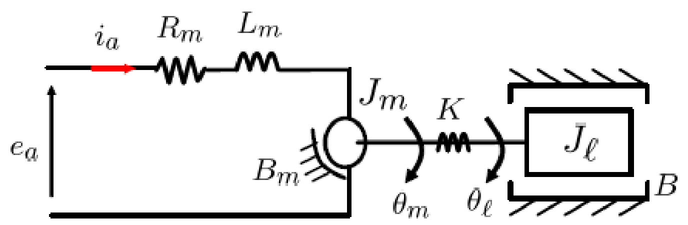
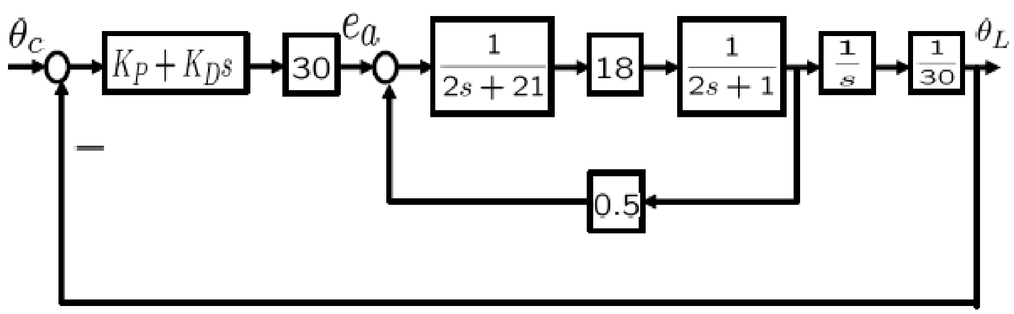
Mech-465 Automatic Control
Lecture 3 Assignment

1. Consider the servomotor with a shaft which can be modelled as a torsional spring. Denote the back EMF constant and the torque constants by $K_b$and K_t, respectively.



   (a) Write the system differential equations.
   (b) Draw a system block diagram.
   (c) Find the transfer function from E_a to θ_m.
   (d) Find the transfer function from $E_a$to θ_l.
   (e) Find the transfer function from $\theta_m$to θ_l.

2. Consider the block diagram of the servo-control system for one of the joints of a robot.



   (a) Find the transfer function from E_a to θ_l.
   (b) Find the transfer function from $\theta_c$to θ_l.
   (c) Find the transfer function from $\theta_c$to E_a.

3. Apply the Routh-Hurwitz criterion to the following polynomials.

   (a) $Q(s) = s^3 + 2s^2 + 3s + 1$
   (b) $Q(s) = s^3 + 2s^2 + 3s - 1$
   (c) $Q(s) = s^4 + s^3 + s + 2$
   (d) $Q(s) = s^4 - 1$
   (e) $Q(s) = s^5 + 2s^4 + 5s^3 + 4s^2 + 6s$
   (f) $Q(s) = s^4 + 2s^2 + 1$

Lecture 3 Assignment Solution

① (a) Armature:

$$e_a(t) = R_m i_a(t) + L_m i_a'(t) + K_b \theta_m'(t)$$

Mechanical part:

$$J_m \theta_m''(t) = K_i i_a(t) - B_m \theta_m'(t) - K(\theta_m(t) - \theta_\ell(t))$$

$$J_\ell \theta_\ell''(t) = -B\theta_\ell'(t) - K(\theta_\ell(t) - \theta_m(t))$$

(b) By taking Laplace transform

$$E_a(s) = R_m I_a(s) + s L_m I_a(s) + s K_b \theta_m(s)$$

$$s^2 J_m \theta_m(s) = K_i I_a(s) - s B_m \theta_m(s) - K(\theta_m(s) - \theta_\ell(s))$$

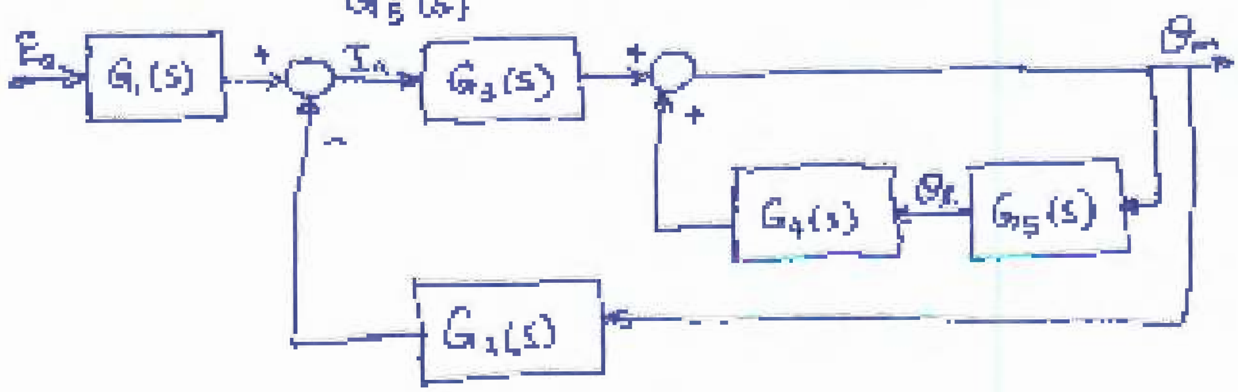
$$s^2 J_\ell \theta_\ell(s) = -sB\theta_\ell(s) - K(\theta_\ell(s) - \theta_m(s))$$

