Mech-465 Automatic Control
Lecture 2 Assignment

1. Solve the differential equation

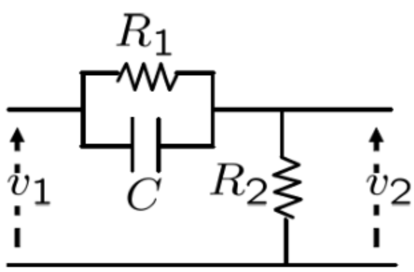
$$\frac{d^2x(t)}{dt^2} + 5\frac{dx(t)}{dt} + 4x(t) = 10u(t)$$

   with the following initial conditions:
   (a) $x(0) = \acute{x}(0) = 0$

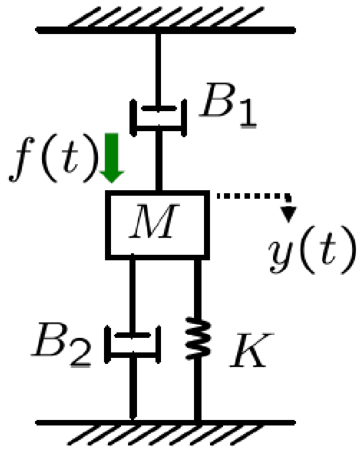
2. For the following $G(s)$, transform the relation $C(s) = G(s)R(s)$ into the differential equation.

$$G(s) = \frac{60}{s^2+10s+60}$$

3. For the following circuit, find the transfer function from $V_1(s)$ to $V_2(s)$. If we input a constant voltage 10V, what is the steady state value of the output voltage?
   (Hint: Use $v_1(t) = 10 \Rightarrow V_1(s) = 10/s$ then find the final value theorem for $v_2(t)$)



4. For the following mechanical systems, obtain the transfer function from the input force f to the mass displacement y



1)

$$\frac{d^2 x(t)}{dt^2} + 5\frac{dx(t)}{dt} + 4\cdot x(t) = 10\,u(t)$$

Laplace transform

$$\left(s^2 X(s) - s\,x(0) - \dot{x}(0)\right) + 5\left(sX(s) - x(0)\right) + 4X(s) = \frac{10}{s}$$

$$\Rightarrow X(s) = \frac{1}{s^2+5s+4}\left(\frac{10}{s} + (s+5)\,x(0) + \dot{x}(0)\right) \quad \text{—— } 2$$

a) $x(0) = \dot{x}(0) = 0$

$$\Rightarrow X(s) = \frac{10}{s(s^2+5s+4)} = \boxed{\frac{10}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}} \quad \text{—— } 1$$

(1 × s) to obtain A

$$\left.\frac{10}{(s+1)(s+4)}\right|_{s\to 0} = A + 0 + 0 \Rightarrow A = \frac{10}{4}$$

((1) * (s+1)) to obtain B :

$$\left.\frac{10}{s(s+4)}\right|_{s\to -1} = 0 + B + 0 \Rightarrow B = \frac{-10}{3}$$

((1) × (s+4)) to obtain C

$$\left.\frac{10}{s(s+1)}\right|_{s\to -4} = 0 + 0 + C \Rightarrow C = \frac{10}{-3(-4)} = \frac{10}{12} = \frac{5}{6}$$

$$X(s) = \frac{10}{s(s+1)(s+4)} = \frac{\cancel{10}^{5}}{\cancel{4}_{2}}\cdot\frac{1}{s} - \frac{10}{3}\cdot\frac{1}{s+1} + \frac{5}{6}\cdot\frac{1}{s+4}$$

$$\Rightarrow x(t) = \frac{5}{2} - \frac{10}{3}e^{-t} + \frac{5}{6}e^{-4t}$$

2)

