جامعة بيشة
UNIVERSITY OF BISHA

VISION رؤية 2030
المملكة العربية السعودية
KINGDOM OF SAUDI ARABIA

معتمد ACCREDITED

بسم الله الرحمن الرحيم

«قال ياقوم أرأيتم إن كنت على بينة من ربى ورزقنى
منه رزقا حسنا وما أريد أن أخالفكم إلى ما أنهاكم عنه إن
أريد إلا الإصلاح ما أستطعت **وما توفيقي إلا بالله**
عليه توكلت وإليه أنيب» صدق الله العظيم (هود ٨٨)

Prof. Ashraf A. Ali
PhD Drexel University, USA 2002
Visiting Scientist, MIT, USA 2007

تصميم أجزاء ماكينات (١) – همك ٤١١

Design of Machine Elements –1 ME 411

Prof. Ashraf A. Ali

PhD Materials Engineering Department, Drexel University, USA 2002

Egyptian Fulbright Awarded Grantee and Visiting Scientist, ISN, MIT, USA 2007
Assistant professor, KSU, Saudi Arabia (2007-2010)
Associate Professor, PSAU, Saudi Arabia (2010-2012)

Tenure Track Professor, Mechanical Design and Production Engineering Department, Faculty of Engineering, Zagazig University,
Founder and Director of NTC (Established 2003), Zagazig, Egypt

Professor
Vice Dean for Development and Quality (2017-2020), College of Engineering 2017-2020
Part Time Consultant at Vice Presidency for Graduate Studies and Scientific Research (2018-2020)
University of Bisha

College of Engineering
Mechanical Engineering Department

Design of Machine Elements-1
ME 411
Week No. 5 Lect. No. 7&8

Design of Beams

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الاسبوع الخامس

الانحناء فى الكمرات

Beam Deflection

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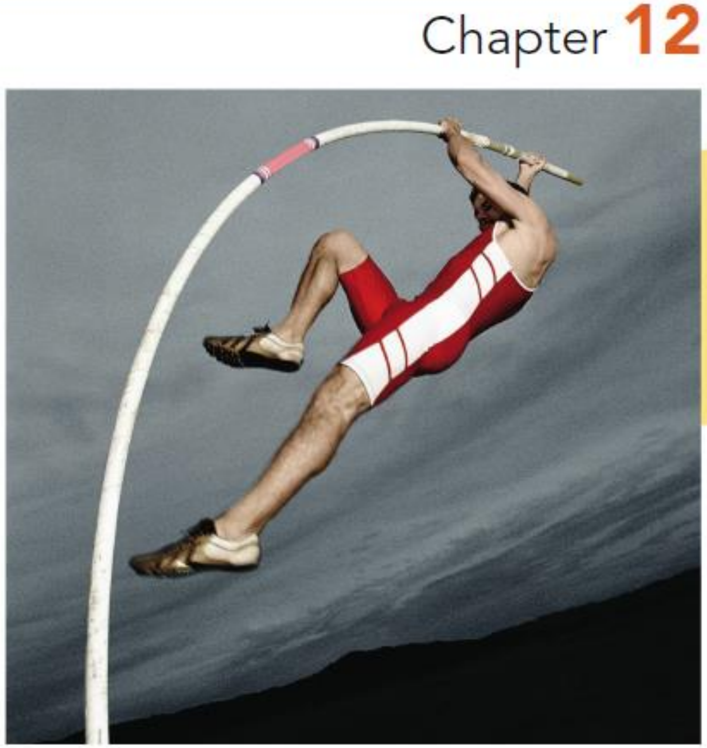
Design Concepts

- ✓ **Design based on Stresses**
- ✓ **Design based on Stiffness**



If the curvature of this pole is measured, it is then possible to determine the bending stress developed within it.

