University of BISHA
College of Engineering

Engineering Mechanics

GE 221

Course Description

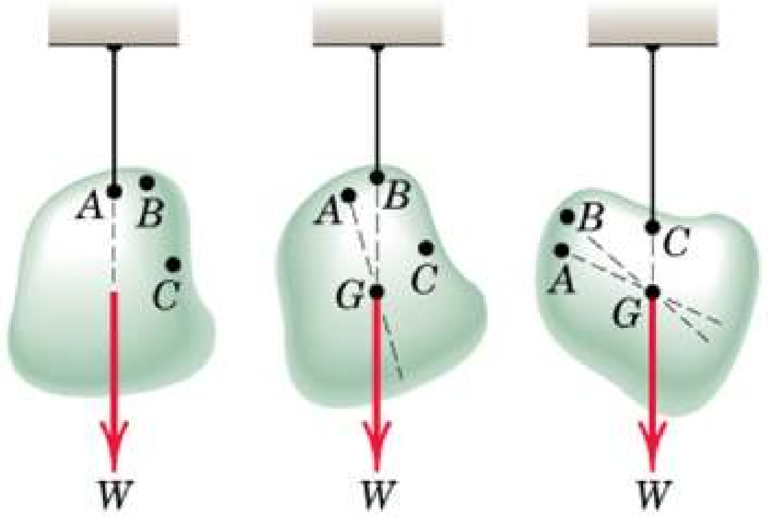
- W1: Introduction and statics of particles
- W2: Newton's laws
- W3: Lami's theorem. Parallélogram law of forces
- W4: How forces are represented, addition of forces
- W5: Law of triangle of forces. Vectorial representation of forces
- W6: Dot product. Resultant of three force vectors
- W7: Equation of equilibrium. Forces in space
- **W8: Mid-term Exam N°1**
- W9: Equilibrium of a particle in space. The principle of transmissibility
- W10: Single equivalent force
- W11: Rigid bodies: equivalent systems of forces
- W12: Distributed forces: centroids and centers of gravity
- **W13: Mid-term Exam N°2**
- W14: Analysis of structures
- W15: Internal forces and moments
- **W16: Final Exam**

Center of Gravity and Centroid

a) Understand the concepts of **center of gravity**, **center of mass**, and **centroid**.

b) Be able to determine the location of these points (CG & Centroid) for a system of particles or a body.

Center of Gravity (CoG) of a body



A body of mass *m* in equilibrium under the action of tension in the cord, and resultant *W* of the gravitational forces acting on all particles of the body.

- The resultant is collinear with the cord

Suspend the body from different points on the body

- Dotted lines show lines of action of the resultant force in each case.
- These lines of action will be concurrent at a single point G

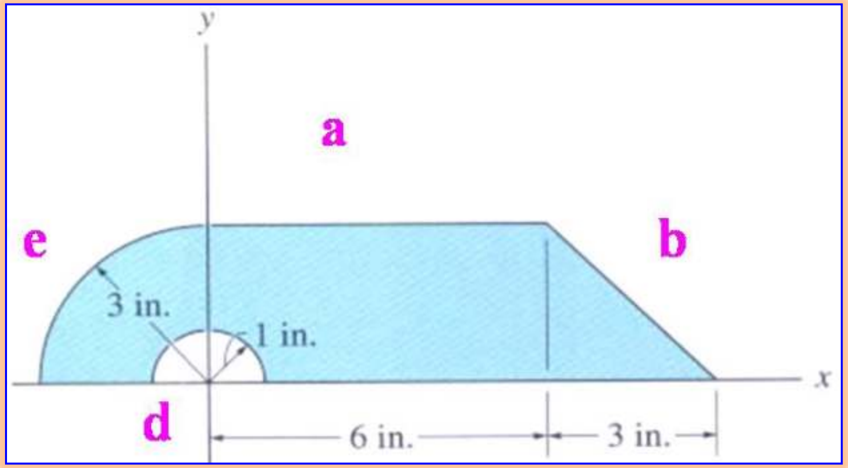
  As long as dimensions of the body are smaller compared with those of the earth.

  - we assume uniform and parallel force field due to the gravitational attraction of the earth.

The unique **Point G** is called the Center of Gravity of the body (CG)

Concept of composite body

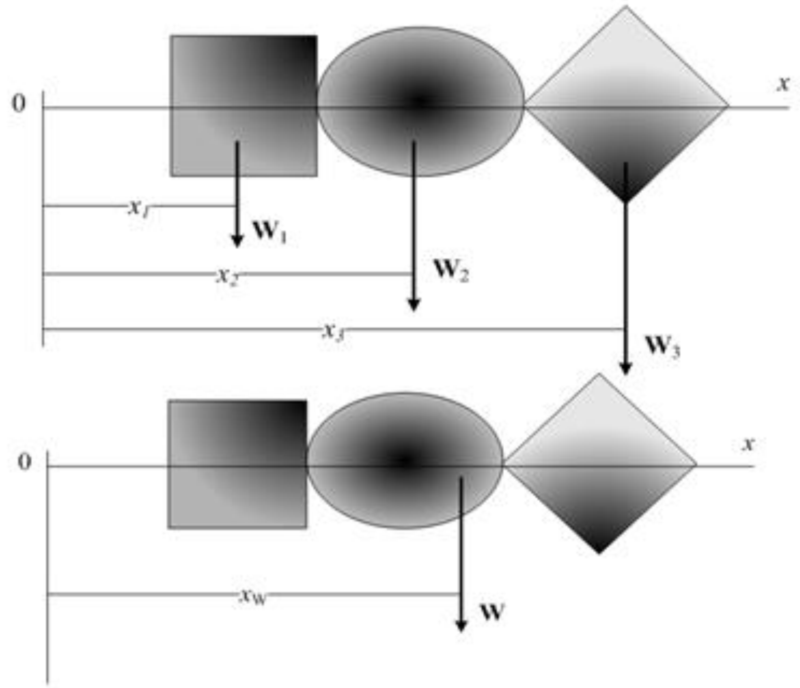
❖ Many industrial objects can be considered as composite bodies made up of a series of connected "simpler" shaped parts or holes, like a rectangle, triangle, and semicircle.



❖ Knowing the location of the center of gravity, **G**, of the simpler shaped parts, we can easily determine the location of the CoG for the more complex composite body.

Center of Gravity (CoG) of composite body

- The center of gravity (G) is a point which locates the resultant weight of a system of particles or body.
- If object rotates when thrown, the CoG is the center of rotation.



The position of the center of the gravity?

Considering the system of the 3 shapes with weight W_1 W_2 W_3

The system is in equilibrium,

Equivalent weight: W=W1+W2+W3

Moment about O: $W.x_W = W_1.x_1 + W_2.x_2 + W_3.x_3$

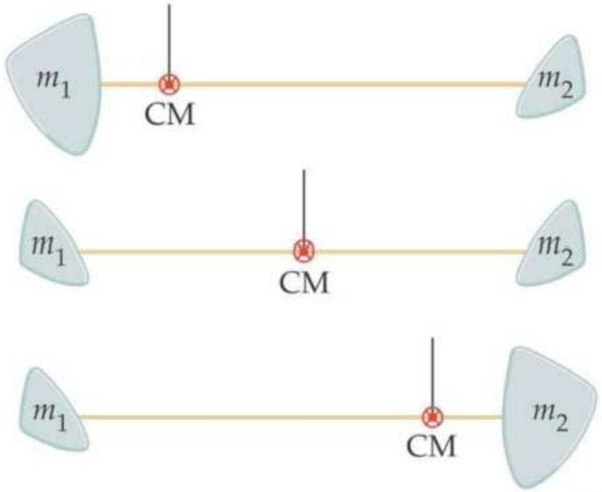
$x_W = (W_1.x_1 + W_2.x_2 + W_3.x_3)/W$ $\Rightarrow$ $x_W = \frac{\sum x_i W_i}{\sum W_i}$

in a similar way for 3D problems

$$x_W = \frac{\sum x_i \, W_i}{\sum W_i} \qquad y_W = \frac{\sum y_i \, W_i}{\sum W_i} \qquad z_W = \frac{\sum z_i \, W_i}{\sum W_i}$$

Similarly, the center of mass is a point which locates the resultant mass of a system of particles or body. Generally, its location is the same as that of G.

$$x_m = \frac{\sum x_i \, m_i}{\sum m_i} \qquad y_m = \frac{\sum y_i \, m_i}{\sum m_i} \qquad z_m = \frac{\sum z_i \, m_i}{\sum m_i}$$



Example

Where is the center of mass on this hammer?

The balancing center of mass bird

Conclusion

Composite Bodies

Centers of Gravity

$$x_W = \frac{\sum x_i W_i}{\sum W_i} \qquad y_W = \frac{\sum y_i W_i}{\sum W_i} \qquad z_W = \frac{\sum z_i W_i}{\sum W_i}$$

Centers of Mass (for constant gravity acceleration)

$$x_m = \frac{\sum x_i m_i}{\sum m_i} \qquad y_m = \frac{\sum y_i m_i}{\sum m_i} \qquad z_m = \frac{\sum z_i m_i}{\sum m_i}$$

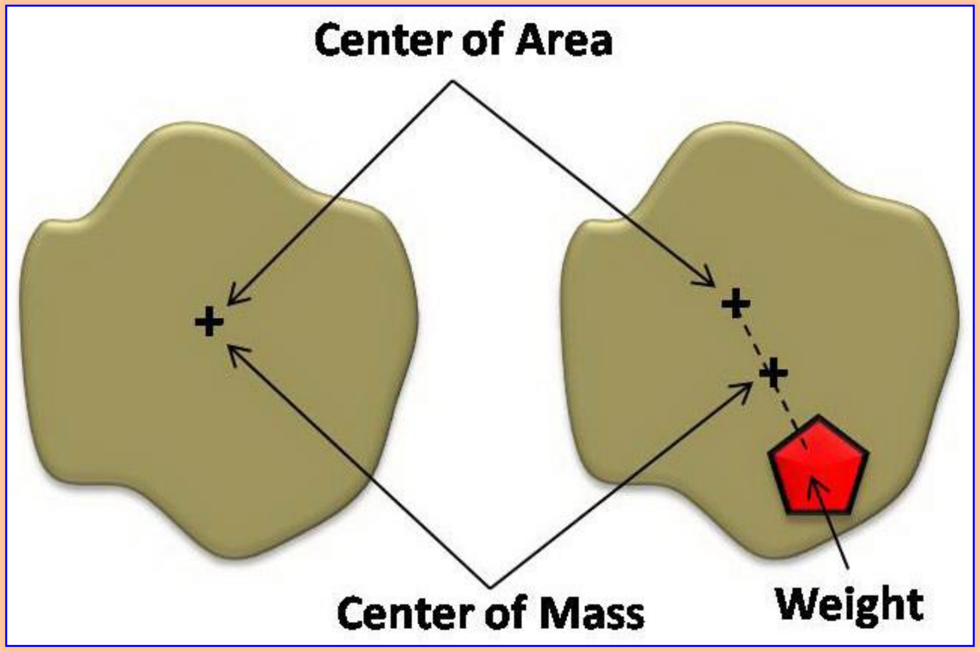
Centroid

Centroid	Center of Gravity
• It is defined as a point about which the entire line, area or volume is assumed to be concentrated. • It is related to distribution of length, area and volume.	• It is defined as a point about which the entire weight of the body is assumed to be concentrated. • Center of mass. • It is related to distribution of mass.

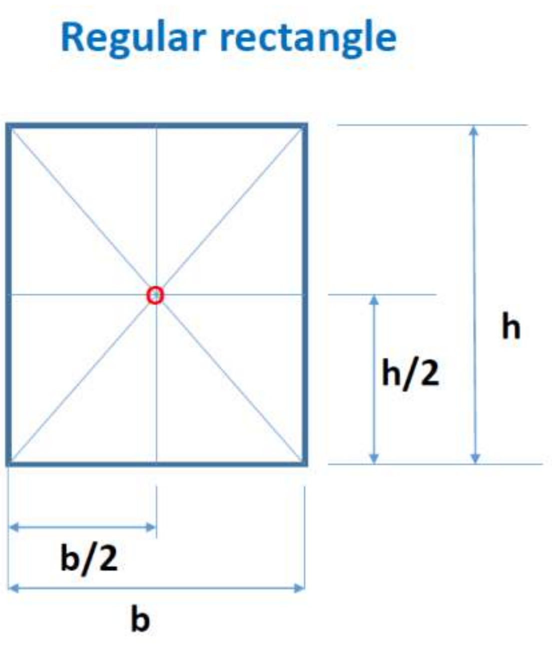
What is the difference between Center of Area and Center of Mass?

