-point Topology

Terminology (3)

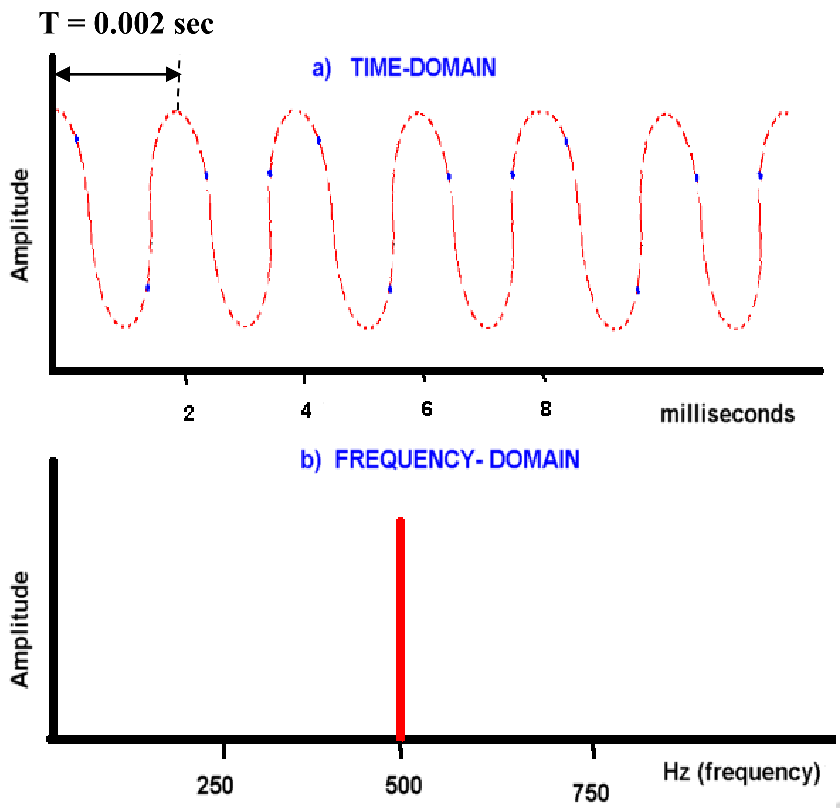
Communication Modes:

- **Simplex**
  - One direction
    - e.g. Television
- **Half duplex**
  - Either direction, but only one way at a time
    - e.g. police radio
- **Full duplex**
  - Both directions at the same time
    - e.g. telephone



Electromagnetic Signals

- **Time-Domain**
  - **Analog** (varies smoothly over time)
  - **Digital** (constant level over time, followed by a change to another level)

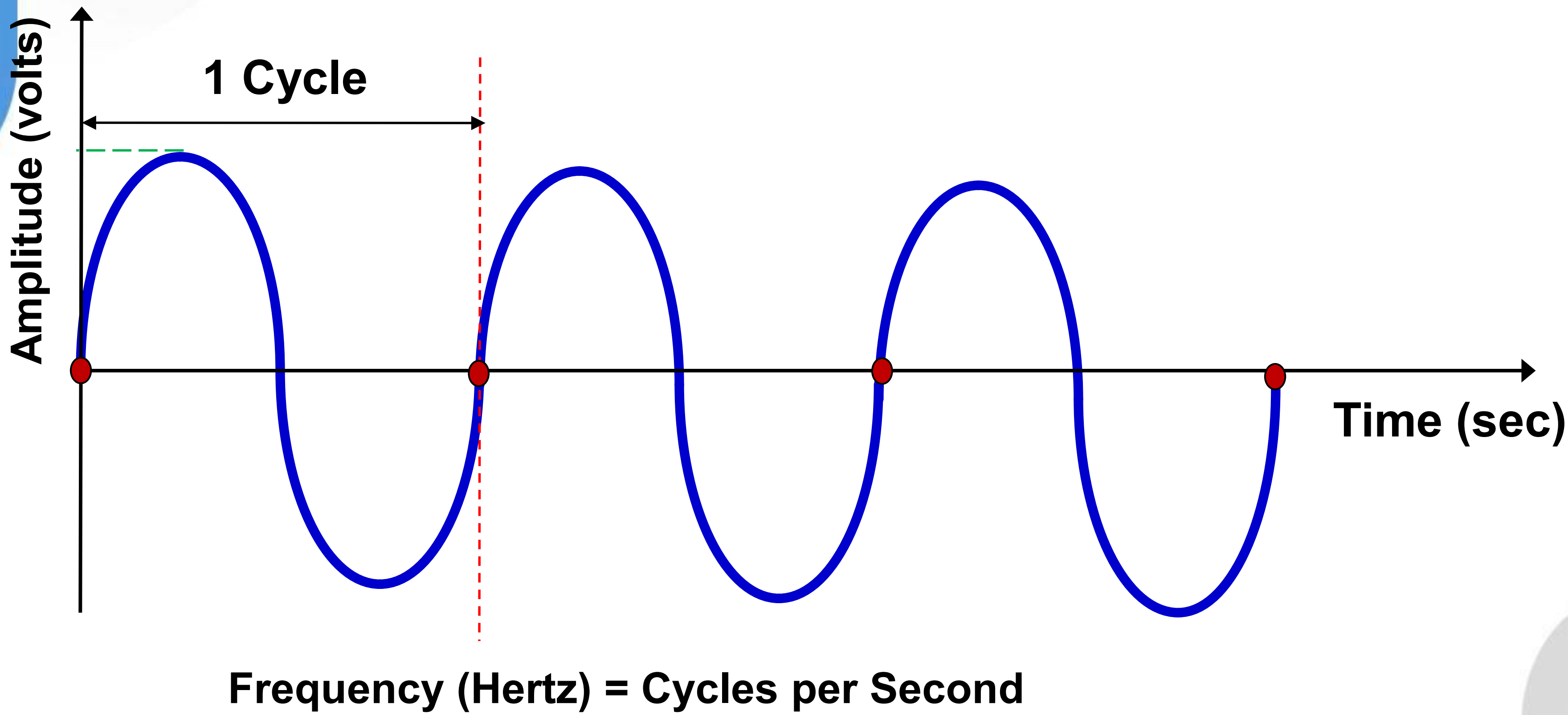


- **Frequency-Domain**
  - **Spectrum** (range of frequencies)
  - **Bandwidth** (width of the spectrum)

T = 0.002 sec ⇒ $f = \frac{1}{T} = \frac{1}{0.002} = 500\,Hz$

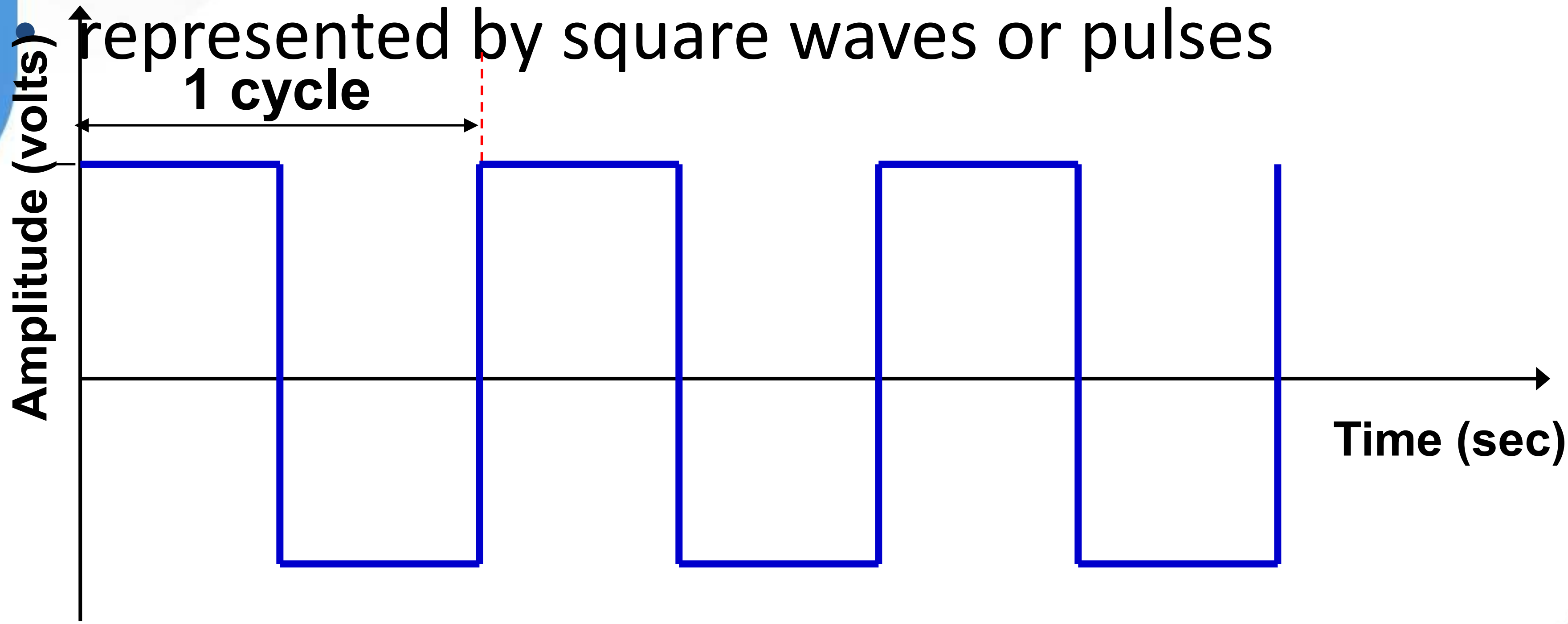
Analog Signaling

- represented by sine waves



Digital Signaling

- represented by square waves or pulses



Frequency (Hertz) = Cycles per Second

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Frequency, Spectrum and Bandwidth

- **Time domain concepts**
  - **Continuous signal**
    - Various in a smooth way over time
  - **Discrete signal**
    - Maintains a constant level then changes to another constant level
  - **Periodic signal**
    - Pattern repeated over time
  - **Aperiodic signal**
    - Pattern not repeated over time

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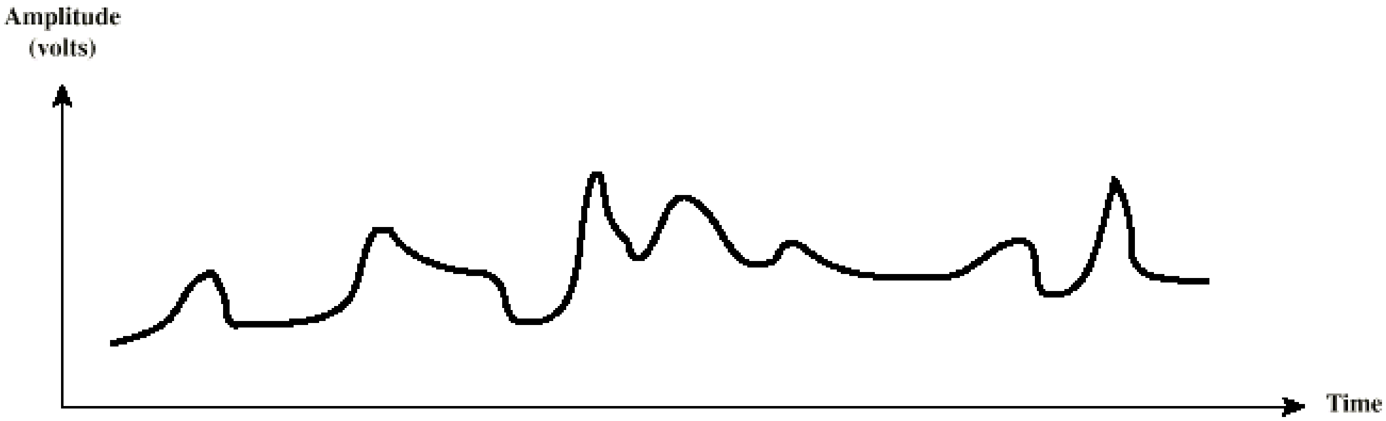
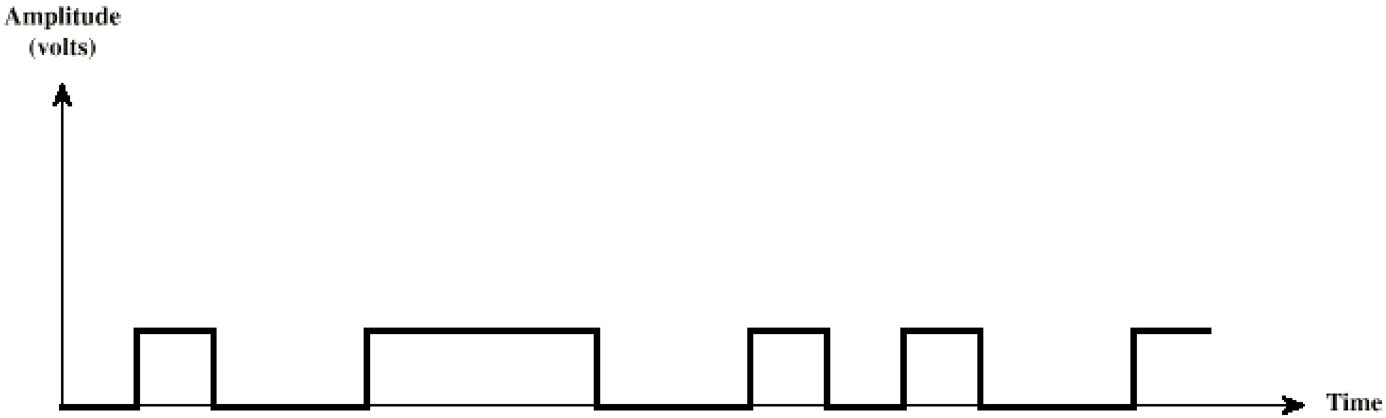
Continuous & Discrete Signals



(a) Continuous



(b) Discrete

Sine Wave Mathematical Expression

$$x(t) = A\sin(2\pi ft + \phi)$$

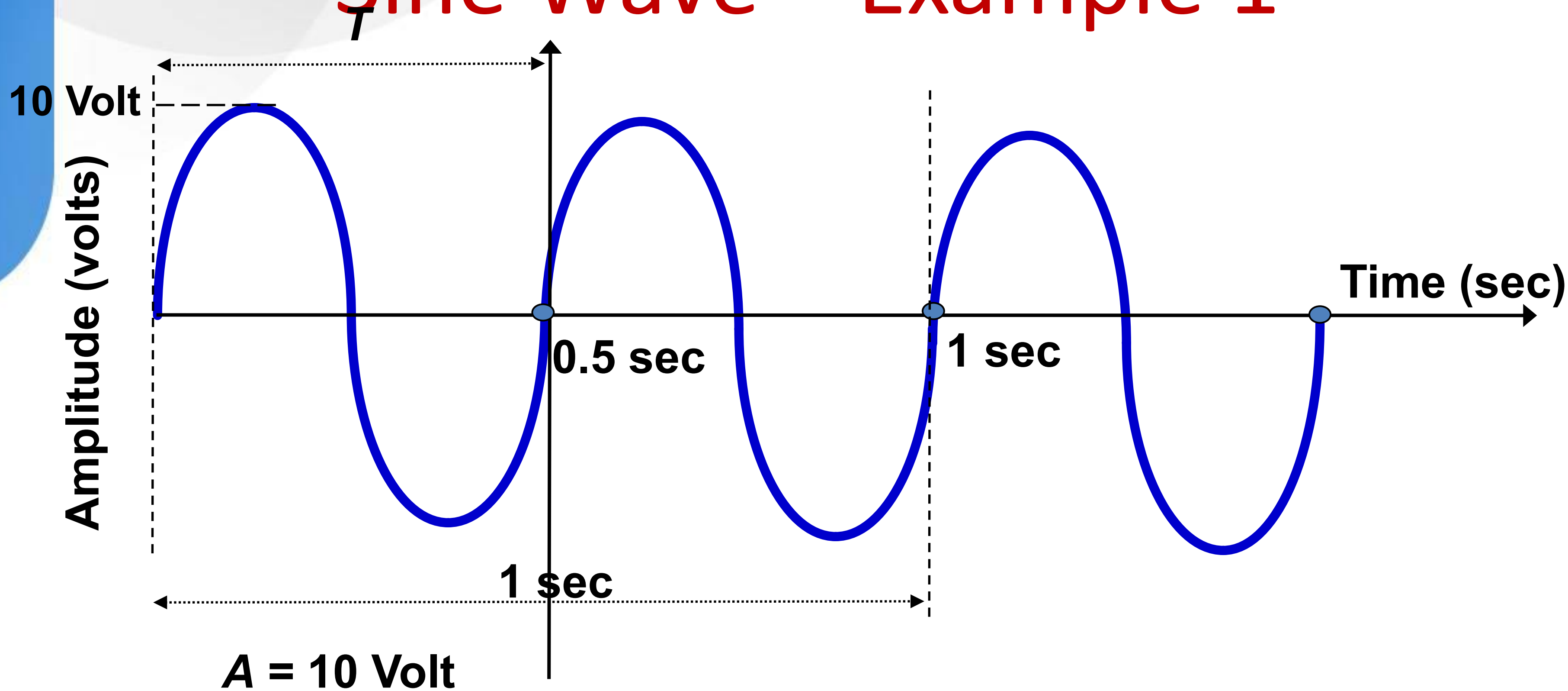
where $x(t)$ is the signal at time t,

A is the maximum amplitude of the signal,

f represents the number of cycles per second, and

ϕ defines the phase of the signal.

Sine Wave – Example 1

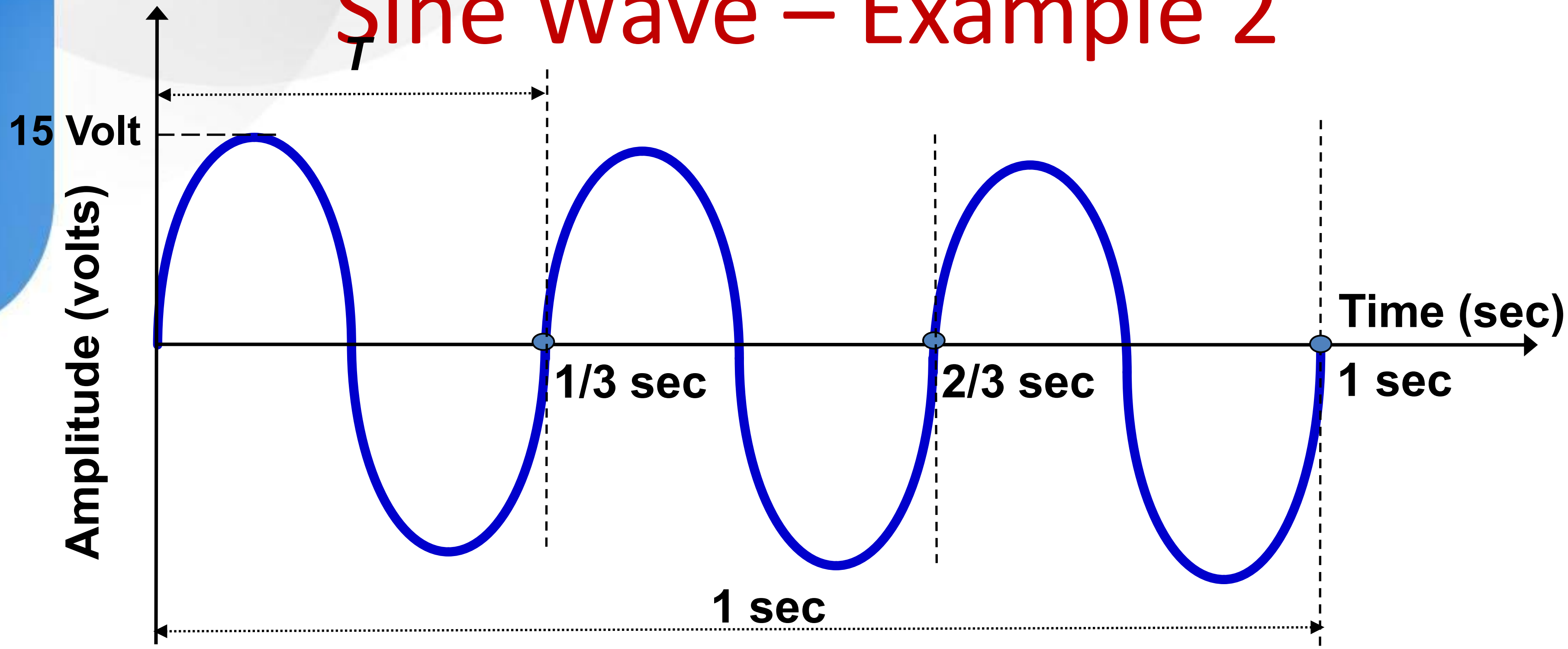


A = 10 Volt

f = 2 Hz

ϕ = 0 Radian

Sine Wave – Example 2

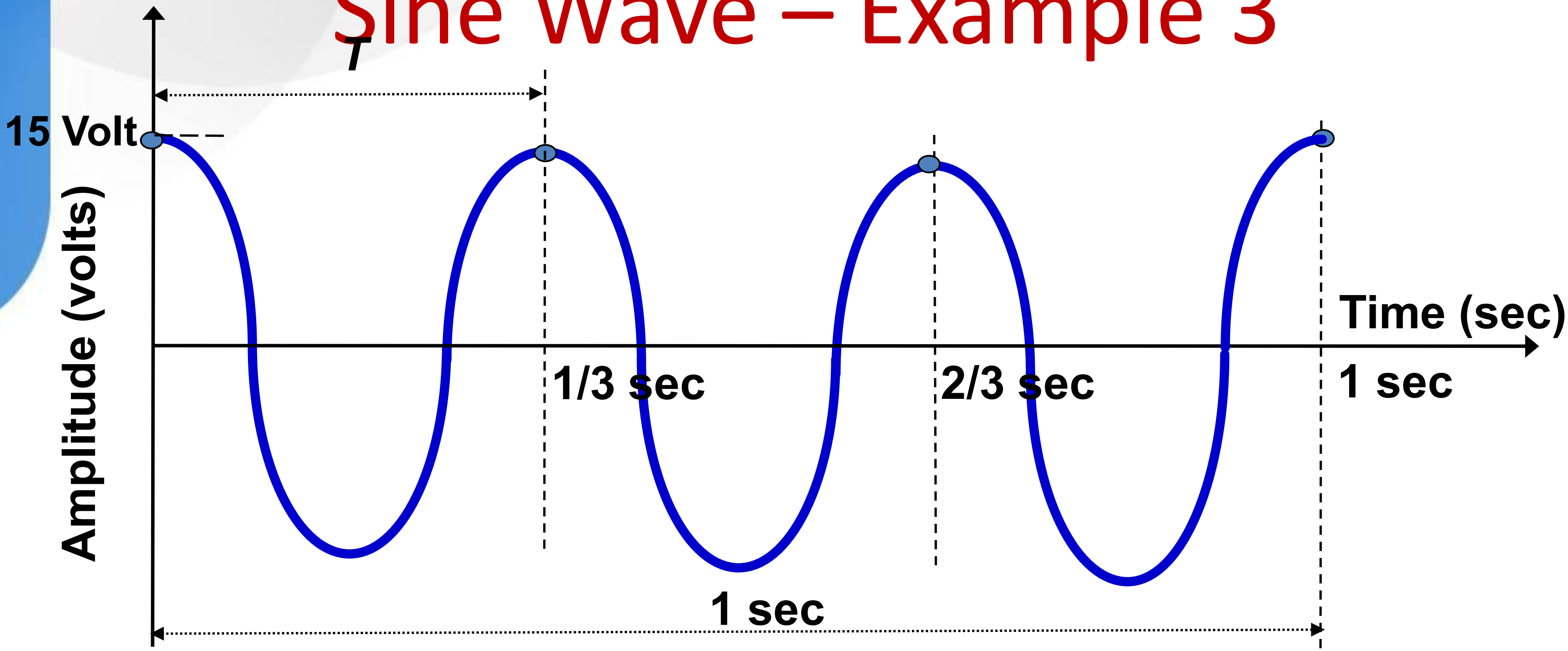


A = 15 Volt

f = 3 Hz

ϕ = 0 Radian

Sine Wave – Example 3



A = 15 Volt

f = 3 Hz

$\phi = \pi/2$ Radian = 90^{0}

Sine Wave

Amplitude:

- The amplitude is the instantaneous value of a signal at any time.

