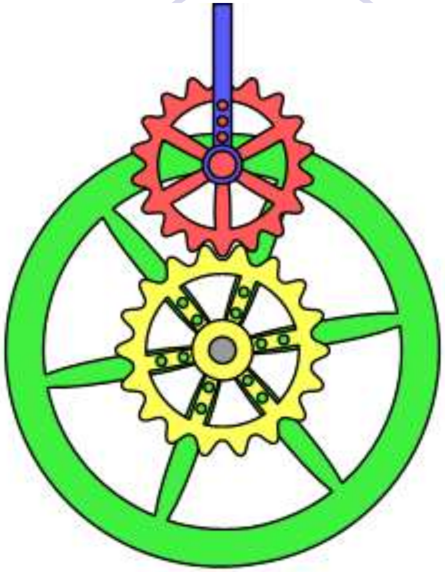
Mechanical Design II ME 352

By S.G.Khan

Chapter 13 Force Analysis

Spur Gear and Helical Gears

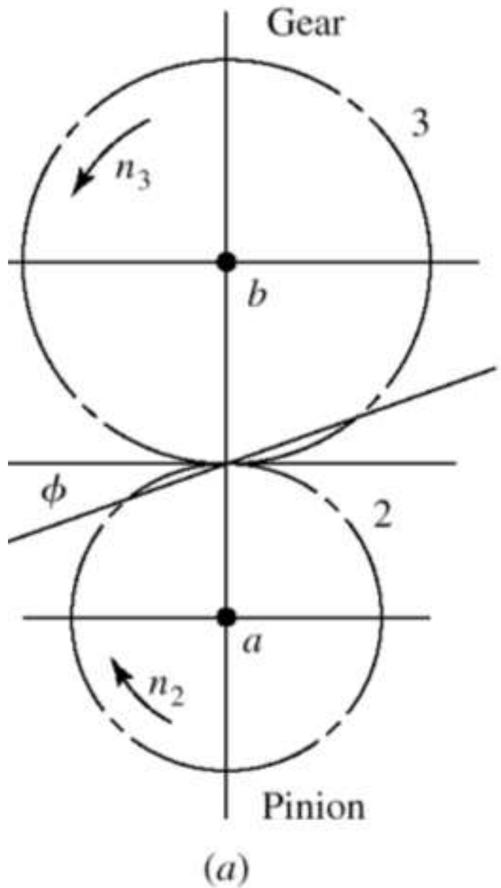


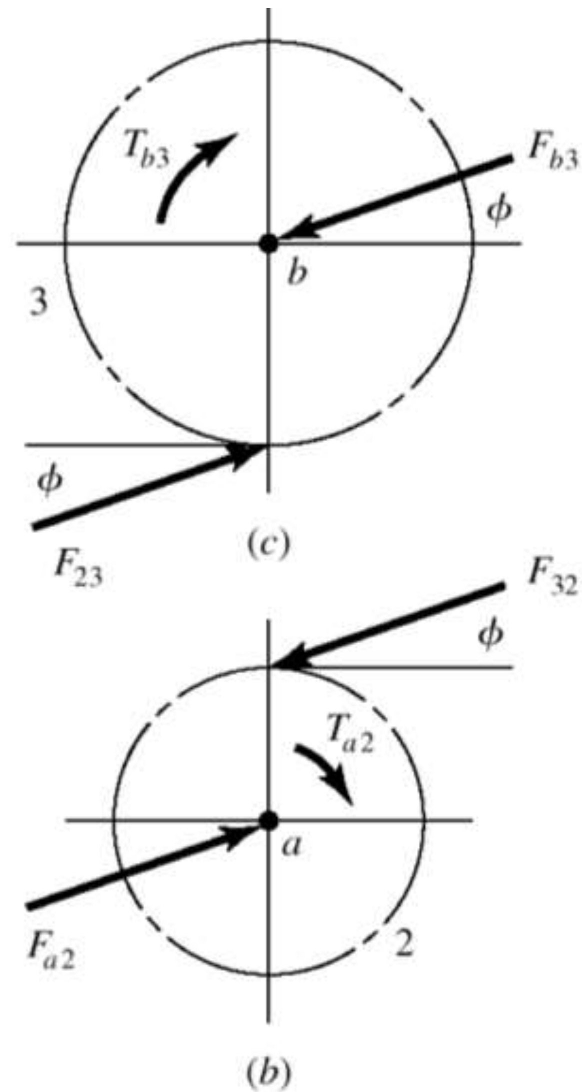
Notation to be used in Force Analysis

- Numerical 1 will be designated to the frame, input gear will thus be designated 2 and then gears will be numbered successively 3, 4 etc, until the last gear in the train is arrived
- There may be several shafts involved, and usually one or two gears are mounted on each shaft as well as other elements; designates the shafts using lowercase letters of the alphabet, a, b, c, etc.
- The forces exerted by gear 2 on 3 will be F_{23}
- Superscript notations will be used to indicate radial and tangential directions and coordinate directions

Force Analysis-Spur Gearing

