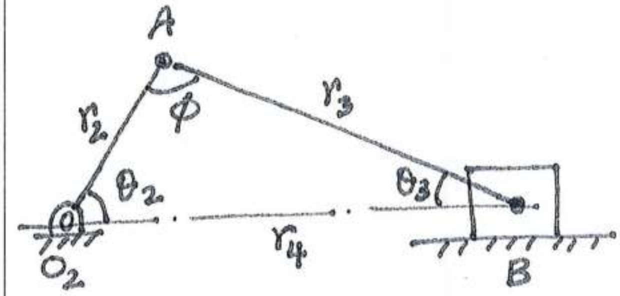
SLIDER-CRANK

This is a single degree-of-freedom mechanism with four links: Ground 1, crank 2, coupler 3, and slider 4.



Link lengths r_2 and r_3 are known and fixed.
One variable – either angle of crank θ_2 or position of slider r_4 will be given and we wish to find the other as well as θ_3.

Case I: Given θ_2, find θ_3, r_4

From triangle O_2AB, using Sine Law

$$\frac{r_2}{\sin\theta_3} = \frac{r_3}{\sin\theta_2} \Rightarrow \theta_3 = \sin^{-1}\left[\frac{r_2 \sin\theta_2}{r_3}\right]$$

The $\sin^{-1}$ function gives two answers, θ and $180° - \theta$. Choose the answer that matches the figure.

Next, ~~we~~ calculate $\phi = 180° - \theta_2 - \theta_3$.

Then apply Sine Law again

$$\frac{r_4}{\sin\phi} = \frac{r_2}{\sin\theta_3} \Rightarrow r_4 = \frac{r_2 \sin\phi}{\sin\theta_3}$$

Case II: Given r_4, find θ_3 and θ_2

Apply Cosine Law to triangle O_2AB:

$$r_3^2 = r_2^2 + r_4^2 - 2r_2 r_4 \cos\theta_2$$

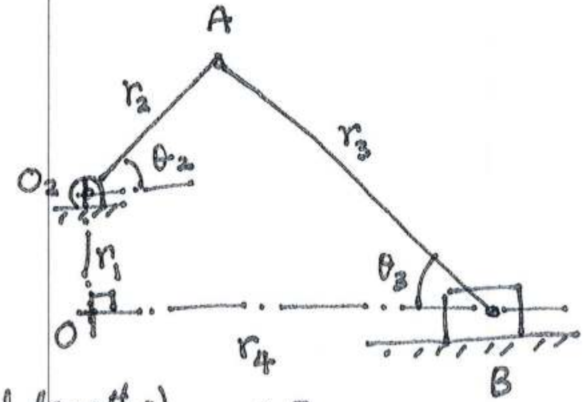
$$\Rightarrow \theta_2 = \cos^{-1}\left[\frac{r_2^2 + r_4^2 - r_3^2}{2r_2 r_4}\right]$$

Then Sine Law gives

$$\frac{r_2}{\sin\theta_3} = \frac{r_3}{\sin\theta_2} \Rightarrow \theta_2 = \sin^{-1}\left[\frac{r_2 \sin\theta_2}{r_3}\right]$$

<u>Slider-Crank with Offset</u>

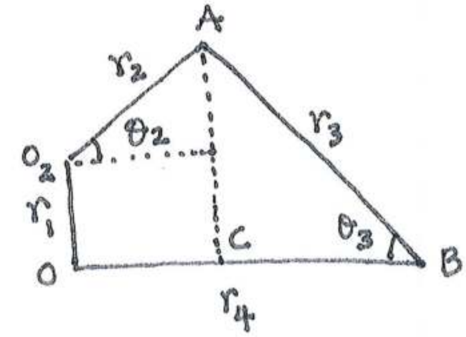
Sometimes, the axis of slider is offset by a distance r_1 from the pivot O_2.