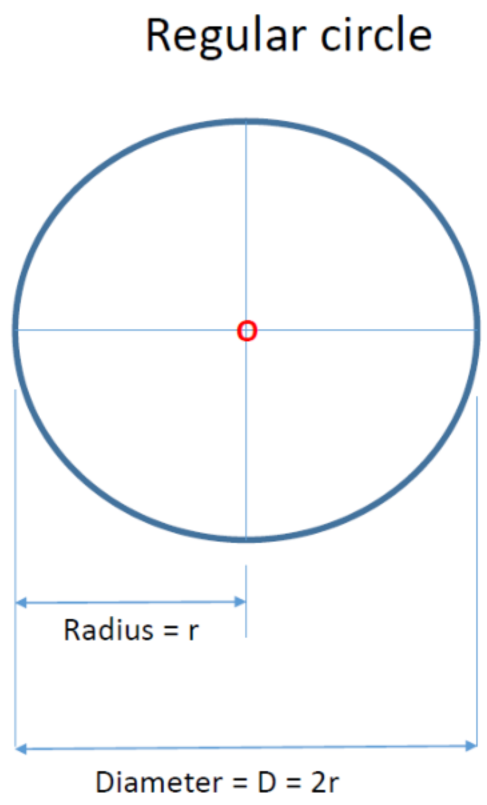
To understand the difference between Center of Area and Center of Mass:

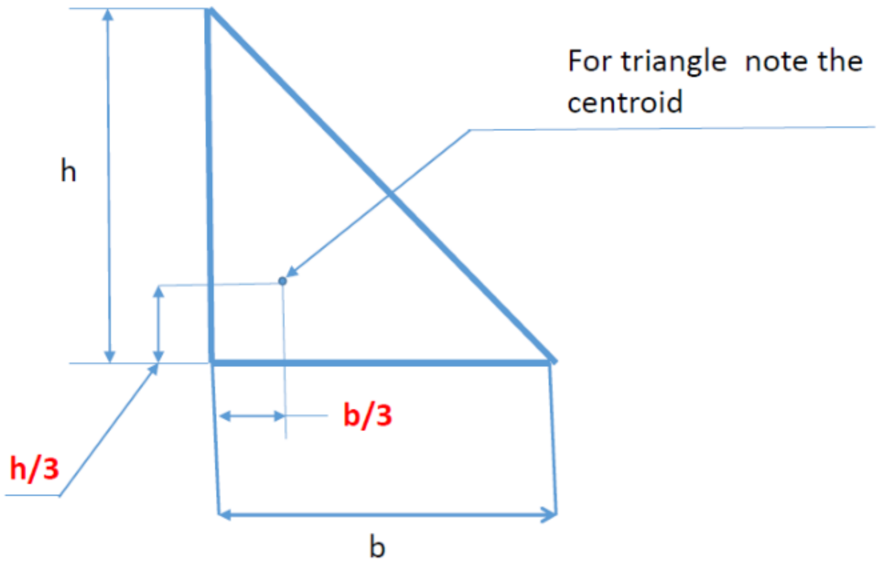
- *If we have two object having the same shape, weight and mass, these objects, then have the same center of area and center of mass.*
- *By sticking another object have a weight on this surface as shown in the figure.*
- *Then the center of mass will be changed, but the center of area will still the same.*



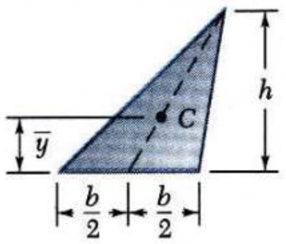
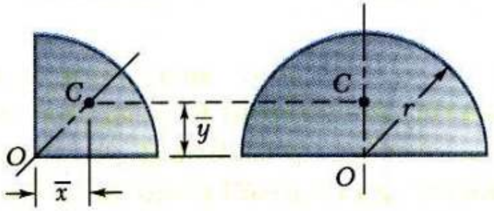
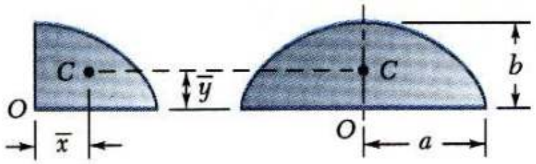
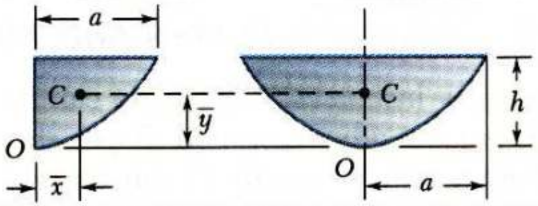
Centroid of Common of Areas



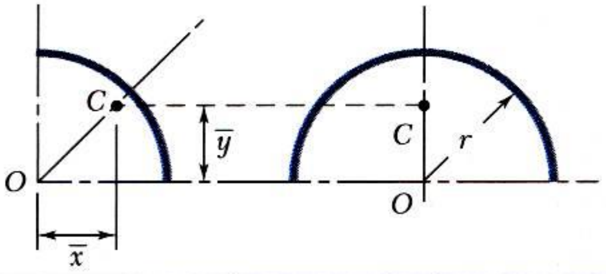
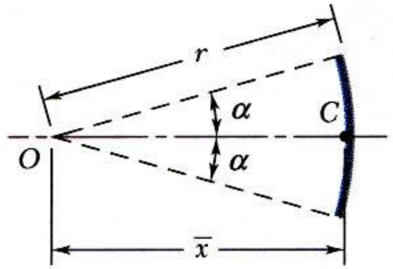




Centroid of Common of Areas

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$

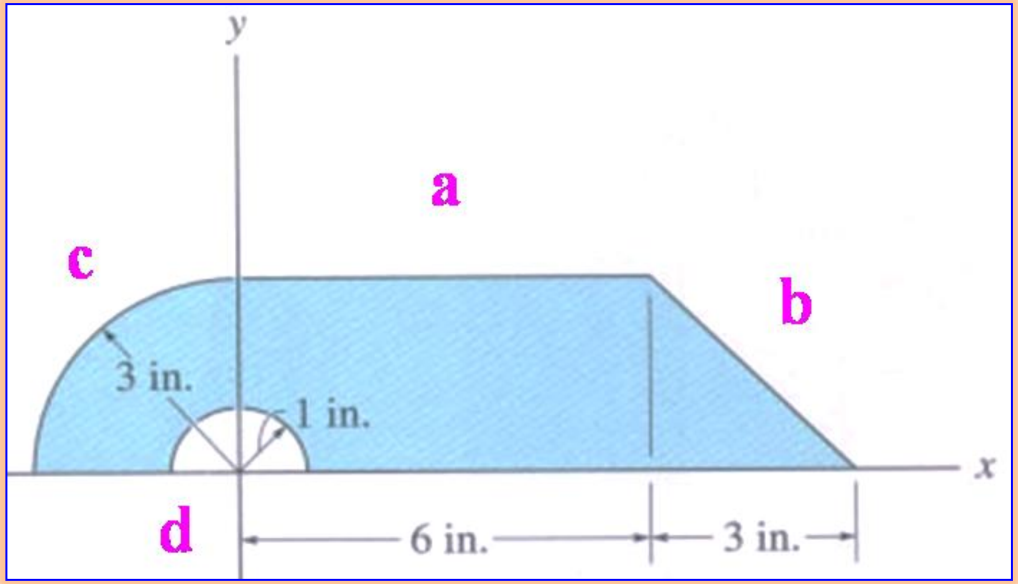
Centroid of Common of Areas

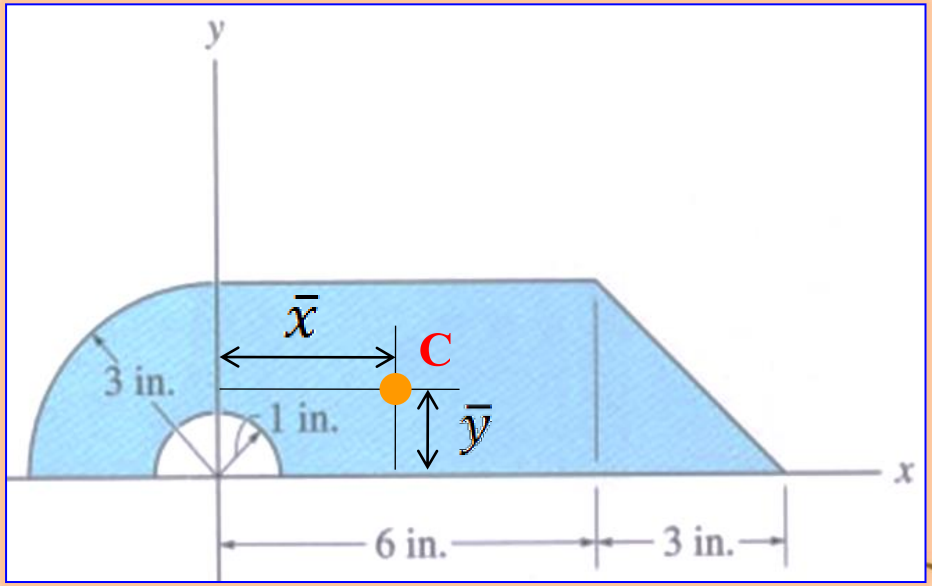
Shape		$\overline{x}$	$\overline{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Centroid of composite bodies (C)

$$\bar{x} = \frac{\sum_{i=0}^{n} A_i \cdot x_i}{\sum_{i=0}^{n} A_i} \qquad \bar{y} = \frac{\sum_{i=0}^{n} A_i \cdot y_i}{\sum_{i=0}^{n} A_i}$$

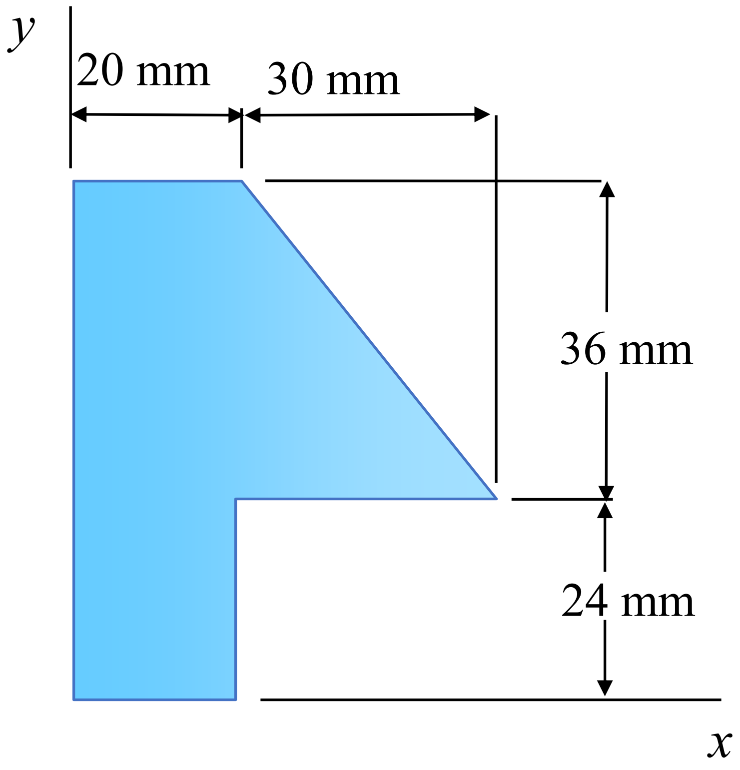
Example: composite bodies: a+b+c-d; Centroid (C) ($\bar{x}$, $\bar{y}$)????????

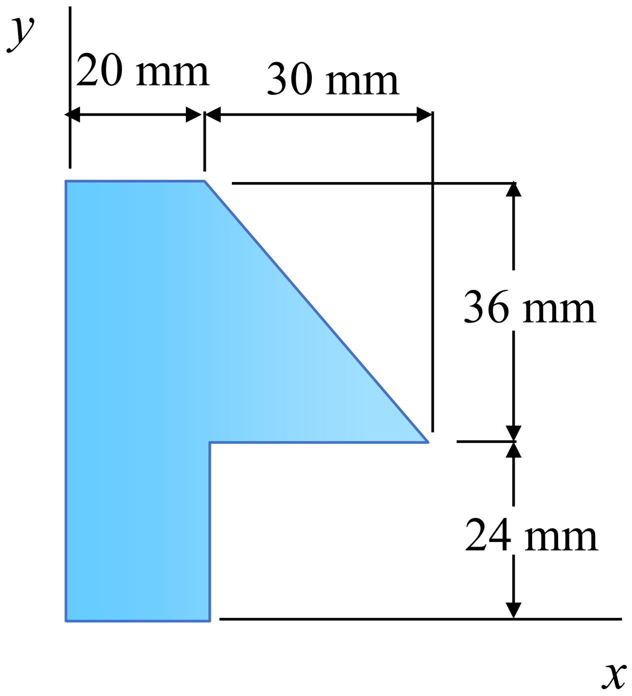




Problem

Locate the centroid of the plane area shown.



