Taibah University – Yanbu Branch

College of Engineering at Yanbu

Mechanical Engineering Department

ME 341
Mechanics of Machines

Dr. S G Khan

engr_ghani@hotmail.com

Degree of Freedom or Mobility

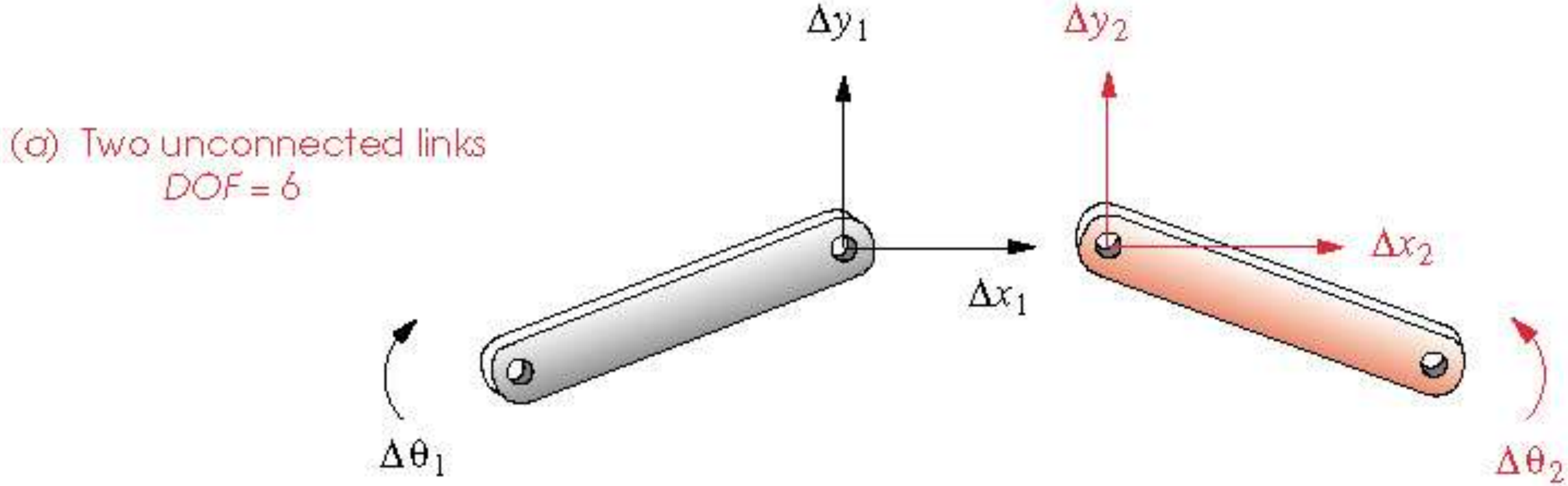
- The number of inputs that need to be provided in order to create a predictable output

Also

- The number of independent coordinates required to define its position

Degree of freedom in Planar Mechanism

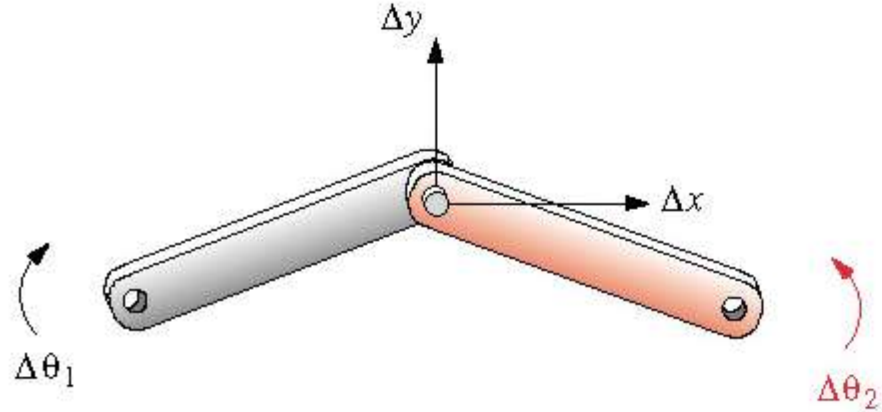
- For determining DOF of a system, one must account for the number of links and joints
- The DOF of any assembly of links can be predicted from an investigation of the *Gruebler condition*
- Any link in a plane has 3 DOF
- A system of L unconnected links have $3L$ DOF