Peak Amplitude (A):

- Maximum strength of a signal. Its unit is Volt.

Frequency (f)

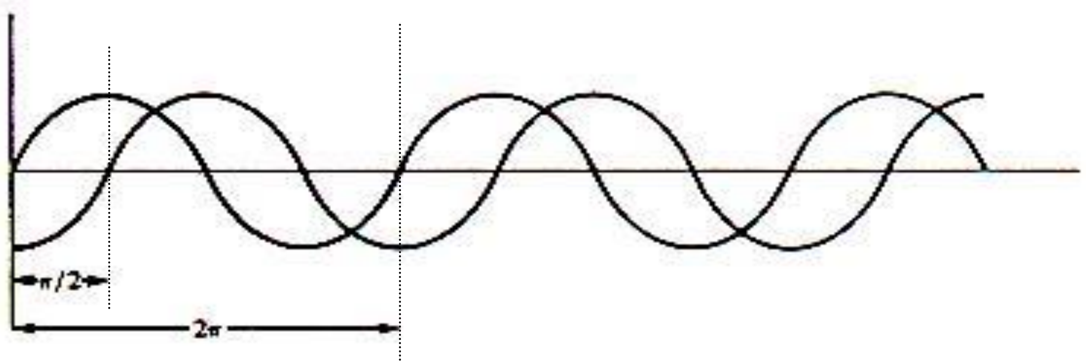
- Rate of change of signal
- Hertz (Hz) or cycles per second
- Period = time for one repetition (T)
- T = 1/f

- **Phase (ϕ)**
  - Position of the waveform relative to time zero.

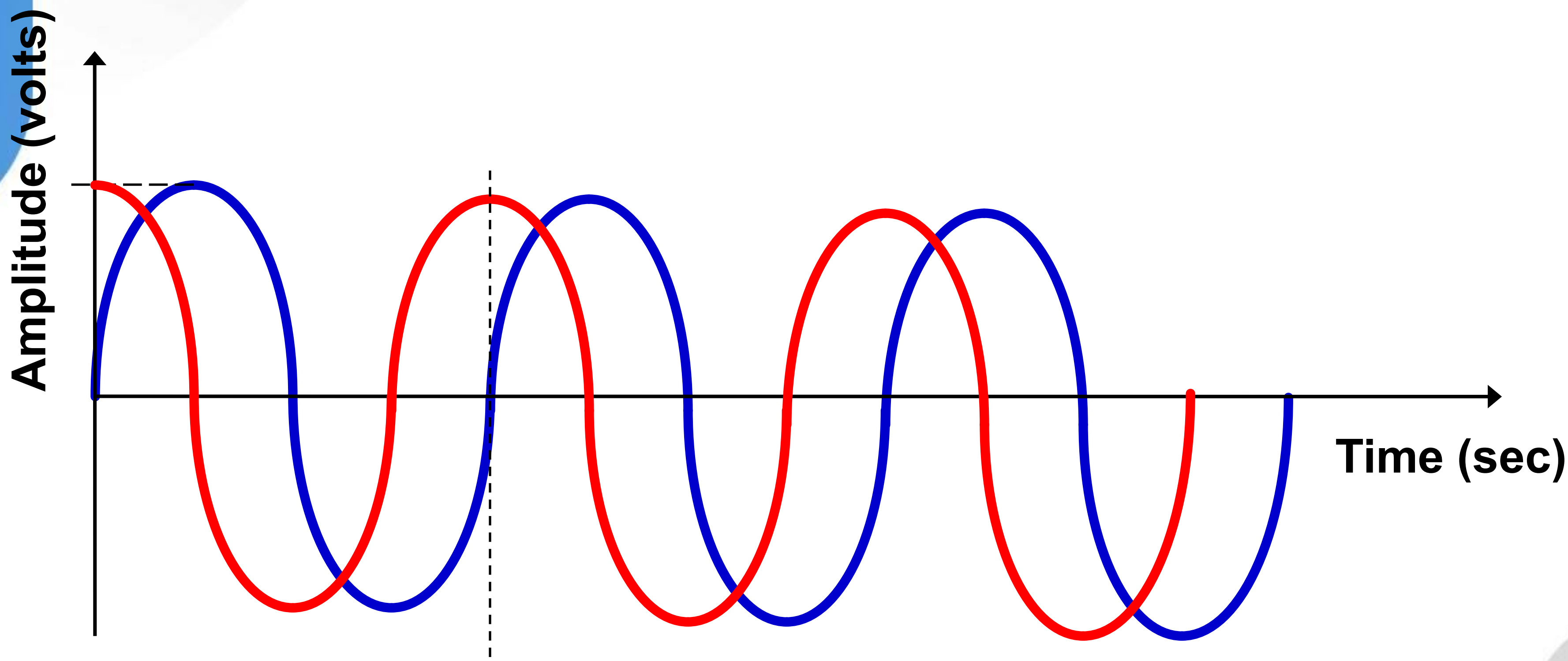
Sine Wave

Phase:

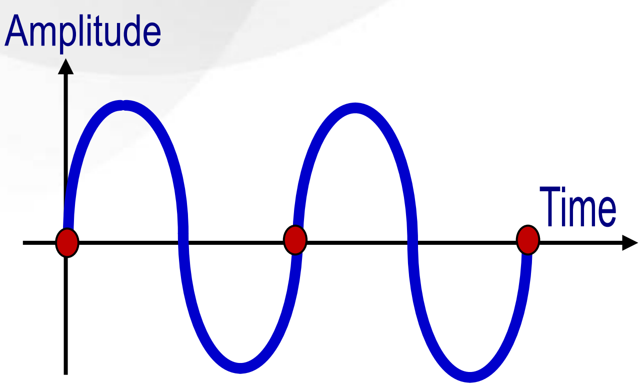
- The phase describes the position of the waveform relative to time zero. The range of shift is within a single period of a signal.
- The phase is a measure in degree or radian (2π = 360^o).
- The figure shows two signals that are out of phase by $\pi/2$ radians



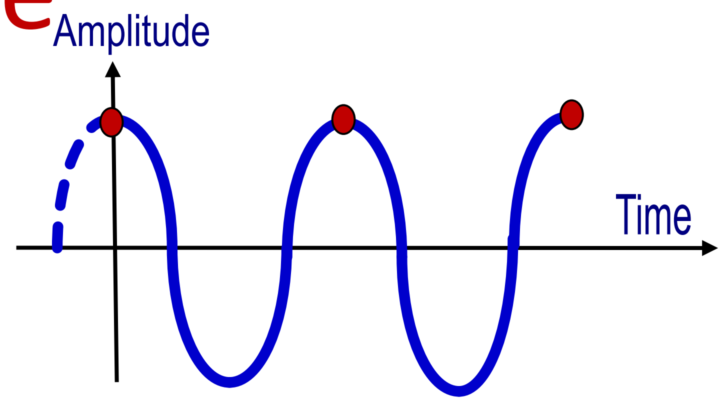
Phase



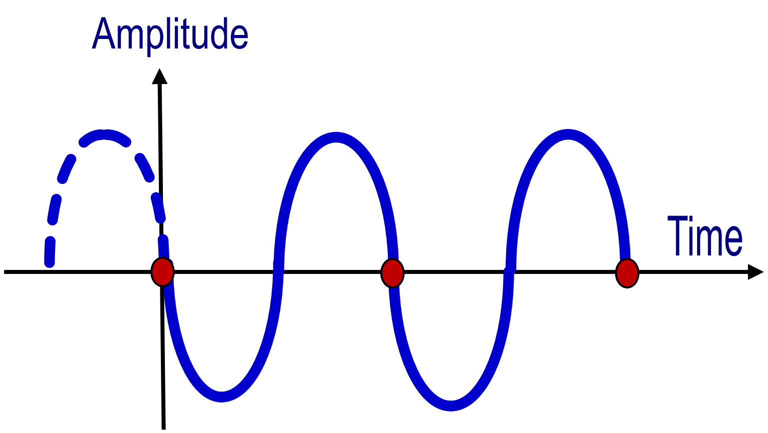
Phase



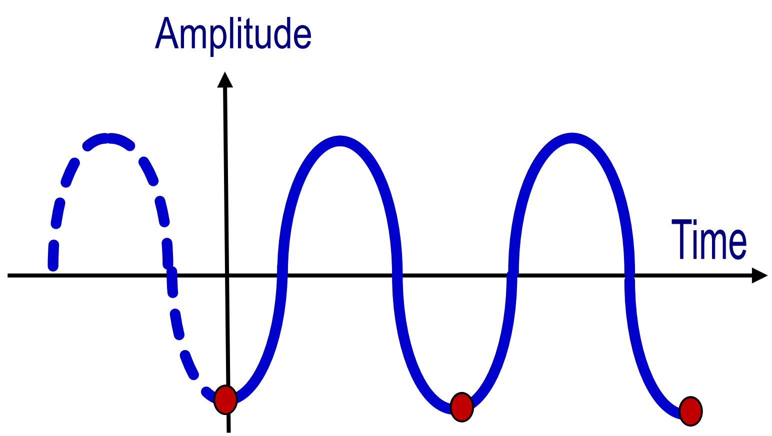
Phase = 0



Phase = $\pi/2$ radian

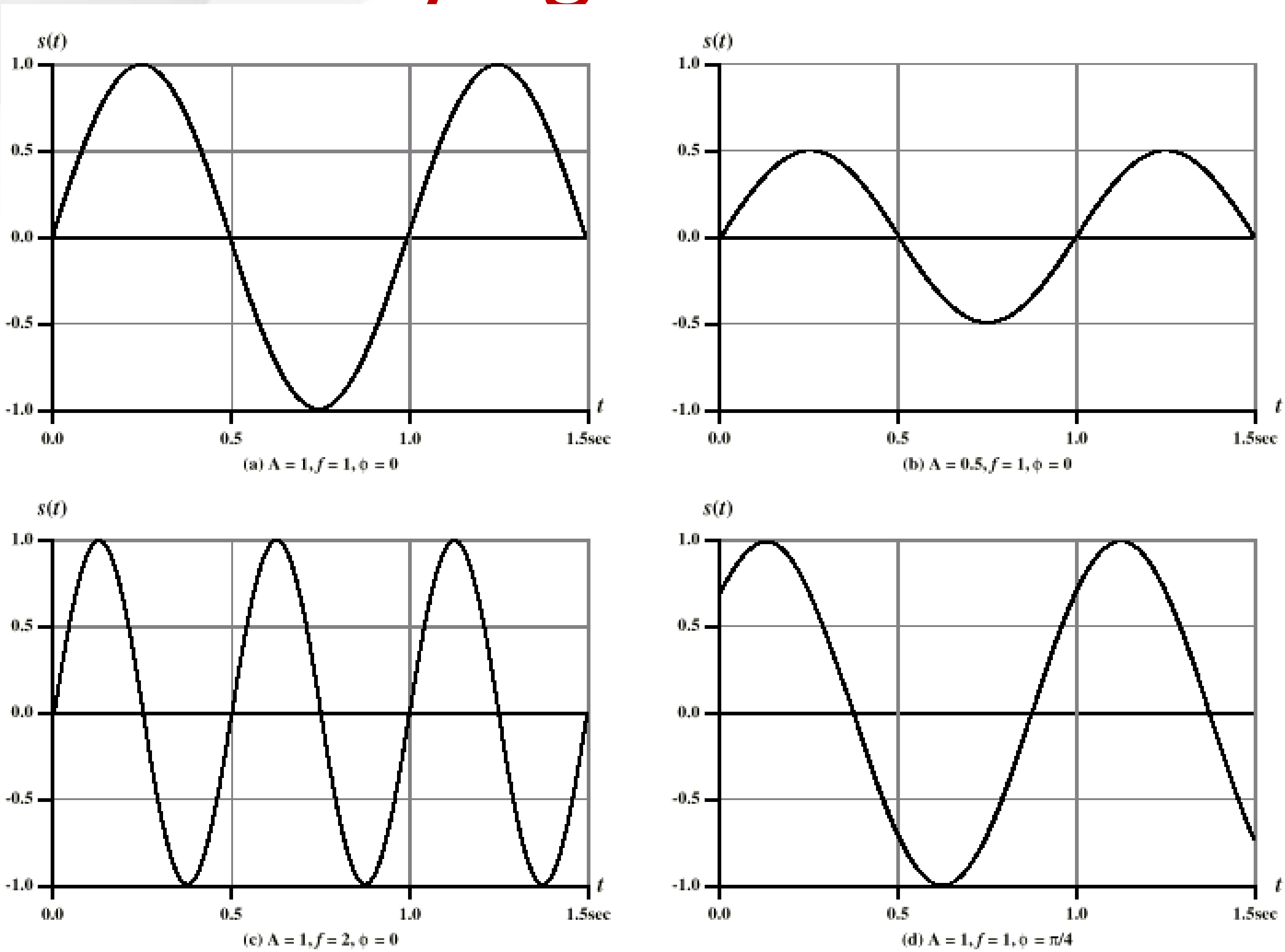


Phase = π radian



Phase = $3\pi/2$ radian

Varying Sine Waves



(a) A = 1, f = 1, ϕ = 0

(b) A = 0.5, f = 1, ϕ = 0

(c) A = 1, f = 2, ϕ = 0

(d) A = 1, f = 1, ϕ = $\pi/4$

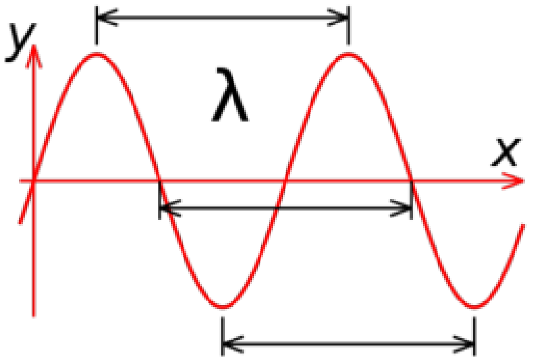
Wavelength

- Distance occupied by one cycle
- Distance between two points of corresponding phase in two consecutive cycles
- Wavelength is commonly designated by λ
- The wavelength is related to the period as follows:

$$\lambda = vT$$

where v : signal velocity

T : signal period



- Equivalently,

$$v = \lambda f$$

where $v = c = 3*10^8\ ms^{-1}$ (speed of light in free space)

Example

Your voice is a summation of sine waves, each sine wave having its own frequency, phase, and amplitude. The range of frequencies is normally between 300 and 3300 Hz. Give a general equation.

$$x(t) = A_1 \sin(2\pi f_0 t + \phi_1) + A_2 \sin(2\pi f_2 t + \phi_2) + \ldots + A_n \sin(2\pi f_n t + \phi_n)$$

with 300 Hz < f_i < 3300 Hz. f_0 is called the *fundamental frequency*, and f_2, f_3 … f_n are called the *harmonics*.

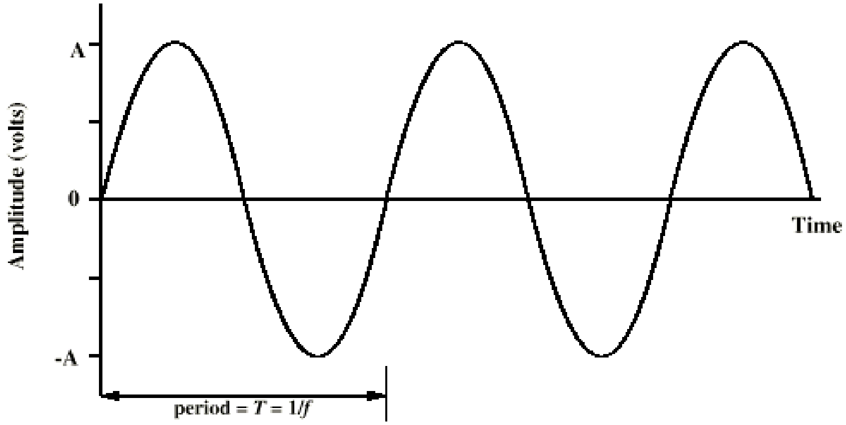
Periodic Signal

- **Periodic signal:** a signal that repeats itself at equal time interval.

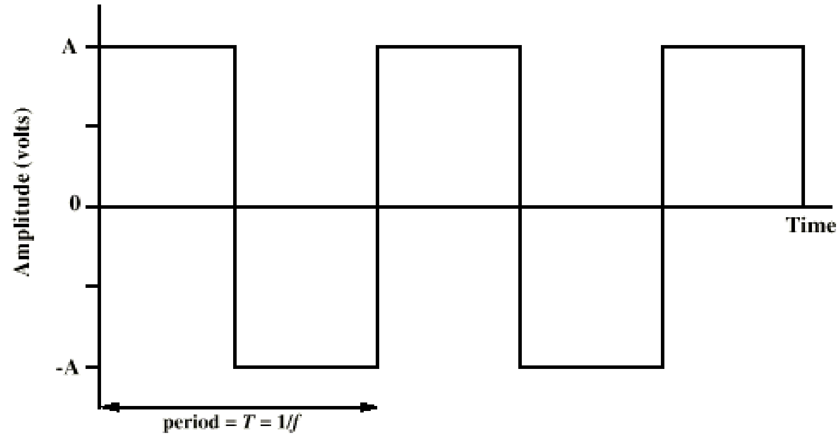
- **It is made up of a infinite series of sinusoidal frequency components.**
- A signal is **periodic** if and only if:

$$s(t+T)=s(t) \qquad -\infty < t < +\infty$$

Periodic Signals



(a) Sine wave



(b) Square wave

Periodic Signal

- Mathematically, we can express any periodic waveform as follows:

$$v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

where:

$v(t)$ is the voltage signal representation as a function of time
ω_0 is the fundamental frequency component in radians per second
$T = 2\pi/\omega_0$ is the period of the waveform in seconds

The terms a_0, a_n and b_n are known as the **Fourier coefficients** and, for a particular waveform, we can derive them from the following set of integrals:

$$a_0 = \frac{1}{T}\int_0^T v(t)\mathrm{d}t$$

$$a_n = \frac{2}{T}\int_0^T v(t)\cos(n\omega_0 t)\mathrm{d}t$$

$$b_n = \frac{2}{T}\int_0^T v(t)\sin(n\omega_0 t)\mathrm{d}t$$

We can deduce from the first integral that a_0 is the mean of the signal over the period T and is known as the **DC component**.

Frequency Domain Concepts

- Signal usually made up of many frequencies
- Components are sine waves
- Can be shown (**Fourier analysis**) that any signal is made up of component sine waves
- Can plot frequency domain functions

Frequency Components of Square Waves

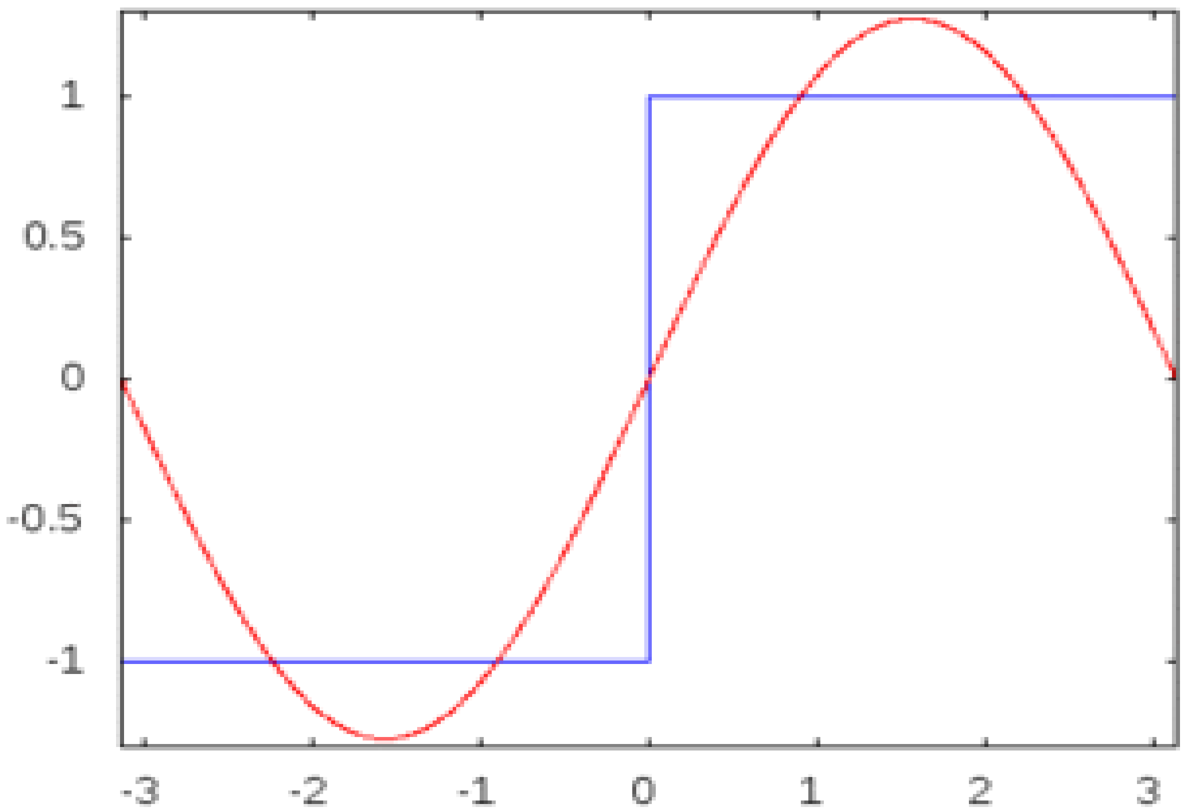
- The Frequency components of a square wave with amplitude *A* and *–A* can be expressed as follows:

$$s(t) = A \times \frac{4}{\pi} \times \sum_{\substack{k \text{ odd} \\ k=1}}^{\infty} \frac{\sin(2\pi k f t)}{k}$$

- This waveform has an infinite number of frequency components, and hence an infinite bandwidth.