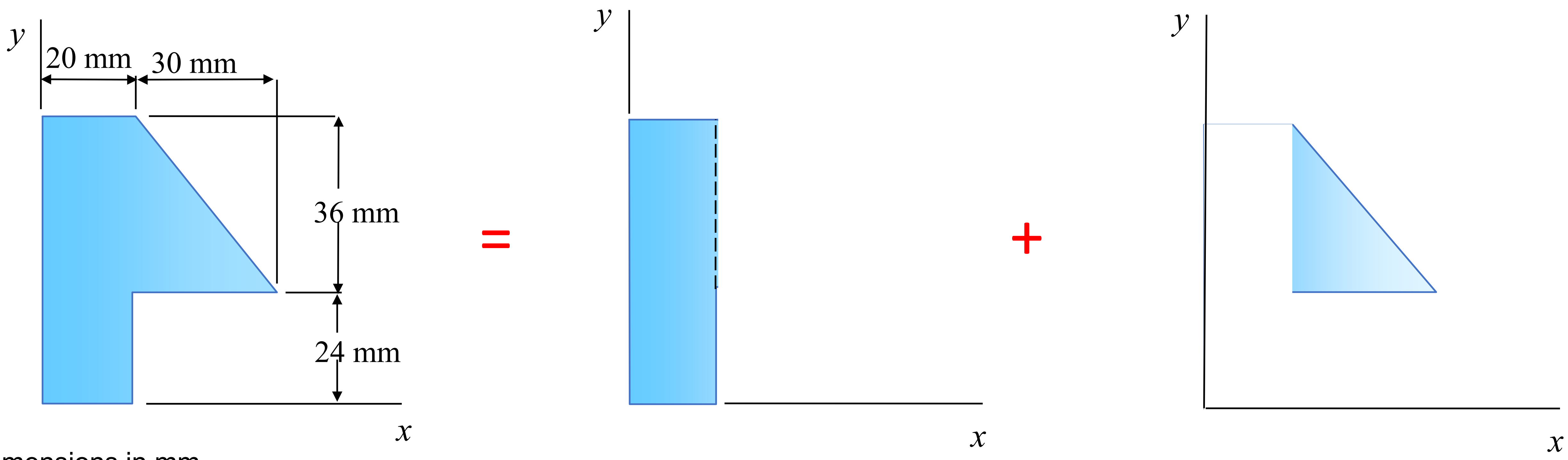


1. *Decide how to construct the given area from common shapes.*

2. It is strongly recommended that you *construct a table containing areas or length and the respective coordinates of the centroids.*

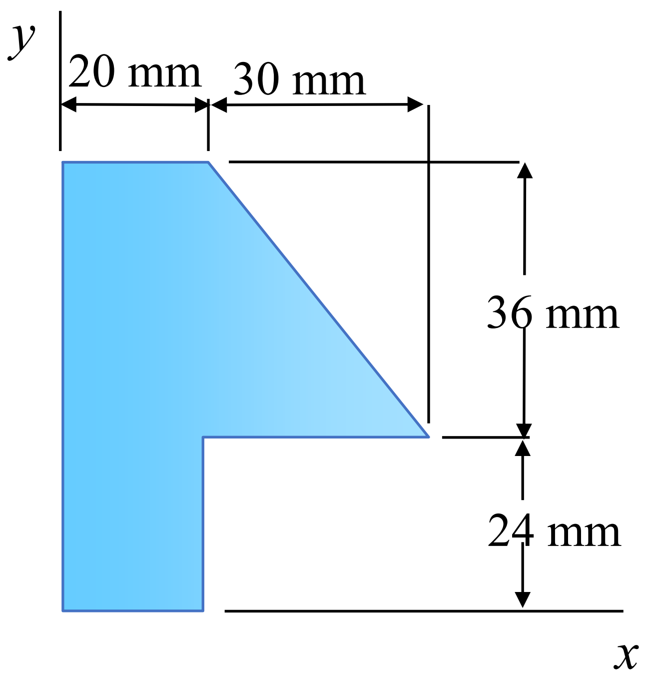
3. When possible, *use symmetry to help locate the centroid.*

1.Decide how to construct the given area from common shapes.

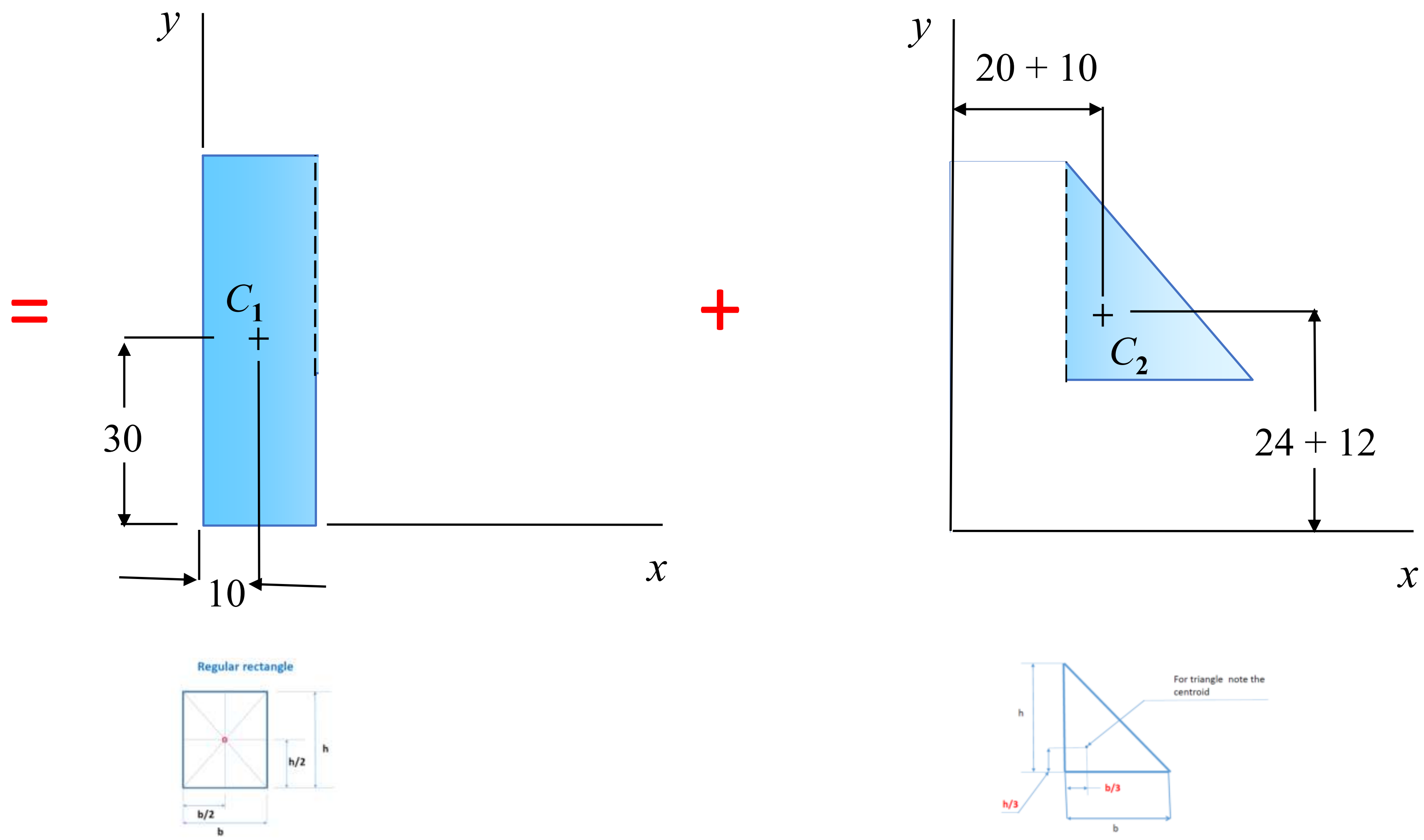


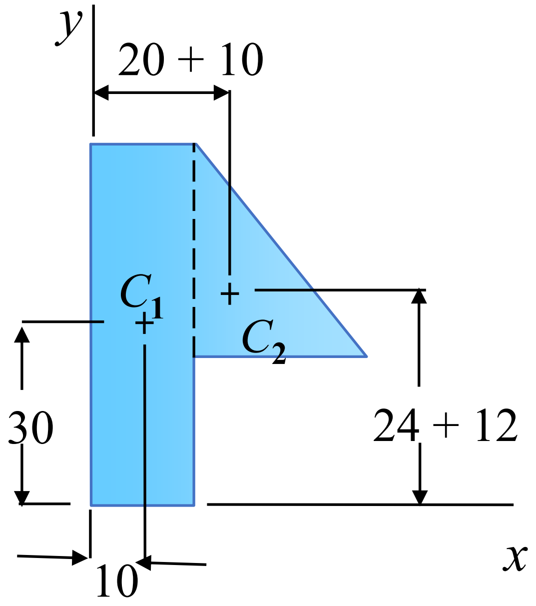
Dimensions in mm

1.Decide how to construct the given area from common shapes.



Dimensions in mm



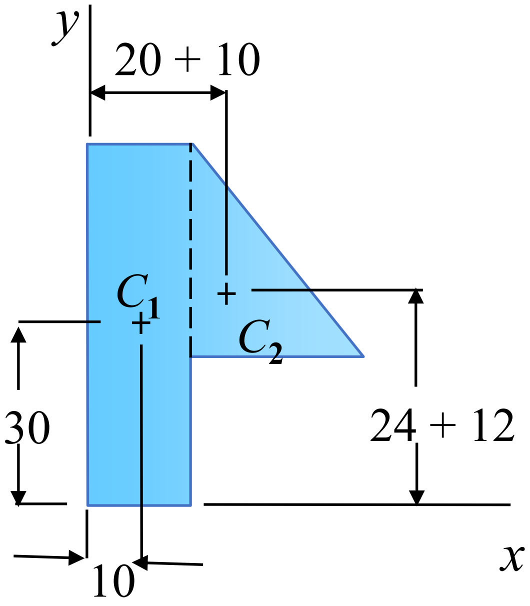


2. *Construct a table containing areas and respective coordinates of the centroids.*

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
R					
T					
Σ					

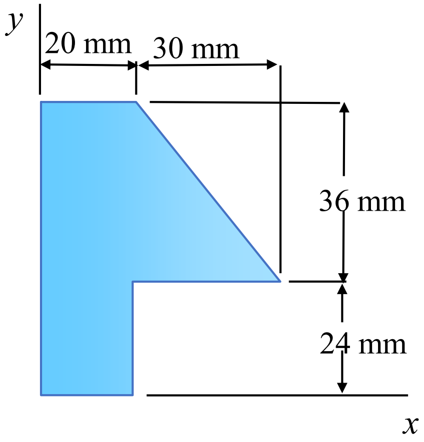
$$\bar{x} = \frac{\sum_{i=0}^{n} A_i \, . x_i}{\sum_{i=0}^{n} A_i} \qquad \bar{y} = \frac{\sum_{i=0}^{n} A_i \, . y_i}{\sum_{i=0}^{n} A_i}$$

Dimensions in mm

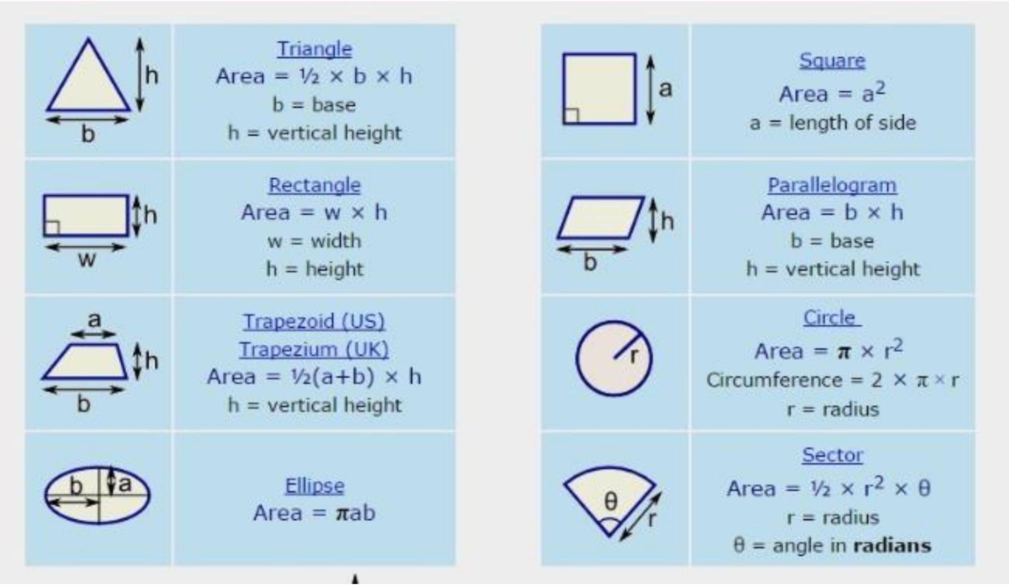


3. Coordinates of the centroids of each body and the addition

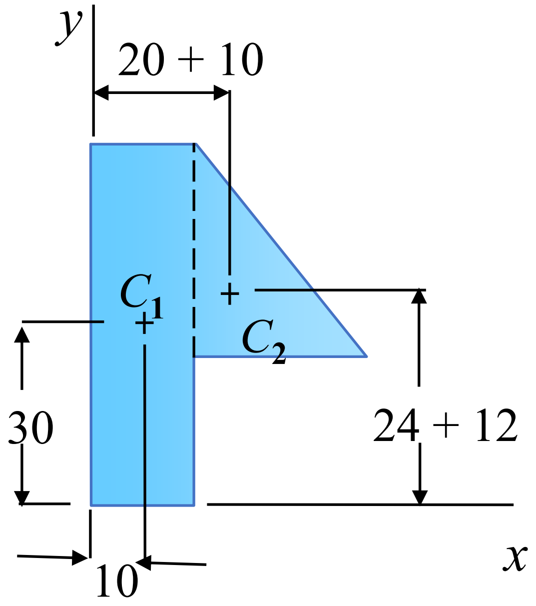
	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
R	20 x 60 =1200	10	30	12 000	36 000
T	(1/2) x 30 x 36 =540	30	36	16 200	19 440
Σ	1740			28 200	55 440



Dimensions in mm



4. $\bar{X}$; $\bar{Y}$



Dimensions in mm

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
R	20 x 60 =1200	10	30	12 000	36 000
T	(1/2) x 30 x 36 =540	30	36	16 200	19 440
Σ	1740			28 200	55 440

