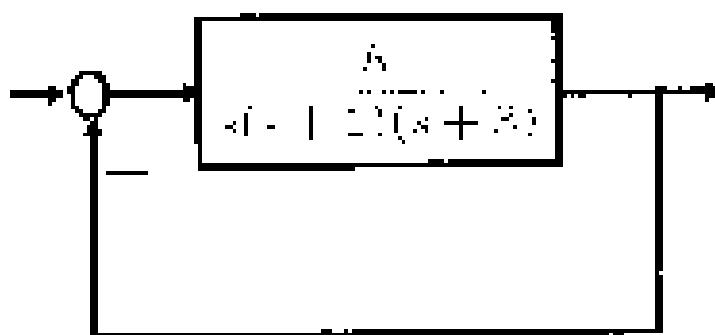


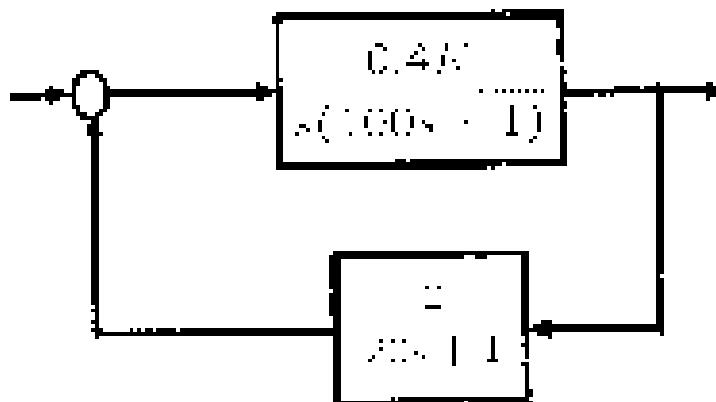
Mech-465 Automatic Control

Lecture 4 Assignment

1. Consider the feedback system below. For what range of K is the system stable?



2. Consider the feedback system below. For what value of K is the system marginally stable? For that value of K, what is the frequency of the oscillation?

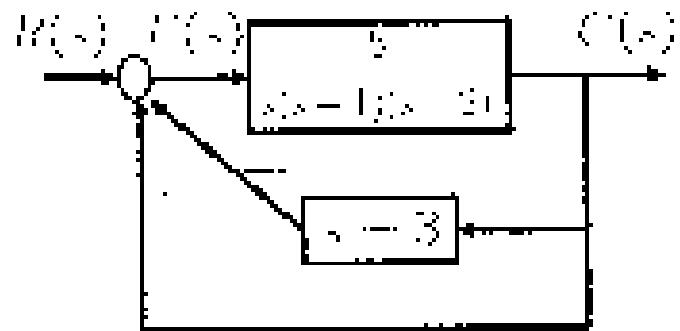


3. For the unity feedback system with

$$G(s) = \frac{500}{(s + 20)(s^2 + 4s + 10)}$$

Find the steady-state error if the input is $40u(t)$ and $70u(t)$

A. For the feedback system below:



- Find K_p , K_v and K_a .
- Find the steady state error for an input of $50u(t)$ and $50t^2u(t)$.
- State the system type.

$$(1) \quad \frac{d}{dt} \left(\frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \right) = \frac{d}{dt} \left(\frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \right)$$

$$C_R = \sigma^2 \lambda^2 \left(1 + \frac{K}{\lambda^2 \sigma^2 \lambda^2 \delta^2 \gamma^2} \right)^{-1/2}$$

$$\mathbb{E}_t \{ \phi_t(\theta_t(t)) \} \leq C_0 - C_1$$

$$x^3+4y^3+z^3+xy^2z=0$$

Using Routh's method,

$$\begin{array}{c|cc} \text{Cof} & 1 & 0 \\ \hline 0 & 3 & 2 \end{array}$$

The condition for the first column to be non-zero

$\Rightarrow 3 \neq 0$ is clearly true. $\therefore 3 \neq 0$

$\therefore \text{Cof} \neq 0$

$$\frac{1}{3}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$$(2) \quad \frac{d}{dt} \left(\frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \right) = \frac{d}{dt} \left(\frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \right)$$

$$\frac{d}{dt} \left(\frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \right) = \frac{d}{dt} \left(\frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \frac{\partial}{\partial t} \right)$$

$$C_R = \sigma^2 \lambda^2 \left(1 + \frac{K}{\lambda^2 \sigma^2 \lambda^2 \delta^2 \gamma^2} \right)^{-1/2}$$