- The peak amplitude of the k^{th} frequency component is only 1/k, so most of the energy in this waveform is in the first few frequency components

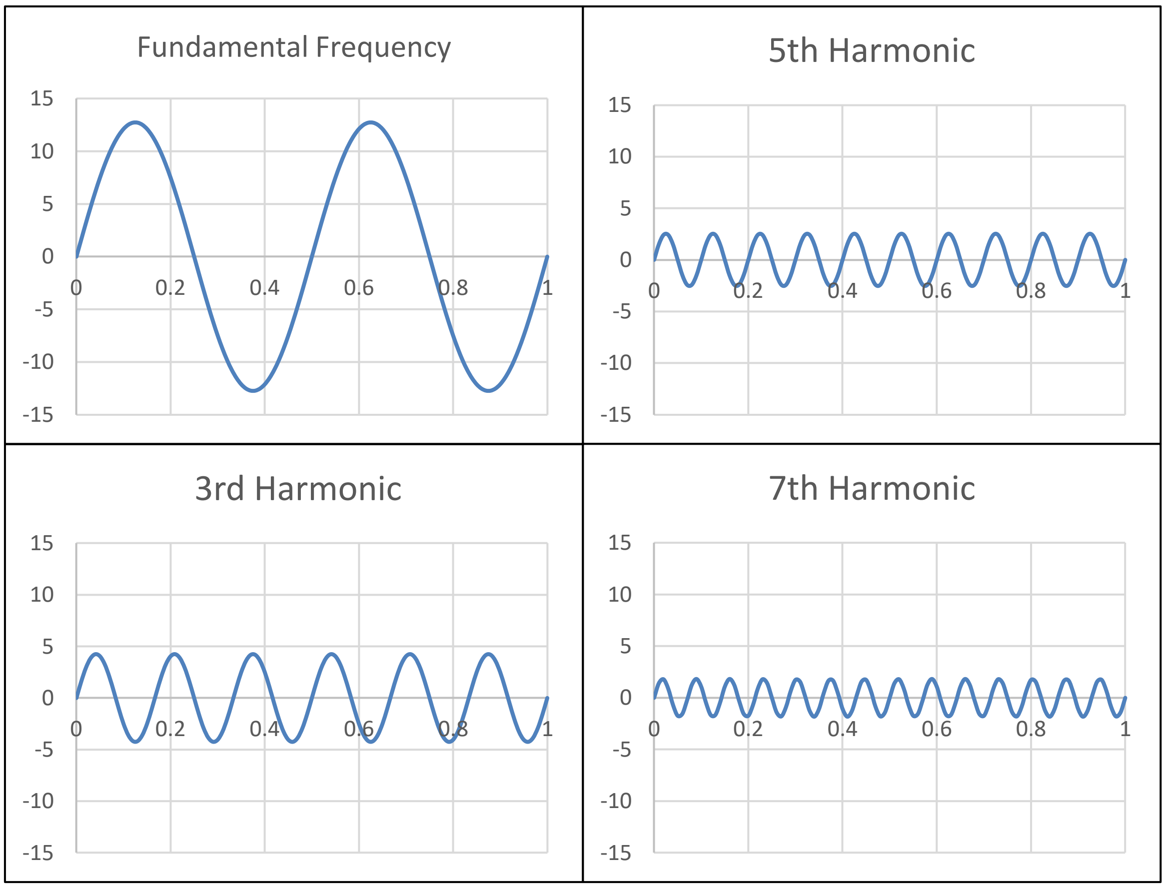
Frequency Components of Square Waves



Frequency Components of Square

$$s(t) = A \times \frac{4}{\pi} \times \sum_{k\ odd\ k=1}^{\infty} \frac{\sin(2\pi k f t)}{k}$$

A = 10 Volt , *f* = 2 Hz



Frequency Components of Square

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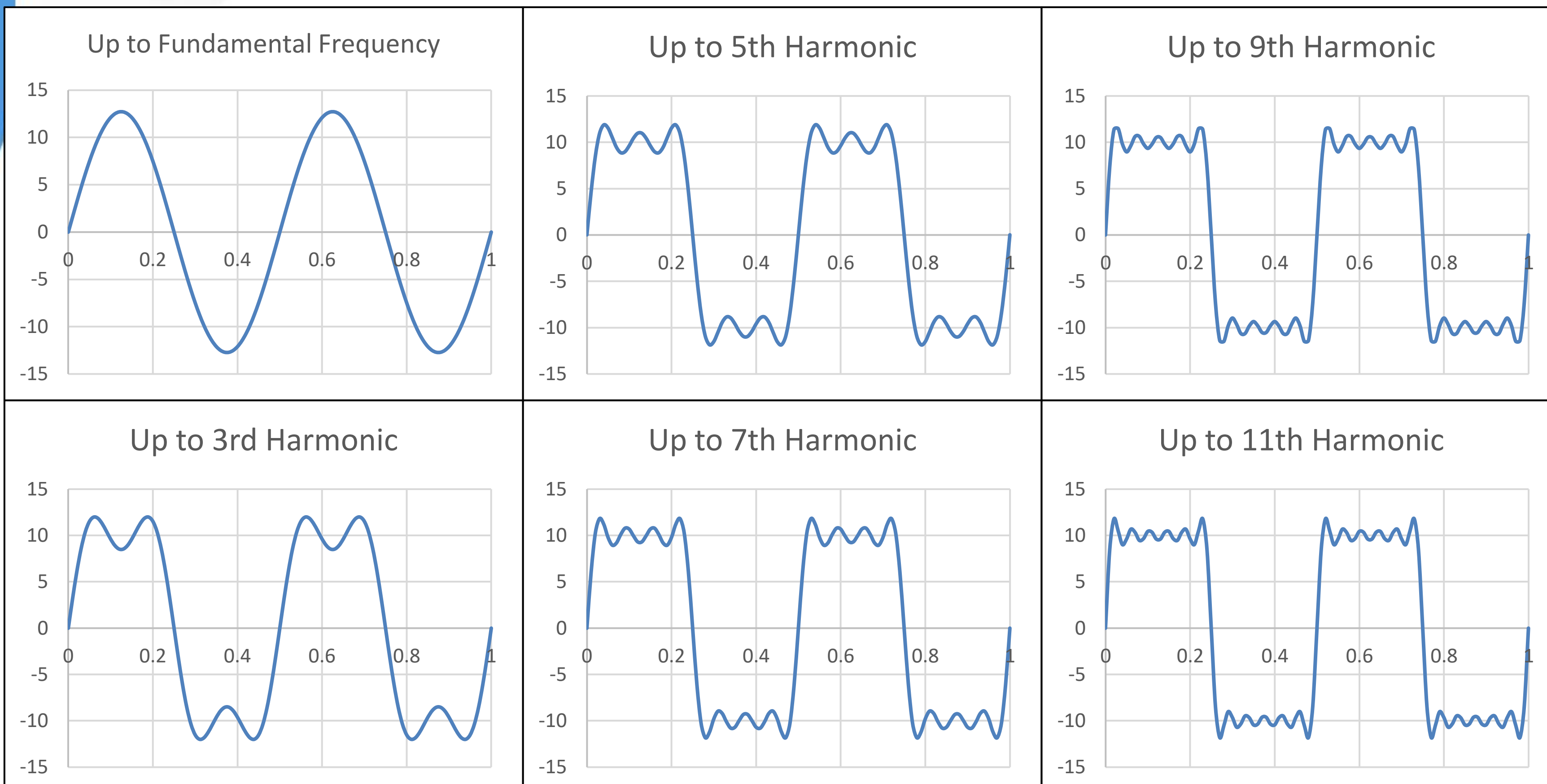
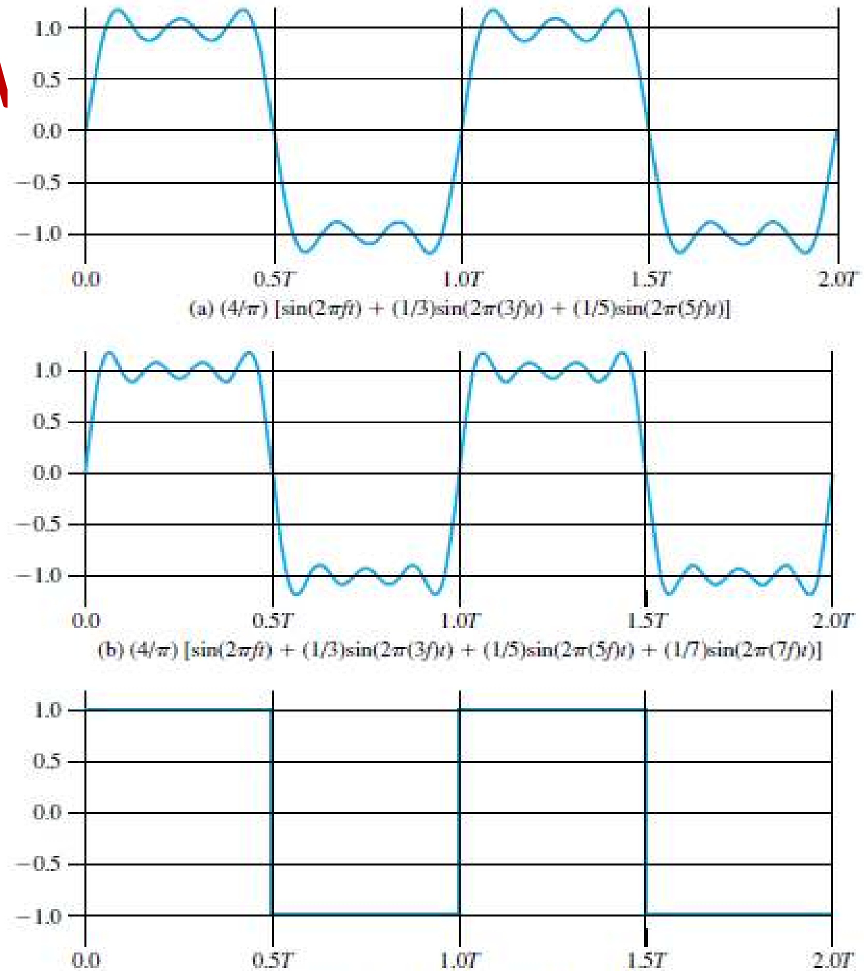


Frequency Components of Square W

Fundamental Frequency + Third harmonic + Fifth Harmonic:

Fundamental Frequency + Third harmonic + Fifth Harmonic + Seventh Harmonic:

Square waveform with an infinite number of frequency components



(a) $(4/\pi)\,[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t)]$

(b) $(4/\pi)\,[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t) + (1/7)\sin(2\pi(7f)t)]$

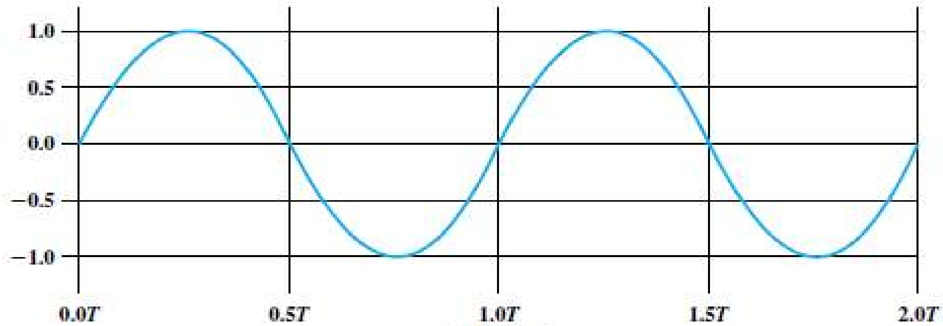
(c) $(4/\pi)\sum (1/k)\sin(2\pi(kf)t)$, for k odd

Frequency Components of Square Wave ($T = 1/f$)

Frequency Components of Square Waves

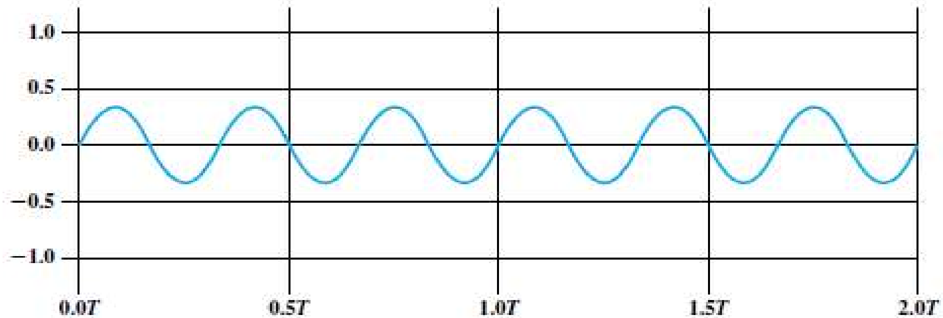
Addition of Frequency Components

Fundamental Frequency Component:



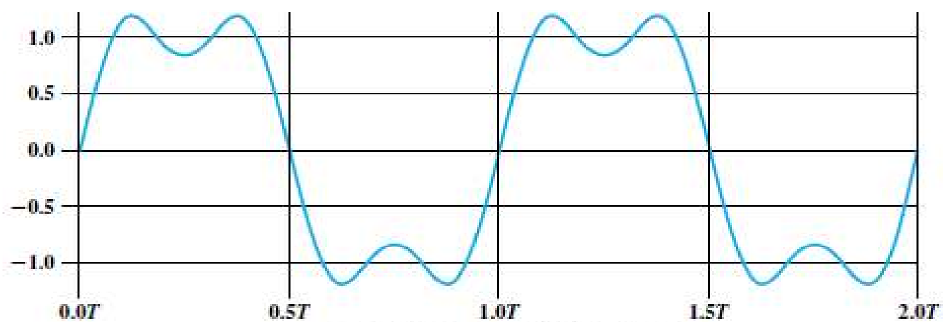
(a) $\sin(2\pi ft)$

Third Harmonic:



(b) $(1/3)\sin(2\pi(3f)t)$

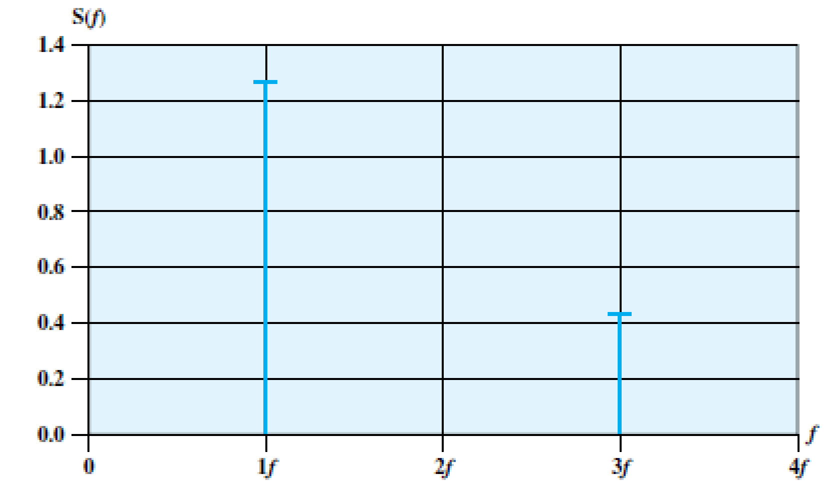
Fundamental Frequency + Third Harmonic Components:



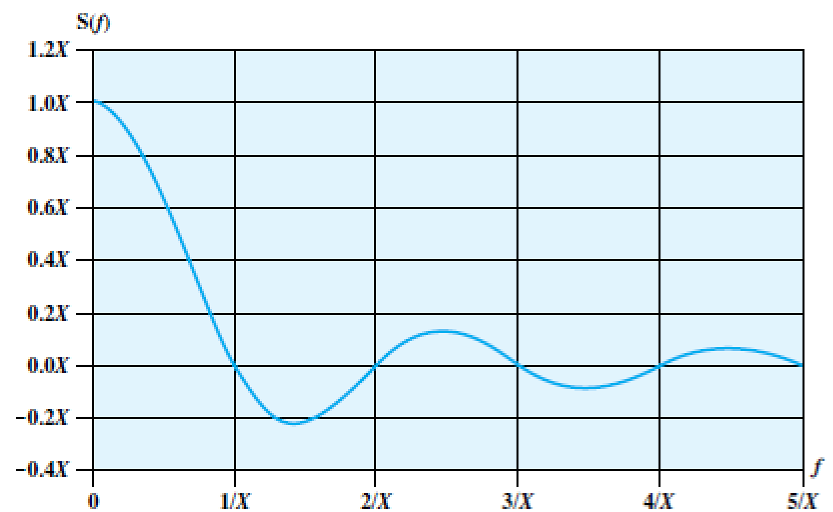
(c) $(4/\pi)\,[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t)]$

Frequency-Domain

- For square waves, only odd harmonics exist (plus the fundamental component of course)
- Figure (a) is discrete because the time domain function is **Periodic**.
- Figure (b) is continuous because the time domain function is **Aperiodic**.



(a) $s(t) = (4/\pi)[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t)]$

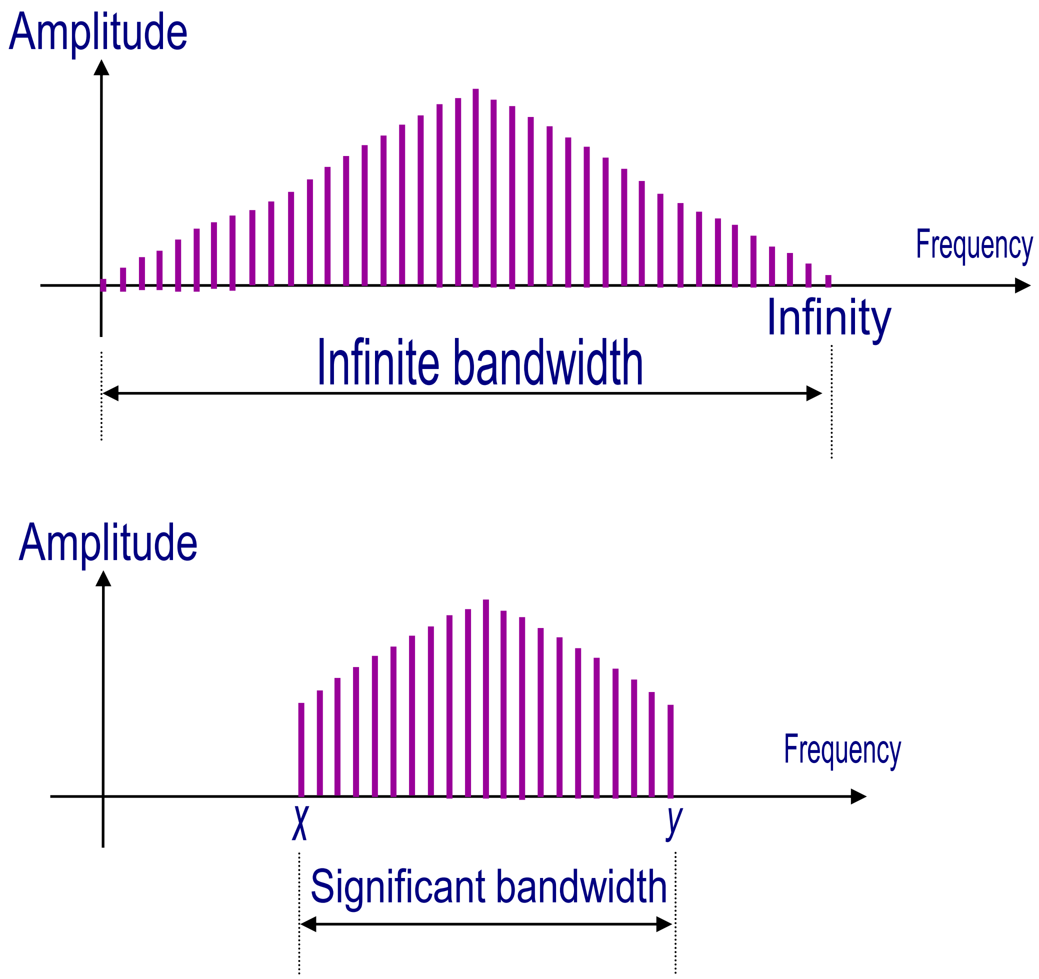


(b) $s(t) = 1 \quad -X/2 \le t \le X/2$

Spectrum & Bandwidth

- **Spectrum**
  - range of frequencies contained in signal
- **Absolute bandwidth**
  - width of spectrum
- **Effective bandwidth**
  - Often just *bandwidth*
  - Narrow band of frequencies containing most of the energy
- **DC Component**
  - Component of zero frequency

Absolute and Effective Bandwidth



Bandwidth
Example 1

- If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900 Hz, what is the bandwidth?

- Let f_H be the highest frequency, f_L be the lowest frequency, and B be the bandwidth. Then,

$$B = f_H - f_L = 900 – 100 = 800 \text{ Hz}$$

Bandwidth
Example 2

- A signal has a bandwidth of 20 KHz. The highest frequency is 60 KHz. What is the lowest frequency?

