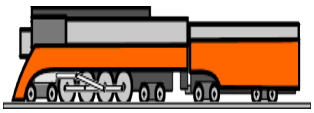
Umm Al-Qura University

College of Engineering & Islamic Architecture

Mechanical Engineering Department

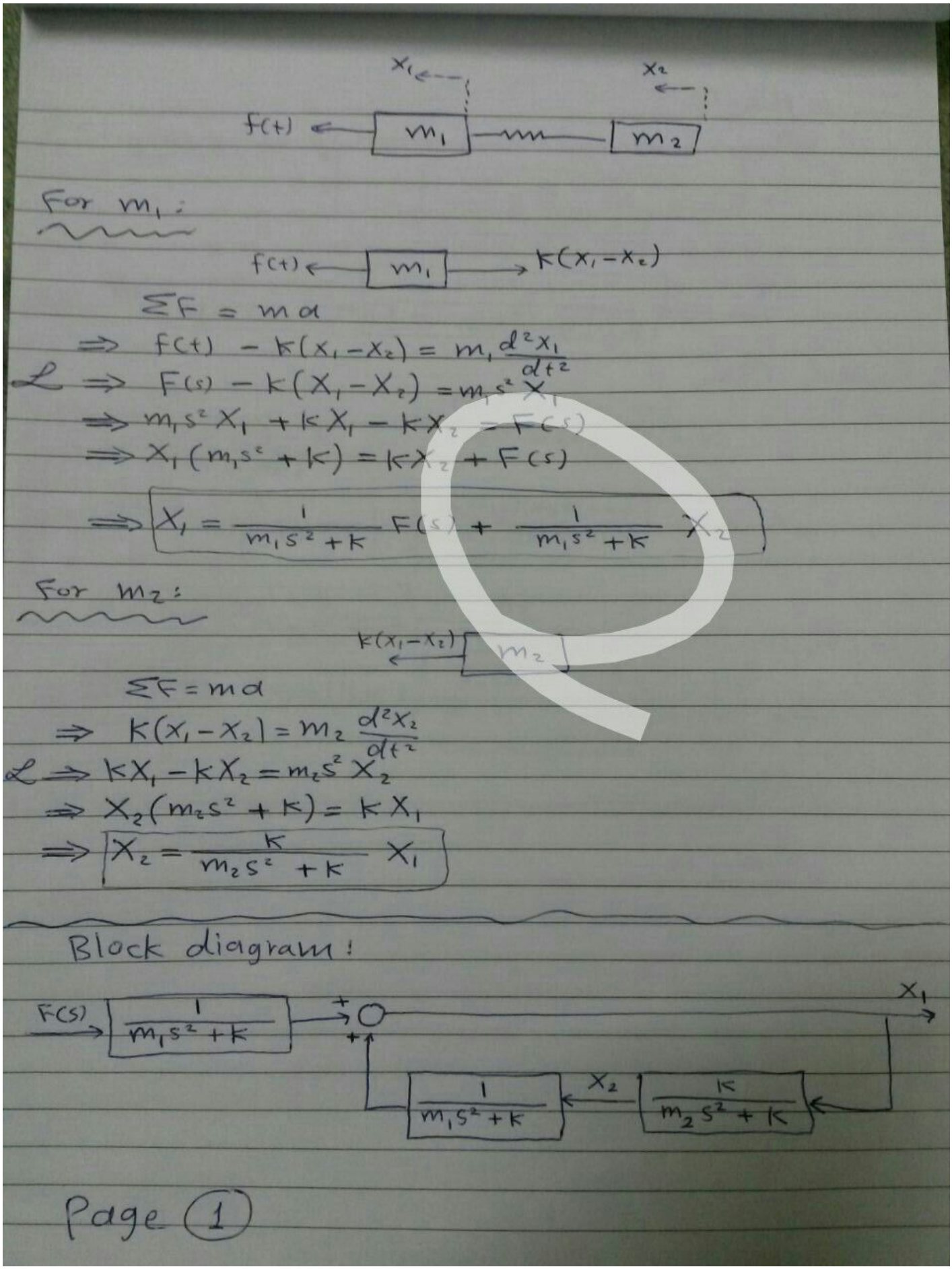
Course #: 804465–Automatic Control Lab # 3 - Assignment

Student's name:_____ **ID #:**_____ **Group #:**___



Consider a toy train consisting of an engine and a car. Assume that the train only travels in one dimension (along the track). The mass of the engine and the car will be represented by M_1 and M_2, respectively and Engine is moving with a Force F_1 and Car F_2 respectively. Furthermore, the engine and car are connected via a spring with stiffness k. Determine the following:

a. Free Body Diagram.
b. Write force equation of the system.
c. Determine Transfer Function of the system with F_1 as input and X_1 as output.
d. Draw block-diagram of the complete system.
e. Build Simulink Model of the system using differential equations and plot displacement (X_1) of the engine after applying step input as F_1.