$$\mathbb{E}_t \{ \phi_t(\theta_t(t)) \} \leq C_0 - C_1$$

$$g_{\alpha\beta\gamma\delta} \phi_t^{\alpha}(x) \phi_t^{\beta}(y) \phi_t^{\gamma}(z) \phi_t^{\delta}(w) = 0$$

$$\begin{array}{c} \text{Let } \alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}, \alpha_i > 0 \\ \text{and } \beta_1, \beta_2, \dots, \beta_n \in \mathbb{R}, \beta_i > 0 \end{array} \quad \begin{array}{c} \frac{\partial^k}{\partial x_1^k} \left(\prod_{i=1}^n x_i^{\alpha_i} \right) = \prod_{i=1}^n x_i^{\alpha_i} \\ \text{and } \frac{\partial^k}{\partial x_1^k} \left(\prod_{i=1}^n x_i^{\alpha_i} \right) \geq 0 \end{array}$$

$$\begin{array}{c} \text{The second term is equal to } \int_{\mathbb{R}^n} \left(\prod_{i=1}^n x_i^{\alpha_i} \right)^{\beta_i} dx = \\ \text{so we have } \text{Int}_1 \leq \int_{\mathbb{R}^n} \left(\prod_{i=1}^n x_i^{\alpha_i} \right)^{\beta_i} dx \end{array}$$

$$(\alpha_1 - 1)\alpha_1 \beta_1 + \dots + (\alpha_n - 1)\alpha_n \beta_n$$

$$\left(\prod_{i=1}^n x_i^{\alpha_i} \right)^{\beta_i} = \prod_{i=1}^n x_i^{\alpha_i \beta_i} = x_1^{\alpha_1 \beta_1} \cdots x_n^{\alpha_n \beta_n}$$

Now let's prove that $\text{Int}_1 \leq \int_{\mathbb{R}^n} \left(\prod_{i=1}^n x_i^{\alpha_i} \right)^{\beta_i} dx$

$$\left(\prod_{i=1}^n x_i^{\alpha_i} \right)^{\beta_i} = x_1^{\alpha_1 \beta_1} \cdots x_n^{\alpha_n \beta_n} \leq x_1^{\alpha_1} \cdots x_n^{\alpha_n} \cdot \beta_1^{\alpha_1} \cdots \beta_n^{\alpha_n} \left(\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n} \right)^{\beta_1} \cdots \left(\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n} \right)^{\beta_n}$$

$$\text{so } \text{Int}_1 \leq \int_{\mathbb{R}^n} \left(\prod_{i=1}^n x_i^{\alpha_i} \right)^{\beta_i} dx \leq \int_{\mathbb{R}^n} \left(\prod_{i=1}^n x_i^{\alpha_i} \right)^{\beta_i} dx = \text{Int}_2$$

So $\text{Int}_1 \leq \text{Int}_2 \leq \text{Int}_3$ and $\text{Int}_1 \leq \text{Int}_2$ so $\text{Int}_1 \leq \text{Int}_3$

$$\text{so } \text{Int}_1 \leq \text{Int}_3 \leq \text{Int}_2 \leq \text{Int}_1 \text{ so } \text{Int}_1 = \text{Int}_3$$

$$\begin{array}{c} \text{Let } \alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}, \alpha_i > 0 \\ \text{and } \beta_1, \beta_2, \dots, \beta_n \in \mathbb{R}, \beta_i > 0 \\ \text{then } \frac{\partial^k}{\partial x_1^k} \left(\prod_{i=1}^n x_i^{\alpha_i} \right)^{\beta_i} = \prod_{i=1}^n x_i^{\alpha_i \beta_i} \geq 0 \end{array}$$

Now we can say that $\left(\prod_{i=1}^n x_i^{\alpha_i} \right)^{\beta_i} \geq 0$ so $\text{Int}_1 \geq 0$

$$\text{Int}_1 = \sqrt[n]{\left(\prod_{i=1}^n x_i^{\alpha_i} \right)^{\beta_i}} = \sqrt[n]{x_1^{\alpha_1 \beta_1} \cdots x_n^{\alpha_n \beta_n}} = \sqrt[n]{\frac{x_1^{\alpha_1}}{\alpha_1!} \cdots \frac{x_n^{\alpha_n}}{\alpha_n!} \cdot \frac{\alpha_1^{\alpha_1}}{\alpha_1!} \cdots \frac{\alpha_n^{\alpha_n}}{\alpha_n!}}$$