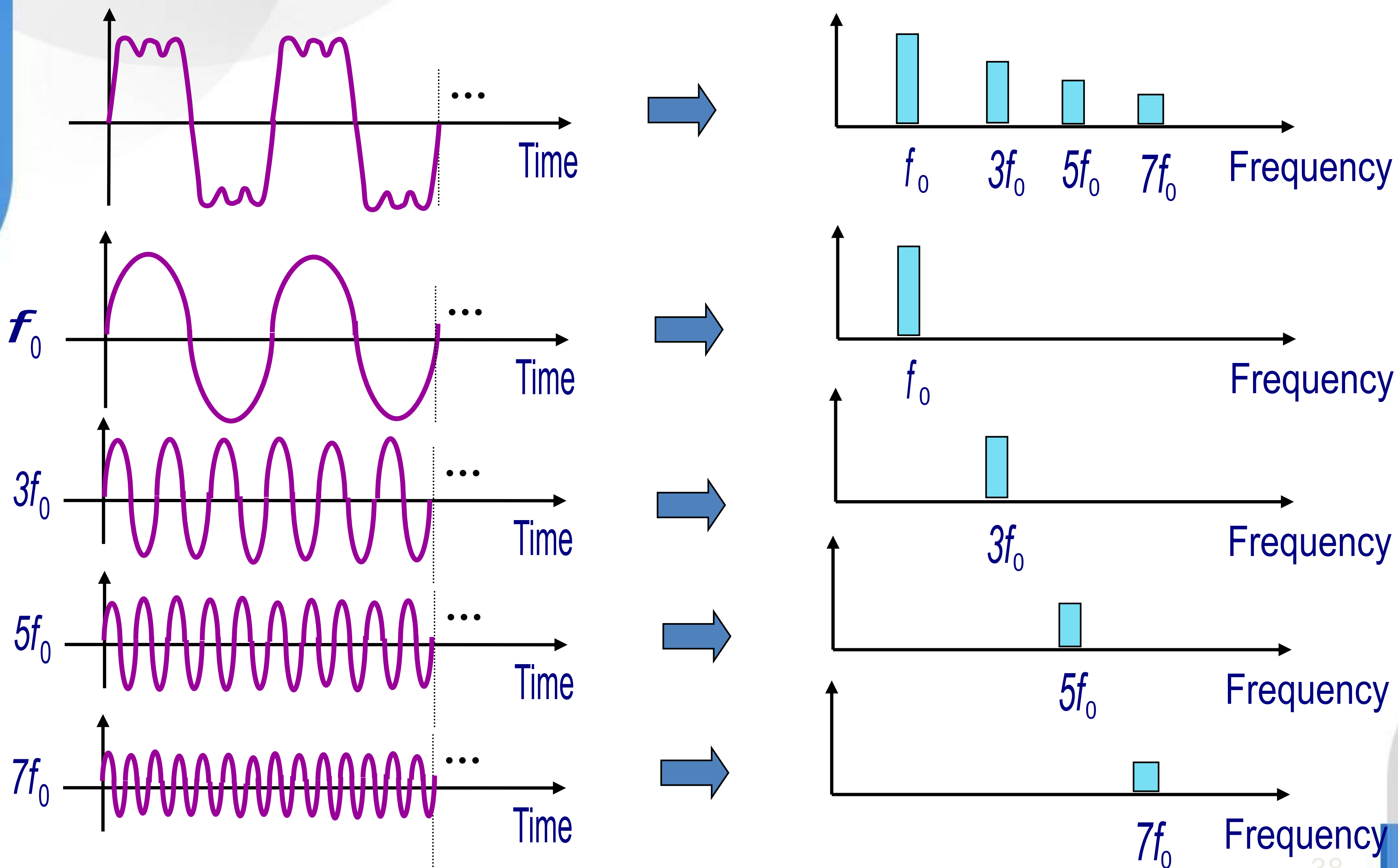
- Let f_H be the highest frequency, f_L be the lowest frequency, and B be the bandwidth. Then,

$$B = f_H - f_L$$

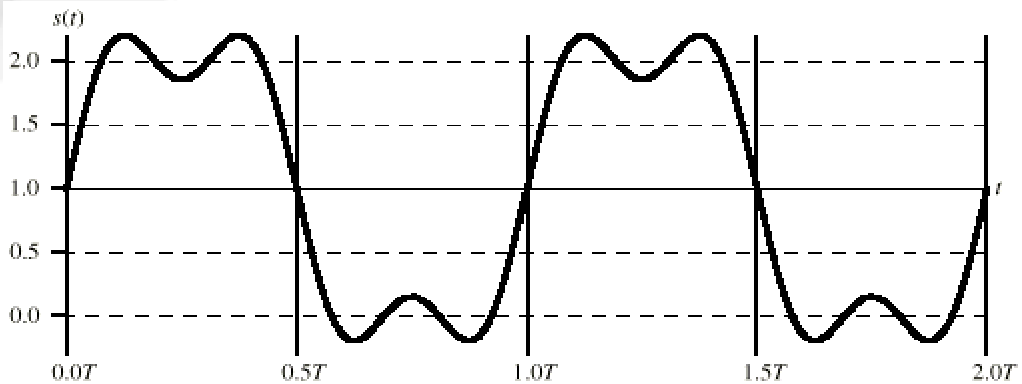
$$\rightarrow \quad f_L = f_H - B$$

$$\rightarrow \quad f_L = 60\text{ KHz} - 20\text{ KHz} = 40\text{ KHz}$$

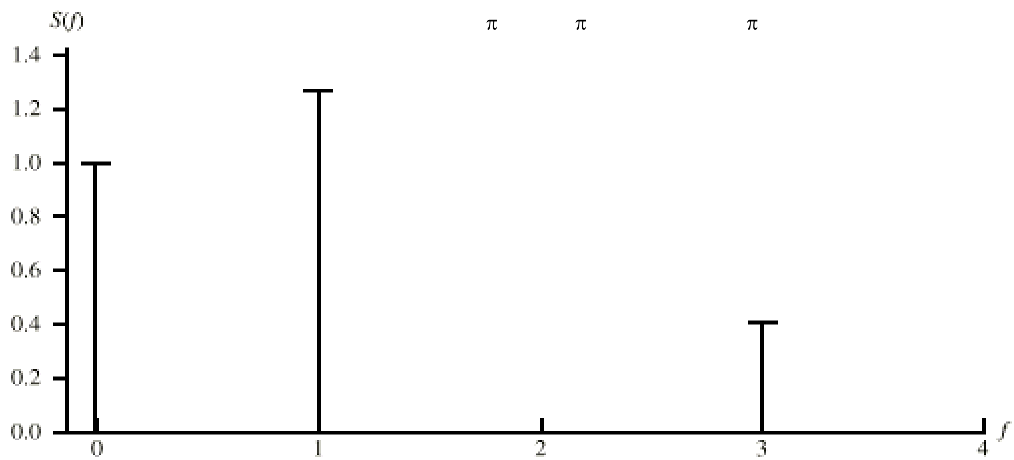
Decomposition of a Digital Signal



Signal with DC Component

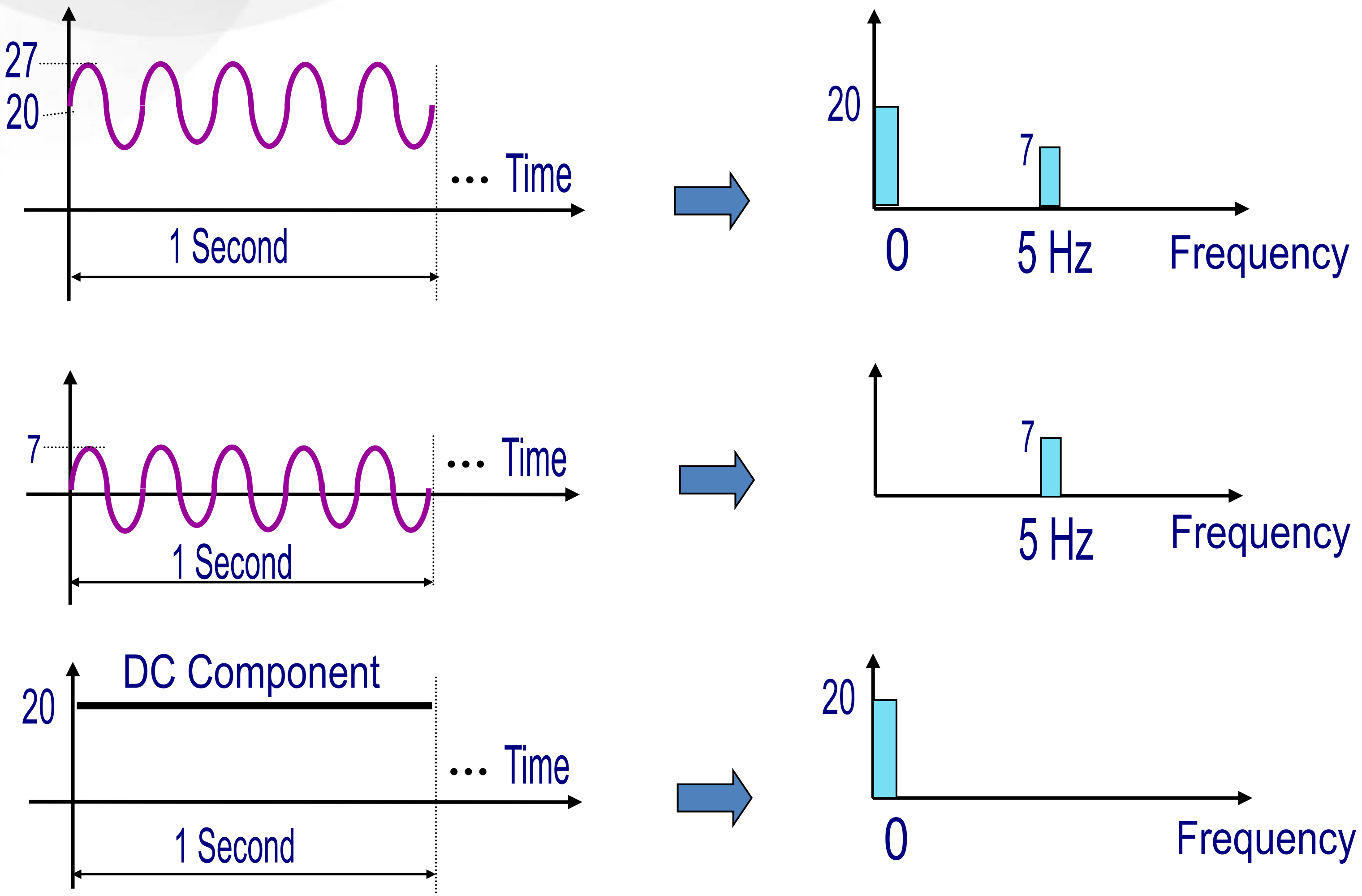


(a) s(t) = 1 + (4/) [sin (2 ft) + (1/3) sin (2 (3f)t)]



(b) S(f)

Time Domain and Frequency Domain

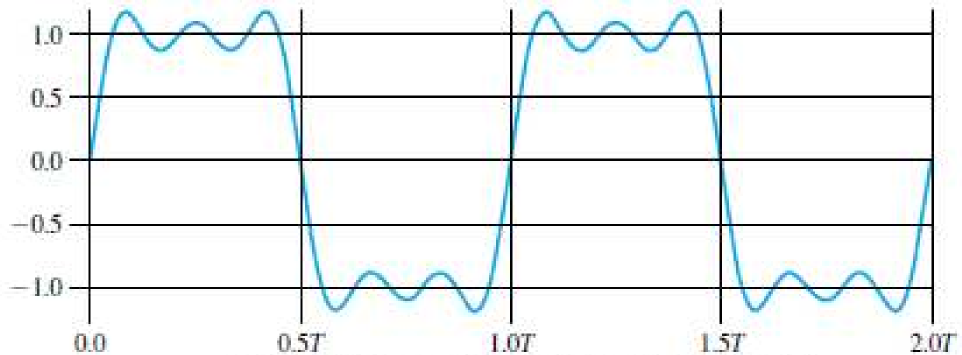


Data Rate and Bandwidth

- Any transmission system has a limited band of frequencies
- This limits the data rate that can be carried
- For economic and practical reasons, digital information must be **approximated by the signal of limited bandwidth**.

Data Rate and Bandwidth Example 1:

- f = 1 MHz

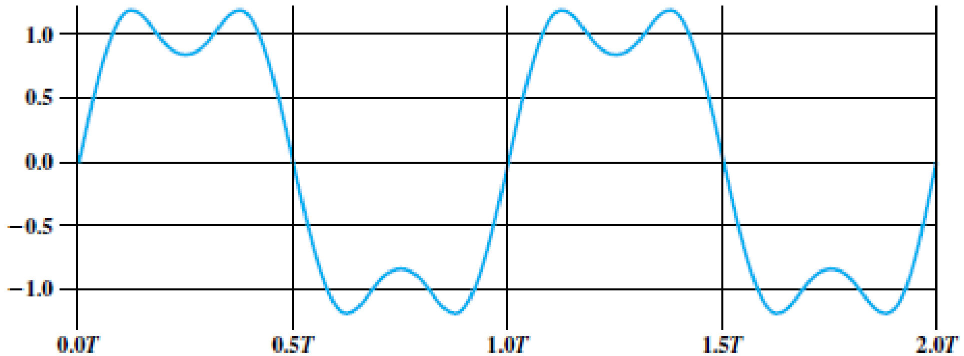


(a) $(4/\pi)\,[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t) + (1/5)\sin(2\pi(5f)t)]$

- Bandwidth = $f_H - f_L$ = 5MHz – 4 MHz = 1 MHz
- Period of the signal = $T = 1/f$ = 1 μsec
- The signal is a bit string of 1s and 0s
- One bit occurs every T_b=0.5 μsec → frequency 2 MHz (R= $1/T_b$)
- Thus, **a bandwidth of 1 MHz, a data rate of 2 Mbps is achieved.**

Data Rate and Bandwidth Example 3:

- f = 2 MHz



(c) $(4/\pi)\,[\sin(2\pi ft) + (1/3)\sin(2\pi(3f)t)]$

- Bandwidth = $f_H - f_L$ = 6MHz – 4 MHz = 2 MHz
- Period of the signal = $T = 1/f$ = 0.5 μsec
- The signal is a bit string of 1s and 0s
- One bit occurs every 0.25 μsec → Data Rate of 4 Mbps

Analog and Digital Data Transmission

- **Data**
  - Entities that convey meaning
- **Signals**
  - Light , Electric or electromagnetic representations of data
- **Transmission**
  - Communication of data by propagation and processing of signals

Data

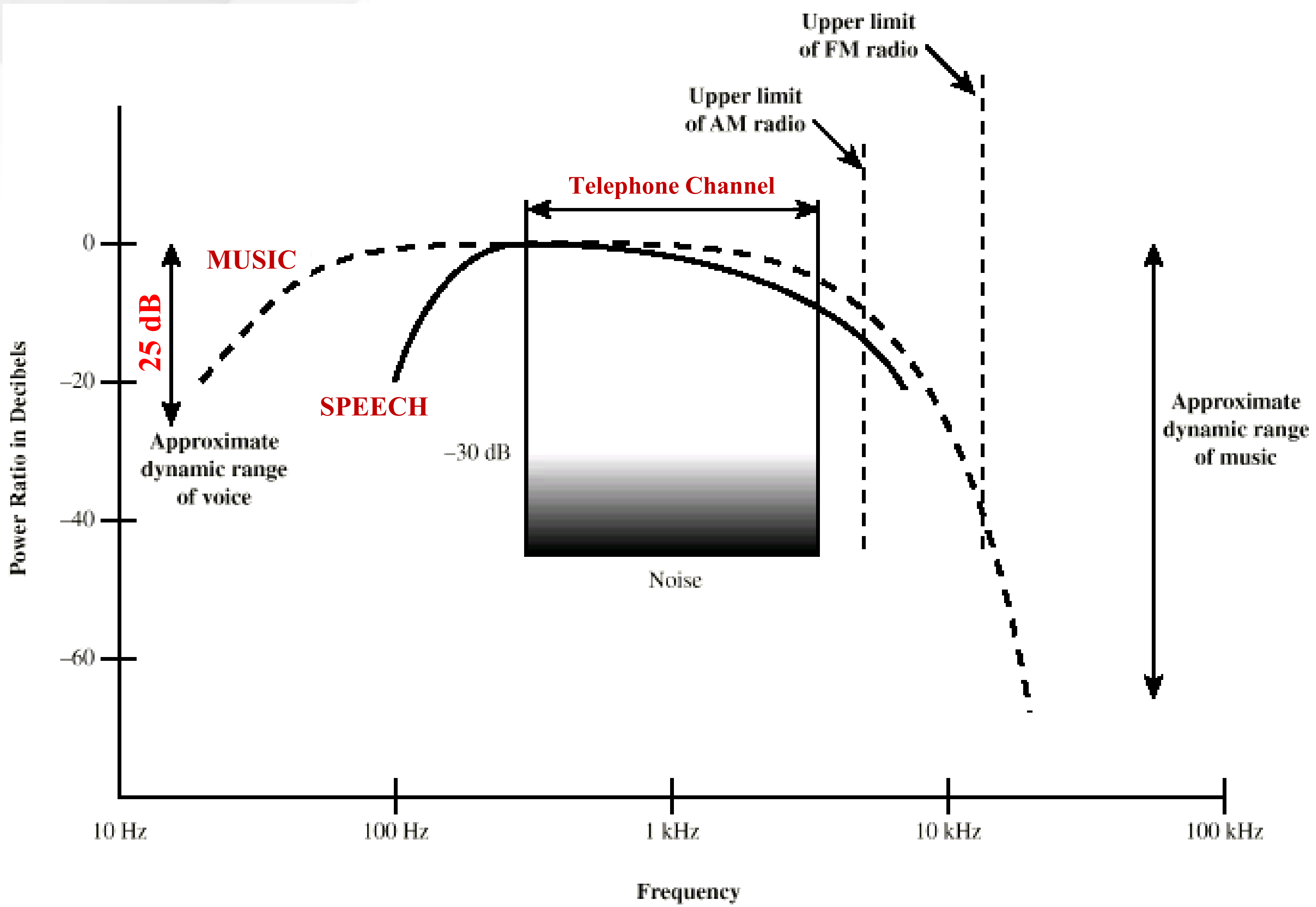
- **Analog**
  - Continuous values within some interval
  - e.g. **sound, video**

- **Digital**
  - Discrete values
  - e.g. **text, integers**

Signals

- Means by which data are propagated
- **Analog**
  - Continuously variable
  - Various media: wire, fiber optic, space
  - **Speech** bandwidth 100Hz to 7kHz
  - **Telephone** bandwidth 300Hz to 3300Hz
  - **Video** bandwidth 4MHz
- **Digital**
  - Use **two DC components**

Acoustic Spectrum (Analog)



Acoustic Spectrum (Analog)

- The power ratio of typical speech has a dynamic range of about **25 dB (decibels)**
- The power produced by the loudest shout may be as much as 300 times greater than the least whisper.
- 25 dB = 10 $\log_{10}X$

  → $\log_{10}X = 2.5$

  → $10^{\log_{10}X} = 10^{2.5}$

  → $X = 10^{2.5} = 316$

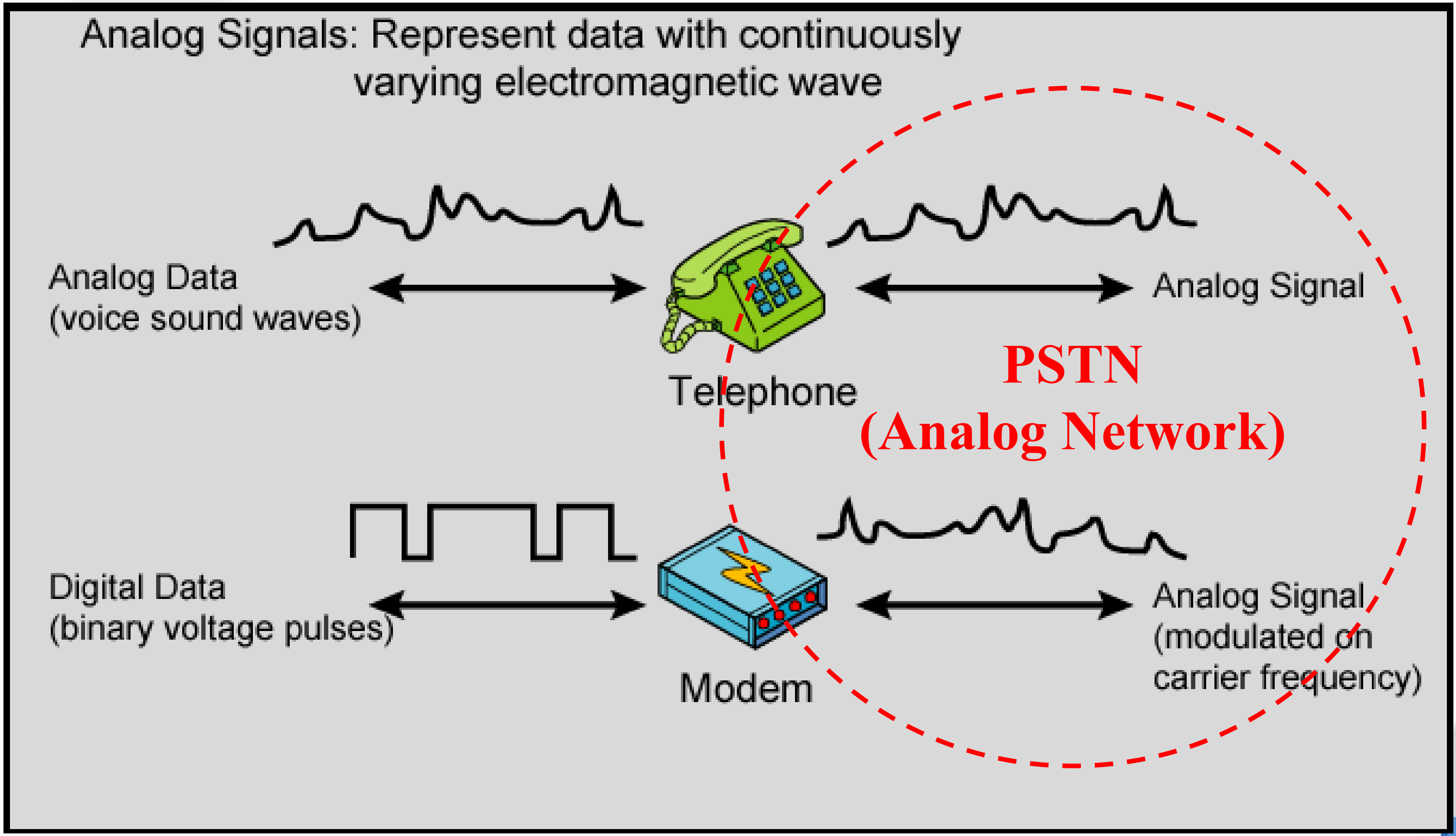
Data and Signals

- Usually use **digital signals for digital data** and **analog signals for analog data**