(a) Two unconnected links
DOF = 6
Δy1
Δy2
Δx1
Δx2
Δθ1
Δθ2

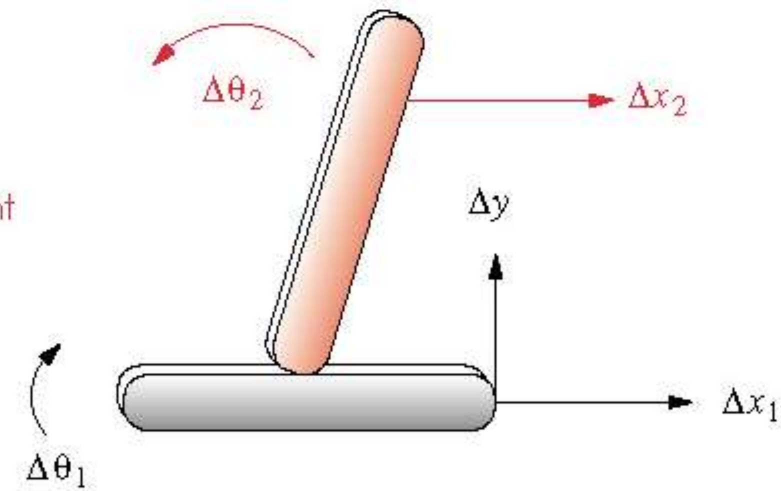
DOF

- Connecting the links by a full joint, Δy_1 and Δy_2 are combined as Δy, and Δx_1 and Δx_2 are combined as Δx. this removed two DOF, leaving four DOF
- Half joint removes only one DOF from the system, leaving the system of two links connected by a half joint with a total of five DOF.

(*b*) Connected by a full joint
DOF = 4



(*c*) Connected by a roll-slide (half) joint
DOF = 5



DOF

- When any link is grounded or attached to the reference frame, all three of its DOF will be removed
- This reasoning lead to Gruebler's equation

$$M = 3L - 2J - 3G$$

Where: M = degree of freedom or mobility

L = number of links

J = number of joints

G = number of grounded links

DOF

- In any real mechanism, even if more than one link of the kinematic chain is grounded, the net effect will be to create one larger, higher-order ground link, as there can be only one ground plane.
- Thus G is always one, and Gruebler's equation becomes:

$$M = 3\,(L - 1) - 2\,J$$

DOF

- The value of J must reflect the value of all joints in the mechanism. That is, half joints count as ½ because they only remove one DOF. It is less confusing to use Kutzbach’s modification of Gruebler’s question in this form:

  $M = 3\,(L - 1) - 2J_1 - J_2$

Where: M = mobility

L = number of links

J_1 = number of 1 DOF (full) joints

J_2 = number of 2 DOF (half) joints

DOF

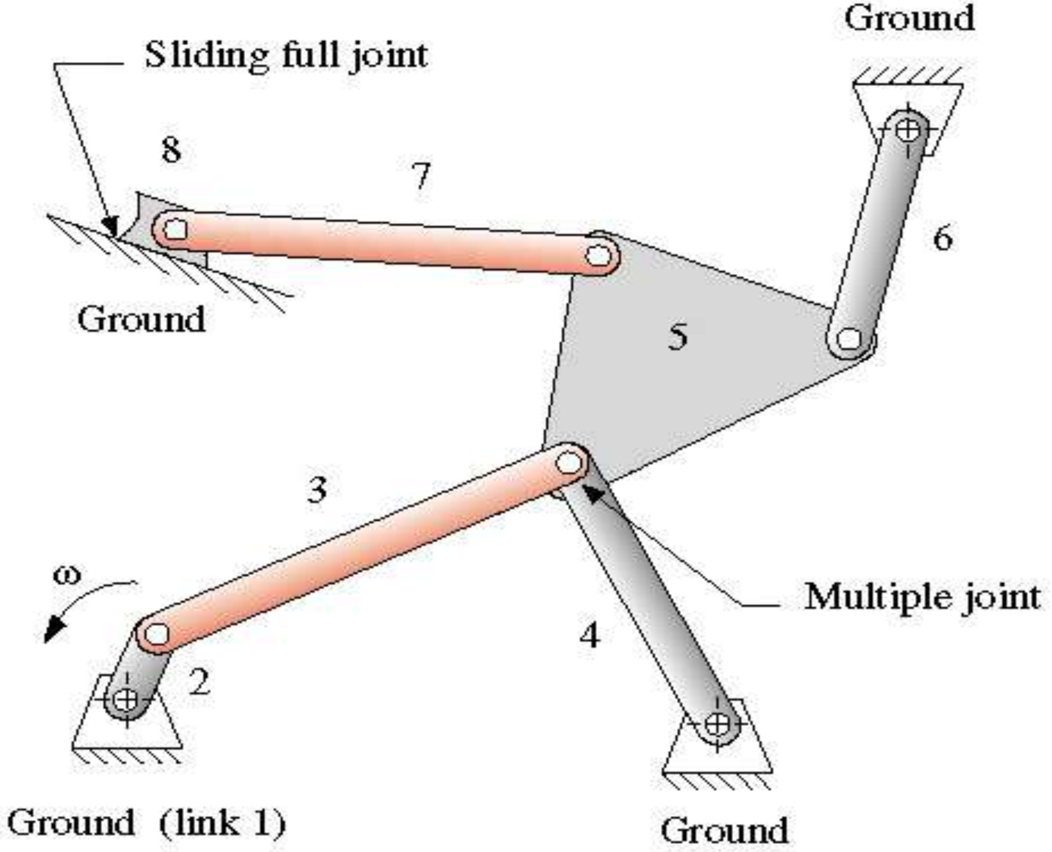
- The value of J_1 and J_2 in these equations must still be carefully determined to account for all full, half and multiple joints in any linkage.
- Multiple joints count as one less than the number of links joined and add to the “full” (J_1) category
- The DOF of any proposed mechanism can be quickly ascertained from this expression before investing any time in more detailed design
- It is interesting to note that this equation has no information in it about link size or shapes, only their quantity

Determine DOF ?