For m_1:

$$\Sigma F = ma$$

$$\Rightarrow f(t) - K(x_1 - x_2) = m_1 \frac{d^2 x_1}{dt^2}$$

$$\mathcal{L} \Rightarrow F(s) - K(X_1 - X_2) = m_1 s^2 X_1$$

$$\Rightarrow m_1 s^2 X_1 + K X_1 - K X_2 = F(s)$$

$$\Rightarrow X_1 (m_1 s^2 + K) = K X_2 + F(s)$$

$$\Rightarrow \boxed{X_1 = \frac{1}{m_1 s^2 + K} F(s) + \frac{1}{m_1 s^2 + K} X_2}$$

For m_2:

$$\Sigma F = ma$$

$$\Rightarrow K(x_1 - x_2) = m_2 \frac{d^2 x_2}{dt^2}$$

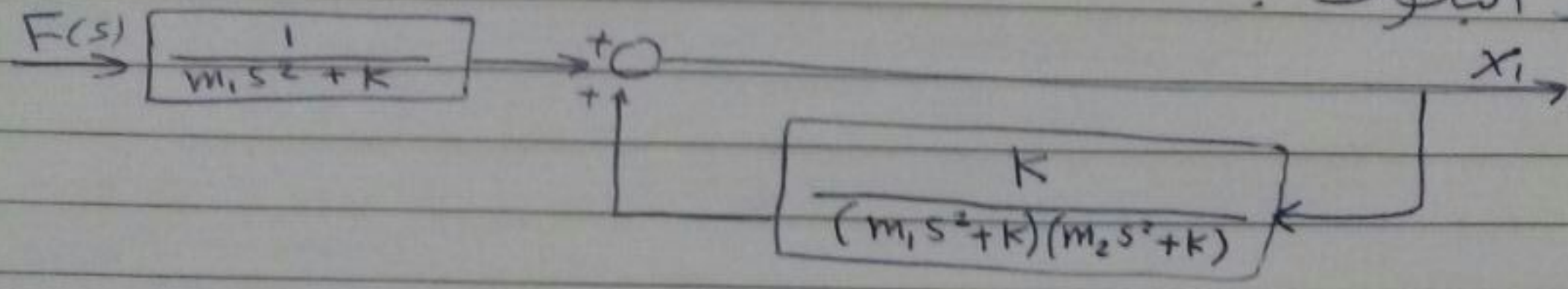
$$\mathcal{L} \Rightarrow K X_1 - K X_2 = m_2 s^2 X_2$$

$$\Rightarrow X_2 (m_2 s^2 + K) = K X_1$$

$$\Rightarrow \boxed{X_2 = \frac{K}{m_2 s^2 + K} X_1}$$

Block diagram:

نبسط البلوك :



$$\Rightarrow \quad F(s) \longrightarrow \boxed{\frac{1}{m_1 s^2 + k}} \longrightarrow \boxed{\frac{1}{1 - \frac{K}{(m_1 s^2+k)(m_2 s^2+k)}}} \longrightarrow x_1$$

نبسط الكسر :

$$\frac{1}{\frac{(m_1 s^2+k)(m_2 s^2+k) - K}{(m_1 s^2+k)(m_2 s^2+k)}} = \frac{(m_1 s^2+k)(m_2 s^2+k)}{(m_1 s^2+k)(m_2 s^2+k) - K}$$

$$\Rightarrow \quad F(s) \longrightarrow \boxed{\frac{1}{m_1 s^2 + k}} \longrightarrow \boxed{\frac{(m_1 s^2+k)(m_2 s^2+k)}{(m_1 s^2+k)(m_2 s^2+k) - K}} \longrightarrow x_2$$

$$\Rightarrow \quad F(s) \longrightarrow \boxed{\frac{m_2 s^2 + k}{(m_1 s^2+k)(m_2 s^2+k) - K}} \longrightarrow x_2$$