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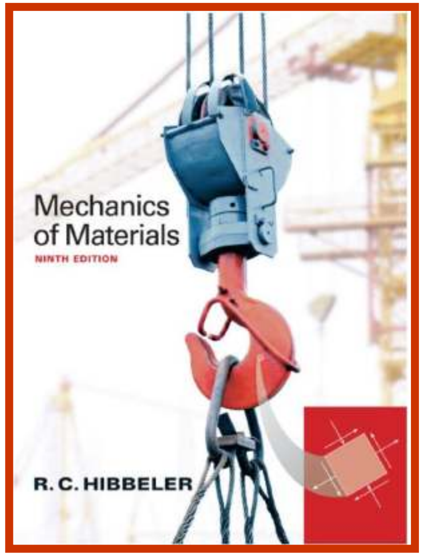

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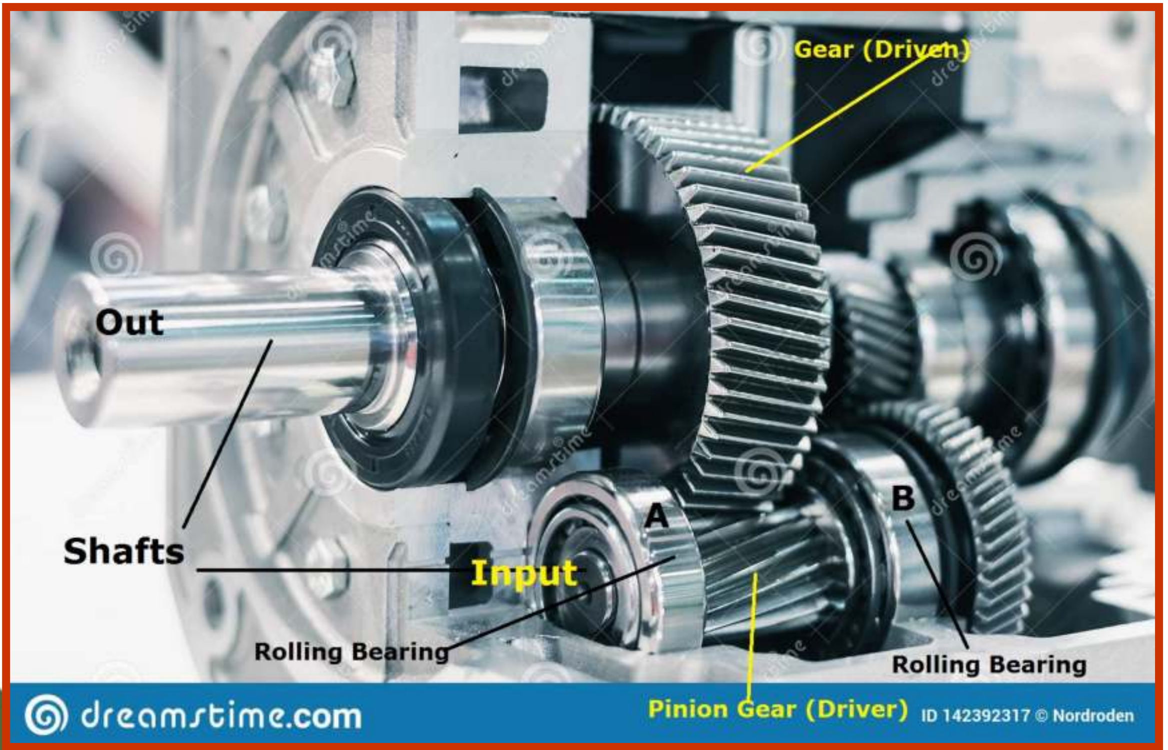
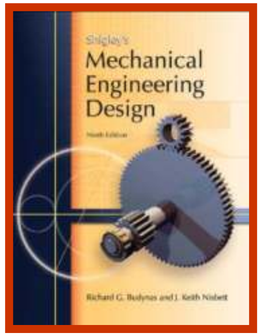
Design Concepts

- ✓ Design based on Stresses
- ✓ Design based on Stiffness





Shafts and Shaft Components | 379

Table 7–2
Typical Maximum Ranges for Slopes and Transverse Deflections

Slopes	
Tapered roller	0.0005–0.0012 rad
Cylindrical roller	0.0008–0.0012 rad
Deep-groove ball	0.001–0.003 rad
Spherical ball	0.026–0.052 rad
Self-align ball	0.026–0.052 rad
Uncrowned spur gear	< 0.0005 rad
Transverse Deflections	
Spur gears with $P < 10$ teeth/in	0.010 in
Spur gears with $11 < P < 19$	0.005 in
Spur gears with $20 < P < 50$	0.003 in