Free-body diagrams of the forces and moments acting upon two gears of a simple gear train.



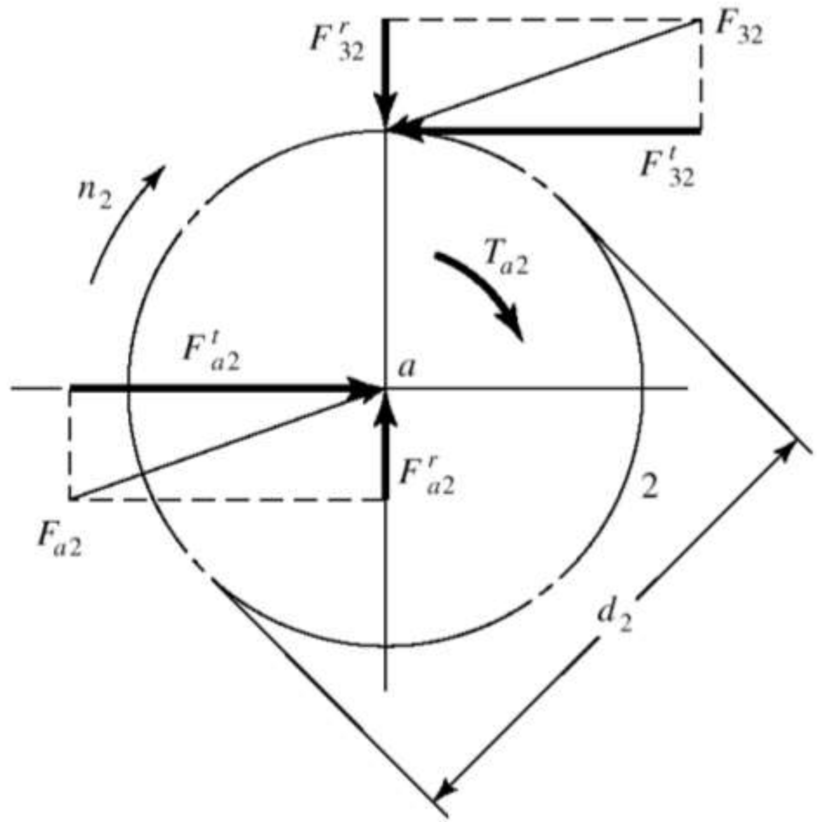


Force Analysis-Spur Gearing

Resolution of gear forces

$$W_t = F_{32}^t$$

$$T = \frac{d}{2} W_t$$



Force Analysis-Spur Gearing

If next we designate the pitch-line velocity be V, where $V = \pi dn/12$ and is in feet per minute, the power H may be obtained from the equation

$$H = \frac{W_t V}{33\,000} \tag{13–26}$$

The corresponding equation in SI is

$$W_t = \frac{60(10)^3 H}{\pi d n} \tag{13–27}$$

where W_t = transmitted load, kN
H = power, kW
d = gear diameter, mm
n = speed, r/min

Example 13.7

Pinion 2 in Fig. 13–34*a* runs at 1750 rev/min and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20° full-depth system and have a module of $m = 2.5$ mm. Draw a free-body diagram of gear 3 and show all the forces that act upon it.