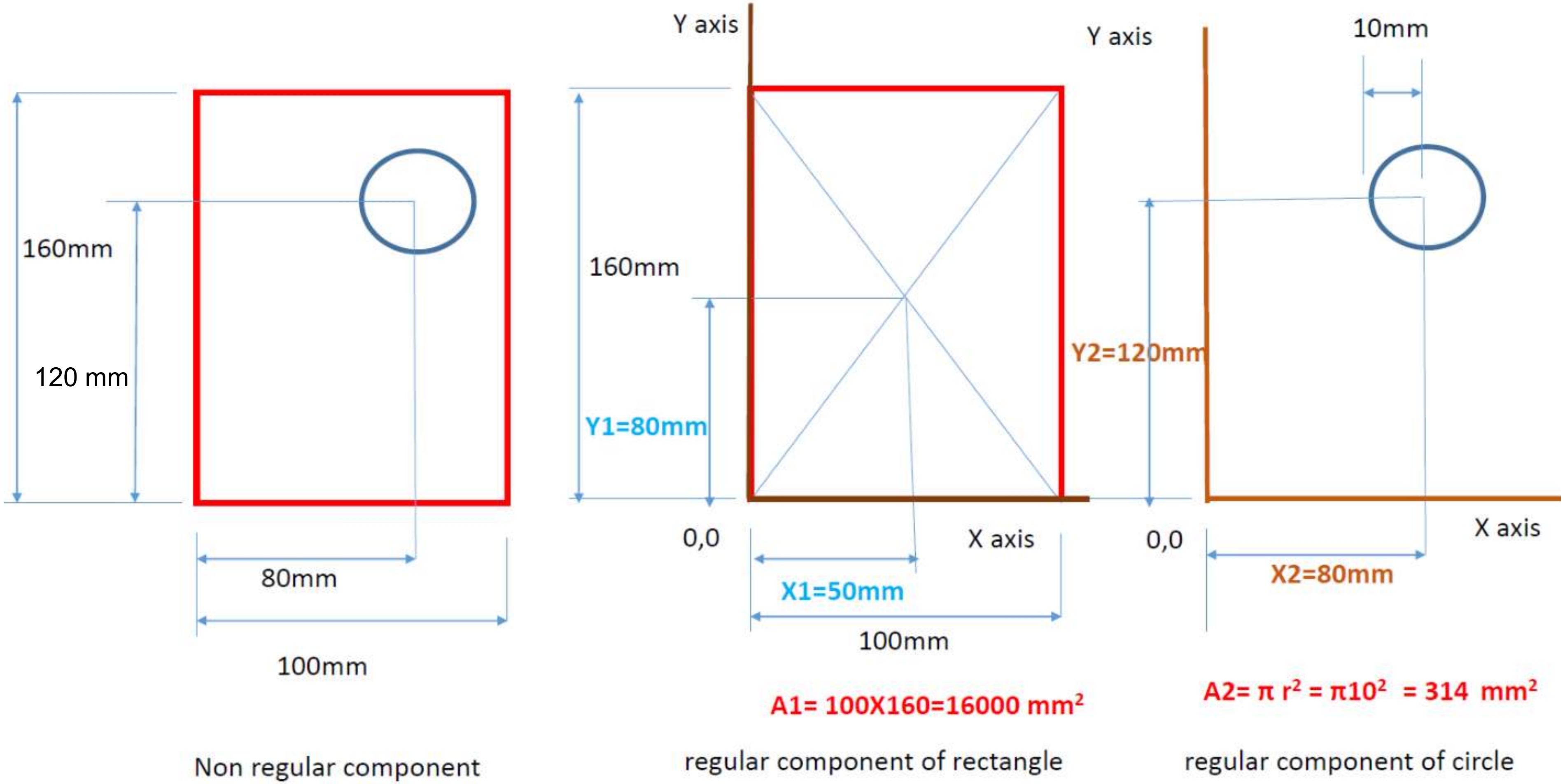
$$\bar{X} = \frac{\sum \bar{x} A}{\sum A}$$

$\bar{X}$ = (28200/1740)=16.21 mm

$$\bar{Y} = \frac{\sum \bar{y} A}{\sum A}$$

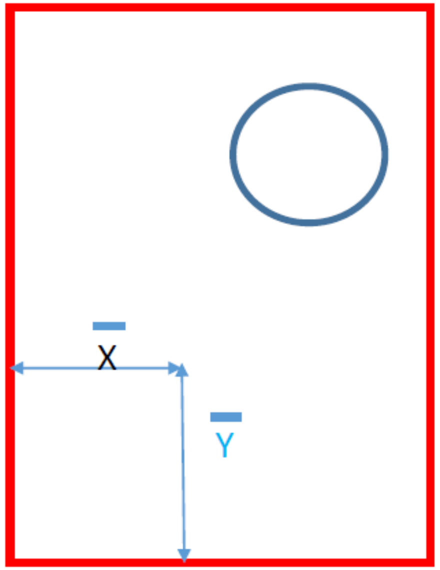
$\bar{Y}$ = (55440/1740)=31.9 mm

Problem 2



160mm
120 mm
80mm
100mm
Non regular component
Y axis
160mm
Y1=80mm
0,0
X axis
X1=50mm
100mm
A1= 100X160=16000 mm²
regular component of rectangle
Y axis
10mm
Y2=120mm
0,0
X axis
X2=80mm
A2= π r² = π10² = 314 mm²
regular component of circle

Area	X cordinate	Y cordinate
A1= 100X160=16000 mm^2	X1=50mm	Y1=80mm
A2= $\pi r^2 = \pi 10^2$ = 314 mm^2	X2=80mm	Y2=120mm



$$\bar{X} = \frac{A1X1 - A2X2}{A1 - A2} = \frac{16000X50 - 314X80}{16000 - 314} = \frac{800000 - 25120}{15686} = 49.3mm$$

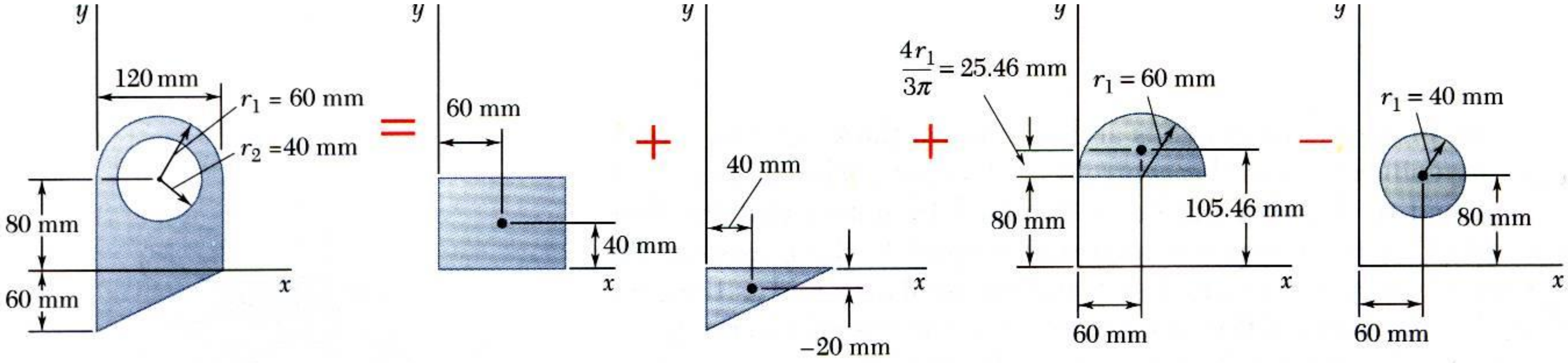
$$\bar{Y} = \frac{A1Y1 - A2Y2}{A1 - A2} = \frac{16000X80 - 314X120}{16000 - 314} = \frac{1280000 - 37680}{15686} = 79.2mm$$

$\bar{X}$ = 49.3mm

$\bar{Y}$ = 79.2mm

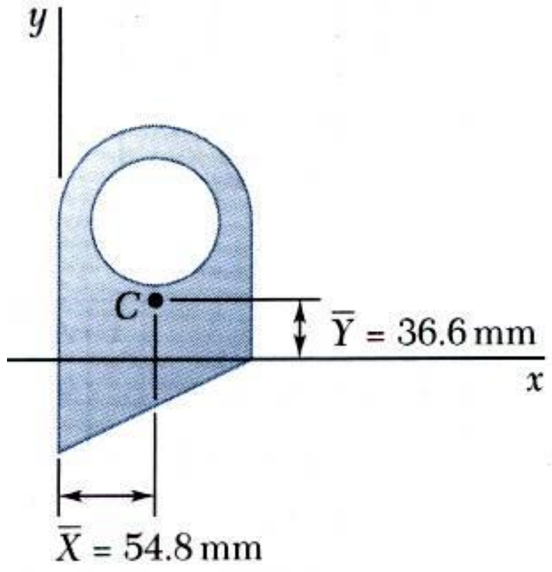
Problem 3

Find the centroid location (x , y) for the part with shaded area:



Component	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

•Compute the coordinates of the area centroid by dividing the first moments by the total area.



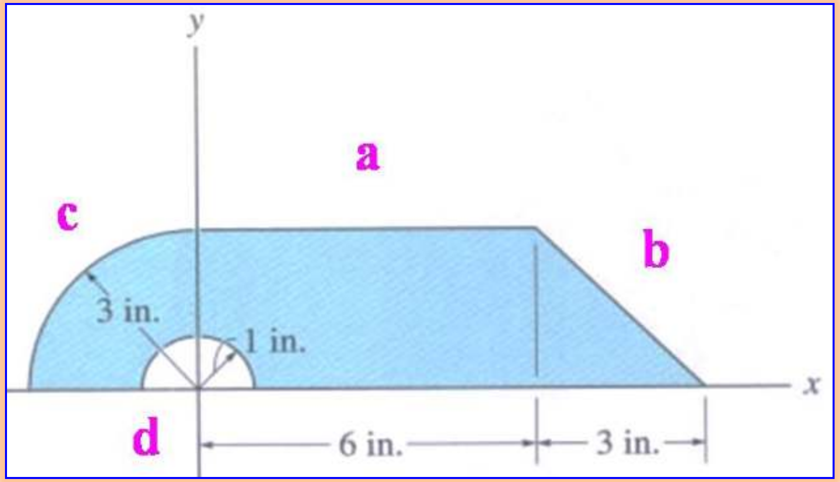
$$\bar{X} = \frac{\sum \bar{x} A}{\sum A} = \frac{+757.7 \times 10^3 \text{ mm}^3}{13.828 \times 10^3 \text{ mm}^2}$$

$$\bar{X} = 54.8 \text{ mm}$$

$$\bar{Y} = 36.6 \text{ mm}$$

Example 3:

Find the centroid location (x , y) for the part with shaded area:



Solution:

1. This body can be divided into the following pieces: rectangle (a) + triangle (b) + quarter circular (c) – semicircular area (d)
2. Make up the table using parts a, b, c, and d.

Segment	Area A (in^2)	$\tilde{x}$ (in)	$\tilde{y}$ (in)	$A\tilde{x}$ (in^3)	$A\tilde{y}$ (in^3)
Rectangle **Triangle** **Q. Circle** **Semi-Circle**					
Σ					

3. Start to fill the table using the data given for each parts a, b, c, and d, let take for example part (**a**) the rectangular:

The rectangular area (A) = 3 * 6 = 18 in^2

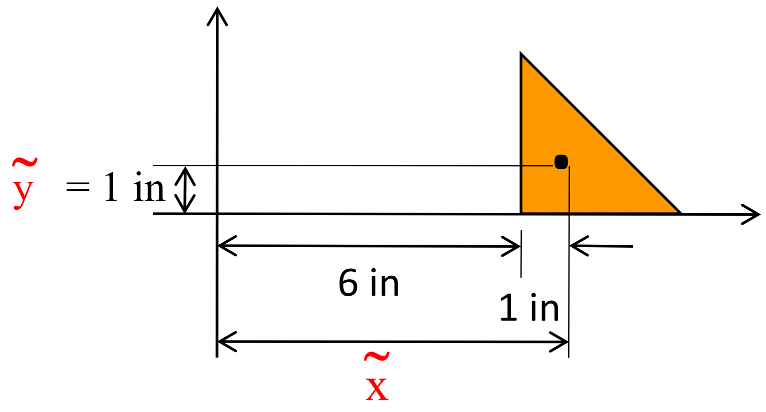
$\tilde{x} = 6 / 2 = 3$ in , $\tilde{y} = 3 / 2 = 1.5$ in

Continue of Example 3:

$A.\tilde{x} = 18 * 3 = 54 \ in^2$,

$A.\tilde{y} = 18 * 1.5 = 27 \ in$

Same step for the triangular:



The triangular area (A) $= 3 * 3 / 2 = 4.5 \ in^2$

$\tilde{x} = (3 / 3) + 6 = 7 \ in$,

$\tilde{y} = 3 / 3 = 1 \ in$

$A.\tilde{x} = 4.5 * 7 = 31.5 \ in^2$, $A.\tilde{y} = 4.5 * 1 = 4.5 \ in$

Do the same step for the other shapes to obtain the following table:

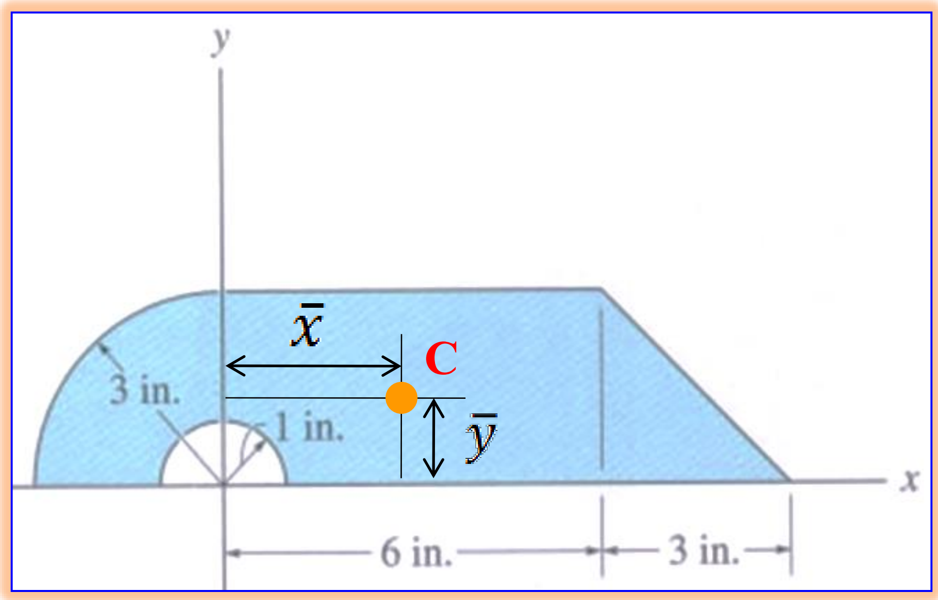
Segment	Area A (in^2)	$\tilde{x}$ (in)	$\tilde{y}$ (in)	$A\tilde{x}$ (in^3)	$A\tilde{y}$ (in^3)
Rectangle	18	3	1.5	54	27
Triangle	4.5	7	1	31.5	4.5
Q. Circle	$9\pi / 4$	$-4(3)/(3\pi)$	$4(3)/(3\pi)$	-9	9
Semi-Circle	$-\pi / 2$	0	$4(1)/(3\pi)$	0	$-2/3$
Σ	28.0			76.5	39.83