After: https://www.dreamstime.com/mechanical-gearbox-cross-section-helical-gear-close-up-image142392317

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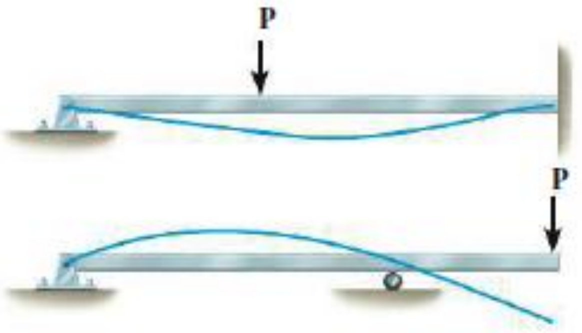
Slope and Deflection vs. $M_{(x)}$

Elastic Curve

where ρ is the radius of curvature. From studies in mathematics we also learn that the curvature of a plane curve is given by the equation

$$\frac{1}{\rho} = \frac{d^2y/dx^2}{[1+(dy/dx)^2]^{3/2}}$$

$$\frac{1}{\rho} = \frac{M}{EI}$$



where the interpretation here is that y is the lateral deflection of the centroidal axis of the beam at any point x along its length. The slope of the beam at any point x is

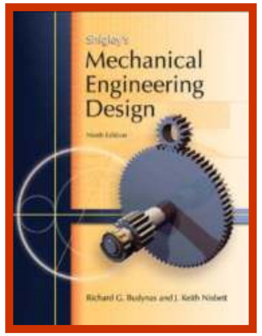
$$\theta = \frac{dy}{dx}$$

For many problems in bending, the slope is very small,

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$EI\frac{dy}{dx} = \int M\,dx$$ → **Slope = θ**

$$EIy = \iint M\,dx$$ → **Deflection = y = δ**



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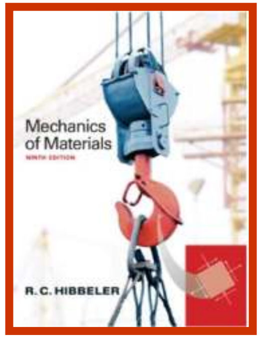

Ch. 12 p.573

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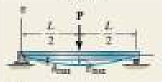
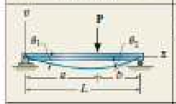
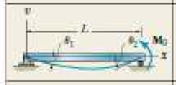
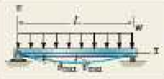
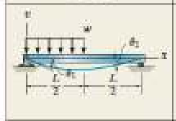
Deflection and Slope Correlations in Different Types of Supported and Loaded Beams

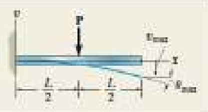
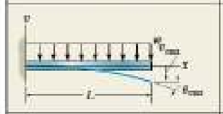
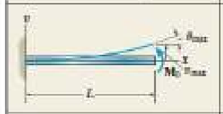
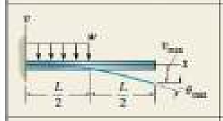
APPENDIX **C**