$$\text{so } \text{Int}_1 = \sqrt[n]{\frac{x_1^{\alpha_1}}{\alpha_1!} \cdots \frac{x_n^{\alpha_n}}{\alpha_n!} \cdot \frac{\alpha_1^{\alpha_1}}{\alpha_1!} \cdots \frac{\alpha_n^{\alpha_n}}{\alpha_n!}}$$



$$\hat{A}_{11}(t) = \frac{\sqrt{2} \sqrt{t} \sqrt{t+2\sqrt{t}}}{(1+t)^2 \sqrt{2\sqrt{t}+2\sqrt{t+2\sqrt{t}}}} \cdot \frac{\sqrt{2} \sqrt{t+2\sqrt{t}}}{\sqrt{2\sqrt{t}+2\sqrt{t+2\sqrt{t}}}} = \frac{2}{(1+t)^2}$$

$$\begin{aligned} \hat{A}_{12}(t) &= \frac{\sqrt{2} \sqrt{t} \sqrt{t+2\sqrt{t}}}{(1+t)^2 \sqrt{2\sqrt{t}+2\sqrt{t+2\sqrt{t}}}} \\ &= \frac{1}{(1+t)^2} \end{aligned}$$

$$\begin{aligned} \hat{A}_{21}(t) &= \frac{\sqrt{2} \sqrt{t} \sqrt{t+2\sqrt{t}}}{(1+t)^2 \sqrt{2\sqrt{t}+2\sqrt{t+2\sqrt{t}}}} \\ &= \frac{1}{(1+t)^2} \end{aligned}$$

$$C_1 \geq -\lambda_1^2 + \lambda_1^2 + \lambda_2^2 = \lambda_2^2 \geq 0 \quad \forall t \in [0,1]$$

$$C_2 \leq k_{\text{min}}^{-1} \cdot \left(\lambda_1^2 + \lambda_2^2 \right) \leq k_{\text{min}}^{-1} \cdot \lambda_1^2 + k_{\text{min}}^{-1} \cdot \lambda_2^2$$

$$\begin{aligned} \frac{\lambda_1^2}{k_{\text{min}}^{-1}} &= \frac{1}{2} \cdot \frac{1}{1+t} \cdot \frac{1}{1+t+2\sqrt{t}} \cdot \frac{1}{1+t+2\sqrt{t+2\sqrt{t}}} \\ &\leq \frac{1}{2} \cdot \frac{1}{1+t} \cdot \frac{1}{1+t+2\sqrt{t}} \cdot \frac{1}{1+t+2\sqrt{t+2\sqrt{t}}} \cdot \frac{1}{1+t+2\sqrt{t+2\sqrt{t+2\sqrt{t}}}} \\ &\leq \frac{1}{2} \cdot \frac{1}{1+t} \cdot \frac{1}{1+t+2\sqrt{t}} \cdot \frac{1}{1+t+2\sqrt{t+2\sqrt{t}}} \cdot \frac{1}{1+t+2\sqrt{t+2\sqrt{t+2\sqrt{t}}}} \end{aligned}$$

$$(1) \quad S_{11} = \sqrt{\int_0^1 \left(\lambda_1^2(t) - \lambda_1^2 \right)^2 dt} \leq \sqrt{k_{\text{min}}^{-1}} \cdot C_1$$

$$(2) \quad S_{12} = \left(\lambda_1^2(t) - \lambda_1^2 \right) \cdot \left(\lambda_2^2(t) - \lambda_2^2 \right) = \frac{1}{2} \frac{\lambda_1^2}{k_{\text{min}}^{-1}} \cdot \lambda_2^2 \leq \frac{1}{2} C_1 \cdot \lambda_2^2$$

$$(3) \quad S_{21} = \left(\lambda_2^2(t) - \lambda_2^2 \right) \cdot \left(\lambda_1^2(t) - \lambda_1^2 \right) = \frac{1}{2} \frac{\lambda_2^2}{k_{\text{min}}^{-1}} \cdot \lambda_1^2 \leq \frac{1}{2} C_1 \cdot \lambda_1^2$$

Es gilt also $S_{11} \leq \sqrt{k_{\text{min}}^{-1}} \cdot C_1$ und entsprechend $S_{12} \leq \frac{1}{2} C_1 \cdot \lambda_2^2$, $S_{21} \leq \frac{1}{2} C_1 \cdot \lambda_1^2$.

$$S_{11} + S_{12} + S_{21} \leq \frac{3}{2} C_1 \cdot \lambda_2^2$$