Note:
There are no roll-slide (half) joints in this linkage

$L = 8, \quad J = 10$

$DOF = 1$

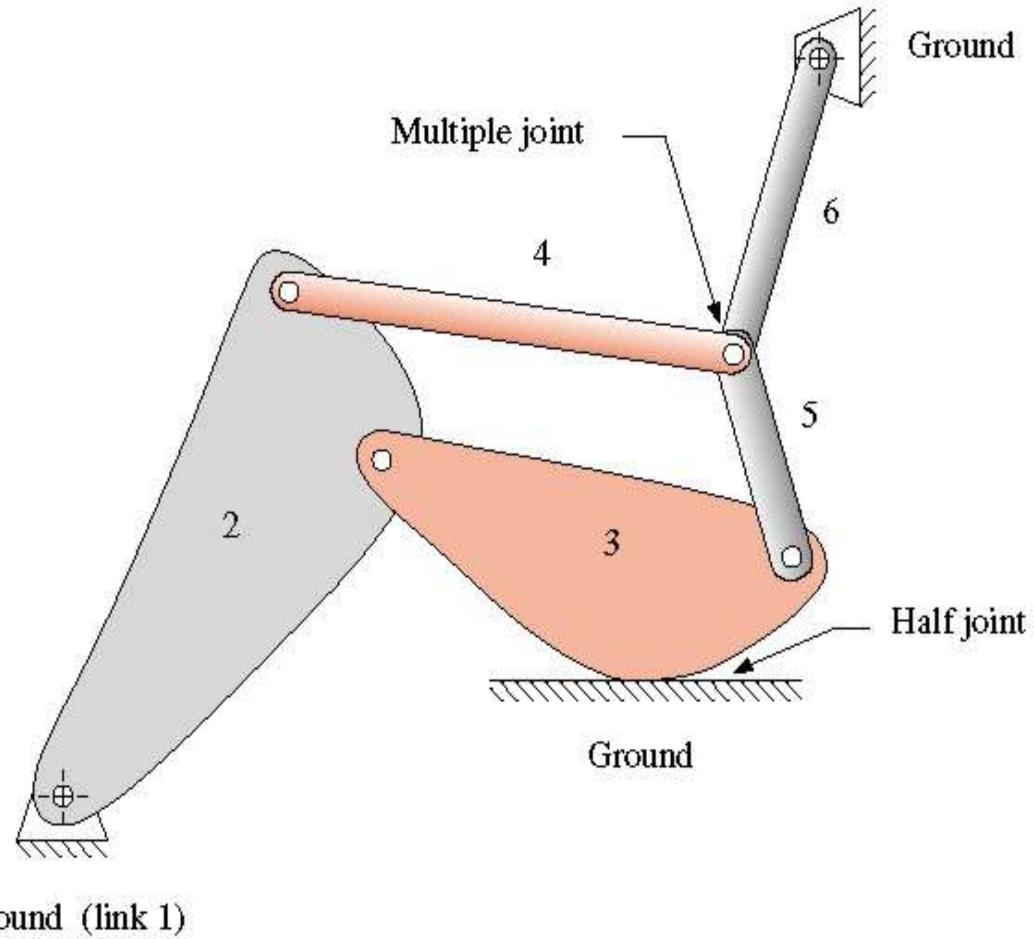


(a) Linkage with full and multiple joints

Determine DOF ?

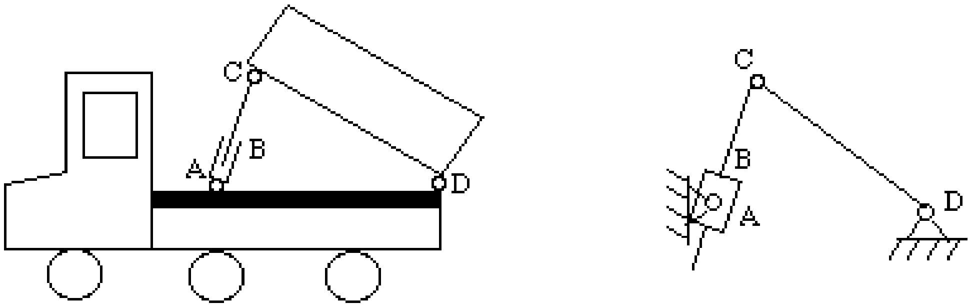
$L = 6, \quad J = 7.5$

$DOF = 0$



Determine DOF?

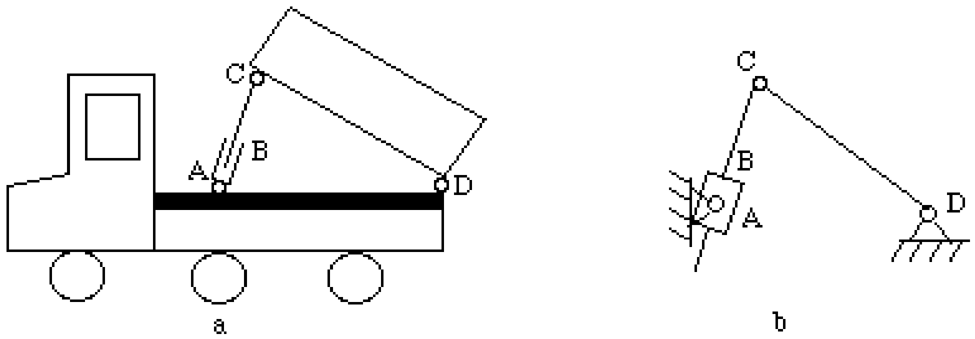
- Calculate the degrees of freedom of the mechanisms (Dump truck) shown in Figure (b).
- Figure (a) is an application of the mechanism.
- Hint: $M = 3(L-1) - 2J_1 - J_2$



a b

Determine DOF?