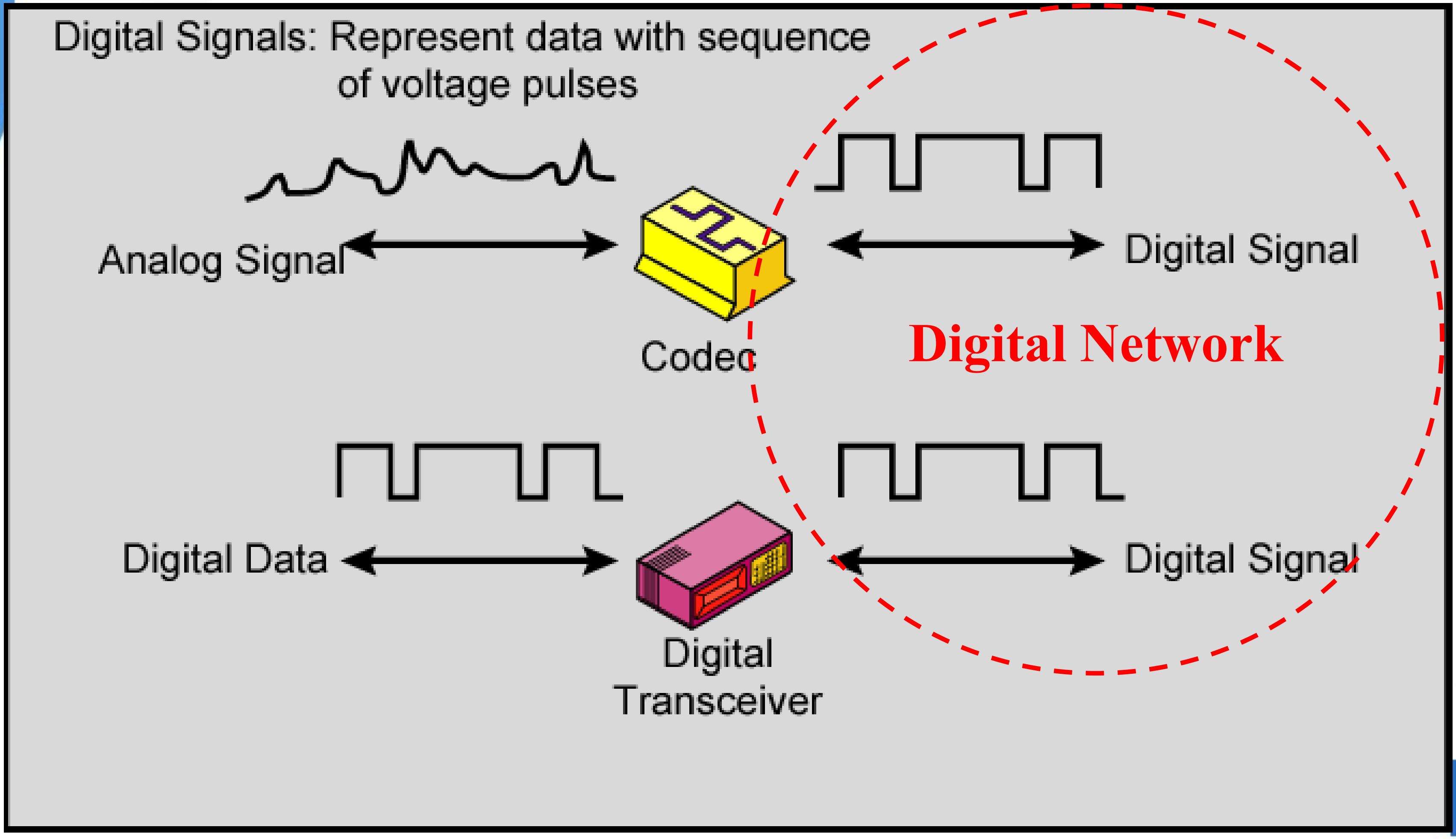
- Can use **analog signal to carry digital data**
  - Modem

- Can use **digital signal to carry analog data**
  - Compact Disc audio

Analog Signals Carrying Analog and Digital Data



Digital Signals Carrying Analog and Digital Data



Analog Transmission

- Analog signal transmitted without regard to content
- May be analog or digital data
- Attenuated over distance
- Use **amplifiers** to boost signal
- Also **amplifies noise**

Digital Transmission

- Concerned with content
- Integrity endangered by noise, attenuation etc.
- **Repeaters** used
  - Repeater receives signal
  - Extracts bit pattern
  - Retransmits
- Attenuation is overcome
- **Noise is not amplified**

Advantages of Digital Transmission

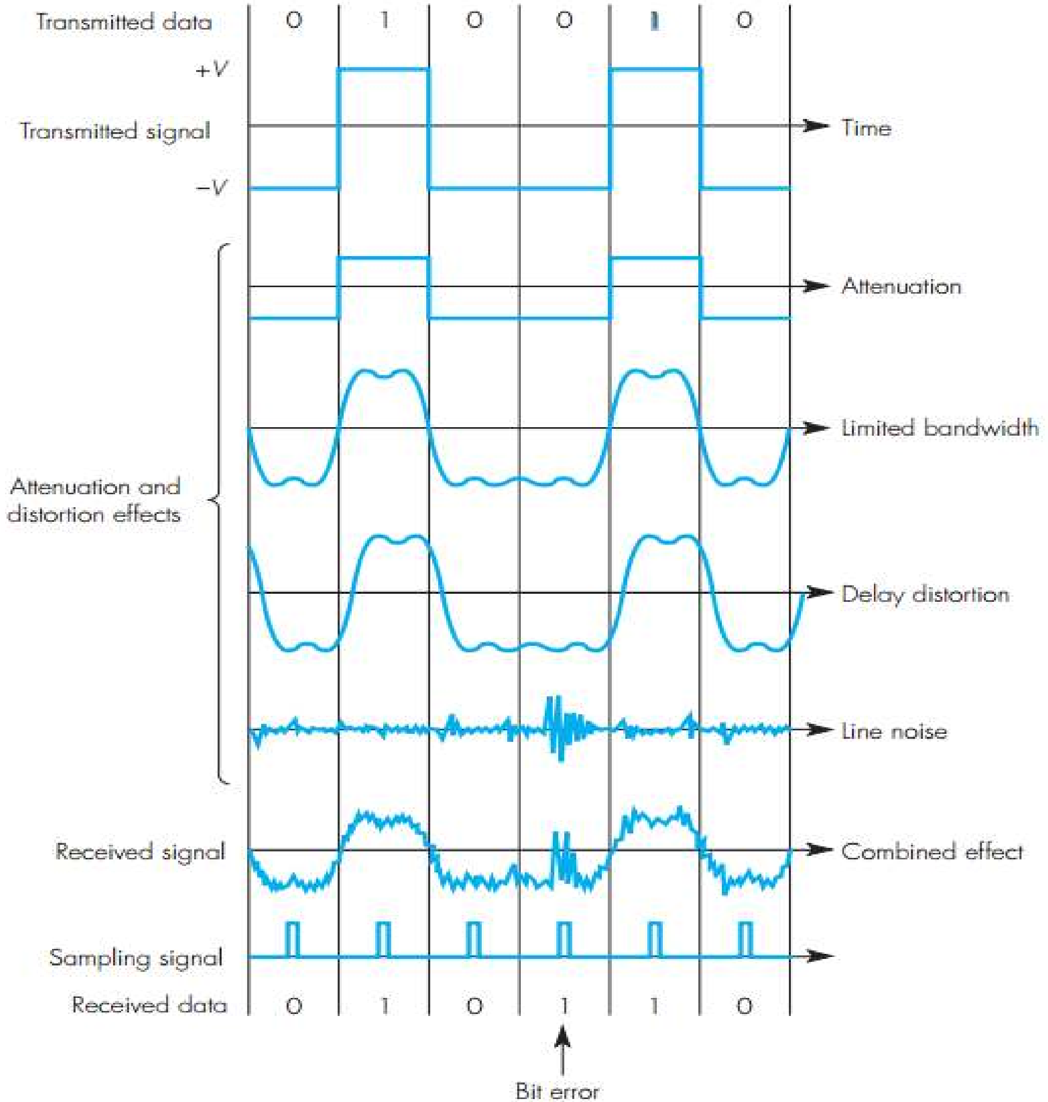
- **Digital technology is cheaper**
  - Low cost LSI/VLSI technology
- **Data integrity**
  - Longer distances over lower quality lines
- **Capacity utilization**
  - High bandwidth links economical
  - High degree of multiplexing easier with digital techniques
- **Security & Privacy**
  - Encryption
- **Integration**
  - Can treat analog and digital data similarly

Transmission Impairments

- Signal received may differ from signal transmitted
- **Analog** - **degradation of signal quality**
- **Digital** - **bit errors**

- Caused by
  1. **Attenuation and attenuation distortion**
  2. **Limited Bandwidth**
  3. **Delay distortion**
  4. **Noise**

Transmission Impairments

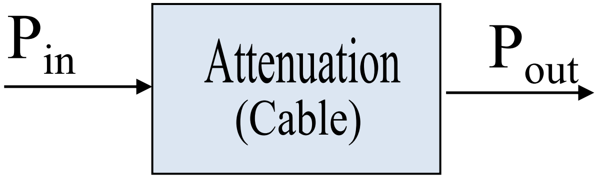


1. Attenuation

- **Signal strength falls off with distance**
- Depends on medium (Chapter4)
- Received signal strength:
  - must be enough to be detected
  - must be sufficiently higher than noise to be received without error
- Attenuation is an increasing function of frequency (Chapter4)

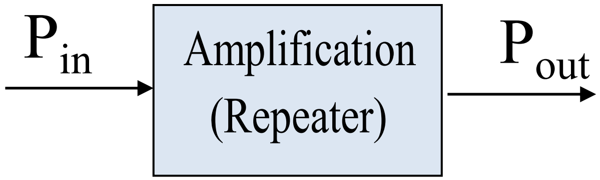
1. Attenuation

- If we denote transmitted signal power level by P_1 and the received power by P_2, then



$$P_{in} > P_{out}$$

$$Attenuation = 10\ log_{10} \frac{P_{in}}{P_{out}} \quad \text{dB}$$



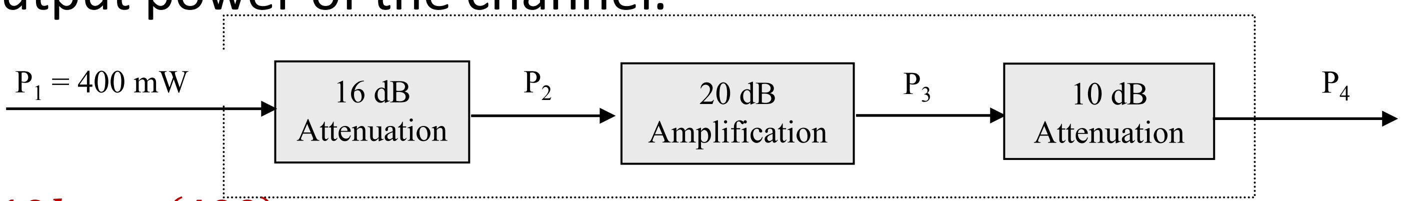
$$P_{out} > P_{in}$$

$$Amplification = 10\ log_{10} \frac{P_{out}}{P_{in}} \quad \text{dB}$$

1. Attenuation

Example:

A transmission channel between two DTEs is made up of three section. The first introduces an attenuation of 16 dB, the second an amplification of 20 dB, and the third an attenuation of 10 dB. Assuming a mean transmitted power level of 400 mW, determine the mean output power of the channel.



$$\boldsymbol{P_1 = 10\log_{10}(400)}$$

$$\rightarrow \boldsymbol{P_1 = 26.02\ dBm}$$

1. Attenuation

Solution:

First section:

$$Attenuation = 10\ log_{10}\frac{P_1}{P_2}$$

→ $16 = 10\ log_{10}\frac{400}{P_2}$

→ $1.6 = \ log_{10}\frac{400}{P_2}$

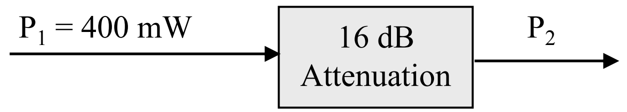
→ $10^{1.6} = 10^{log_{10}\frac{400}{P_2}}$

→ $39.81 = \frac{400}{P_2}$

→ $P_2 = \frac{400}{39.81}$

→ $P_2 = 10.0475\ mW$

→ $P_2 = 10log_{10}\ (10.0475) = 10.02\ \text{dBm}$



$\boldsymbol{P_1 = 26.02\ dBm}$ $\boldsymbol{P_2 = 10.02\ dBm}$

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1. Attenuation

Solution:

Second section:

$$Amplification = 10\ log_{10} \frac{P_3}{P_2}$$

→ $20 = 10\ log_{10} \frac{P_3}{10.0475}$

→ $2 = \ log_{10} \frac{P_3}{10.0475}$

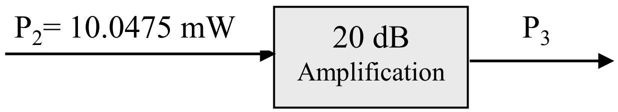
→ $10^2 = 10^{log_{10} \frac{P_3}{10.0475}}$

→ $100 = \frac{P_3}{10.0475}$

→ $P_3 = 100\ \times 10.0475$

→ $P_3 = 1004.75\ mW$

→ $P_3 = 10log_{10}\ (1004.75) = 30.02$ dBm



$\boldsymbol{P_2 = 10.02\ dBm}$ $\boldsymbol{P_3 = 30.02\ dBm}$

1. Attenuation

Solution:

Third section:

$$Attenuation = 10\ log_{10}\frac{P_3}{P_4}$$

→ $10 = 10\ log_{10}\frac{1004.75}{P_4}$

→ $1 = \ log_{10}\frac{1004.75}{P_4}$

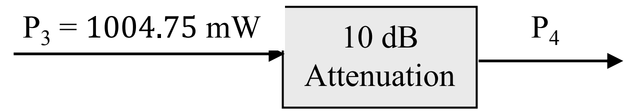
→ $10^1 \ = 10^{log_{10}\frac{1004.75}{P_4}}$

→ $10 = \frac{1004.75}{P_4}$

→ $P_4 = \frac{1004.75}{10}$

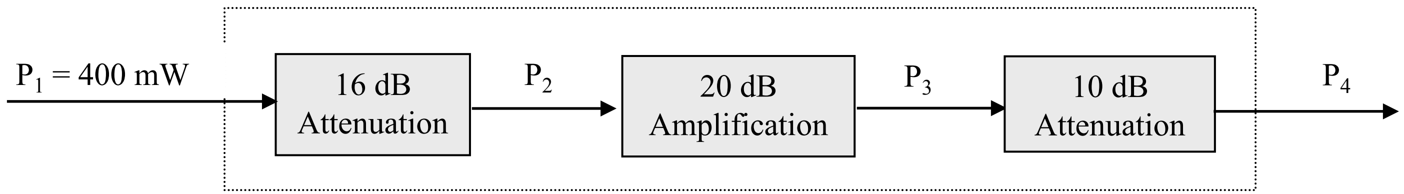
→ $P_4 = 100.475\ mW$

→ $P_4 = 10log_{10}\ (100.475) = 20.02$ dBm



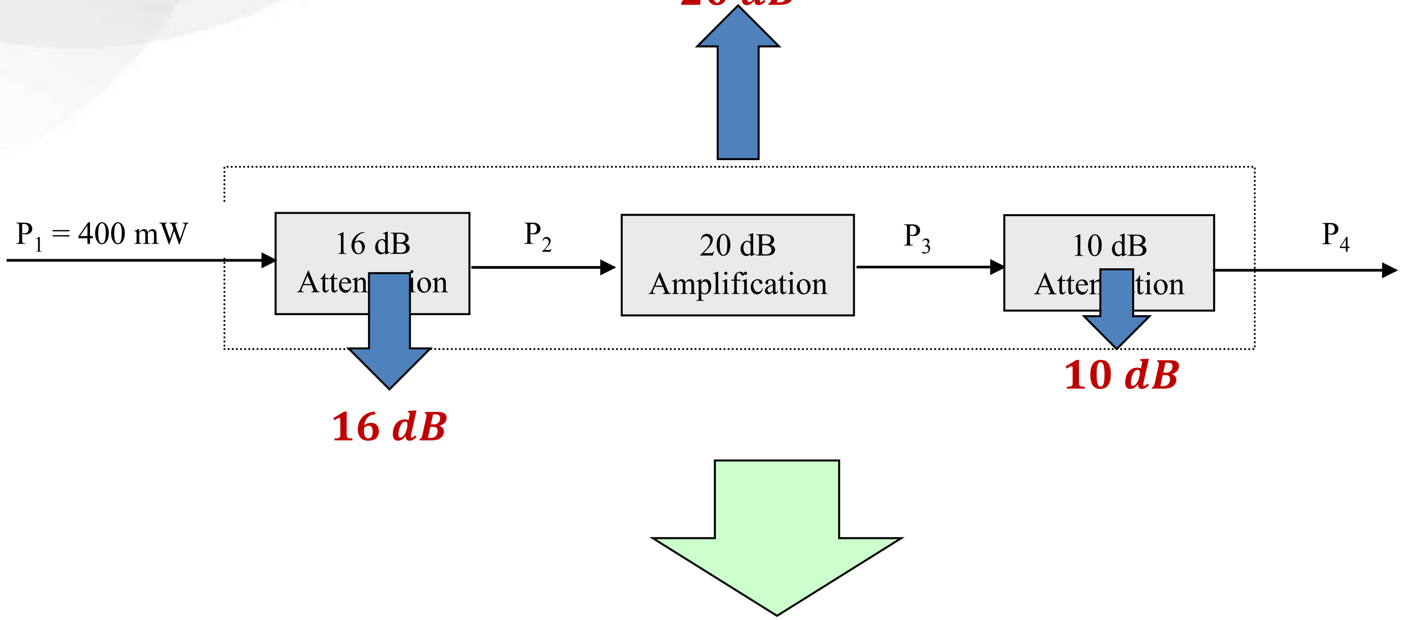
$\boldsymbol{P_3 = 30.02\ dBm}$ $\boldsymbol{P_4 = 20.02\ dBm}$

