

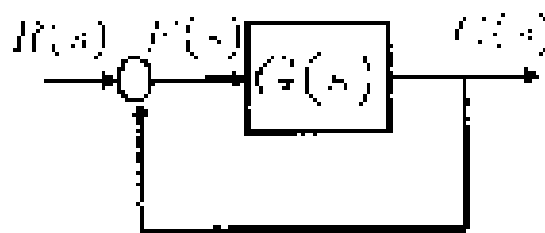
Mech-465 Automatic Control
Lecture 5 Assignment

1. Sketch the unit step response of the following systems.

(a) $\frac{3}{s+1.5}$

(b) $\frac{s}{s^2+2s+11}$

2. Consider the feedback system in the figure below:



For a given first order plant, $G(s) = \frac{K}{Ts+1}$, what are the time-constant and DC gain of the closed-loop transfer function from $R(s)$ to $C(s)$?

3. Consider the system

$$G(s) = \frac{600}{(s+20)(s^2+8s+20)}$$

- (a) If the step input $5u(t)$ is applied to $G(s)$, what is the steady state output value?
- (b) Approximately how many seconds are required for the system to reach steady-state (2% settling time)?
- (c) Do you expect the transient response to be oscillatory? Justify your answer.

4. Consider a unity feedback system with a forward transfer function

$$\frac{K}{s(s+5)}$$

- (a) Find the range of K for which the system is underdamped, critically damped and overdamped.
- (b) Find the value of K that will result in the system having minimum settling time.
5. Consider a system

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

Find the region of allowable s -plane pole locations such that the system 2% settling time is less than 2 seconds and an overshoot for a step response less than 16 percent.

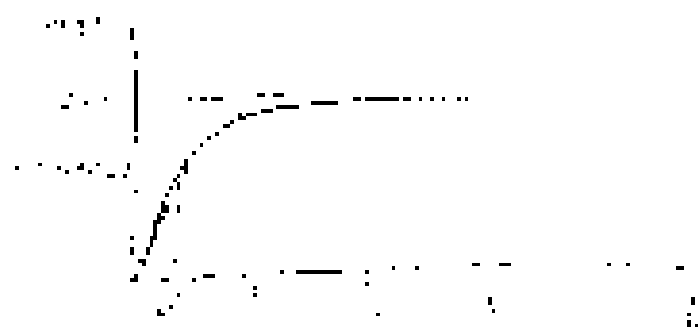
①

(a)

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} = \frac{d^2y}{dt^2} + 1$$

$$x(0) = 0, \quad \dot{x}(0) = 0, \quad y(0) = 0$$

$$\text{Time constant } \tau = \frac{1}{\lambda} = 1 \text{ sec.}$$



(b)

$$\frac{d^2x}{dt^2} + 4\dot{x} + 4x = \frac{d^2y}{dt^2} + 3y$$

$$x(0) = 0, \quad \dot{x}(0) = 0, \quad y(0) = 0$$

$$\text{Characteristic equation: } \lambda^2 + 4\lambda + 4 = 0 \Rightarrow \lambda = -2 \Rightarrow \tau = \frac{1}{2} \text{ sec.}$$

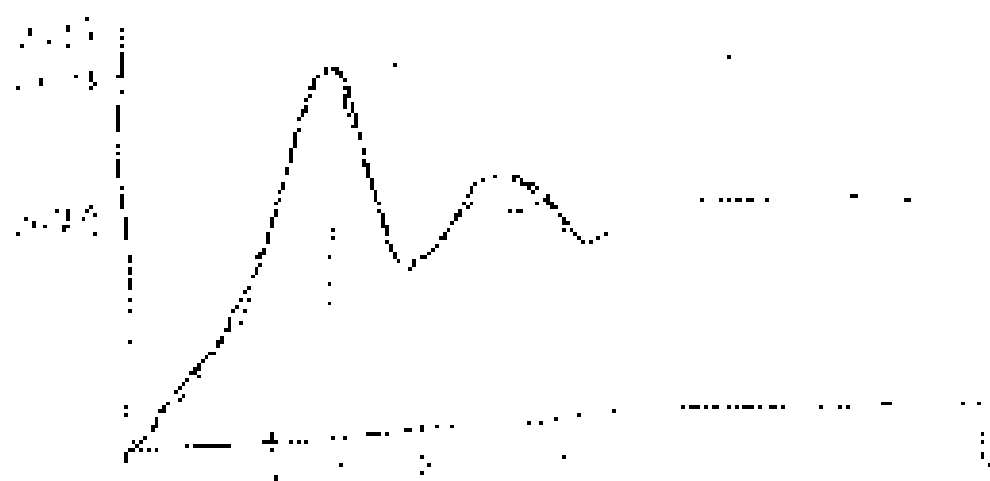
$$\text{Natural period } T = \frac{2\pi}{\omega_n} = \frac{2\pi}{2} = \pi \text{ sec.}$$

$$\text{Time constant } \tau = \frac{1}{\lambda} = \frac{1}{2} \text{ sec.}$$

$$\text{Damping is overdamped because } \zeta = \frac{2}{2} = 1 > 0.5 \text{ (or } \tau < \frac{T}{4} \text{).}$$

$$\text{Time to rise } t_r = \frac{0.707}{\lambda} = 0.3535 \text{ sec.}$$

$$\text{Rise time } t_{90\%} = \frac{2.2}{\lambda} = 1.1 \text{ sec.}$$



②

$$\begin{aligned} \text{GDP} &= \text{GDP} - \text{GDP} + \text{GDP} \\ &= \text{GDP} - \text{GDP} + \text{GDP} \end{aligned}$$

→ Savings

$$\text{GDP} = \frac{\text{GDP}}{1 + \frac{\text{GDP}}{\text{GDP}}} = \frac{\text{GDP}}{1 + \frac{\text{GDP}}{\text{GDP}}}$$

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③

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(4) (a) The characteristic function of $T(x)$ is $\frac{f(x)}{1 - F(x)}$ for $x \in \mathbb{R}$

$$\phi_T(x) = \int_{-\infty}^{\infty} e^{itx} \frac{f(t)}{1 - F(t)} dt$$

(b) $\phi_T(x) = 0$ is not bounded at $x = 0$

$$\Rightarrow \phi_T(x) \text{ is not a characteristic function}$$

(c) $\phi_T(x) = 0$ is not a probability density function

$$\Rightarrow \phi_T(x) \text{ is not a characteristic function}$$

(d) $\phi_T(x) = 0$ is not a probability density function

$$\Rightarrow \phi_T(x) \text{ is not a characteristic function}$$

(5) (a) \mathbb{R}^n is a separable metric space and hence compact

$$\mathbb{R}^n = \bigcup_{k=1}^{\infty} K_k \text{ where } K_k \text{ is compact}$$

(b) \mathbb{R}^n is a separable metric space and hence compact

$$\Rightarrow \mathbb{R}^n \text{ is compact}$$

(c) \mathbb{R}^n is a separable metric space and hence compact
 (d) \mathbb{R}^n is a separable metric space and hence compact

⑧

Given: $\frac{1}{2} \times 2 \times 2 = \frac{1}{2} \times 2 \times 2$ (Area of triangle)

$$\Rightarrow \frac{1}{2} \times 2 \times 2 = \frac{1}{2} \times 2 \times 2$$

Given: The area of triangle is 2

$$\Rightarrow \frac{1}{2} \times 2 \times 2 = 2$$