- Calculate the degrees of freedom of the mechanisms (Dump truck) shown in Figure (b).
- Figure (a) is an application of the mechanism.
- Hint: $M = 3(L-1) - 2J_1 - J_2$



L= 4, J_1 = 4 (at A, B, C, D), J_2 = 0

$$m = 3(4-1) - 2 \times 4 - 1 \times 0 = 1$$

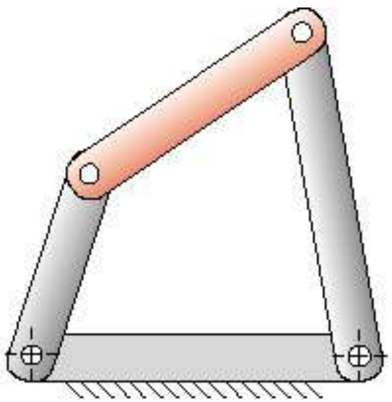
m = 1

DOF in Spatial (3D) Mechanisms

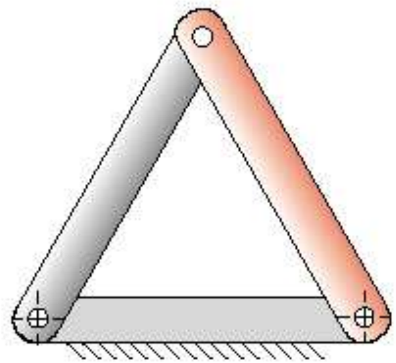
- The approach used to determine the mobility of a planar mechanism can be easily extended to three dimensions
- Each unconnected link in three-space has 6 DOF, and any one of the six lower pairs can be used to connect them, as can higher pairs with more freedom
- A one-freedom removes 5 DOF, a two-freedom joint removed 4 DOF
- Grounding a link removes 6 DOF
- We will limit our study to 2-D (Planar) mechanisms

Mechanisms and Structures

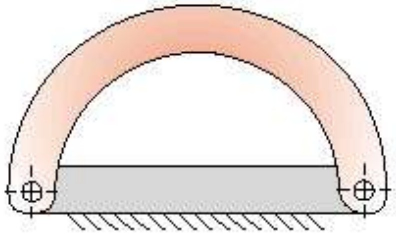
- If the DOF is positive, it will be a mechanism, and the links will have relative motion
- If the DOF is exactly zero, then it will be a structure, and no motion is possible
- If the DOF is negative, then it will be a preloaded structure, which means that no motion is possible and some stresses may also be present at the time of assembly



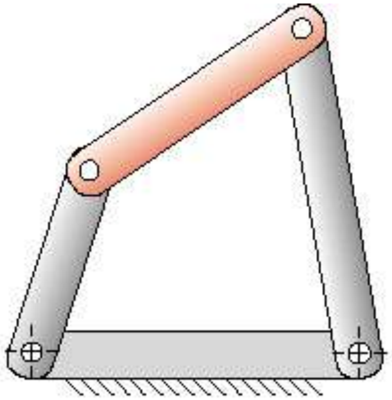
Mechanism?
Structure??



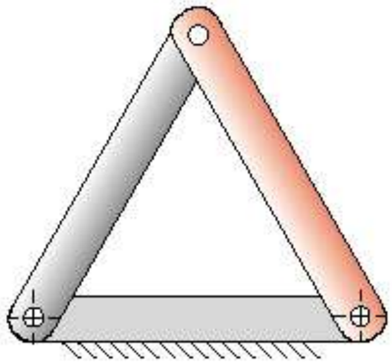
Stucture with no motion possible??