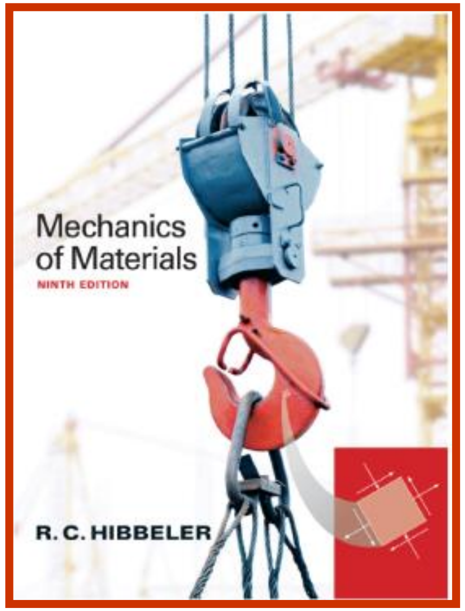
Slopes and Deflections of Beams

Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{16EI}$	$v_{max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \frac{-Pab(L+b)}{6EIL}$ $\theta_2 = \frac{Pab(L+a)}{6EIL}$	$v\Big\vert_{x=a} = \frac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \frac{-M_0L}{6EI}$ $\theta_2 = \frac{M_0L}{3EI}$	$v_{max} = \frac{-M_0L^2}{9\sqrt{3}EI}$ at $x = 0.5774L$	$v = \frac{-M_0x}{6EIL}(L^2 - x^2)$
	$\theta_{max} = \frac{-wL^3}{24EI}$	$v_{max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \frac{-3wL^3}{128EI}$ $\theta_2 = \frac{7wL^3}{384EI}$	$v\Big\vert_{x=L/2} = \frac{-5wL^4}{768EI}$ $v_{max} = -0.006563\frac{wL^4}{EI}$ at $x = 0.4598L$	$v = \frac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \frac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \frac{-7w_0L^3}{360EI}$ $\theta_2 = \frac{w_0L^3}{45EI}$	$v_{max} = -0.00652\frac{w_0L^4}{EI}$ at $x = 0.5193L$	$v = \frac{-w_0x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

Cantilevered Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{2EI}$	$v_{max} = \frac{-PL^3}{3EI}$	$v = \frac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \frac{-PL^2}{8EI}$	$v_{max} = \frac{-5PL^3}{48EI}$	$v = \frac{-Px^2}{12EI}(3L - 2x) \quad 0 \le x \le L/2$ $v = \frac{-PL^2}{48EI}(6x - L) \quad L/2 \le x \le L$
	$\theta_{max} = \frac{-wL^3}{6EI}$	$v_{max} = \frac{-wL^4}{8EI}$	$v = \frac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \frac{M_0L}{EI}$	$v_{max} = \frac{M_0L^2}{2EI}$	$v = \frac{M_0x^2}{2EI}$
	$\theta_{max} = \frac{-wL^3}{48EI}$	$v_{max} = \frac{-7wL^4}{384EI}$	$v = \frac{-wx^2}{24EI}\left(x^2 - 2Lx + \frac{3}{2}L^2\right)$ $0 \le x \le L/2$ $v = \frac{-wL^3}{384EI}(8x - L)$ $L/2 \le x \le L$
	$\theta_{max} = \frac{-w_0L^3}{24EI}$	$v_{max} = \frac{-w_0L^4}{30EI}$	$v = \frac{-w_0x^2}{120EIL}(10L^3 - 10L^2x + 5Lx^2 - x^3)$



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Beam Deflection Derivation

Case No. (1) Tabulated

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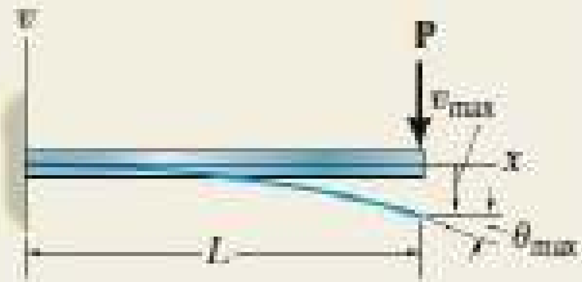
Cantilevered Beam Slopes and Deflections

Beam	Slope √	Deflection √	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{2EI}$	$v_{max} = \frac{-PL^3}{3EI}$	$v = \frac{-Px^2}{6EI}(3L - x)$

δ
Delta

$$E\,I\,(\theta) = \frac{Px^2}{2} - PLx$$

$$E\,I(\delta) = \frac{Px^3}{6} - \frac{PLx^2}{2}$$

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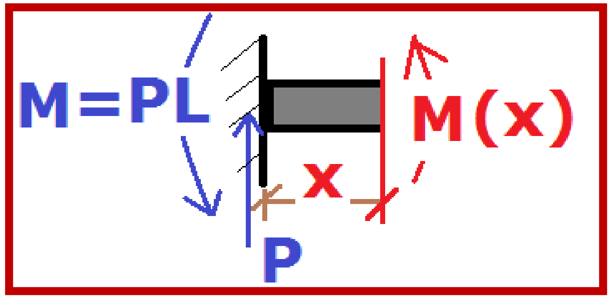
Beam Deflection Derivation