1. Attenuation

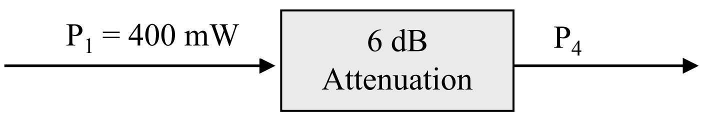


$P_1 = 26.02\ dBm$ $P_2 = 10.02\ dBm$ $P_3 = 30.02\ dBm$ $P_4 = 20.02\ dBm$

1. Attenuation



Overall Attenuation Channel = 16 - 20 + 10 = 6 dB



$P_1 = 26.02\ dBm$

$P_4 = 20.02\ dBm$

1. Attenuation

Overall Attenuation Channel = 16 - 20 + 10 = 6 dB

$\boldsymbol{P_1 = 26.02\ dBm}$ $\boldsymbol{P_4 = 20.02\ dBm}$

$$Attenuation = 10\ log_{10}\frac{P_1}{P_4}$$

→ $6 = 10\ log_{10}\frac{400}{P_4}$

→ $0.6 = \ log_{10}\frac{400}{P_4}$

→ $10^{0.6} = 10^{log_{10}\frac{400}{P_4}}$

→ $3.981 = \frac{400}{P_4}$

→ $P_4 = \frac{400}{3.981}$

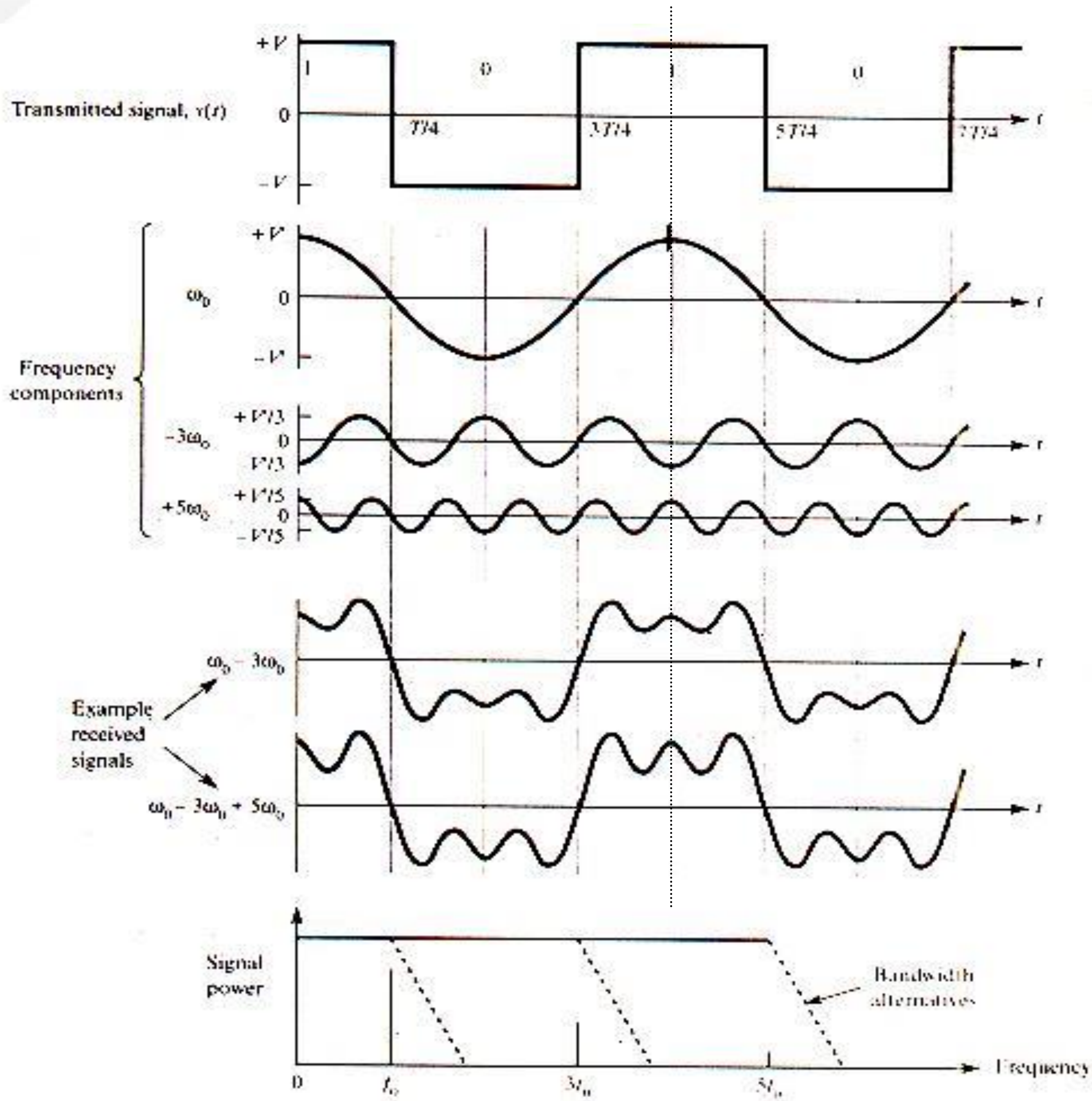
→ $P_4 = 100.475\ mW$

→ $P_4 = 10log_{10}\ (100.475) = 20.02$ dBm

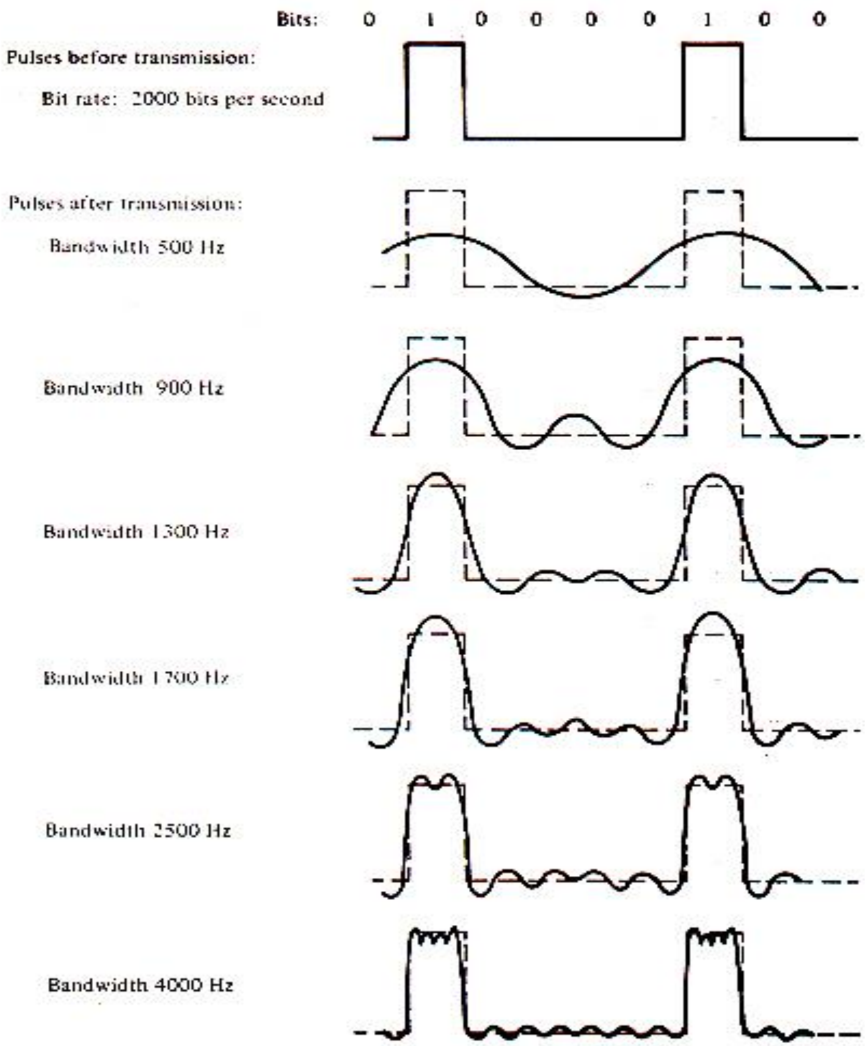
2. Limited Bandwidth

- **Channel Bandwidth specifies the sinusoidal frequency components from 0 up to some frequency f_c that will be transmitted by the channel undiminished.** All frequencies above this cutoff frequency are strongly attenuated.
- In general, channel bandwidth refers to the width of the range of frequencies that channel can transmit, and not the frequency themselves.
- **If the lowest frequency a channel can transmit is f_1 and the highest is f_2, then the bandwidth is: $f_2 - f_1$.**
- Because the **telephone line** can transmit frequencies from approximately 300 to 3300 Hz, its **bandwidth is 3 KHz**.

2. Limited Bandwidth



2. Limited Bandwidth



Effect of bandwidth on a digital signal.

2. Limited Bandwidth

- **The sequence 101010… generates the highest-frequency components, while a sequence of all 1s or all 0s is equivalent to a zero frequency of the appropriate amplitude.**
- The **channel capacity** is the data rate, in bit per second (bps), at which data can be communicated.
- In 1928, **Nyquest** developed the relationship between bandwidth (*B*) and the channel capacity (*R*) in *noise-free environment*. The **Nyquest relationship** is:

$$R = 2B$$

2. Limited Bandwidth

Example:

A binary signal of rate 500 bps is to be transmitted over a communication channel. Derive the minimum bandwidth required assuming:

(a) The fundamental frequency only,

(b) The fundamental and third harmonic, and

(c) The fundamental, third, and fifth harmonic of the worst-case sequence are to be received.

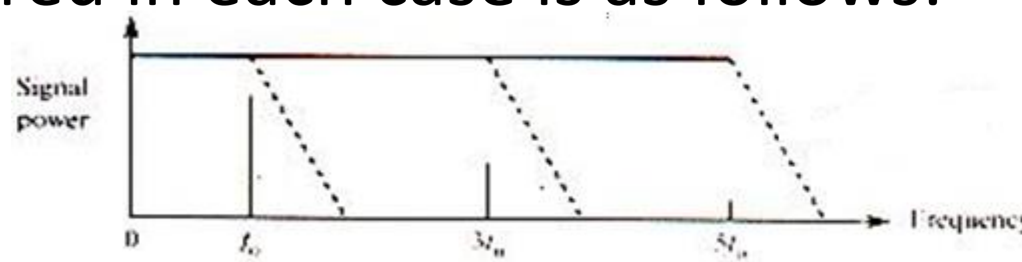
2. Limited Bandwidth

Solution:

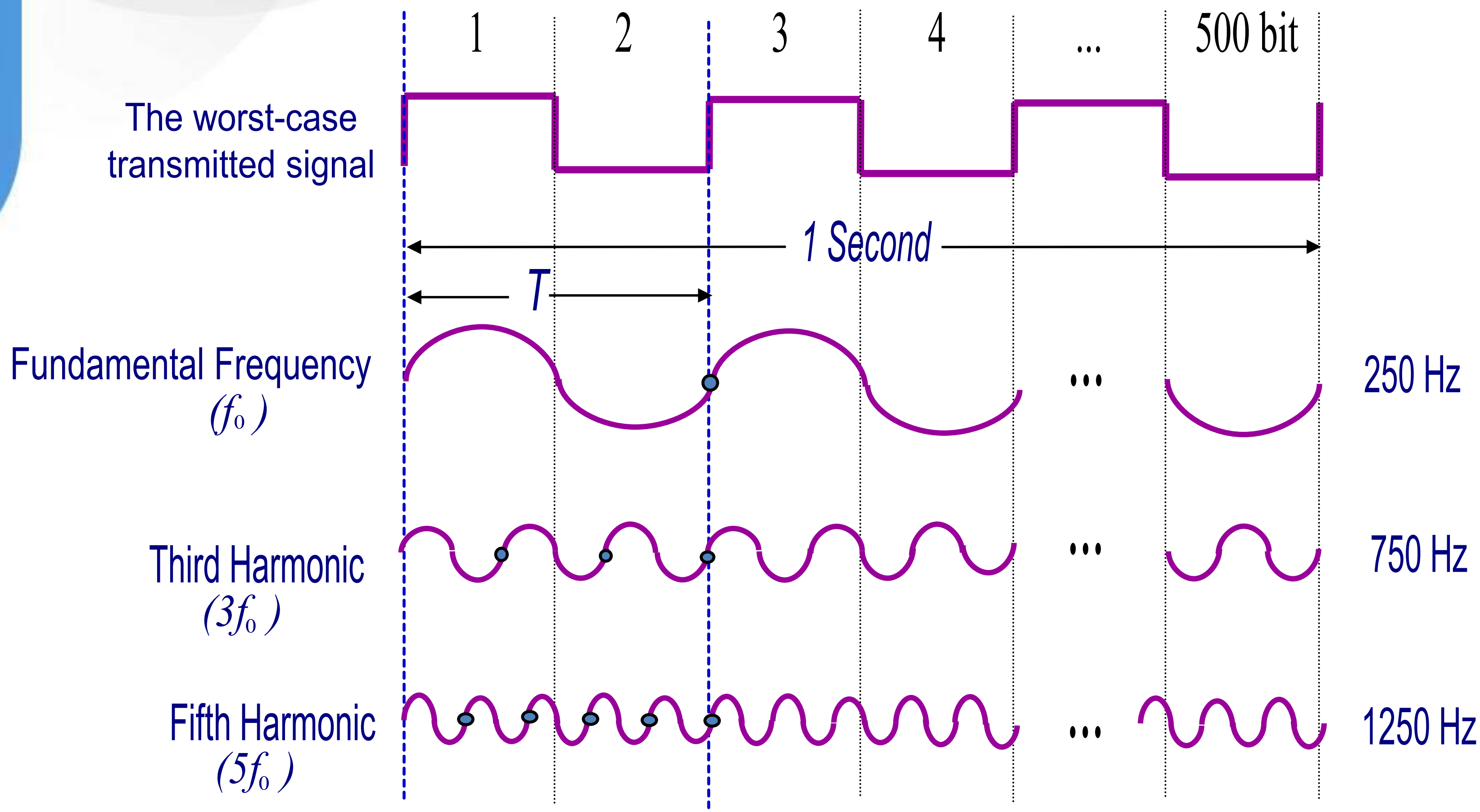
The worst case sequence 101010… at 500 bps has a fundamental frequency component of 250 Hz. Hence the third harmonic is 750 Hz and the fifth harmonic is 1250Hz.

The bandwidth required in each case is as follows:

(a) 0-250 Hz.
(b) 0-750 Hz.
(c) 0-1250 Hz.



2. Limited Bandwidth



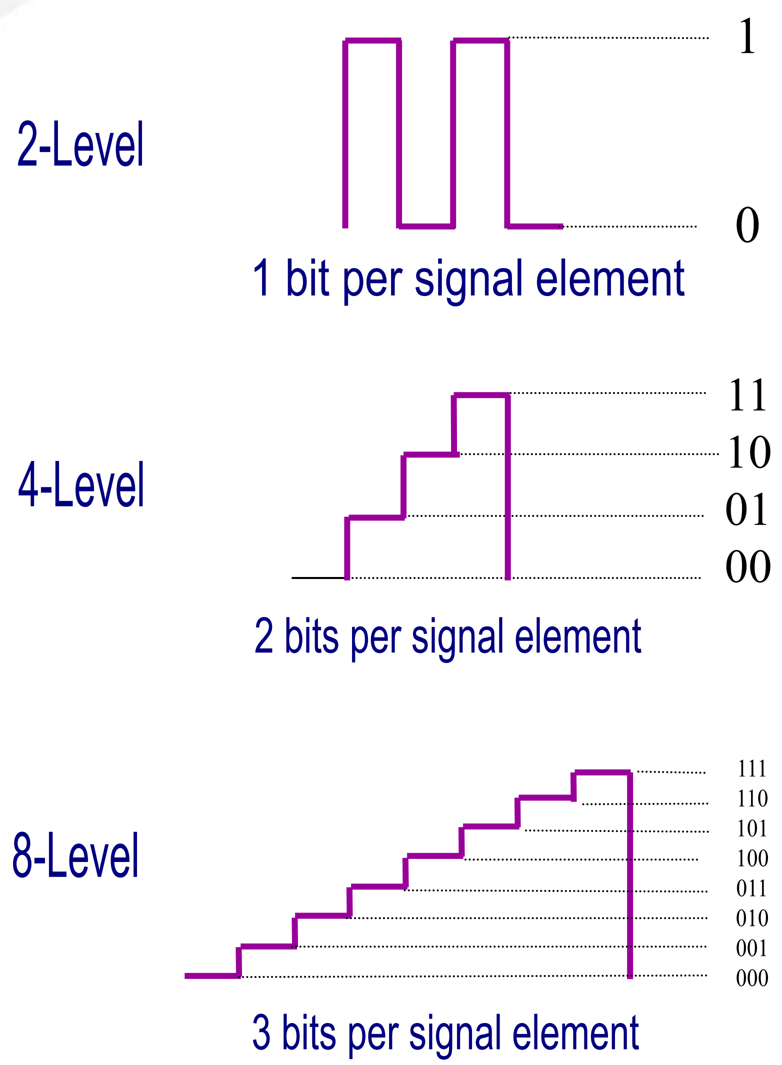
2. Limited Bandwidth

- We can transmit more than one bit with each change in the signal amplitude, therefore increasing the data bit rate.
- With multilevel signaling in noise-free environment, the Nyquest formulation becomes:

$$R = 2B\log_2 M$$

  - Where *R* is the channel capacity in bps.
    B is the bandwidth of the channel in Hz.
    M is the number of levels per signaling elements.

2. Limited Bandwidth



2. Limited Bandwidth

- For Limited-bandwidth channel such as PSTN, we can often use more than two levels. This means that each signal element can represent more than a single binary digit.

- In general, if the number of signal levels is *M*, the number of bits per signal element *m*, is given by:

$$n = \log_2 M$$

- The rate of change of signal is known as the **signaling rate (Baud rate)** (R_s), and measures in **baud**.

$$R_s = 2B$$

2. Limited Bandwidth

- It is related to the data bit rate, *R*, by the following expression:

$$R = R_s n$$

- The signaling element time period, T_s, is given by:

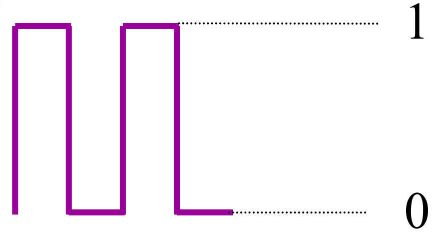
$$T_s = \frac{1}{R_s}$$

- The time duration of each bit, T_b, is:

$$T_b = \frac{1}{R}$$

2. Limited Bandwidth

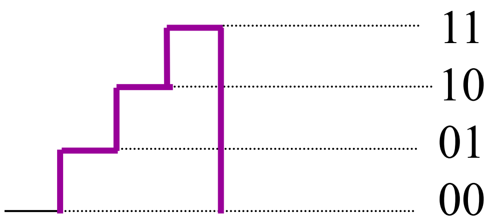
2-Level



1 bit per signal element

$$M = 2 \Rightarrow n = 1 \Rightarrow R = R_s$$

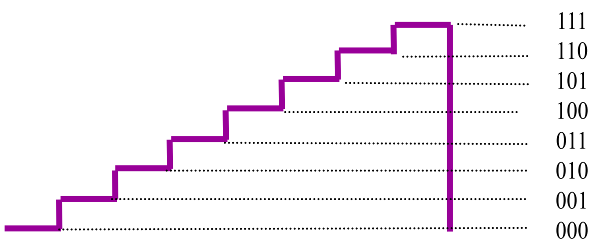
4-Level



2 bits per signal element

$$M = 4 \Rightarrow n = 2 \Rightarrow R = 2 \times R_s$$

8-Level



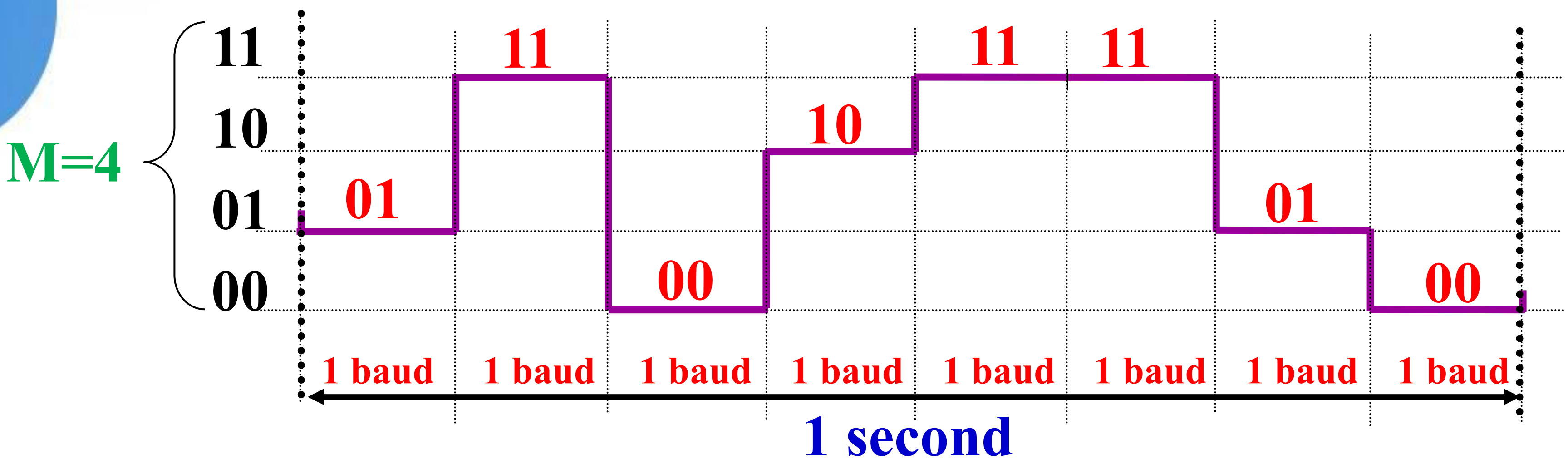
3 bits per signal element

$$M = 8 \Rightarrow n = 3 \Rightarrow R = 3 \times R_s$$

2. Limited Bandwidth

Example 1: (Theoretical)

Data = 01 11 00 10 11 11 01 00



M=4 → n=2

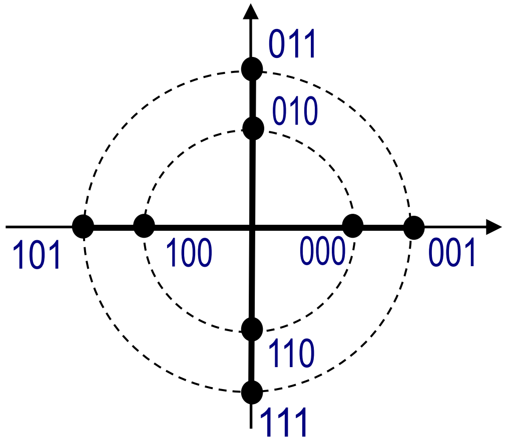
Baud Rate $= R_s =$ **8 baud** $\rightarrow T_s = \frac{1}{8}$ sec

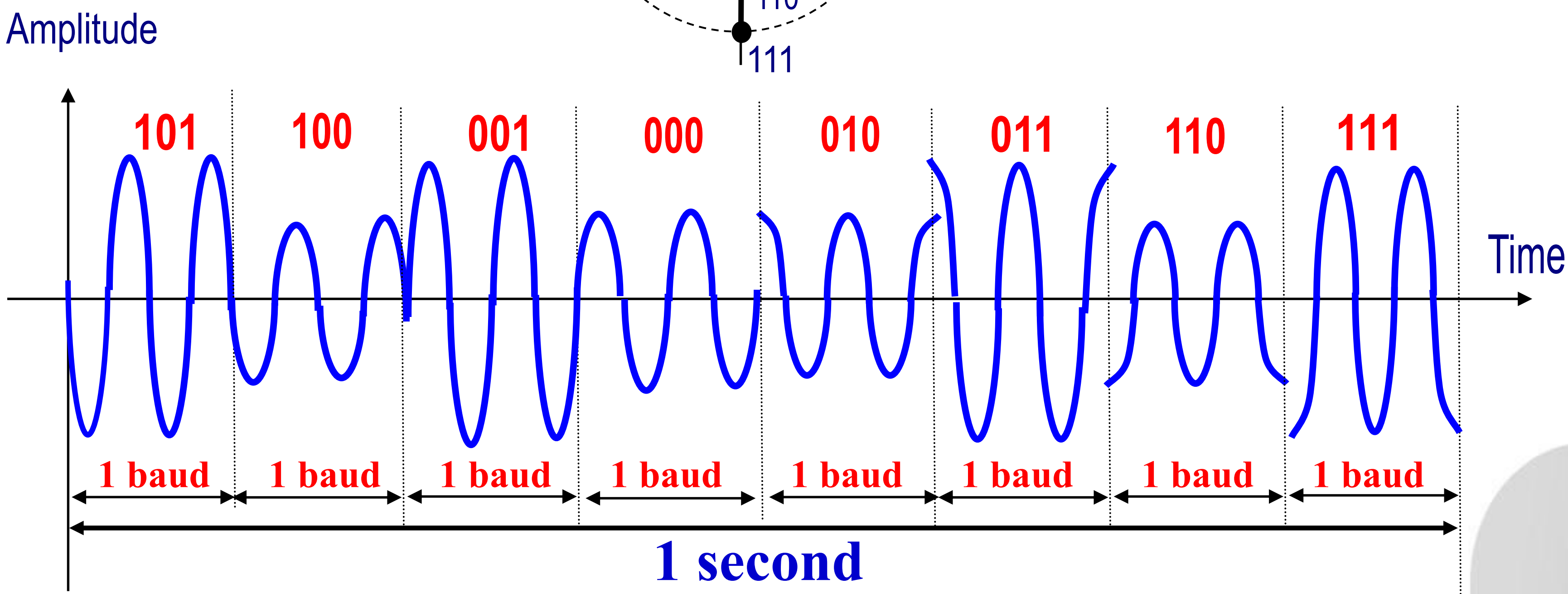
$R = R_s \times m =$ **16 bps** $\rightarrow T_b = \frac{1}{16}$ sec

2. Limited Bandwidth

Example 2: (Practical)