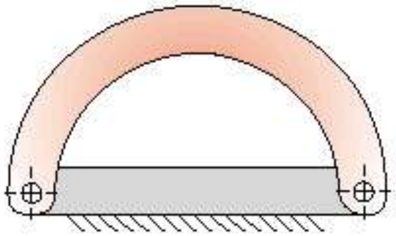
Preloaded



(a) Mechanism—*DOF* = +1



(b) Structure—*DOF* = 0



(c) Preloaded structure—*DOF* = −1

Links, Joints, DOF?

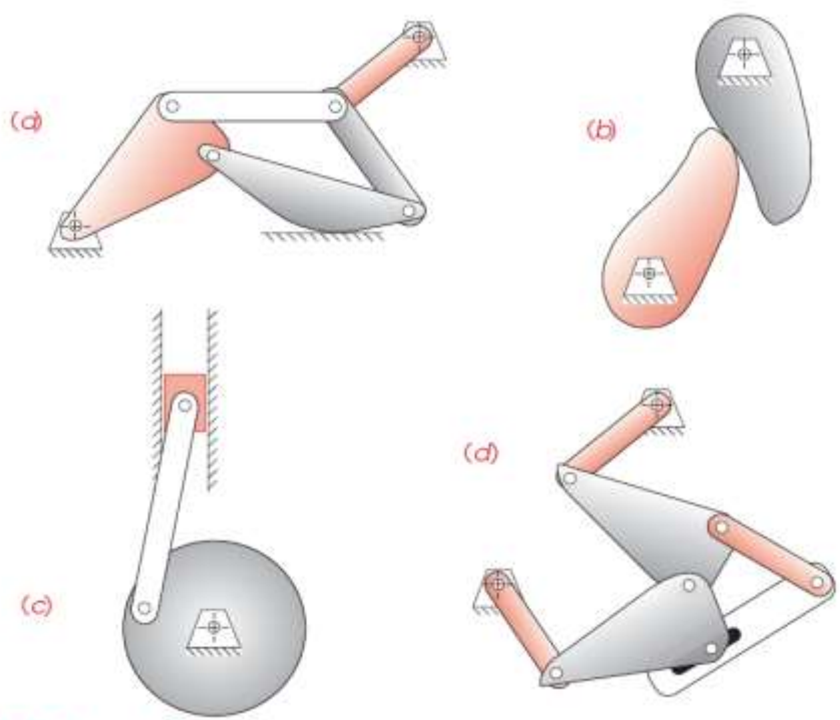


FIGURE P2-1 part 1

Number synthesis

- The determination of the number and order of links and joints necessary to produce motion of a particular DOF
- Link order in this context refers to the number of nodes per link, i.e., binary, ternary, quaternary, etc.
- The value of number synthesis is to allow the exhaustive determination of all possible combinations of link that will yield any chosen DOF

Hypothesis

- If all joints are full joints, an odd number of DOF requires an even number of links and vice versa

Proof: **Given:** All even integers can be denoted by $2m$ or by $2n$, and all odd integers can be denoted by $2m - 1$ or by $2n - 1$, where n and m are any positive integers. The number of joints must be a positive integer.

Let : L = number of links, J = number of joints, and $M = DOF = 2m$ (i.e., all even numbers)

Then: rewriting Gruebler's equation 2.1b to solve for J,

$$J = \frac{3}{2}(L-1) - \frac{M}{2} \tag{2.3a}$$

Try: Substituting $M = 2m$, and $L = 2n$ (i.e., both any even numbers):

$$J = 3n - m - \frac{3}{2} \tag{2.3b}$$

This cannot result in J being a positive integer as required.