Case No. (1)

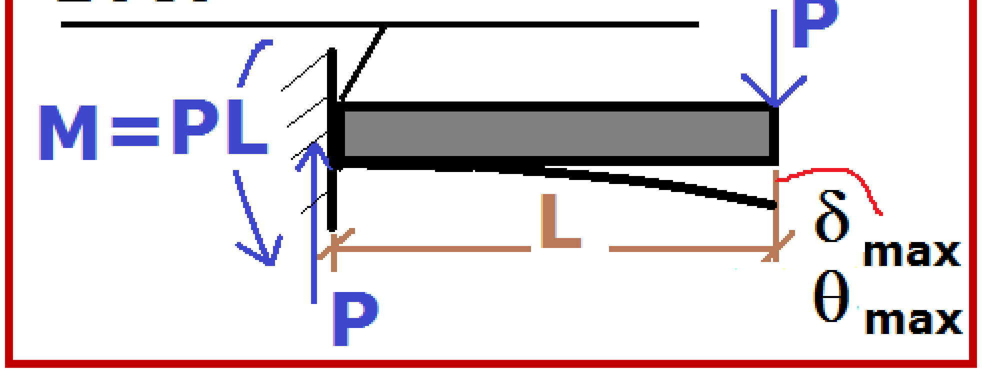
Derived by Integration Method

$$E\,I\,(\theta) = \int M(x)dx$$

$$E\,I\,(\delta) = \iint M(x)dx$$



$$M(x) = Px - PL$$



$$E\,I\,(\theta) = \int (Px - PL)\,dx = \frac{Px^2}{2} - PLx + C1$$ → @ $x=0;\ \theta=0$ gives $C1=0$

$$E\,I(\delta) = \iint M(x)dx = \frac{Px^3}{6} - \frac{PLx^2}{2} + C1x + C2$$ → @ $x=0;\ \delta=0$ gives $C2=0$

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Beam Design on Both Stresses and Deflection

Solved Problem (Practice No.1)

1. Design standard two cross-sections: Solid square (section I), and Box square (Section II) (Use static flexural stress formula only).

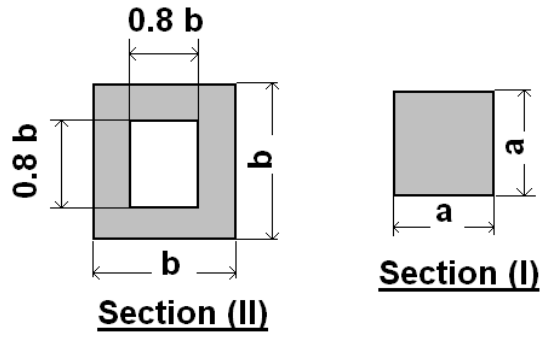
2. Calculate the maximum deflection in (mm) and slope in (rad) of the loaded beam for each cross-section and identify its position.

3. Comment on the previous results and state one conclusion.

<u>Given:</u>

Young's modulus = 210 GPa, yield strength = 390 MPa and Design factor = 3, P = 10 kN, L = 1 m

Cantilevered Beam



<u>Two cross-sections</u>

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Beam Design on Both Stresses and Deflection

Solved Problem (Practice No.1)

$M_{(max)}$ = PL = 10x1= 10 (kN.m) at the support as [$M_{(max)}$/ Z] = [σ_y/n_d]
Z = (10x1000X1000) N.mm/130 (N/mm^2) = 76.9231x1000 (mm^3)
For solid square (Z = a3/6), a = 77.28 mm, for standard **use $a_{st.}$ = 80 mm**
For box square (Z = 0.0813 b3), b = 98.16 mm, for standard **use $b_{st.}$ = 100 mm**

I (moment of Inertia) for solid square = $a_{st}^4/12$ = 3413333.333 mm^4
I (moment of Inertia) for Box square = $(b_{st}^4-(0.8b)^4)/12$ = 4920000 mm4