Continue of Example 3:

4. Use the table data and these formulas to find the coordinates of the centroid.

$$\bar{x} = \frac{\sum_{i=0}^{n} A_i \, . x_i}{\sum_{i=0}^{n} A_i}$$

$$\bar{y} = \frac{\sum_{i=0}^{n} A_i \, . y_i}{\sum_{i=0}^{n} A_i}$$



$$\bar{x} = (\Sigma \, \tilde{x} \, A) / (\Sigma \, A) = 76.5 \text{ in}^3 / 28.0 \text{ in}^2 = 2.73 \text{ in}$$

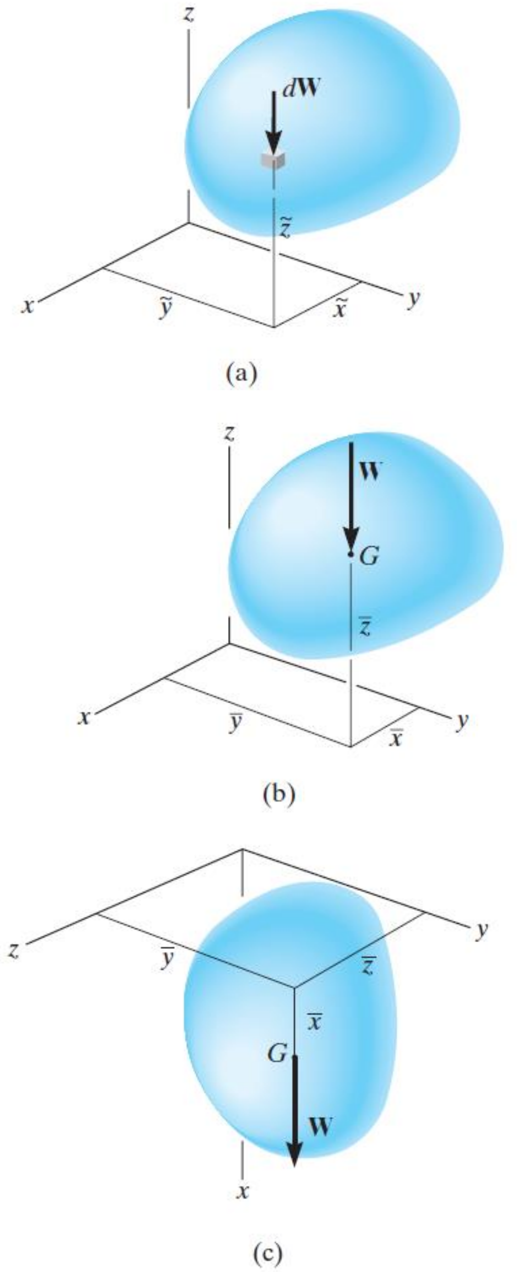
$$\bar{y} = (\Sigma \, \tilde{y} \, A) / (\Sigma \, A) = 39.83 \text{ in}^3 / 28.0 \text{ in}^2 = 1.42 \text{ in}$$

Center of Gravity

In this section we will first show how to locate the center of gravity for a body, and then we will show that the center of mass and the centroid of a body can be developed using this same method.

A body is composed of an infinite number of particles of differential size, and so if the body is located within a gravitational field, then each of these particles will have a weight *dW*, Fig. *a*.

These weights will form an approximately parallel force system, and the resultant of this system is the total weight of the body, which passes through a single point called the *center of gravity, G*, Fig *b*.



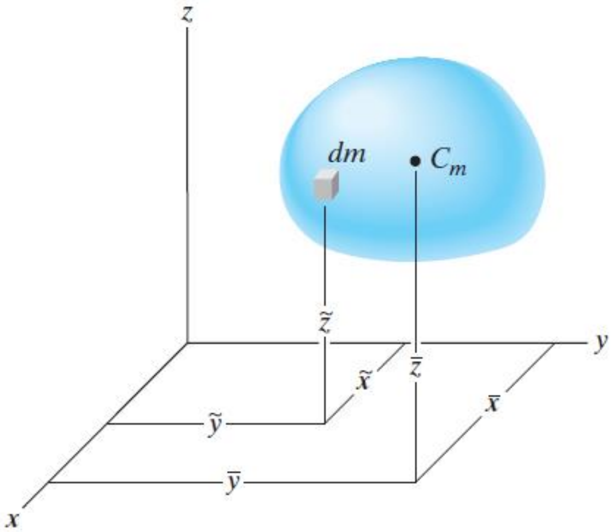
Therefore, the location of the center of gravity *G* with respect to the *x*, *y*, *z* axes becomes:

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

Here

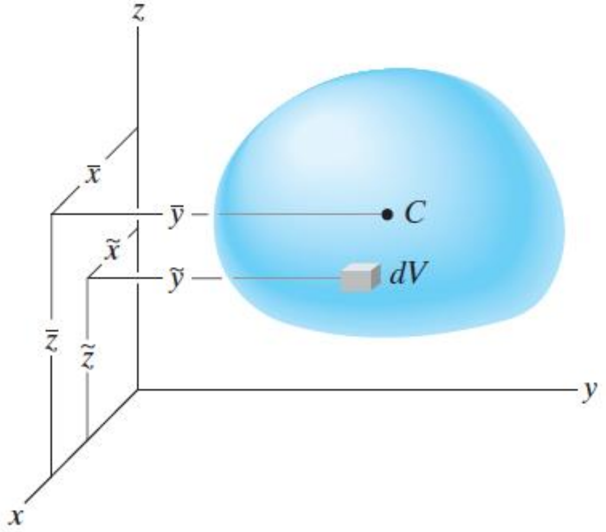
$\bar{x}, \bar{y}, \bar{z}$ are the coordinates of the center of gravity *G*, Fig. 9–1*b*.

$\tilde{x}, \tilde{y}, \tilde{z}$ are the coordinates of each particle in the body, Fig. 9–1*a*.



Center of Mass of a Body.

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm} \qquad \bar{y} = \frac{\int \tilde{y}\, dm}{\int dm} \qquad \bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$$

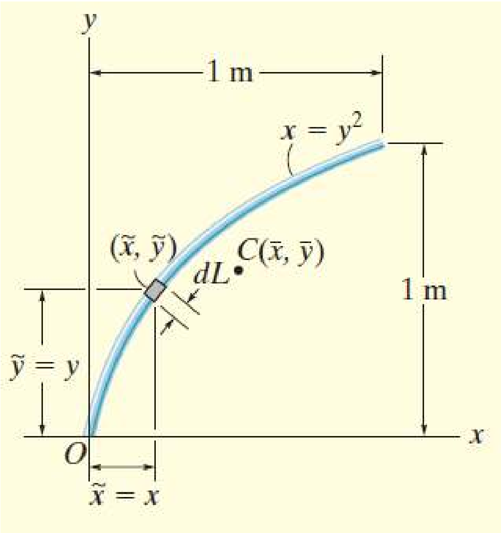


Centroid of a Volume.

$$\bar{x} = \frac{\int_V \tilde{x}\, dV}{\int_V dV} \qquad \bar{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} \qquad \bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV}$$

EXAMPLE

Locate the centroid of the rod bent into the shape of a parabolic arc as shown in Fig.



Solution

Differential Element. The differential element is shown in Fig. It is located on the curve at the *arbitrary point* (*x, y*).

Area and Moment Arms. The differential element of length *dL* can be expressed in terms of the differentials *dx* and *dy* using the Pythagorean theorem.

$$dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\, dy$$

Since $x = y^2$, then $dx/dy = 2y$. Therefore, expressing dL in terms of y and dy, we have

$$dL = \sqrt{(2y)^2 + 1}\, dy$$

As shown in Fig. 9–8, the centroid of the element is located at $\tilde{x} = x$, $\tilde{y} = y$.

Integrations. Applying Eqs. 9–5, using the formulas in Appendix A to evaluate the integrals, we get

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} = \frac{\int_0^{1\text{m}} x\sqrt{4y^2 + 1}\, dy}{\int_0^{1\text{m}} \sqrt{4y^2 + 1}\, dy} = \frac{\int_0^{1\text{m}} y^2\sqrt{4y^2 + 1}\, dy}{\int_0^{1\text{m}} \sqrt{4y^2 + 1}\, dy}$$

$$= \frac{0.6063}{1.479} = 0.410 \text{ m} \qquad \textit{Ans.}$$

$$\bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} = \frac{\int_0^{1\text{m}} y\sqrt{4y^2 + 1}\, dy}{\int_0^{1\text{m}} \sqrt{4y^2 + 1}\, dy} = \frac{0.8484}{1.479} = 0.574 \text{ m} \qquad \textit{Ans.}$$

Composite Bodies

- Consists of a series of connected "simpler" shaped bodies, which may be rectangular, triangular or semicircular
- A body can be sectioned or divided into its composite parts
- Accounting for finite number of weights

$$\bar{x} = \frac{\sum \tilde{x} W}{\sum W} \quad \bar{y} = \frac{\sum \tilde{y} W}{\sum W} \quad \bar{z} = \frac{\sum \tilde{z} W}{\sum W}$$

Composite Bodies

Procedure for Analysis

Composite Parts

- Divide the body or object into a finite number of composite parts that have simpler shapes
- Treat the hole in composite as an additional composite part having negative weight or size

Moment Arms

- Establish the coordinate axes and determine the coordinates of the center of gravity or centroid of each part

Summations

- Determine the coordinates of the center of gravity by applying the center of gravity equations
- If an object is symmetrical about an axis, the centroid of the objects lies on the axis

Structures Analysis

Introduction

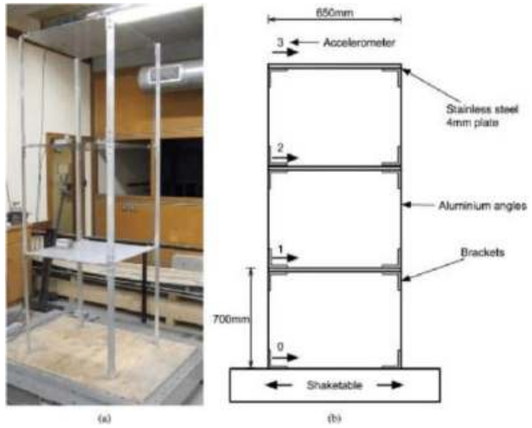
Some of the most common structures we see around us are buildings & bridges. In addition to these, one can also classify a lot of other objects as "structures."

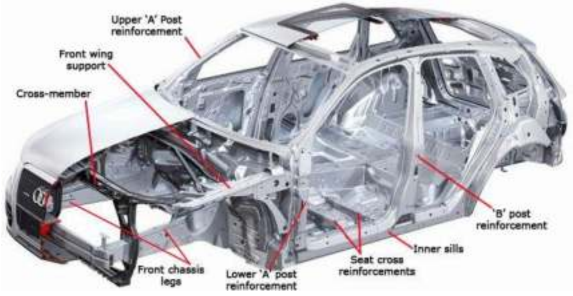
The space station

Chassis of your car

Your chair, table, bookshelf

Truss, bridge, etc. etc.