Simplified as:

$$I_a(s) = \underbrace{\frac{1}{L_m s + R_m}}_{G_1(s)} E_a(s) - \underbrace{\frac{K_b s}{L_m s + R_m}}_{G_2(s)} \theta_m(s)$$

$$\theta_m(s) = \underbrace{\frac{K_i}{J_m s^2 + B_m s + K}}_{G_3(s)} I_a(s) + \underbrace{\frac{K}{J_m s^2 + B_m s + K}}_{G_4(s)} \theta_\ell(s)$$

$$\theta_\ell(s) = \underbrace{\frac{K}{J_\ell s^2 + Bs + K}}_{G_5(s)} \theta_m(s)$$



(c) $$\frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{G_3(s)}{1 - G_4(s)G_5(s)}}{1 + \dfrac{G_2(s)\,G_3(s)}{1 - G_4(s)\,G_5(s)}} \cdot G_1(s)$$

(d) $$\frac{\theta_L(s)}{E_a(s)} = \frac{\theta_L(s)}{\theta_m(s)} \cdot \frac{\theta_m(s)}{E_a(s)} = G_5(s)\,\frac{\dfrac{G_3(s)}{1 - G_4(s)\,G_5(s)}}{1 + \dfrac{G_2(s)\,G_3(s)}{1 - G_4(s)\,G_5(s)}} \cdot G_1(s)$$

(e) $$\frac{\theta_L(s)}{\theta_m(s)} = G_5(s)$$

② (a) $$\frac{\theta_L(s)}{E_a(s)} = \frac{1}{30s} \cdot \frac{\dfrac{18}{(2s+1)(2s+21)}}{1 - 0.5 \cdot \dfrac{18}{(2s+1)(2s+21)}} = \frac{\frac{3}{20}}{s(s^2+11s+8)} = P(s)$$

(b) $$\frac{\theta_L(s)}{\theta_c(s)} = \frac{30\,P(s)\,(K_P + K_D(s))}{1 + 30\,P(s)\,(K_P + K_D(s))}$$

(c) $$\frac{E_a(s)}{\theta_c(s)} = \frac{30\,(K_P + K_D s)}{1 + 30\,P(s)\,(K_P + K_D s)}$$

③ (a) $Q(s) = s^3 + 2s^2 + 3s + 1$

s^3	1	3
s^2	2	1
s^1	5/2	
s^0	1	

- No sign change in first column ⇒
  ⇒ Q(s) has no RHP root
  ⇒ 3 LHP roots exist (3rd order)

(b) $Q(s) = s^3 + 2s^2 + 3s - 1$

s^3	1	3
s^2	2	-1
s^1	7/2	
s^0	-1	

- One sign change in 1st column
  $\Rightarrow$ Q(s) has one RHP root
- Q(s) also has 2 LHP roots

(c) $Q(s) = s^4 + s^3 + s + 2$

s^4	1	0	2
s^3	1	1	
s^2	-1	2	
s^1	3		
s^0	2		

- 2 sign changes in 1st column
  $\Rightarrow$ Q(s) has 2 RHP roots
- Q(s) has 2 LHP roots as well.

(d) $Q(s) = s^4 - 1$

s^4	1	0	-1
s^3	$0^{(4)}$	0	
s^2	$0^{(\epsilon)}$	-1	
s^1	$4/\epsilon$		
s^0	-1		

- 1 sign change in 1st column
  $\Rightarrow$ Q(s) has one RHP root
- Q(s) has a factor s^2+1
  $\Rightarrow$ 2 roots on the imaginary axis exist
- Q(s) has one LHP root (since Q(s) 4th order)

(e) $Q(s) = s^5 + 2s^4 + 5s^3 + 4s^2 + 6s$

$Q(s) = s(s^4 + 2s^3 + 5s^2 + 4s + 6)$

s^4	1	5	6
s^3	2	4	
s^2	3	6	
s^1	$0^{(6)}$		
s^0	6		

- No sign change in 1st column
  $\Rightarrow$ Q(s) has no RHP roots.
- Due to factor s and auxiliary polynomial
  $3s^2+6$ $\Rightarrow$ 3 roots exist at imaginary axis
- Q(s) has 2 LHP roots (since Q(s) is 5th order)

(g) $Q(s) = s^4 + 2s^2 + 1$

s^4	1	2	1
s^3	$0^{(4)}$	$0^{(4)}$	
s^2	1	1	
s^1	$0^{(2)}$		
s^0	1		

- No sign change in 1st column

  $\Rightarrow$ Q(s) has no RHP root

- Due to auxiliary polynomial s^2+1

  $\Rightarrow$ it has 2 roots on the imaginary axis

- Therefore, 2 LHP roots exist (since Q(s) is fourth order)