Maximum Deflection at free end = PL^3/3EI =
{[(10000(1000)^3 N.mm^3)]/[3(210x1000 N/mm^2)(I)]} = **{15873015.87/(I)} (mm)**

Maximum Slope at free end = PL^2/2EI =
{[(10000(1000)^{2}N.mm^2)]/[2(210x1000 N/mm^2)(I)]} = **{500000/21(I)} (rad.)**

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Beam Design on Both Stresses and Deflection
Solved Problem (Practice No.1)

For **solid square cross-section** (Maximum Deflection) **= 4.7 mm**
For **solid square cross-section** (Maximum Slope) **= 0.00698 rad. = 0.40 degrees**

For **Box square cross-section** (Maximum Deflection) **= 3.2 mm**
For **Box square cross-section** (Maximum Slope) **= 0.00484 rad. = 0.28 degree**

Comments

1- Deflection and Slope of the loaded beam showed **lower values** by using box square cross section instead of solid square cross section.

2- Beams with **box square cross-section** not only lighter in weight but also behave more rigid (stiff) in its elastic curve (deflection and slope) than solid square cross-section.

Conclusion: Cross-section **profile and orientation** is very important parameter in designing of beams; not only to save weight (money) but also to make **firm, stiff and rigid beam** in its applications.

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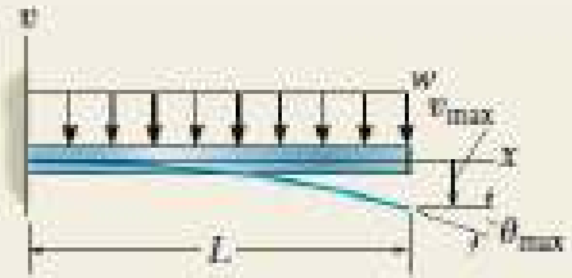
Beam Deflection Derivation

Case No. (2) Tabulated

Cantilevered Beam Slopes and Deflections			
Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-wL^3}{6EI}$	$v_{max} = \frac{-wL^4}{8EI}$	$v = \frac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$

δ

Delta

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Beam Deflection Derivation

Case No. (2)

Derived by Integration Method

(Student Competition No. 1)

one Pones point for any student derived the deflection and slope equations for case 2 in the next lecture (Lecture No. 8).

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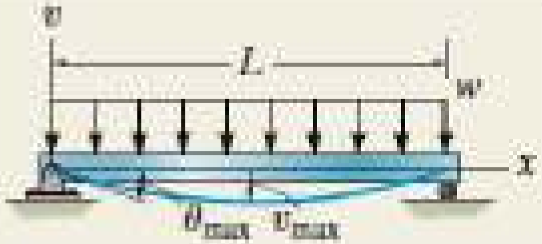
Beam Deflection Derivation

Case No. (3) Tabulated

Simply Supported Beam Slopes and Deflections

Beam	Slope √	Deflection √	Elastic Curve
	$\theta_{max} = \frac{-wL^3}{24EI}$	$v_{max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$

δ

Delta

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Beam Deflection Derivation
Case No. (3)
Derived by Integration Method

EXAMPLE 4–1 For the beam in Fig. 4–2, the bending moment equation, for $0 \le x \le l$, is

$$M = \frac{wl}{2}x - \frac{w}{2}x^2$$

Using Eq. (4–12), determine the equations for the slope and deflection of the beam, the slopes at the ends, and the maximum deflection.

Solution Integrating Eq. (4–12) as an indefinite integral we have

$$EI\frac{dy}{dx} = \int M\,dx = \frac{wl}{4}x^2 - \frac{w}{6}x^3 + C_1 \tag{1}$$

where C_1 is a constant of integration that is evaluated from geometric boundary conditions. We could impose that the slope is zero at the midspan of the beam, since the beam and loading are symmetric relative to the midspan. However, we will use the given boundary conditions of the problem and verify that the slope is zero at the midspan. Integrating Eq. (1) gives