Try: $M = 2m - 1$ and $L = 2n - 1$ (i.e., both any odd numbers):

$$J = 3n - m - \frac{5}{2} \tag{2.3c}$$

This also cannot result in J being a positive integer as required.

Try: $M = 2m - 1$, and $L = 2n$ (i.e., odd-even):

$$J = 3n - m - 2 \tag{2.3d}$$

This is a positive integer for $m \geq 1$ and $n \geq 2$.

Try: $M = 2m$ and $L = 2n - 1$ (i.e., even-odd):

$$J = 3n - m - 3 \tag{2.3e}$$

This is a positive integer for $m \geq 1$ and $n \geq 2$.

So, for our example of one-*DOF* mechanisms, we can only consider combinations of 2, 4, 6, 8 . . . links.

Link combinations for one DOF

- Deriving all possible link combinations for one DOF, including sets of up to eight links, and link orders up to and including hexagonal links
- For simplicity assume that the links will be connected with only single, full rotating joints (i.e., a pin connecting two links)

Link combinations for one DOF

- From the hypothesis proved, it is evident that for one-DOF mechanisms, only combinations of 2, 4, 6, 8, … links can be considered. Let

  B = number of binary links

  T = number of ternary links

  Q = number of quaternaries

  P = number of pentagonals

  H = number of hexagonals

The total number of links in any mechanism will be:

$$L = B + T + Q + P + H + \ldots\ldots\ldots$$

Link combinations for one DOF

- Since two link nodes are needed to make one joint:

  J = nodes / 2

- And nodes = order of link X no. of links of that order
- Then $$J = \frac{(2B + 3T + 4Q + 5P + 6H + ...)}{2}$$
- Gruebler equation will become

$$M = 3(B + T + Q + P + H - 1) - 2\left(\frac{2B + 3T + 4Q + 5P + 6H}{2}\right)$$

$$M = B - Q - 2P - 3H - 3$$

Link combinations for one DOF

- Note that ternary link is missing from final equation and it shows that DOF is independent of the number of ternary links in the mechanism
- In order to determine all possible combinations of links for a particular DOF, lets combine the equation of J with its previous value from gruebler's equation

$$\frac{3}{2}(L-1)-\frac{M}{2}=\frac{(2B+3T+4Q+5P+6H)}{2}$$

$$3L-3-M=2B+3T+4Q+5P+6H$$

- Combining with L = B+T+Q+P+H+…. To eliminate B we get

$$L-3-M=T+2Q+3P+4H$$

- Now solve this equation and L=B+T+Q+P+H+….. Simultaneously to determine all possible combinations of links for DOF =1, up to eight links

Link combinations for one DOF

- **Case 1**
  $L = 2$
  $L\text{-}4=T+2Q+3P+4H=\text{-}2$

This requires a negative number of links, so $L = 2$ is impossible

- **Case 2**
  $L = 4$
  $L\text{-}4=T+2Q+3P+4H=0$, so : $T = Q = P = H = 0$
  $L = B + 0 = 4$; $B = 4$

The simplest one-DOF linkage is four binary link- the fourbar linkage

- **Case 3**
  $L = 6$
  $L\text{-}4=T+2Q+3P+4H=2$; so: $P = H = 0$
  T may be 0 or 2; Q may only be 0 or 1
  If $Q = 0$, then T must be 2, and
  $L = B + 2T + 0Q = 6$; $B = 4$, $T= 2$
  If Q= 1, then T must be 0 and:
  $L = B + 0T+ 1Q = 6$; $B = 5$, $Q = 1$

There are then two possibilities for L = 6

Link combinations for one DOF

- Same can be applied for L = 8
- In tabular form, 1-DOF planar mechanism with revolute joints and up to 8 links

Total Links	Binary	Ternary	Quaternary	Pentagonal	Hexagonal
4	4	0	0	0	0
6	4	2	0	0	0
6	5	0	1	0	0
8	7	0	0	0	1
8	4	4	0	0	0
8	5	2	1	0	0
8	6	0	2	0	0
8	6	1	0	1	0