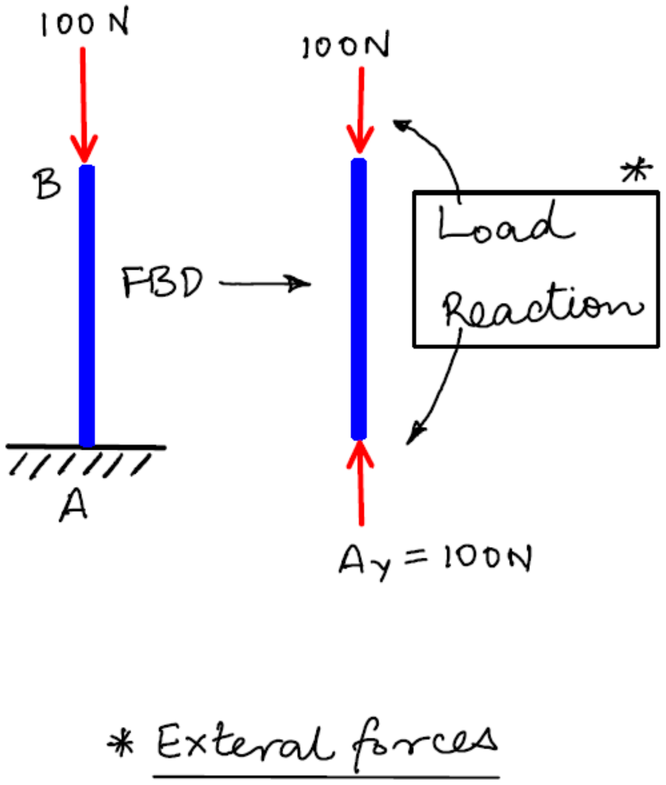


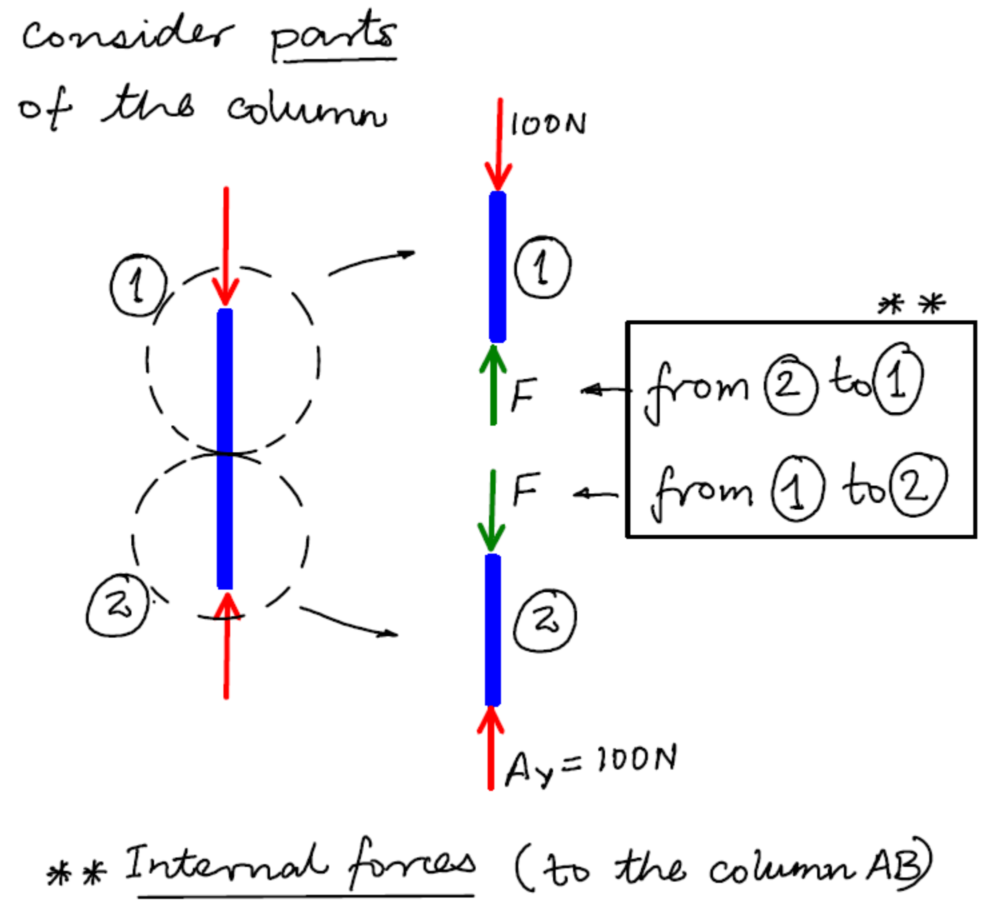


The objective of this chapter is to figure out the forces being carried by these structures so that as an engineer, you can decide whether the structure can sustain these forces or not.

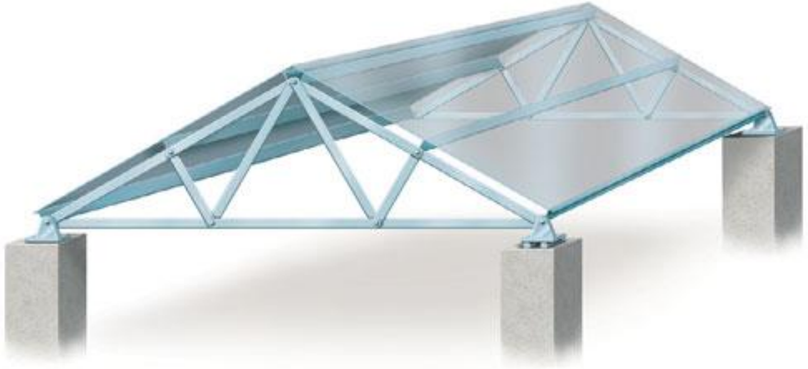
Recall

- External forces: "Loads" acting **on** your structure.
  Note: this includes "reaction" forces from the supports as well.
- Internal forces: Forces that develop within every structure that keep the different parts of the structure together.





Simple Trusses

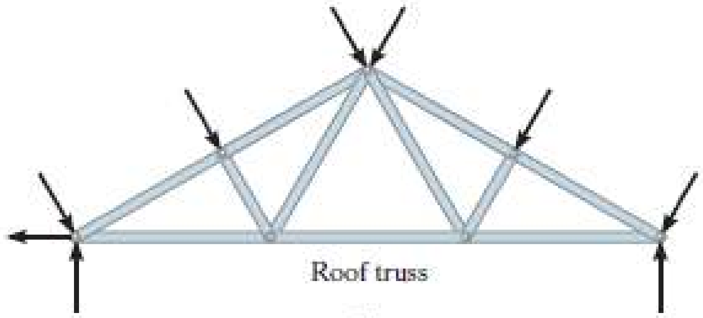


A *truss* is a structure composed of

- slender members
- joined together at their end points with pin joints.

So, all external forces (loads & reactions) must be applied only at the joints.

external forces



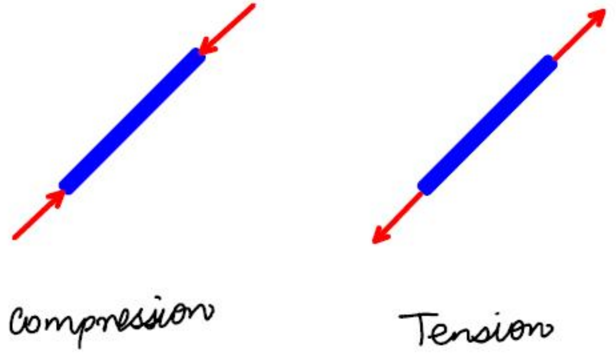
(b)

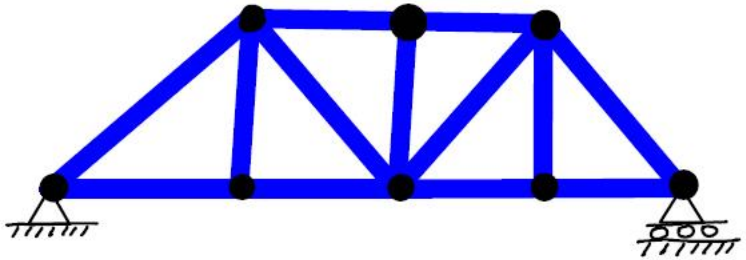
The truss shown in Fig. (*a)* is an example of a typical roof-supporting truss. In this figure, the roof load is transmitted to the truss *at the joints* by means of a series of *purlins*. Since this loading acts in the same plane as the truss, Fig. (*b*), the analysis of the forces developed in the truss members will be two-dimensional.

Note:

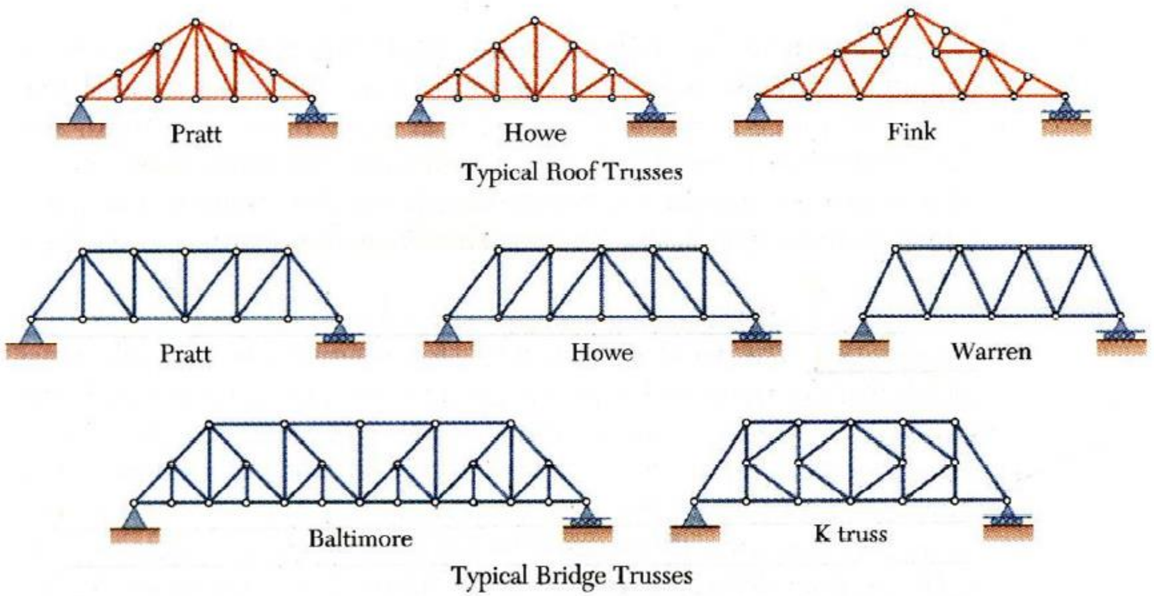
Every member of a truss is a 2 forces member.

Trusses are assumed to be of negligible weight (compared to the loads they carry)

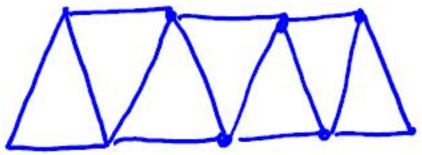




Types of Trusses



Simple Trusses: constructed from a "base" triangle by adding two members at a time.



simple

Note: For Simple Trusses (and in general statically determinate trusses)

$$2n = m + r$$

m: members } (m+r) unknowns
r: reactions
n: joints } 2n equations

Note: This is a necessary condition for statical determinancy

This is not sufficient condition. So even if a truss satisfies the above relation it may not be determinate.

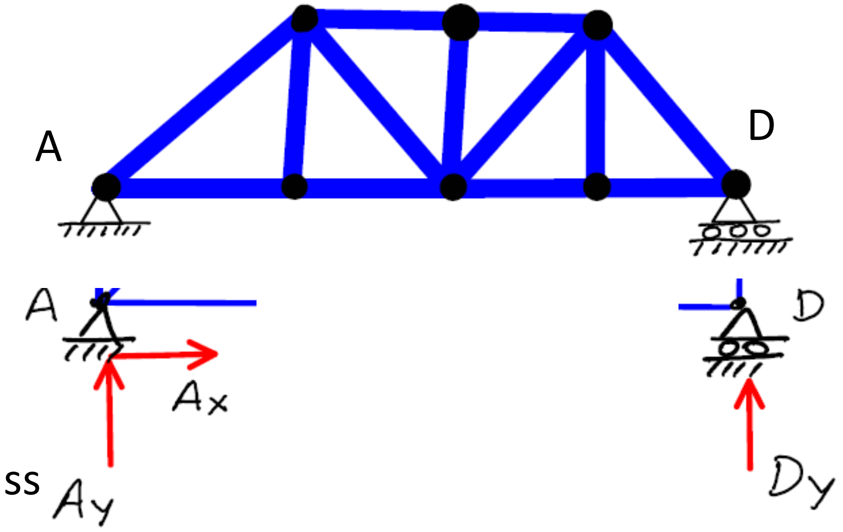
But if it is determinate then it satisfies the above relation.

Example:

m=13

r=3

n=8



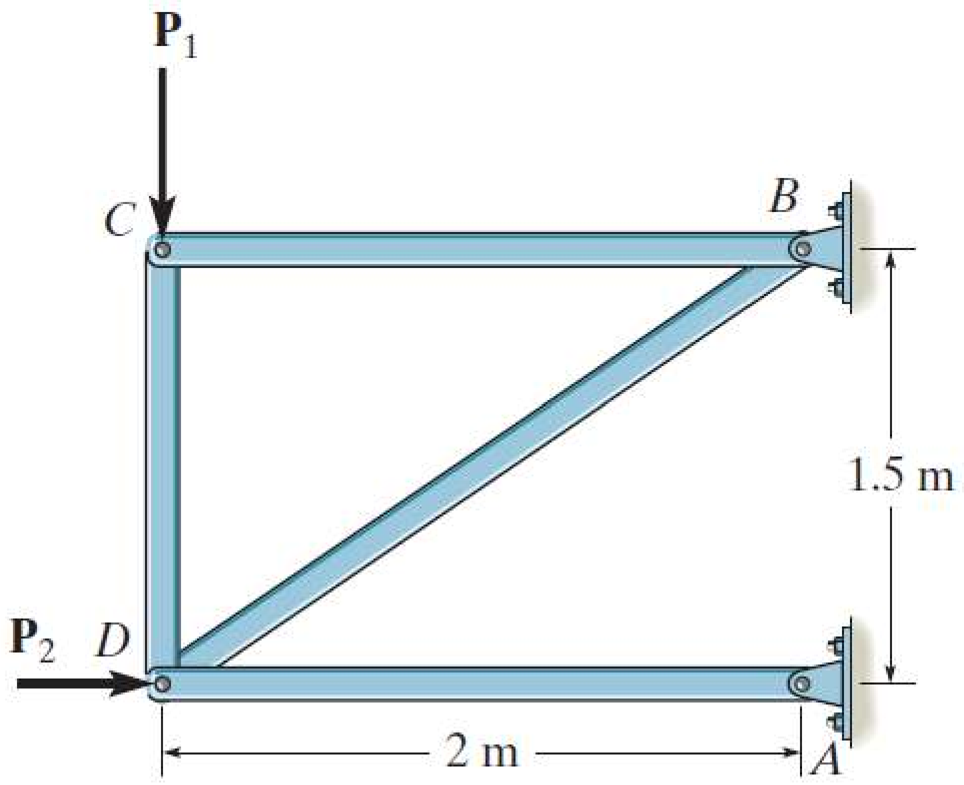
2n=2x8=16 ; m+r=13+3=16 ……> statical determinate truss

The Method of Joints (truss analysis)

In order to analyze or design a truss, it is necessary to determine the force in each of its members. One way to do this is to use the method of joints.