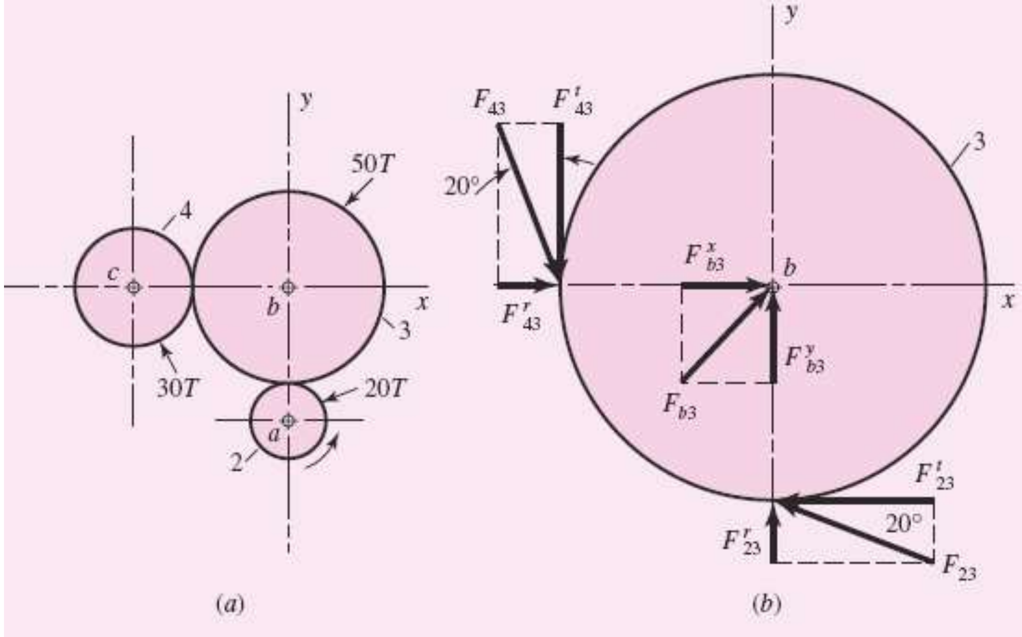


Figure 13–34

A gear train containing an idler gear. (*a*) The gear train. (*b*) Free-body of the idler gear.

Solution

The pitch diameters of gears 2 and 3 are

$$d_2 = N_2 m = 20(2.5) = 50 \text{ mm}$$

$$d_3 = N_3 m = 50(2.5) = 125 \text{ mm}$$

From Eq. (13–36) we find the transmitted load to be

$$W_t = \frac{60\,000 H}{\pi d_2 n} = \frac{60\,000(2.5)}{\pi(50)(1750)} = 0.546 \text{ kN}$$

Thus, the tangential force of gear 2 on gear 3 is $F_{23}^t = 0.546$ kN, as shown in Fig. 13–34*b*. Therefore

$$F_{23}^r = F_{23}^t \tan 20^\circ = (0.546) \tan 20^\circ = 0.199 \text{ kN}$$

and so

$$F_{23} = \frac{F_{23}^t}{\cos 20^\circ} = \frac{0.546}{\cos 20^\circ} = 0.581 \text{ kN}$$

Since gear 3 is an idler, it transmits no power (torque) to its shaft, and so the tangential reaction of gear 4 on gear 3 is also equal to W_t. Therefore

$$F_{43}^t = 0.546\ \text{kN} \qquad F_{43}^r = 0.199\ \text{kN} \qquad F_{43} = 0.581\ \text{kN}$$

and the directions are shown in Fig. 13–34*b*.