a) $G(s) = \dfrac{60}{s^2+10s+60}$

$$C(s) = G(s)\,R(s)$$

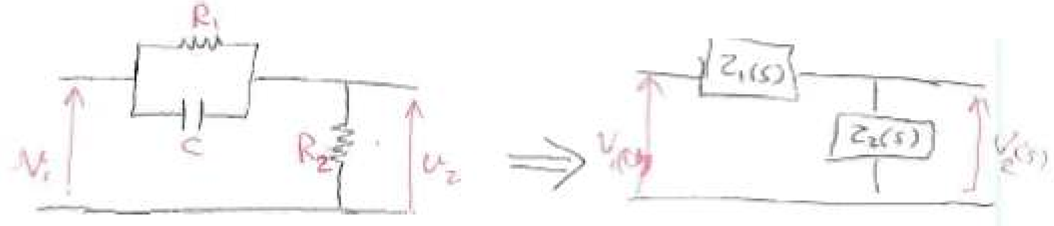
$$C(s) = \frac{60}{s^2+10s+60}\,R(s)$$

$$(s^2+10s+60)\,C(s) = 60\,R(s)$$

$$s^2C(s) + 10sC(s) + 60C(s) = 60\,R(s) \quad \mathcal{L}^{-1}$$

$$\ddot{c}(t) + 10\dot{c}(t) + 60c(t) = 60\,r(t)$$

3)



Using Kirchoff's law

$$V_1(s) = Z_1(s) + Z_2(s) \qquad \left(Z_1(s) = \frac{\frac{R_1}{Cs}}{R_1 + \frac{1}{Cs}} \right)$$

$$V_2(s) = Z_2(s) \qquad \left(Z_2(s) = R_2 \right)$$

$$\therefore TF = \frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

$$= \frac{R_2}{\underbrace{\frac{\frac{R_1}{Cs}}{R_1 + \frac{1}{Cs}}}_{\times \frac{Cs}{Cs}} + R_2} = \frac{R_2}{\frac{R_1}{R_1Cs + 1} + R_2}$$

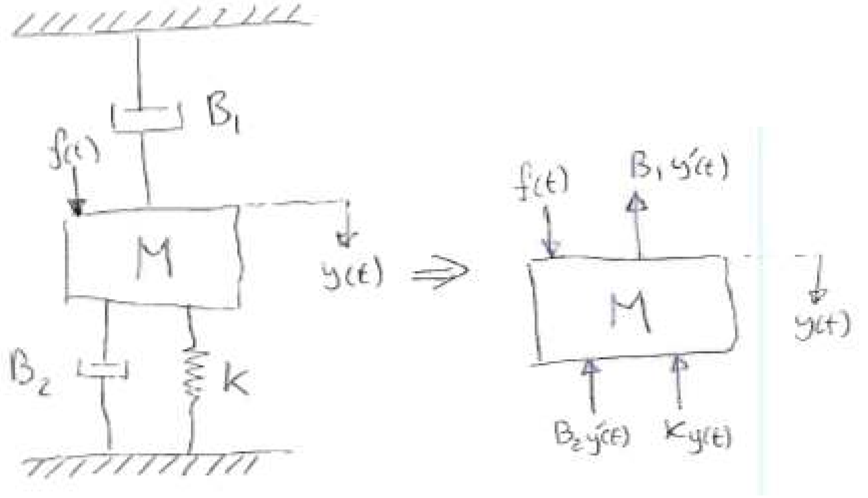
$$= \frac{R_2}{\frac{R_1 + R_2(R_1Cs+1)}{R_1Cs+1}} = \frac{R_1R_2Cs + R_2}{R_1R_2Cs + R_2 + R_1}$$

$$\text{For } V_1(s) = \frac{10}{s} \Rightarrow V_2(s) = \frac{R_1R_2Cs + R_2}{R_1R_2Cs + R_2 + R_1} \cdot \frac{10}{s}$$

$$\lim_{t \to \infty} v_2(t) = \lim_{s \to 0} sV_2(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{R_1R_2Cs + R_2}{R_1R_2Cs + R_2 + R_1} \cdot \frac{10}{\cancel{s}}$$

$$= \frac{10R_2}{R_1 + R_2}$$

4)



$$M\ddot{y}(t) = f(t) - (B_1+B_2)\dot{y}(t) - Ky(t)$$

$$Ms^2 Y(s) = F(s) - (B_1+B_2)s\,Y(s) - K\,Y(s)$$

Laplace

$$\left(Ms^2 + (B_1+B_2)s + K\right)Y(s) = F(s)$$

$$\frac{Y(s)}{F(s)} = \frac{1}{Ms^2+(B_1+B_2)s+K}$$