Amplitude & Phase



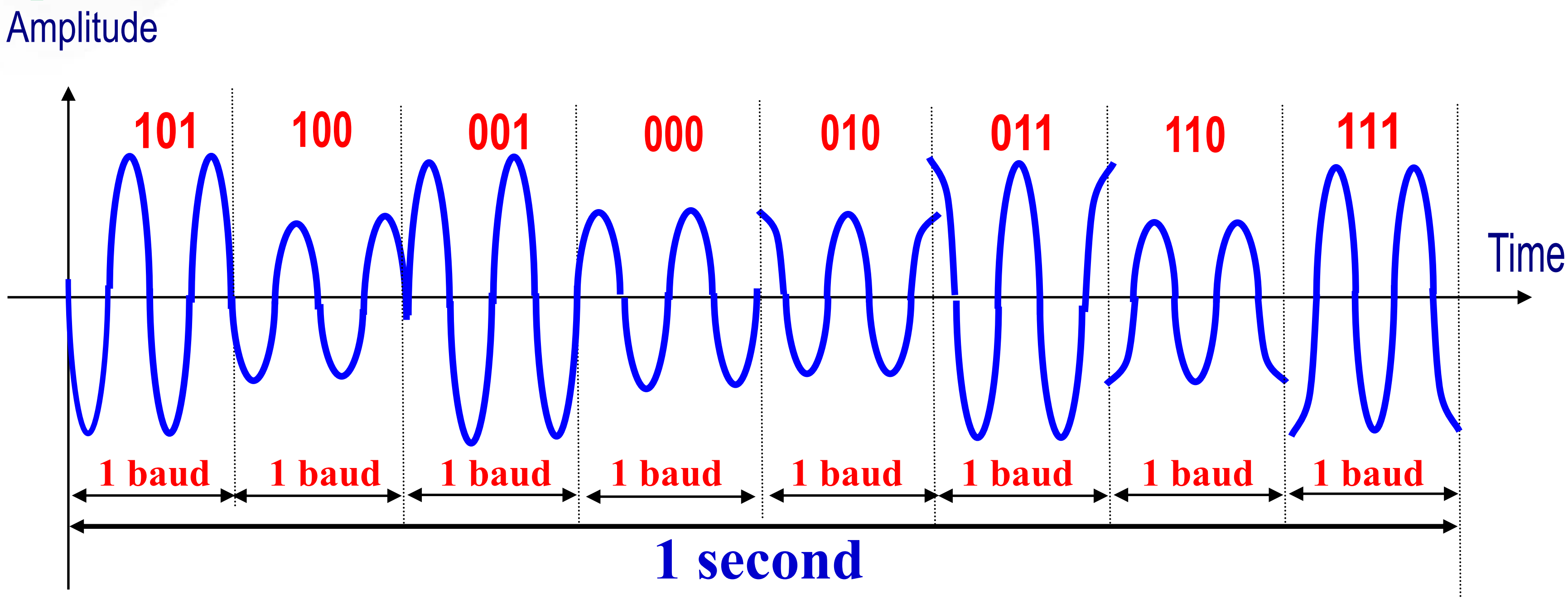


$$R = m \times R_s = 3 \times 8 = 24 \text{ bps}$$

2. Limited Bandwidth

Example 2: (Practical)

Amplitude & Phase



$R_s = 8$ baud $\rightarrow T_s = \frac{1}{R_s} = \frac{1}{8}$ second

$B = \frac{R_s}{2} = 4$ Hz

$R = n \times R_s = 3 \times 8 = 24$ bps $\rightarrow T_b = \frac{1}{R} = \frac{1}{24}$ second

2. Limited Bandwidth

- The **bandwidth efficiency** of transmission channel is defined as:

$$BandwidthEffeciency = \frac{R}{B} = \frac{2Bm}{B} = 2n$$

2. Limited Bandwidth

Example 1:

Data is to be transmitted over the PSTN using a transmission scheme with eight levels per signaling element. If the bandwidth of the PSTN is 3000 Hz, determine the Nyquest maximum data transfer rate (*R*) and the bandwidth efficiency.

Solution:

$$R = 2Blog_2M$$

$$\rightarrow R = 2 \times 3000 \times 3 = 18000 \text{ bps}$$

$$\text{Bandwidth Efficiency} = 2n$$

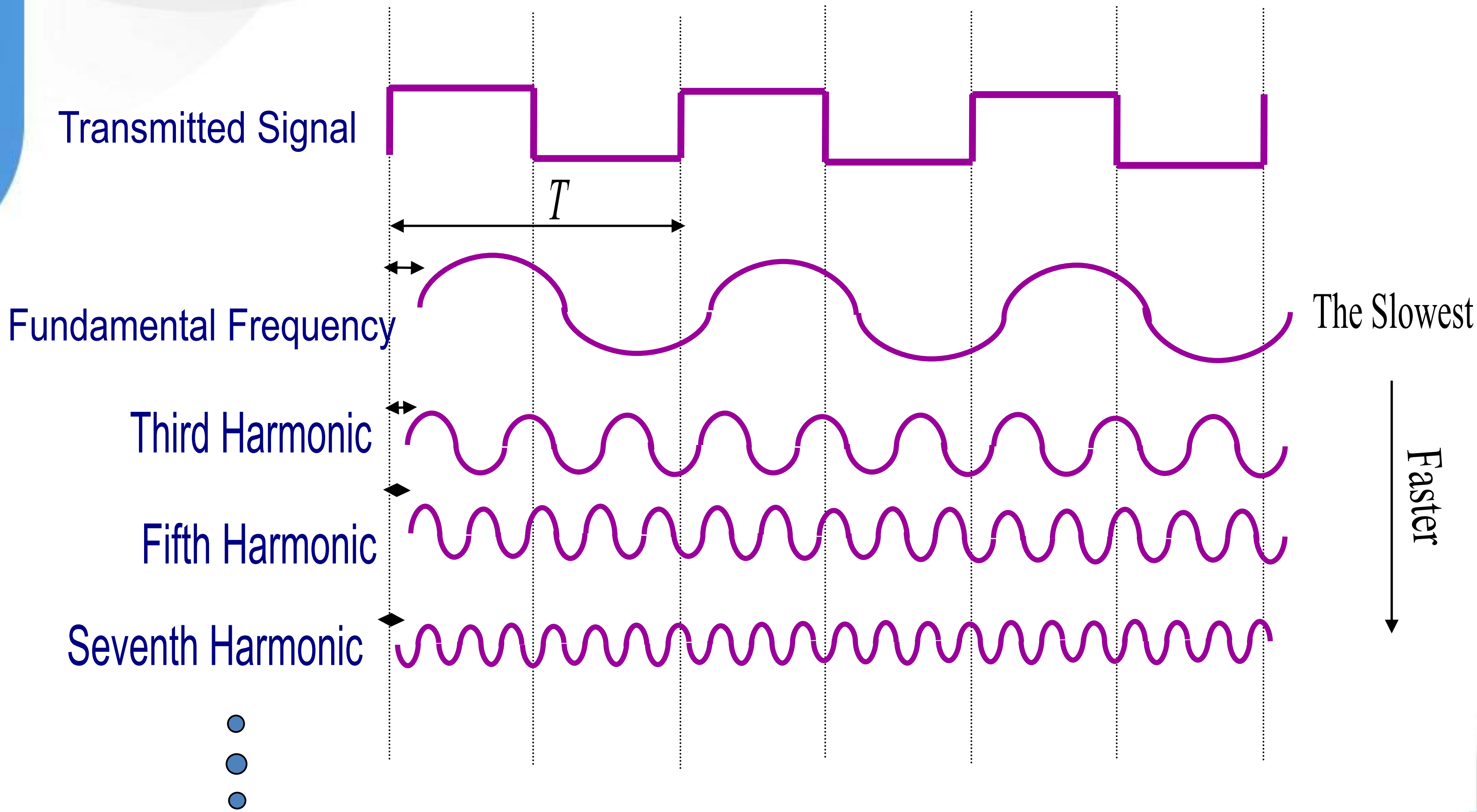
$$\rightarrow \text{Bandwidth Efficiency} = 2 \times 3 = 6 \text{ bps/Hz}$$

3. Delay Distortion

- **The rate of propagation of a sinusoidal signal along a transmission line varies with the frequency of the signal.**
- **When we transmit a digital signal with various frequency components, making up the signal, arrive at the receiver with varying delays, resulting in delay distortion** of the received signal.
- Note that: $$\lambda = \frac{v}{f}$$

$$\Rightarrow \quad v \propto f$$

3. Delay Distortion



4. Noise

- Additional signals inserted between transmitter and receiver
- **A.Thermal Noise**
  - Due to thermal agitation of electrons
  - Uniformly distributed → **White noise**
  - At all temperatures above absolute zero, all transmission media experience thermal noise, where absolute zero = 0 kelvin (K) = - 273° C.
  - Thermal noise cannot be eliminated

4. Noise

- **Thermal Noise**
  - The amount of thermal noise to be found in a bandwidth of **1 Hz** in any conductor is:

$$N_o = kT$$

where N_o is the noise power density for one Hz (watts/Hz),

k is Boltzmann's constant (1.3803×10^{-23} joule K^{-1}), and

T is the temperature in Kelvin (K).

EXAMPLE 3.1 Room temperature is usually specified as T = 17°C, or 290 K. At this temperature, the thermal noise power density is

$$N_0 = (1.38 \times 10^{-23}) \times 290 = 4 \times 10^{-21} \text{ W/Hz} = -204 \text{ dBW/Hz}$$

where dBW is the decibel-watt, defined in Appendix 3A.

4. Noise

- **Thermal Noise**
  - The thermal noise in watts present in a bandwidth of **B Hz** can be expressed by:

$$N = B \times N_o$$

  - or, in decibel-watts,

$$N = 10\log_{10} k + 10\log_{10} T + 10\log_{10} B$$

EXAMPLE 3.2 Given a receiver with an effective noise temperature of 294 K and a 10-MHz bandwidth, the thermal noise level at the receiver's output is

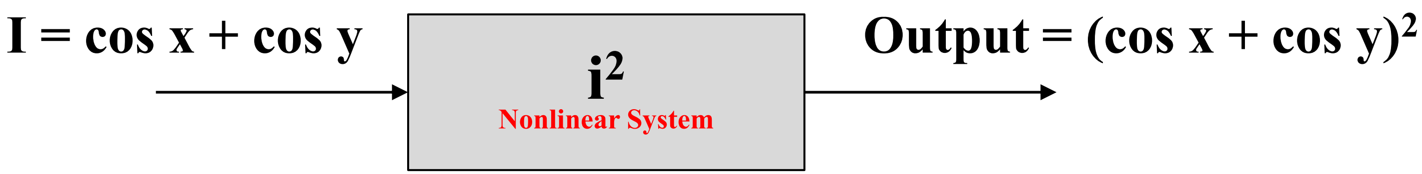
$$\begin{aligned} N &= -228.6 \text{ dBW} + 10\log(294) + 10\log 10^7 \\ &= -228.6 + 24.7 + 70 \\ &= -133.9 \text{ dBW} \end{aligned}$$

4. Noise

- **B. Intermodulation Noise**
  - When signals at different frequencies share the same transmission medium, the resulting may be **Intermodulation Noise**.
  - The effect of intermodulation noise is to produce signals at a frequency that **is the sum of two original frequencies or multiples of those frequencies.**
  - For example, the mixing of signals at frequencies f_1 and f_2 might produce energy at the frequency f_1+f_2.
  - This derived signal could interfere with an intended signal at the frequency f_1+f_2.

4. Noise

- **B. Intermodulation Noise - Example**



$$\boldsymbol{Output} = (\cos \boldsymbol{x} + \cos \boldsymbol{y})^{2}$$

$$\boldsymbol{Output} = \cos \boldsymbol{x} \cos \boldsymbol{x} + 2\cos \boldsymbol{x} \cos \boldsymbol{y} + \cos \boldsymbol{y} \cos \boldsymbol{y}$$

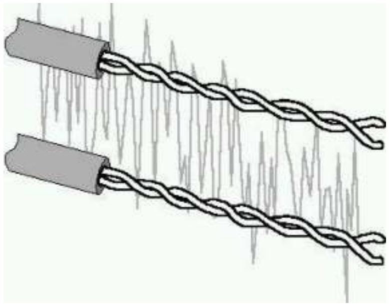
$$\cos A.\cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

$$\boldsymbol{Output} = \frac{1}{2}\cos 2\boldsymbol{x} + \frac{1}{2} + \cos(\boldsymbol{x}+\boldsymbol{y}) + \cos(\boldsymbol{x}-\boldsymbol{y}) + \frac{1}{2}\cos 2\boldsymbol{y} + \frac{1}{2}$$

Multiple of Original Frequency | **Sum of Two Original Frequencies** | **Difference of Two Original Frequencies** | **Multiple of Original Frequency**

4. Noise

- **C. Crosstalk Noise**
  - A signal from one line is picked up by another



- **D. Impulse Noise**
  - Irregular pulses or spikes
  - e.g. External electromagnetic interference
  - Short duration
  - High amplitude

4. Noise

- The **Signal-to-Noise Ratio (SNR)** is expressed in decibels as:

$$SNR = 10\log_{10}\left(\frac{S}{N}\right) \qquad dB$$

where *S* is the average power in a received signal, and

N is noise power.

- **High SNR means a high power signal relative to the prevailing noise level, resulting in a good-quality signal.**

4. Noise

- In 1948, **Shannon** calculated the theoretical *maximum bit rate* of a channel of bandwidth *B* as

$$C = B\log_2(1+\frac{S}{N})$$

where *C* is the maximum channel capacity in bps,
B is the bandwidth of the channel in Hz,
S is the average signal power in watts, and
N is the thermal noise power in watts.

- Note that: $$\log_2 x = \frac{\ln x}{\ln 2}$$

4. Noise

$$SNR_{dB} = 10log\frac{S}{N}$$

$$SNR_{dB} = 10logS - 10logN$$

$$SNR_{dB} = S_{dBW} - N_{dBW}$$

Example:

S is Given

N can be calculated using N=B kT

Method 1:

Calculate $\frac{S}{N}$

Then calculate $SNR_{dB} = 10log\frac{S}{N}$

Method 2:

Calculate $S_{dBW} = 10logS$ and calculate $N_{dBW} = 10logN$

Then calculate $SNR_{dB} = S_{dBW} - N_{dBW}$

4. Noise

Example 1:

Assuming that a PSTN has a bandwidth of 3000 Hz and a signal-to-noise ratio of 20 dB, determine the maximum theoretical data rate that can be achieved.

Solution:

$$SNR = 10\log_{10}(\frac{S}{N}) \Rightarrow 20 = 10\log_{10}(\frac{S}{N}) \Rightarrow \frac{S}{N} = 10^2 = 100$$

$$C = B\log_2(1+\frac{S}{N})$$

$$\Rightarrow C = 3000\log_2(1+100) \Rightarrow C = 3000\frac{\ln 101}{\ln 2} = 19963\,bps$$

2. Limited Bandwidth

Example 2:

EXAMPLE 3.4 Let us consider an example that relates the Nyquist and Shannon formulations. Suppose that the spectrum of a channel is between 3 MHz and 4 MHz and $SNR_{dB} = 24$ dB. Then

$$B = 4\text{ MHz} - 3\text{ MHz} = 1\text{ MHz}$$

$$SNR_{dB} = 24\text{ dB} = 10\log_{10}(SNR)$$

$$SNR = 251$$

Using Shannon's formula,

$$C = 10^6 \times \log_2(1 + 251) \approx 10^6 \times 8 = 8\text{ Mbps}$$

This is a theoretical limit and, as we have said, is unlikely to be reached. But assume we can achieve the limit. Based on Nyquist's formula, how many signaling levels are required? We have

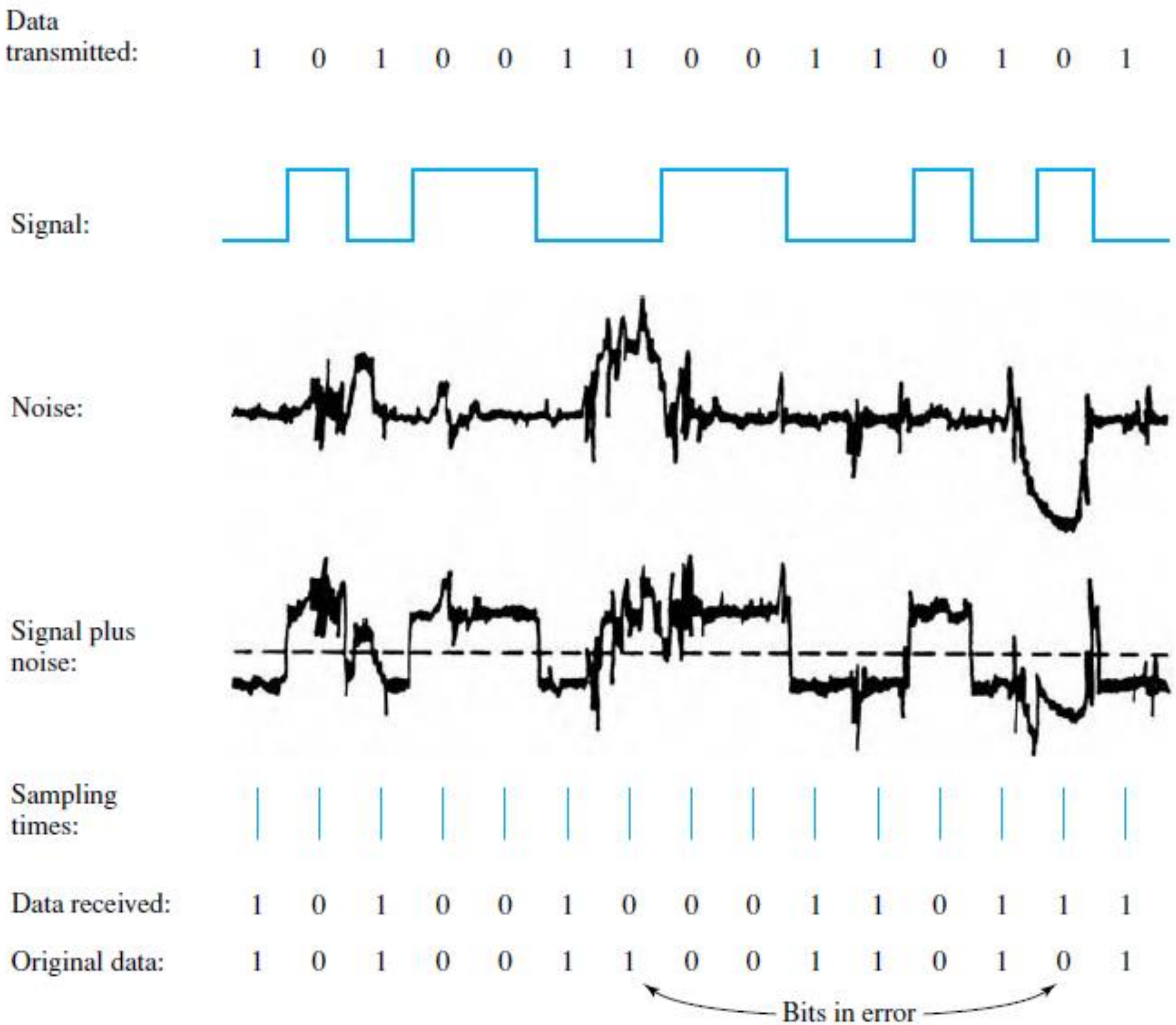
$$C = 2B\log_2 M$$

$$8 \times 10^6 = 2 \times (10^6) \times \log_2 M$$

$$4 = \log_2 M$$

$$M = 16$$

Effect of Noise on a Digital Signal



The Expression E_b / N_0

- The parameter is the **ratio** of **signal energy per bit** to **noise power density per Hertz**

- Consider a signal, digital or analog, that contains binary digital data transmitted at a certain bit rate *R.*

- Recalling that 1 Watt = 1 J/s, the energy per bit in a signal is given by $E_b = ST_b$, where S is the signal power and is the T_b *is the* time required to send one bit.

- The data rate *R* is just $R = 1/T_b$. Thus $$\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{kTR}$$

The Expression E_b / N_0

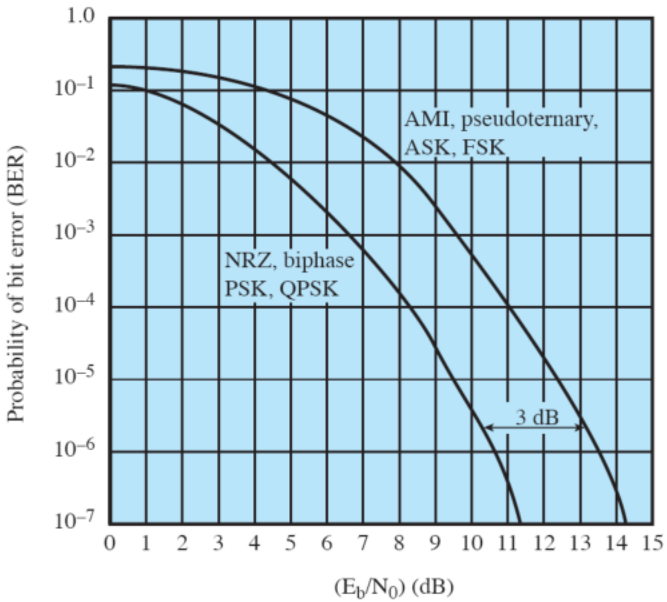
- or, in decibel notation,

$$\left(\frac{E_b}{N_0}\right)_{dB} = 10\log_{10} S - 10\log_{10} R - 10\log_{10} k - 10\log_{10} T$$

$$\left(\frac{E_b}{N_0}\right)_{dB} = S_{dBW} - 10\log_{10} R + 228.6_{dBW} - 10\log_{10} T$$

The Expression E_b / N_0

- The ratio E_b/N_0 is important because the **bit error rate** for digital data is a (decreasing) function of this ratio.

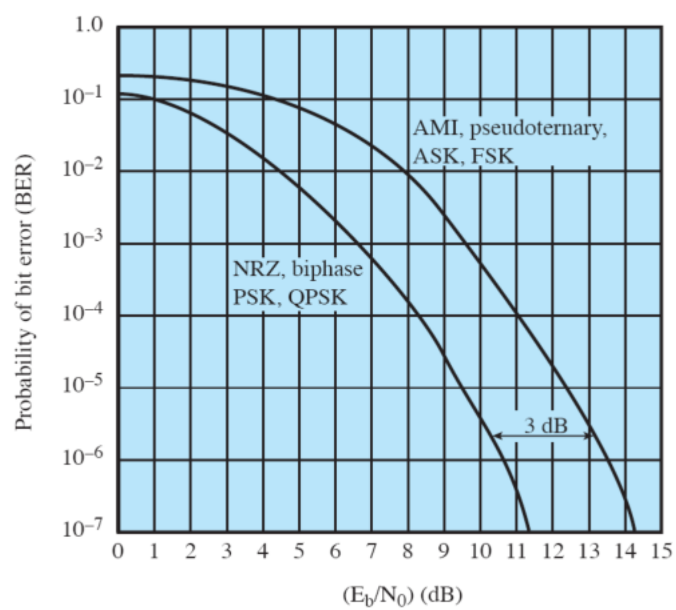


- Given a value of needed to achieve a desired error rate, the parameters in the preceding formula may be selected.
- Note that as the bit rate *R increases, the transmitted signal power, relative to noise, must increase to* maintain the required E_b/N_0 .