This method is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium.

Therefore, if the free-body diagram of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint.



Procedure for Analysis

The following procedure provides a means for analyzing a truss using the method of joints.

- Draw the free-body diagram of a joint having at least one known force and at most two unknown forces. (If this joint is at one of the supports, then it may be necessary first to calculate the external reactions at the support.)

- Use one of the two methods described above for establishing the sense of an unknown force.

- Orient the x and y axes such that the forces on the free-body diagram can be easily resolved into their x and y components and then apply the two force equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Solve for the two unknown member forces and verify their correct sense.

- Using the calculated results, continue to analyze each of the other joints. Remember that a member in *compression* "pushes" on the joint and a member in *tension* "pulls" on the joint. Also, be sure to choose a joint having at most two unknowns and at least one known force.

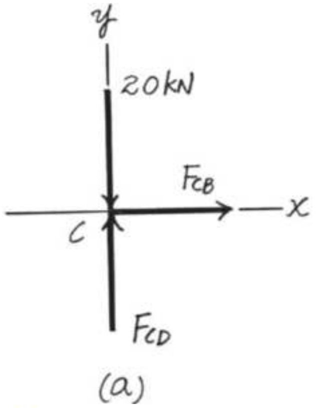
Application

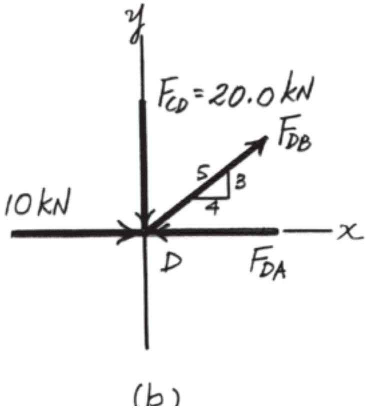
Determine the force in each member and state if the members are in tension (T) or compression (C). Set $P_1 = 20$ kN, $P_2 = 10$ kN.

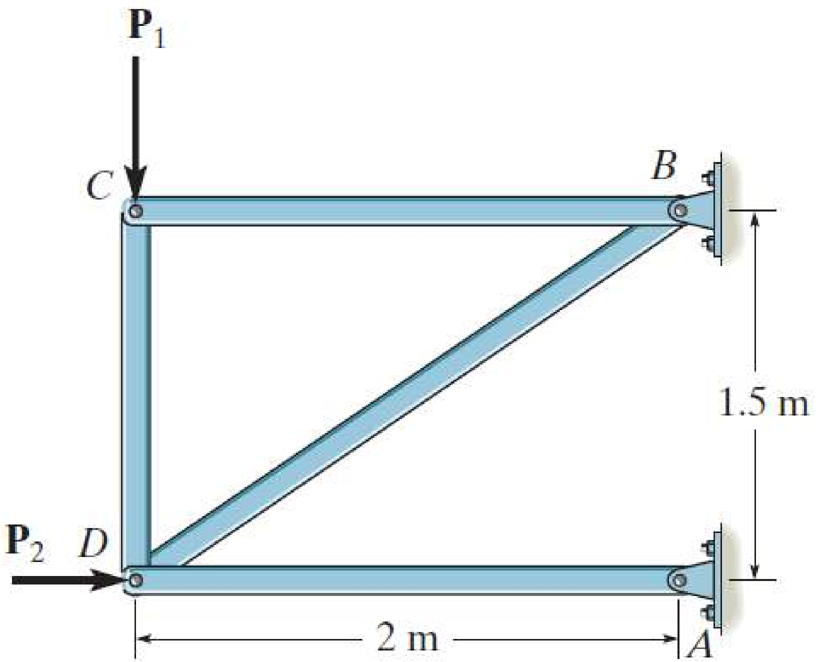
Solution

Method of Joints. Start at joint C and then proceed to join D.

Free body-diagram







Joint C. Fig. a

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{CB} = 0$ **Ans.**

$+\uparrow\Sigma F_y = 0;\quad F_{CD} - 20 = 0 \quad F_{CD} = 20.0\text{ kN (C)}$ **Ans.**

Joint D. Fig. b

$+\uparrow\Sigma F_y = 0;\quad F_{DB}\left(\frac{3}{5}\right) - 20.0 = 0 \quad F_{DB} = 33.33\text{ kN (T)} = 33.3\text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 10 + 33.33\left(\frac{4}{5}\right) - F_{DA} = 0$

$F_{DA} = 36.67\text{ kN (C)} = 36.7\text{ kN (C)}$ **Ans.**

Ans:

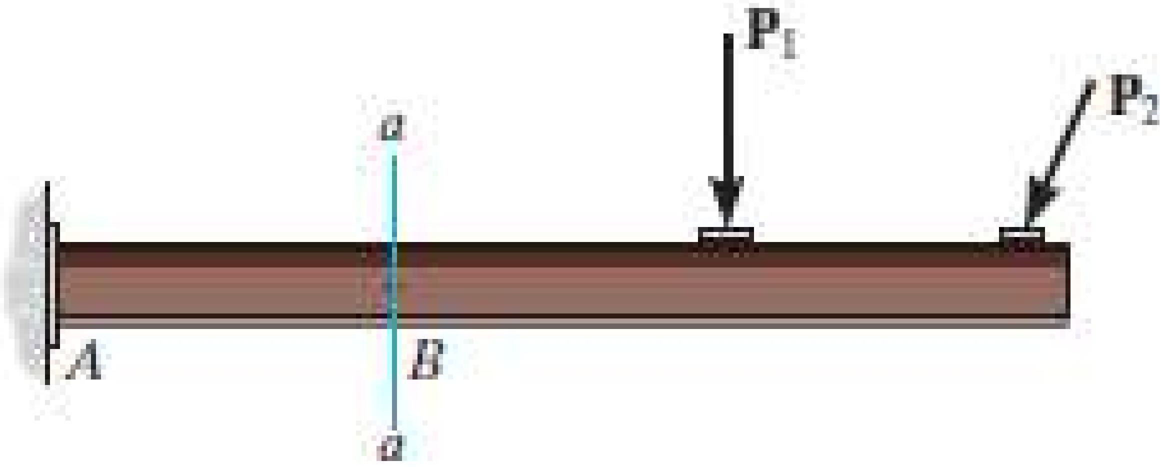
$F_{CB} = 0$
$F_{CD} = 20.0\text{ kN (C)}$
$F_{DB} = 33.3\text{ kN (T)}$
$F_{DA} = 36.7\text{ kN (C)}$

Internal Forces and Moments

Internal Forces Developed in Structural Members

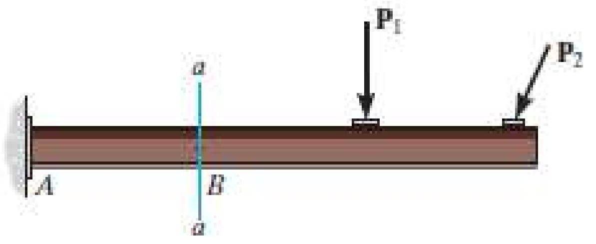
To design a structural or mechanical member it is necessary to know the loading acting within the member in order to be sure the material can resist this loading. Internal loadings can be determined by using the ***method of sections.***

To illustrate this method, consider the cantilever beam in Fig. (*a*). If the internal loadings acting on the cross section at point *B* are to be determined, we must pass an imaginary section *a–a* perpendicular to the axis of the beam through point *B* and then separate the beam into two segments. The internal loadings acting at *B* will then be exposed and become *external* on the free-body diagram of each segment.

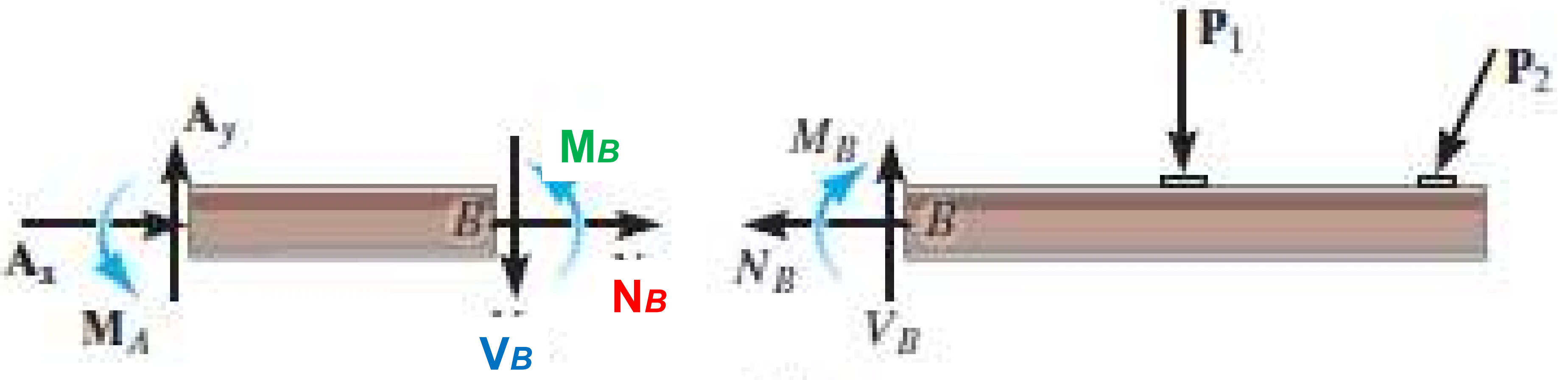


(a)

N_B: normal force
V_B: shear force
M_B :bending moment



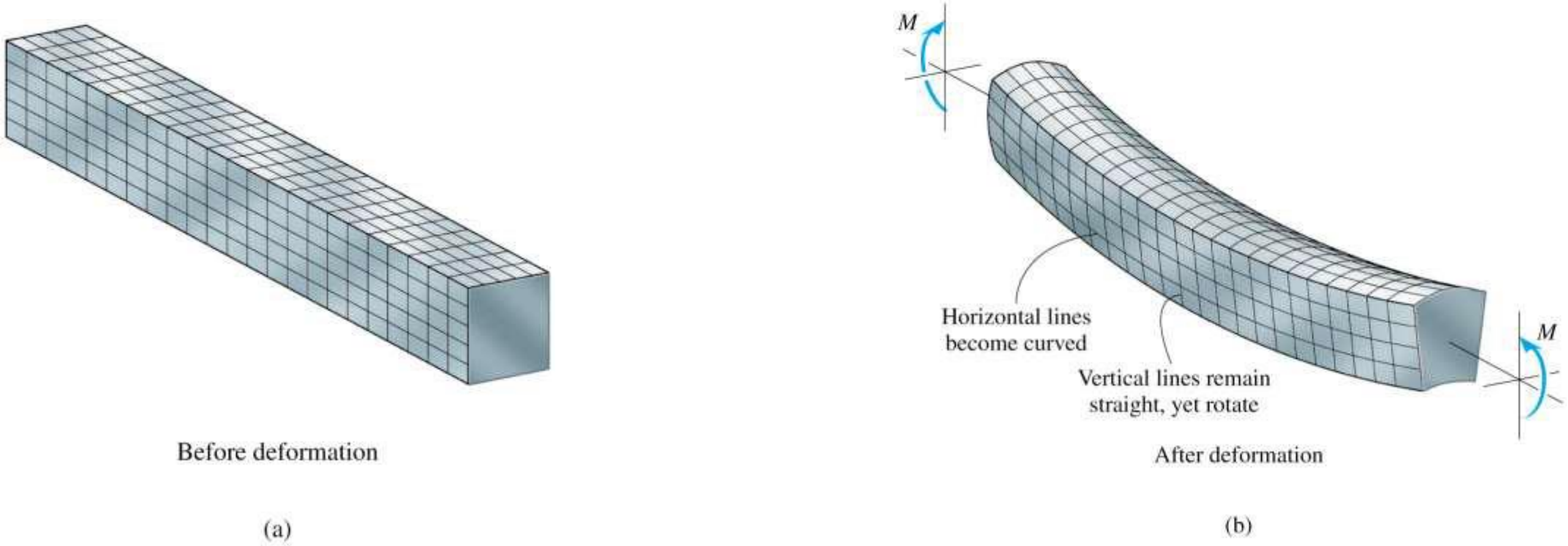
(a)



(b)

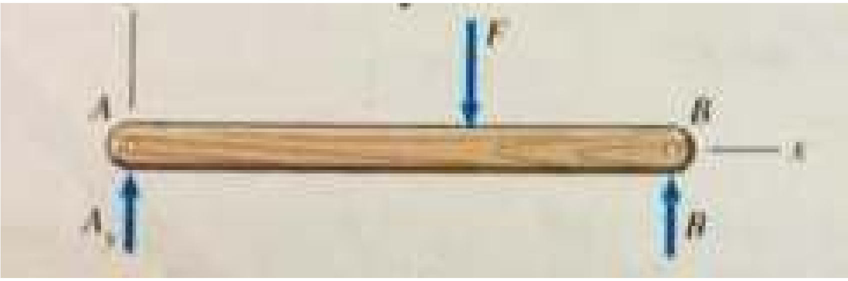
The force **N_B** that acts *perpendicular* to the cross section, is termed the *normal force*. The force **V_B** that is tangent to the cross section is called the *shear force*, and the couple moment **M_B** is referred to as the *bending moment*. The force components prevent the relative translation between the two segments, and the couple moment prevents the relative rotation. According to Newton's third law, these loadings must act in opposite directions on each segment, as shown in Fig. *(b)*. They can be determined by applying the equations of equilibrium to the free-body diagram of either segment.