The shaft reactions in the *x* and *y* directions are

$$F_{b3}^x = -(F_{23}^t + F_{43}^r) = -(-0.546 + 0.199) = 0.347\ \text{kN}$$

$$F_{b3}^y = -(F_{23}^r + F_{43}^t) = -(0.199 - 0.546) = 0.347\ \text{kN}$$

The resultant shaft reaction is

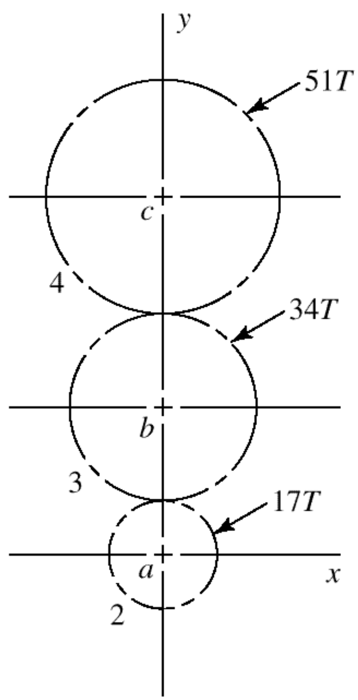
$$F_{b3} = \sqrt{(0.347)^2 + (0.347)^2} = 0.491\ \text{kN}$$

These are shown on the figure.

13–27 Shaft *a* in the figure has a power input of 75 kW at a speed of 1000 r/min in the counterclockwise direction. The gears have a module of 5 mm and a 20° pressure angle. Gear 3 is an idler.

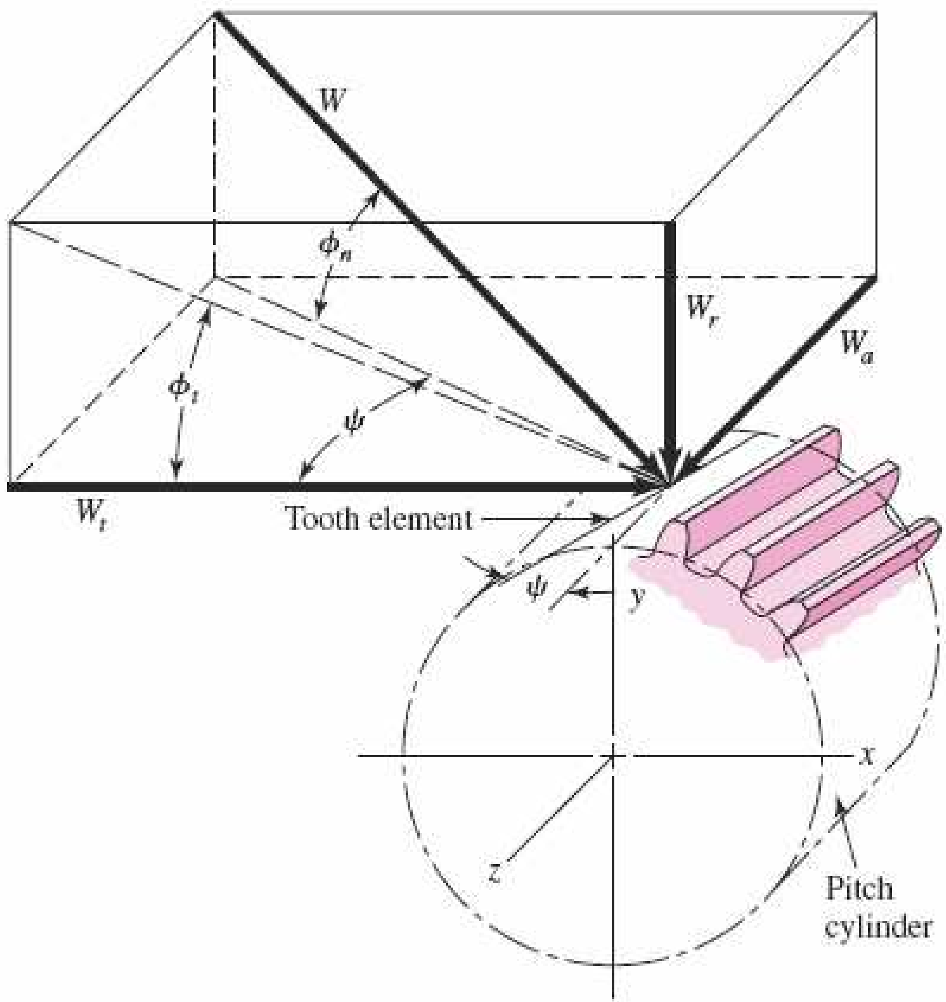
(*a*) Find the force F_{3b} that gear 3 exerts against shaft *b*.

