Department of Electrical engineering
College of Engineering
EE 351 Electromagnetic Fields
Problem Set 4 (Electric Flux Density and Gauss's Law)

Electric Flux Density (Section 4.3, 4.4)

Problem 1

4.21 A ring placed along $y^2 + z^2 = 4, x = 0$ carries a uniform charge of 5 μC/m.

(a) Find **D** at $P(3, 0, 0)$.

(b) If two identical point charges Q are placed at $(0, -3, 0)$ and $(0, 3, 0)$ in addition to the ring, find the value of Q such that **D** = 0 at P.

Total Electric Flux and Divergence (Section 4.4)

Problem 2

4.22 The electric flux density in free space is given by $\mathbf{D} = y^2\mathbf{a}_x + 2xy\mathbf{a}_y - 4z\mathbf{a}_z$ nC/m^2(a). Find the volume charge density. (b) Determine the flux through surface $x = 3, 0 < y < 6, 0 < z < 5$.

Problem 3

$< z < 5$.

.23 In free space, $\mathbf{E} = 12\rho z\cos\phi\mathbf{a}_\rho - 6\rho z\sin\phi\mathbf{a}_\phi + 6\rho^2\cos\phi\mathbf{a}_z$ V/m. Find the electric flux through surface $\phi = 90°$ $0<\rho<2$, $0<z<5$.

Problem 4

4.24 If $\mathbf{D} = \sin\theta\sin\phi\mathbf{a}_r + \cos\theta\sin\phi\mathbf{a}_\theta + \cos\phi\mathbf{a}_\phi$ nC/m^2, find: (a) the charge density at $(2, 30°, 60°)$, (b) the flux through $r = 2, 0 < \theta < 30°, 0 < \phi < 60°$.

Problem 5

4.25 Determine the charge density due to each of the following electric flux densities:

(a) $\mathbf{D} = 8xy\mathbf{a}_x + 4x^2\mathbf{a}_y$ C/m^2

(b) $\mathbf{D} = 4\rho\sin\phi\,\mathbf{a}_\rho + 2\rho\cos\phi\,\mathbf{a}_\phi + 2z^2\mathbf{a}_z$ C/m^2

(c) $\mathbf{D} = \dfrac{2\cos\theta}{r^3}\mathbf{a}_r + \dfrac{\sin\theta}{r^3}\mathbf{a}_\theta$ C/m^2

Problem 6

4.26 A cube with 2 m sides ($0 < x, y, z < 2$ m) carries a charge with density $\rho_v = 12xyz$ mC/m^3. (a) Calculate the total charge. (b) Find the total outward flux from the cube.

Gauss's Law and Applications (Section 4.5, 4.6)

Problem 7

4.27 If spherical surfaces $r = 1$ m and $r = 2$ m, respectively, carry uniform surface charge densities 8 nC/m^2 and -6 mC/m^2, find **D** at $r = 3$ m.

Problem 8

4.28 A sphere of radius a is centered at the origin. If $\rho_v = \begin{cases} 5r^{1/2}, & 0 < r < a \\ 0, & \text{otherwise} \end{cases}$. Determine **E** everywhere.

Problem 9

4.30 In a certain region, the electric field is given by

$$\mathbf{D} = 2\rho(z+1)\cos\phi\,\mathbf{a}_\rho - \rho(z+1)\sin\phi\,\mathbf{a}_\phi + \rho^2\cos\phi\,\mathbf{a}_z\ \mu\text{C/m}^2$$

(a) Find the charge density.

(b) Calculate the total charge enclosed by the volume $0 < \rho < 2$, $0 < \phi < \pi/2$, $0 < z < 4$.

(c) Confirm Gauss's law by finding the net flux through the surface of the volume in (b).

Problem 10

4.32 A long coaxial cable has an inner conductor with radius a and outer conductor with radius b. If the inner conductor has $\rho_s = \rho_o/\rho$, where ρ_o is a constant, determine **E** everywhere.

Problem 11

4.33 Let

$$\rho_v = \begin{cases} \dfrac{10}{r^2}\ \text{mC/m}^3, & 1 < r < 4 \\ 0, & r > 4 \end{cases}$$

(a) Find the net flux crossing surface $r = 2$ m and $r = 6$ m.

(b) Determine **D** at $r = 1$ m and $r = 5$ m.

Problem 12

4.34 A spherical region of radius a has total charge Q. If the charge is uniformly distributed, apply Gauss's law to find **D** both inside and outside the sphere.