The Expression E_b / N_0

- Thus, for constant signal to noise ratio (SNR), **an increase in data rate increases the error rate.**

- The advantage of E_b/N_0 over SNR is tha depends on the bandwidth.



EXAMPLE 3.5 For binary phase-shift keying (defined in Chapter 5), $E_b/N_0 =$ 8.4 dB is required for a bit error rate of 10^{-4} (one bit error out of every 10,000). If the effective noise temperature is 290°K (room temperature) and the data rate is 2400 bps, what received signal level is required?

We have

$$8.4 = S(\text{dBW}) - 10 \log 2400 + 228.6 \text{ dBW} - 10 \log 290$$
$$= S(\text{dBW}) - (10)(3.38) + 228.6 - (10)(2.46)$$
$$S = -161.8 \text{ dBW}$$

The Expression E_b / N_0

- We can relate E_b/N_0 to SNR as follows. We have:

$$\frac{E_b}{N_0} = \frac{S}{N_0 R}$$

- The parameter N_0 is the noise power density in Watts/Hertz. Hence, the noise in a signal with bandwidth B is $\mathbf{N=N_0B}$. Substituting, we have:

$$\frac{E_b}{N_0} = \frac{S}{N} \times \frac{B}{R}$$

$$\frac{S}{N} = \frac{E_b}{N_0} \times \frac{R}{B}$$

The Expression E_b / N_0

- Another formulation of interest relates E_b/N_0 to spectral efficiency.
- Shannon's result can be rewritten as:

$$C = B\log_2(1+\frac{S}{N})$$

$$\frac{S}{N} = 2^{C/B} - 1$$

$$\frac{E_b}{N_0} = \frac{B}{C}(2^{C/B} - 1)$$

- This is a useful formula that relates the achievable spectral efficiency C/B to E_b/N_0 .

EXAMPLE 3.6 Suppose we want to find the minimum E_b/N_0 required to achieve a spectral efficiency of 6 bps/Hz. Then

$$E_b/N_0 = (1/6)(2^6 - 1) = 10.5 = 10.21 \text{ dB}.$$

Required Reading

- Stallings chapter 3

Fourier Series

- Any periodic signal can be represented as sum of sinusoids, known as Fourier Series

$$x(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t) \right]$$

$$A_0 = \frac{2}{T} \int_0^T x(t)\,dt$$

$$A_n = \frac{2}{T} \int_0^T x(t) \cos(2\pi n f_0 t)\,dt$$

If A_0 is not 0, $x(t)$ has a DC component

$$B_n = \frac{2}{T} \int_0^T x(t) \sin(2\pi n f_0 t)\,dt$$

Fourier Series

- Amplitude-phase representation

$$x(t) = \frac{C_0}{2} + \sum_{n=1}^{\infty} \left[C_n \cos(2\pi n f_0 t + \theta_n) \right]$$

$$C_0 = A_0$$

$$C_n = \sqrt{A_n^2 + B_n^2}$$

$$\theta_n = \tan^{-1}\left(\frac{-B_n}{A_n} \right)$$

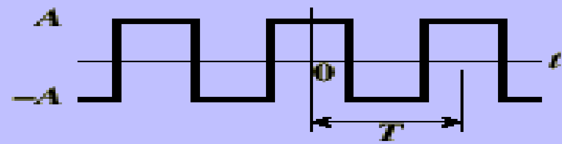
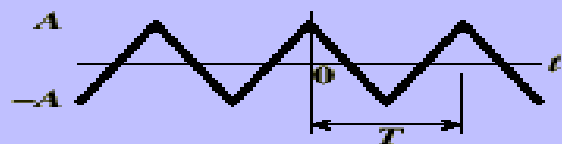
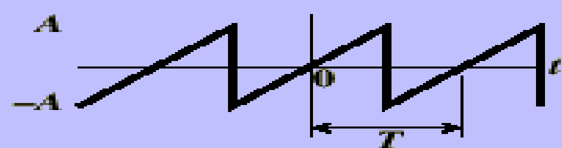
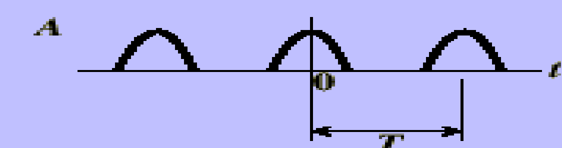
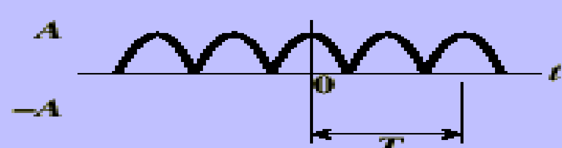
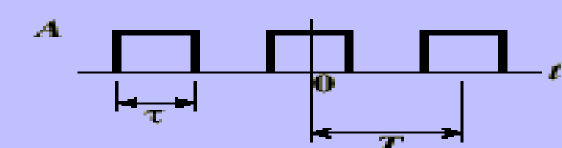
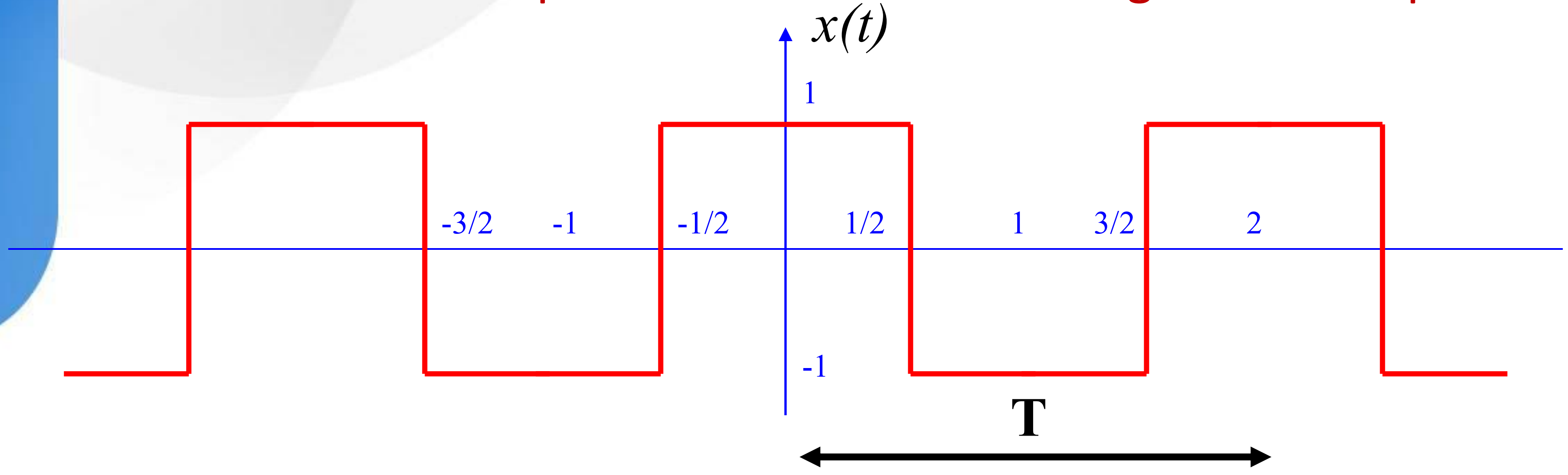
Signal	Fourier Series
Square wave 	$(4A/\ ) \times [\cos(2\ f_1 t) - (1/3)\cos(2\ (3f_1)t) + (1/5)\cos(2\ (5f_1)t) - (1/7)\cos(2\ (7f_1)t) + \ldots]$
Triangular wave 	$(8A/\ ^2) \times [\cos(2\ f_1 t) + (1/9)\cos(2\ (3f_1)t) + (1/25)\cos(2\ (5f_1)t) + \ldots]$
Sawtooth wave 	$(2A/\ ) \times [\sin(2\ f_1 t) - (1/2)\sin(2\ (2f_1)t) + (1/3)\sin(2\ (3f_1)t) - (1/4)\sin(2\ (4f_1)t) + \ldots]$
Half-wave rectified cosine 	$C_0 = A/$ $C_n = 0$ for n odd $C_n = (A/\ ) \times (-1)^{(1+n/2)} \times (2/(n^2 - 1))$ for n even
Full-wave rectified cosine 	$C_0 = 2A/$ $C_n = (2A/\ ) \times (-1)^n \times (1/(4n^2 - 1))$
Pulse Train 	$C_n = A \times \left\lvert \frac{\sin(n\ \tau/T)}{n\ \tau/T} \right\rvert$

Figure 3.15 Some Common Periodic Signals and Their Fourier Series

Fourier Series Representation of Periodic Signals - Example



Note that x(-t)=x(t) => x(t) is an even function

$$A_0 = \frac{2}{T}\int_0^T x(t)dt = \frac{2}{2}\int_0^2 x(t)dt = 2\int_0^1 x(t)dt = 2\int_0^{1/2} 1dt + 2\int_{1/2}^1 -1dt = 1 - 1 = 0$$

Fourier Series Representation of Periodic Signals - Example

$$A_n = \frac{2}{T}\int_0^T x(t)\cos(2\pi n f_0 t)dt = \frac{4}{T}\int_0^{T/2} x(t)\cos(2\pi n f_0 t)dt = 2\int_0^1 x(t)\cos(2\pi n f_0 t)dt$$

$$= 2\int_0^{1/2}\cos(2\pi n f_0 t)dt + 2\int_{1/2}^1 -\cos(2\pi n f_0 t)dt = \frac{4}{n\pi}\sin\frac{n\pi}{2}$$

$$B_n = \frac{2}{T}\int_0^T x(t)\sin(2\pi n f_0 t)dt = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\sin(2\pi n f_0 t)dt$$

$$= \frac{2}{T}\int_{-T/2}^{0} x(t)\sin(2\pi n f_0 t)dt + \frac{2}{T}\int_0^{T/2} x(t)\sin(2\pi n f_0 t)dt$$

$$= -\frac{2}{T}\int_0^{T/2} x(-t)\sin(2\pi n f_0 t)dt + \frac{2}{T}\int_0^{T/2} x(t)\sin(2\pi n f_0 t)dt$$

Replacing t by $-t$ in the first integral

$\sin(-2\pi n f\, t) = -\sin(2\pi n f\, t)$

Fourier Series Representation of Periodic Signals - Example

Since *x(-t)=x(t)* as *x(t)* is an even function, then

$B_n = 0$ for n=1, 2, 3, …

$$x(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left[A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t)\right]$$

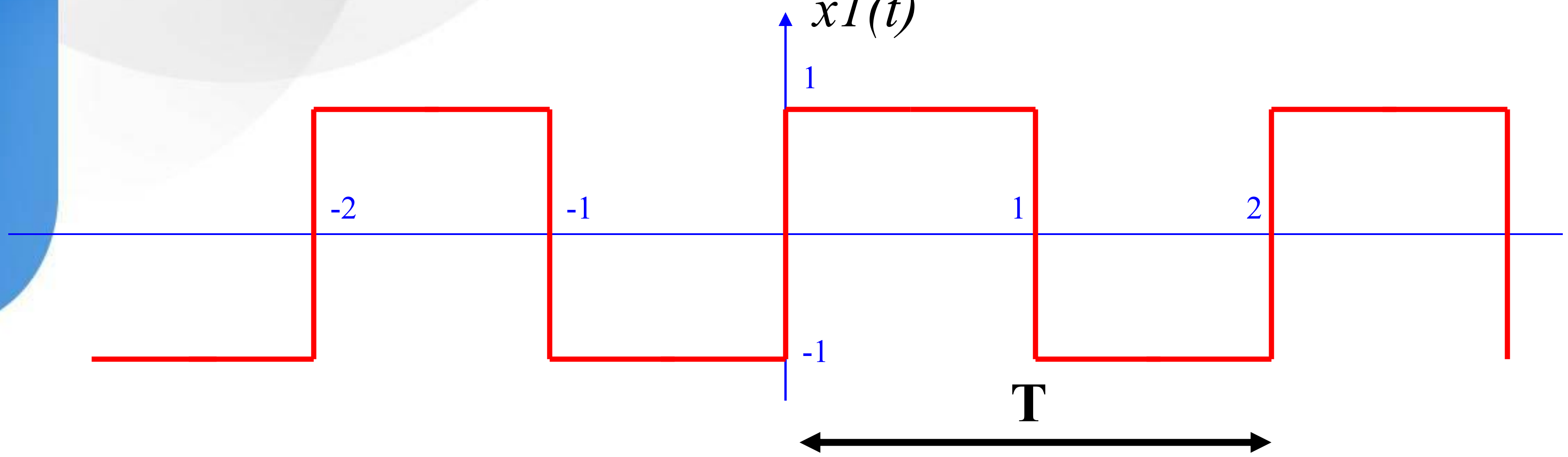
$$x(t) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin\frac{n\pi}{2} \cos n\pi t$$

$$\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0 & n \text{ is even} \\ \pm 1 & n \text{ is odd} \end{cases}$$

$$x(t) = \frac{4}{\pi}\cos \pi t - \frac{4}{3\pi}\cos 3\pi t + \frac{4}{5\pi}\cos 5\pi t - \frac{4}{7\pi}\cos 7\pi t$$

$$x(t) = \frac{4}{\pi}\left[\cos \pi t - \frac{1}{3}\cos 3\pi t + \frac{1}{5}\cos 5\pi t - \frac{1}{7}\cos 7\pi t\right]$$

Another Example



Note that x1(-t)= -x1(t) => x(t) is an odd function

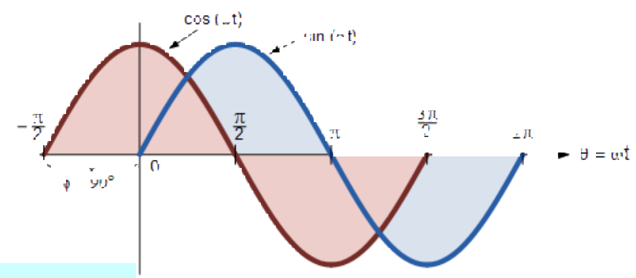
Also, x1(t)=x(t-1/2)

$$x1(t)=\frac{4}{\pi}\left[\cos\pi\left(t-\frac{1}{2}\right)-\frac{1}{3}\cos3\pi\left(t-\frac{1}{2}\right)+\frac{1}{5}\cos5\pi\left(t-\frac{1}{2}\right)-\frac{1}{7}\cos7\pi\left(t-\frac{1}{2}\right)\right]$$

Another Example

$$x1(t) = \frac{4}{\pi}\left[\cos\left(\pi t - \frac{\pi}{2}\right) - \frac{1}{3}\cos\left(3\pi t - \frac{3\pi}{2}\right) + \frac{1}{5}\cos\left(5\pi t - \frac{5\pi}{2}\right) - \frac{1}{7}\cos\left(7\pi t - \frac{7\pi}{2}\right)\right]$$

$$x1(t) = \frac{4}{\pi}\left[\sin \pi t + \frac{1}{3}\sin 3\pi t + \frac{1}{5}\sin 5\pi t + \frac{1}{7}\sin 7\pi t\right]$$



$$\cos\left(\pi t - \frac{\pi}{2}\right) = \sin \pi t$$

$$\cos\left(3\pi t - \frac{3\pi}{2}\right) = -\sin 3\pi t$$

$$\cos\left(5\pi t - \frac{5\pi}{2}\right) = \sin 5\pi t$$

$$\cos\left(7\pi t - \frac{7\pi}{2}\right) = -\sin 7\pi t$$

Fourier Transform

- For a periodic signal, spectrum consists of discrete frequency components at fundamental frequency & its harmonics.
- For an aperiodic signal, spectrum consists of a continuum of frequencies.
  - Spectrum can be defined by Fourier transform
  - For a signal x(t) with spectrum X(f), the following

$$x(t)=\int_{-\infty}^{\infty} X(f)\,e^{j2\pi ft}\,df$$

$$X(f)=\int_{-\infty}^{\infty} x(t)\,e^{-j2\pi ft}\,dt$$

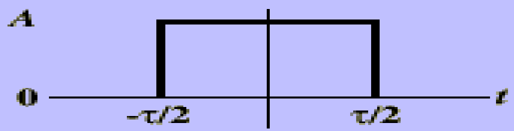
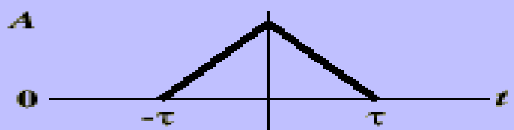
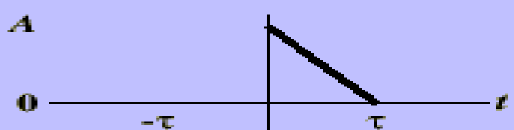
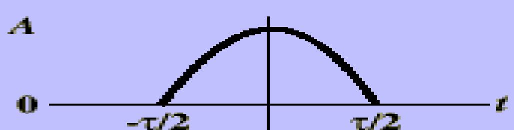
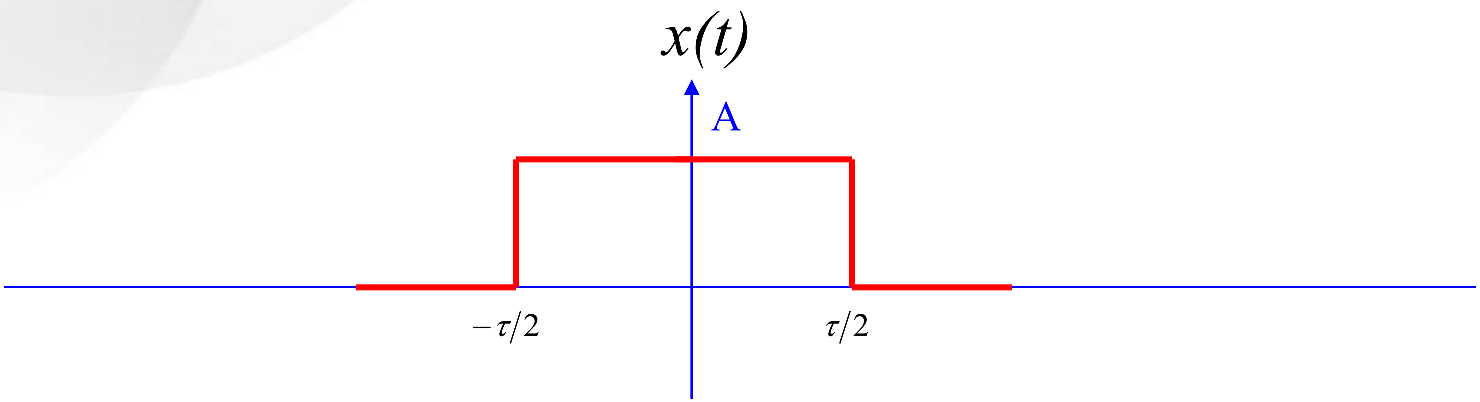
Signal $x(t)$	Fourier Transform $X(f)$
Rectangular Pulse 	$A\tau \frac{\sin(\ f\tau)}{ f\tau}$
Triangular Pulse 	$A\tau \left(\frac{\sin(\ f\tau)}{ f\tau}\right)^2$
Sawtooth Pulse 	$(jA/2 \ f\tau) \times$ $\{[(\sin \ f\tau)/ \ f\tau]\exp(-j \ f\tau) - 1\}$
Cosine Pulse 	$\frac{2A\tau}{ } \times \frac{\cos(\ f\tau)}{1 - (2f\tau)^2}$

Figure 3.16 Some Common Aperiodic Signals and Their Fourier Transforms

Fourier Transform Example



$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt$$

$$X(f) = \int_{-\tau/2}^{\tau/2} A\, e^{-j2\pi ft}\, dt = -\frac{A}{j2\pi f} e^{-j2\pi ft} \Big|_{-\tau/2}^{\tau/2}$$

Fourier Transform Example

$$=\frac{2A}{2\pi f}\left[\frac{e^{j2\pi f\tau/2}-e^{-j2\pi f\tau/2}}{2j}\right]=\frac{2A}{2\pi f}\left(\frac{2\pi f\tau}{2}\right)\left(\frac{\sin(2\pi f\tau/2)}{2\pi f\tau/2}\right)$$

$$X(f)=A\tau\frac{\sin(2\pi f\tau/2)}{2\pi f\tau/2}=A\tau\frac{\sin(\pi f\tau)}{\pi f\tau}$$

$$\sin\theta=\left[\frac{e^{j\theta}-e^{-j\theta}}{2j}\right]$$

$$\cos\theta=\left[\frac{e^{j\theta}+e^{-j\theta}}{2}\right]$$

Signal Power

- A function x(t) specifies a signal in terms of either voltage or current $|x(t)|^2$
- Instantaneous power of a signal is related to $|x(t)|^2$
- Average power of a time limited signal is

$$\frac{1}{t1-t2} \int_{t1}^{t2} |x(t)|^2 \, dt$$

- For a periodic signal, the average power in one period is

$$\frac{1}{T} \int_{0}^{T} |x(t)|^2 \, dt$$

Power Spectral Density & Bandwidth

- Absolute bandwidth of any time-limited signal is infinite.
- Most power in a signal is concentrated in finite band.
- Effective bandwidth is the spectrum portion containing most of the power.
- Power spectral density (PSD) describes power content of a signal as a function of frequency.

Power Spectral Density & Bandwidth

- For a continuous valued function *S(f),* power contained in a band of frequencies $f_1<f<f_2$

$$P = 2\int_{f_1}^{f_2} S(f)\,df$$

- For a periodic waveform, the power through the first *j* harmonics is

$$P = C_0^2 + \frac{1}{2}\sum_{n=1}^{j} C_n^2$$

Power Spectral Density & Bandwidth - Example

- Consider the following signal

$$x(t) = \left[\sin \pi t + \frac{1}{3}\sin 3\pi t + \frac{1}{5}\sin 5\pi t + \frac{1}{7}\sin 7\pi t \right]$$

- The signal power is

$$Power = \frac{1}{2}\left[1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} \right] = 0.586 \text{ watt}$$