(*b*) Find the torque T_{4c} that gear 4 exerts on shaft *c*.



n 13–27

Force Analysis – Helical Gearing



$$W_r = W \sin \phi_n$$

$$W_t = W \cos \phi_n \cos \psi$$

$$W_a = W \cos \phi_n \sin \psi$$

Force Analysis – Helical Gearing

Figure 13–37 is a three-dimensional view of the forces acting against a helical-gear tooth. The point of application of the forces is in the pitch plane and in the center of the gear face. From the geometry of the figure, the three components of the total (normal) tooth force W are

$$W_r = W \sin \phi_n$$
$$W_t = W \cos \phi_n \cos \psi \qquad (13\text{–}39)$$
$$W_a = W \cos \phi_n \sin \psi$$

where W = total force

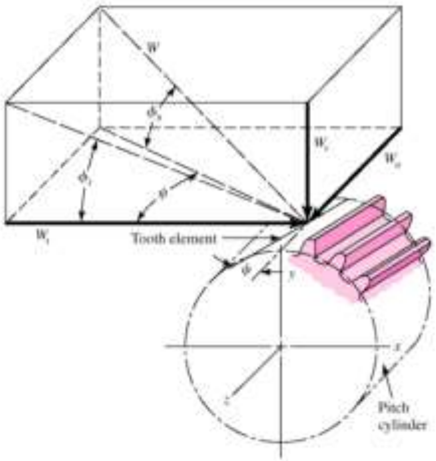
W_r = radial component

W_t = tangential component, also called transmitted load

W_a = axial component, also called thrust load

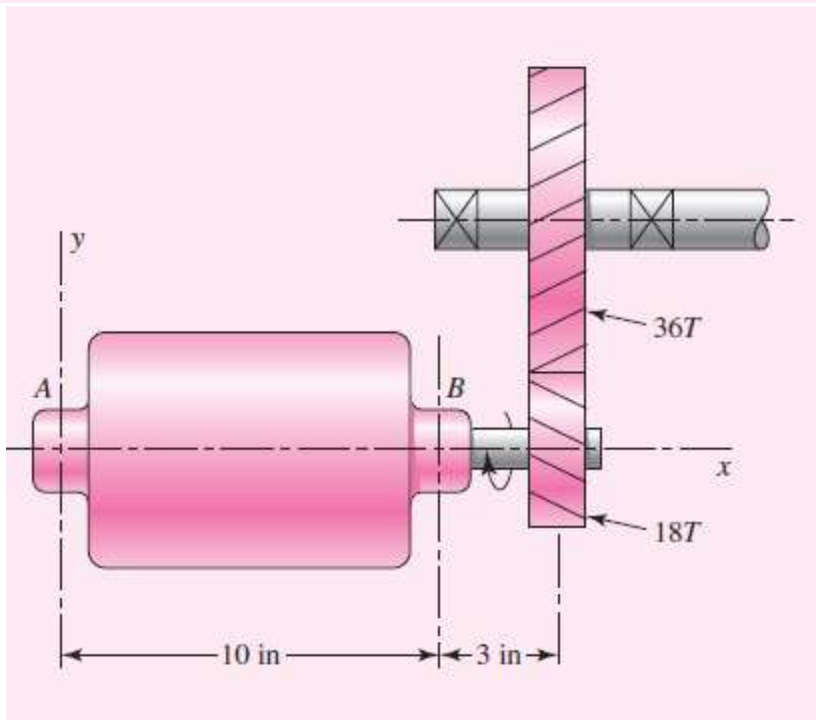
Usually W_t is given and the other forces are desired. In this case, it is not difficult tc discover that

$$W_r = W_t \tan \phi_t$$
$$W_a = W_t \tan \psi \qquad (13\text{–}40$$
$$W = \frac{W_t}{\cos \phi_n \cos \psi}$$