What is bending?



Approach to determine internal forces and moments for beams in bending (5 steps)

1st step: construct a free body diagram (FBD) for the structure under load

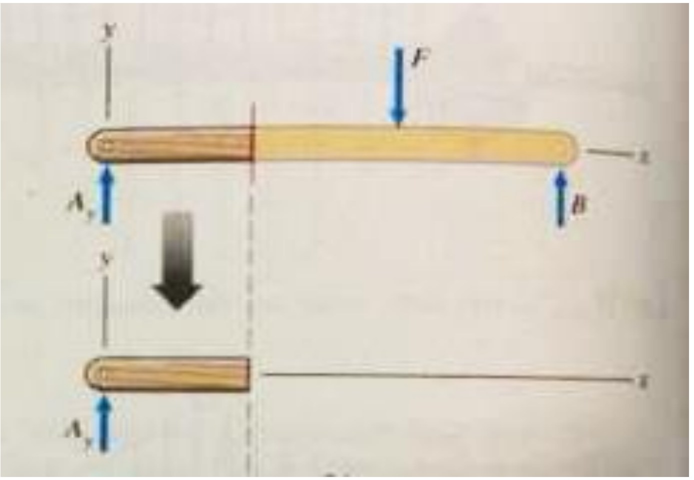


Note: it is clear that A_x=0

2d step: determine the corresponding external reactions

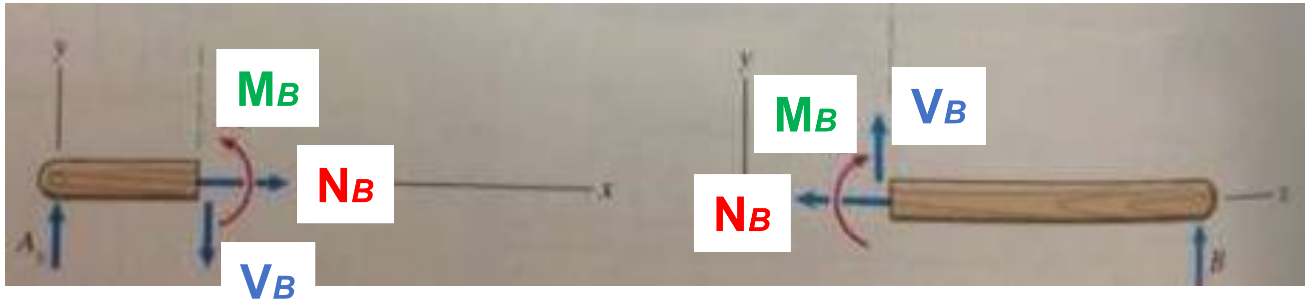
(we choice the left segment, we will only determine A_y)

3d step: make imaginary “cuts” along the beam



Note: cutting before the load

4d step: identify N_B, V_B, M_B for each segment

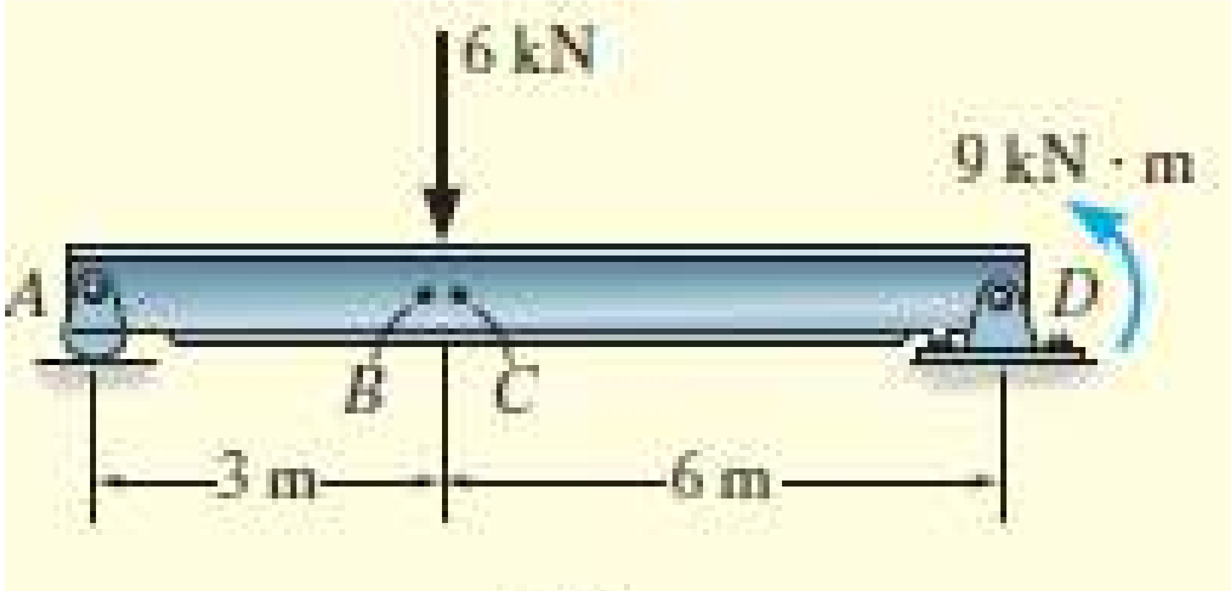


Note: each section becomes a FBD, with its own forces and moments.

5th step: write the equilibrium equations of the left segment

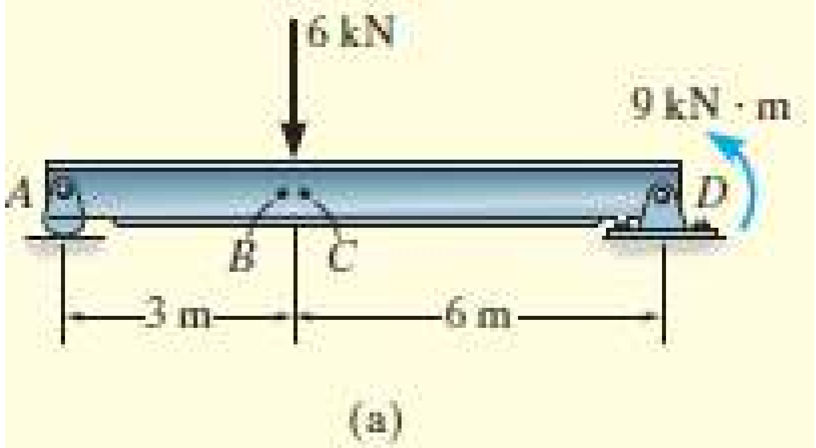
EXAMPLE

Determine the normal force, shear force, and bending moment acting just to the left, point *B*, and just to the right, point *C*, of the 6-kN force on the beam in Fig. (*a*).



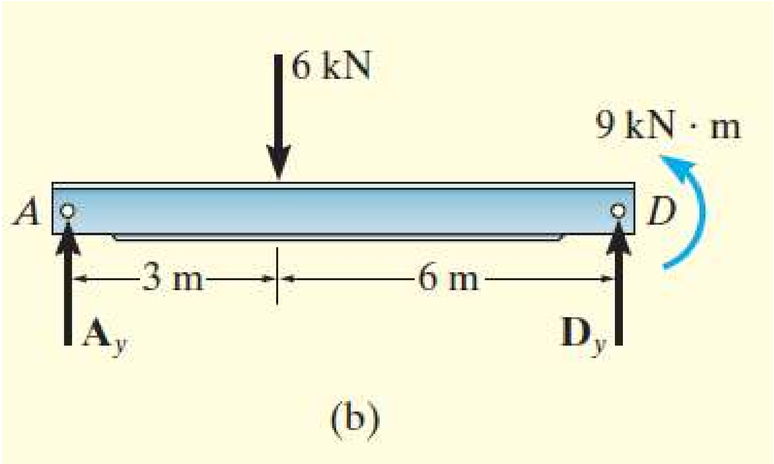
(a)

SOLUTION



(a)

Free body diagram (FBD) of the beam



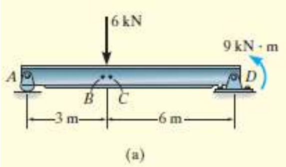
(b)

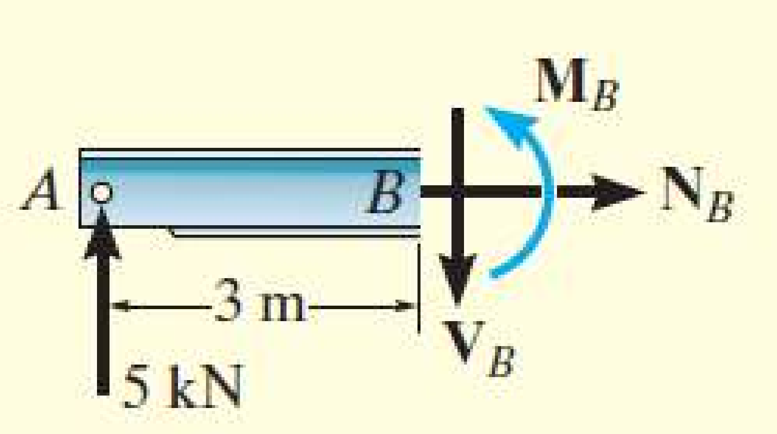
Support Reactions. The free-body diagram of the beam is shown in Fig. 7–4*b*. When determining the *external reactions*, realize that the 9-kN · m couple moment is a free vector and therefore it can be placed *anywhere* on the free-body diagram of the entire beam. Here we will only determine $\mathbf{A}_y$, since the left segments will be used for the analysis.

$$\circlearrowleft + \Sigma M_D = 0; \quad 9\text{ kN}\cdot\text{m} + (6\text{ kN})(6\text{ m}) - A_y(9\text{ m}) = 0$$

$$A_y = 5\text{ kN}$$

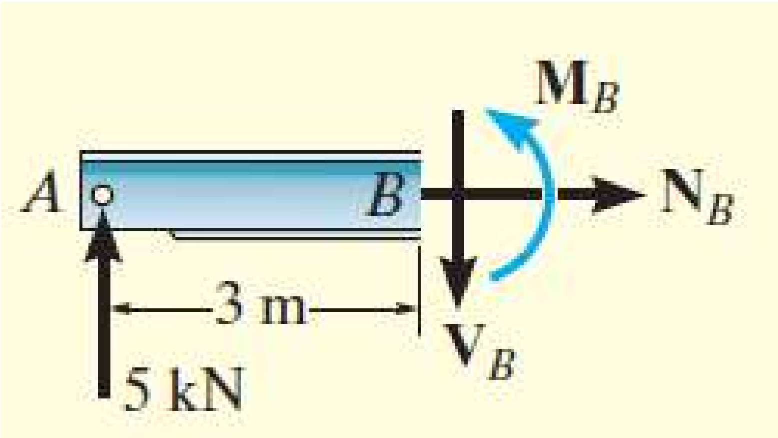
Free body diagram (FBD) of the left segment

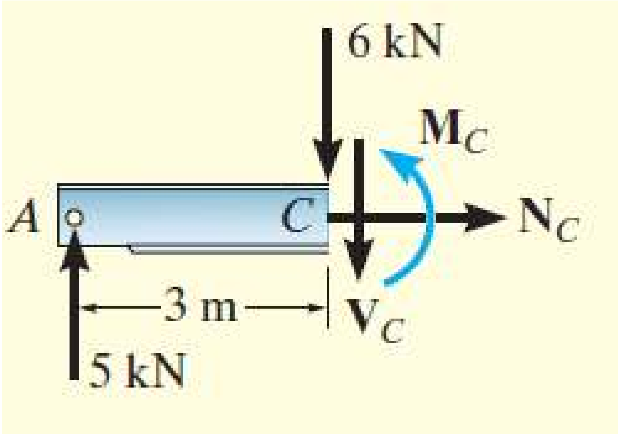




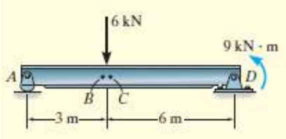
Free-Body Diagrams. The free-body diagrams of the left segments *AB* and *AC* of the beam are shown in Figs. 7–4*c* and 7–4*d*. In this case the 9-kN · m couple moment is *not included* on these diagrams since it must be kept in its *original position* until *after* the section is made and the appropriate segment is isolated.

Equations of equilibrium





(d)



(a)

Equations of Equilibrium.

Segment AB

$\xrightarrow{+} \Sigma F_x = 0;$	$N_B = 0$		*Ans.*
$+\uparrow \Sigma F_y = 0;$	$5\text{ kN} - V_B = 0$	$V_B = 5\text{ kN}$	*Ans.*
$\circlearrowleft + \Sigma M_B = 0;$	$-(5\text{ kN})(3\text{ m}) + M_B = 0$	$M_B = 15\text{ kN}\cdot\text{m}$	*Ans.*

Segment AC

$\xrightarrow{+} \Sigma F_x = 0;$	$N_C = 0$		*Ans.*
$+\uparrow \Sigma F_y = 0;$	$5\text{ kN} - 6\text{ kN} - V_C = 0$	$V_C = -1\text{ kN}$	*Ans.*
$\circlearrowleft + \Sigma M_C = 0;$	$-(5\text{ kN})(3\text{ m}) + M_C = 0$	$M_C = 15\text{ kN}\cdot\text{m}$	*Ans.*

NOTE: The negative sign indicates that **V***C* acts in the opposite sense to that shown on the free-body diagram. Also, the moment arm for the 5-kN force in both cases is approximately 3 m since *B* and *C* are “almost” coincident.