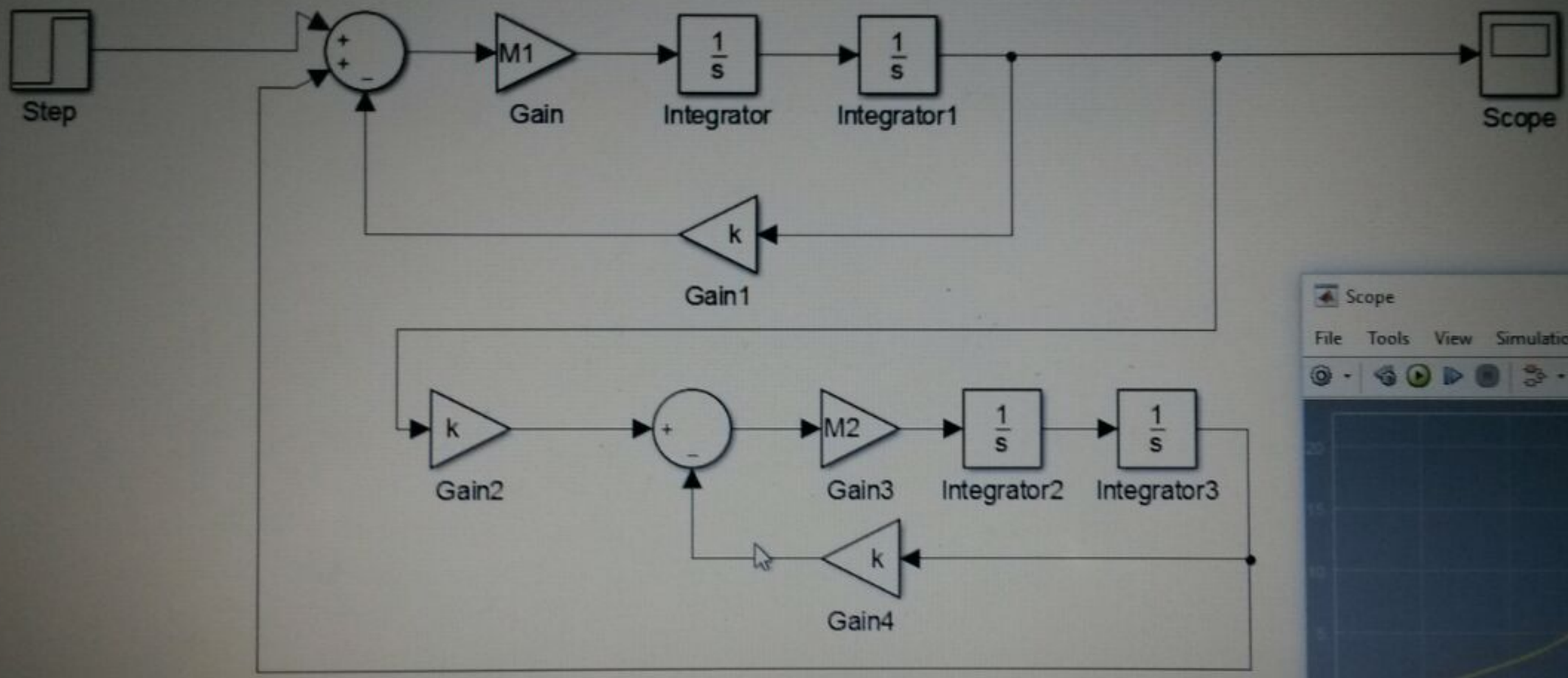
$$\Rightarrow \text{Transfer function} : \quad \frac{X_2(s)}{F(s)} = \frac{m_2 s^2 + k}{(m_1 s^2+k)(m_2 s^2+k) - K}$$

- Simulink student use

View Display Diagram Simulation Analysis Code Tools Help

10.0 Normal

sed



Scope

File Tools View Simulation Help

Ready T=10.000

175%

autoload

ENG ...Using Si ...Laplace T ...ESSMech Scope ... * untitled ... Simulink ... MATLAB شروحاتي