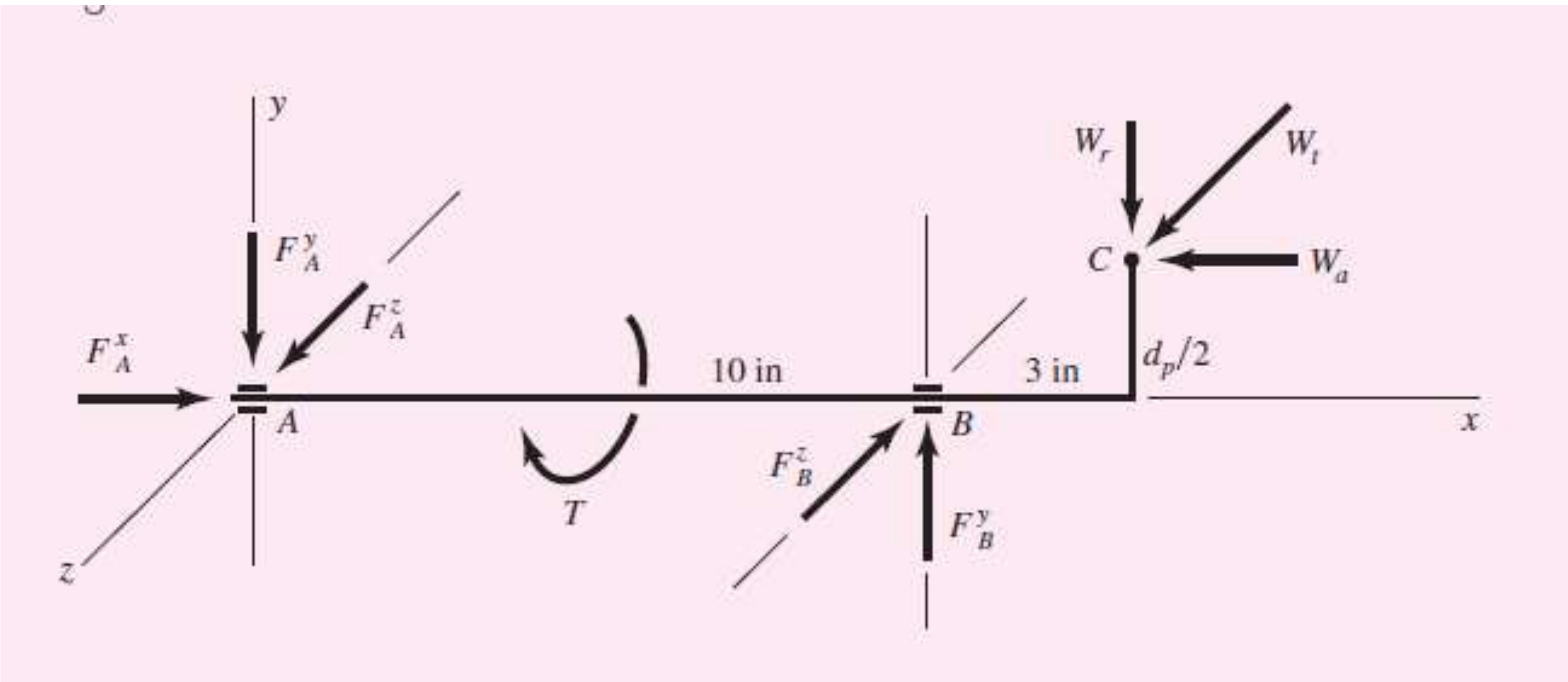


Example 13-9

In Fig. 13–38 a 1-hp electric motor runs at 1800 rev/min in the clockwise direction, as viewed from the positive *x* axis. Keyed to the motor shaft is an 18-tooth helical pinion having a normal pressure angle of 20°, a helix angle of 30°, and a normal diametral pitch of 12 teeth/in. The hand of the helix is shown in the figure. Make a three-dimensional sketch of the motor shaft and pinion, and show the forces acting on the pinion and the bearing reactions at *A* and *B*. The thrust should be taken out at *A*.