$$EIy = \iint M\,dx = \frac{wl}{12}x^3 - \frac{w}{24}x^4 + C_1x + C_2 \tag{2}$$

The boundary conditions for the simply supported beam are $y = 0$ at $x = 0$ and l. Applying the first condition, $y = 0$ at $x = 0$, to Eq. (2) results in $C_2 = 0$. Applying the second condition to Eq. (2) with $C_2 = 0$,

$$EIy(l) = \frac{wl}{12}l^3 - \frac{w}{24}l^4 + C_1l = 0$$

Solving for C_1 yields $C_1 = -wl^3/24$. Substituting the constants back into Eqs. (1) and (2) and solving for the deflection and slope results in

$$y = \frac{wx}{24EI}(2lx^2 - x^3 - l^3) \tag{3}$$

$$\theta = \frac{dy}{dx} = \frac{w}{24EI}(6lx^2 - 4x^3 - l^3) \tag{4}$$

Comparing Eq. (3) with that given in Table A–9, beam 7, we see complete agreement.

For the slope at the left end, substituting $x = 0$ into Eq. (4) yields

$$\theta|_{x=0} = -\frac{wl^3}{24EI}$$

and at $x = l$,

$$\theta|_{x=l} = \frac{wl^3}{24EI}$$

At the midspan, substituting $x = l/2$ gives $dy/dx = 0$, as earlier suspected.

The maximum deflection occurs where $dy/dx = 0$. Substituting $x = l/2$ into Eq. (3) yields

$$y_{max} = -\frac{5wl^4}{384EI}$$

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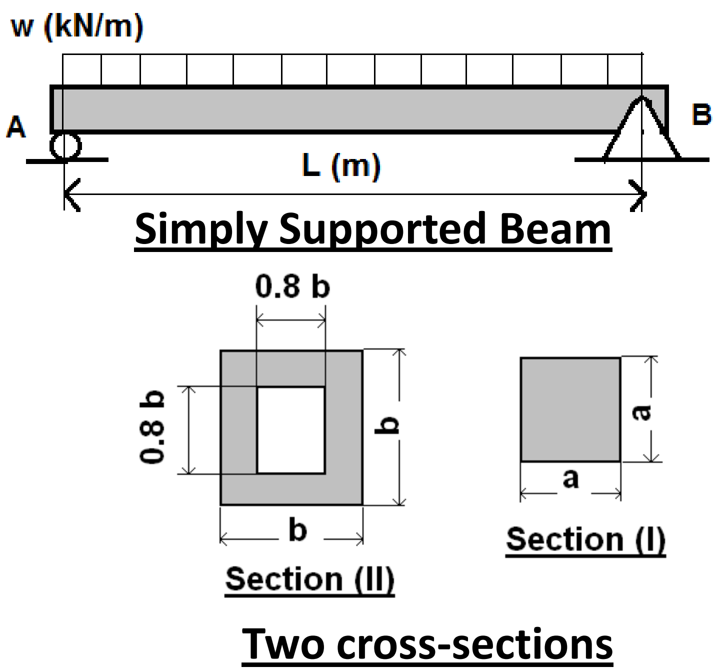
Beam Design on Both Stresses and Deflection

Solved Problem (Drill No. 1)

1. Deduce the deflection and slope equations for the shown simply supported beam shown in Figure (1).
2. Design standard two cross-sections: Solid square (section I), and Box square (Section II) (Use static flexural stress formula only).
3. Get the factor of safety for each standard cross-section to the corrected nearest 5 mm of each side length.
4. Calculate the weight of the beam for each cross-section in (kg) and identify the lighter beam weight?
5. Calculate the maximum deflection in (mm) and slope in (rad) of the loaded beam for each cross-section and identify its position.
6. Comment on the previous results (parts 4 and 5) and state one conclusion.



Assume the loaded beam has the following data
Young's modulus = 210 GPa, yield strength = 390 MPa and Design factor = 3, w = 10 kN/m, L = 1 m and material's density = 7.8 g/cm^3

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بسم الله الرحمن الرحيم

«يوم تجد كل نفس ما عملت من خير
محضرا وما عملت من سوء تود لو ان بينها
وبينه امدا بعيدا ويحذركم اللّٰه نفسه واللّٰه
رءوف بالعباد» صدق الله العظيم

٣٠ آل عمران

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