Free Body Diagram



Solution

From Eq. (13–19) we find

$$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.8^\circ$$

Also, $P_t = P_n \cos\psi = 12\cos 30^\circ = 10.39$ teeth/in. Therefore the pitch diameter of the pinion is $d_p = 18/10.39 = 1.732$ in. The pitch-line velocity is

$$V = \frac{\pi d n}{12} = \frac{\pi(1.732)(1800)}{12} = 816\ \text{ft/min}$$

The transmitted load is

$$W_t = \frac{33\,000H}{V} = \frac{(33\,000)(1)}{816} = 40.4\ \text{lbf}$$

From Eq. (13–40) we find

$$W_r = W_t \tan \phi_t = (40.4) \tan 22.8^\circ = 17.0 \text{ lbf}$$

$$W_a = W_t \tan \psi = (40.4) \tan 30^\circ = 23.3 \text{ lbf}$$

$$W = \frac{W_t}{\cos \phi_n \cos \psi} = \frac{40.4}{\cos 20^\circ \cos 30^\circ} = 49.6 \text{ lbf}$$

These three forces, W_r in the $-y$ direction, W_a in the $-x$ direction, and W_t in the $+z$ direction, are shown acting at point C in Fig. 13–39. We assume bearing reactions at A and B as shown. Then $F_A^x = W_a = 23.3$ lbf. Taking moments about the z axis,

$$-(17.0)(13) + (23.3)\left(\frac{1.732}{2}\right) + 10F_B^y = 0$$

or $F_B^y = 20.1$ lbf. Summing forces in the y direction then gives $F_A^y = 3.1$ lbf. Taking moments about the y axis, next

$$10F_B^z - (40.4)(13) = 0$$

or $F_B^z = 52.5$ lbf. Summing forces in the z direction and solving gives $F_A^z = 12.1$ lbf. Also, the torque is $T = W_t d_p/2 = (40.4)(1.732/2) = 35$ lbf · in.

For comparison, solve the problem again using vectors. The force at C is

$$\mathbf{W} = -23.3\mathbf{i} - 17.0\mathbf{j} + 40.4\mathbf{k} \text{ lbf}$$

Position vectors to B and C from origin A are

$$\mathbf{R}_B = 10\,\mathbf{i} \qquad \mathbf{R}_C = 13\mathbf{i} + 0.866\mathbf{j}$$

Taking moments about A, we have

$$\mathbf{R}_B \times \mathbf{F}_B + \mathbf{T} + \mathbf{R}_C \times \mathbf{W} = \mathbf{0}$$

Using the directions assumed in Fig. 13–39 and substituting values gives

$$10\,\mathbf{i} \times \left(F_B^y \mathbf{j} - F_B^z \mathbf{k}\right) - T\mathbf{i} + (13\mathbf{i} + 0.866\mathbf{j}) \times (-23.3\mathbf{i} - 17.0\mathbf{j} + 40.4\mathbf{k}) = \mathbf{0}$$

When the cross products are formed, we get

$$\left(10F_B^y \mathbf{k} + 10F_B^z \mathbf{j}\right) - T\mathbf{i} + (35\mathbf{i} - 525\mathbf{j} - 201\mathbf{k}) = \mathbf{0}$$

whence $T = 35\,\text{lbf} \cdot \text{in}$, $F_B^y = 20.1\,\text{lbf}$, and $F_B^z = 52.5\,\text{lbf}$.

Next,

$$\mathbf{F}_A = -\mathbf{F}_B - \mathbf{W}, \text{ and so } \mathbf{F}_A = 23.3\mathbf{i} - 3.1\mathbf{j} + 12.1\mathbf{k}\ \text{lbf}.$$