Here, given r_1, r_2, r_3 (all fixed lengths) and θ_2,

find r_4 and θ_3

We cannot apply triangle laws directly, so we divide the figure as shown. Then,



$$\overline{AC} = r_1 + r_2 \sin\theta_2$$

$$= r_3 \sin\theta_3$$

So $$\theta_3 = \sin^{-1}\left[\frac{r_1 + r_2 \sin\theta_2}{r_3}\right]$$

Then, $$r_4 = \overline{OC} + \overline{CB}$$

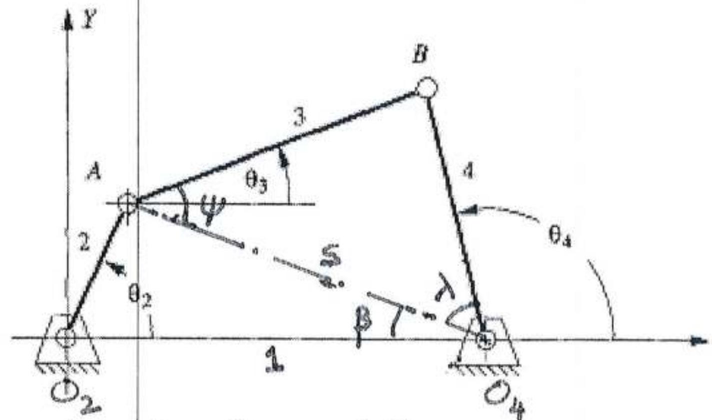
$$= r_2 \cos\theta_2 + r_3 \cos\theta_3$$

FOUR-BAR LINKAGE – POSITION ANALYSIS

We are given the link lengths

$r_1 = \overrightarrow{O_2 O_4}$, $r_2 = \overrightarrow{O_2 A}$

$\vec{r}_3 = \overrightarrow{AB}$, and $r_4 = \overrightarrow{O_4 B}$.



For any given angular position of link 2, θ_2, we wish to find the angular positions of links 3 and 4, ie, θ_3 and θ_4.

Divide the figure into two triangles and assign names to interior angles as shown. Consider the lower triangle first:

Cosine law gives $\quad s^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta_2$

$$\text{or } s = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta_2} \quad -(1)$$

Sine law gives

$$\frac{r_2}{\sin\beta} = \frac{s}{\sin\theta_2} \quad \Rightarrow \quad \beta = \sin^{-1}\left[\frac{r_2 \sin\theta_2}{s}\right] \quad -(2)$$

Now solve upper triangle

Cosine law. $\quad s^2 + r_4^2 - 2 s r_4 \cos\lambda = r_3^2$

$$\therefore \lambda = \cos^{-1}\left[\frac{s^2 + r_4^2 - r_3^2}{2 s r_4}\right] \quad -(3)$$

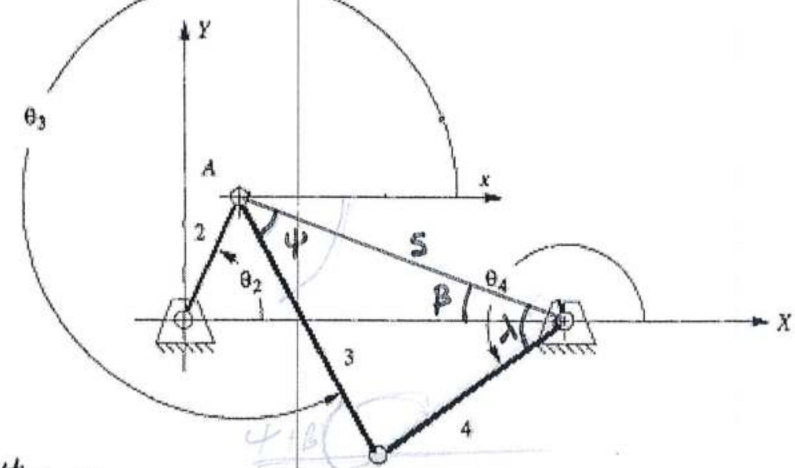
Sine law $\quad \frac{r_4}{\sin\psi} = \frac{r_3}{\sin\lambda} \quad \Rightarrow \quad \psi = \sin^{-1}\left[\frac{r_4 \sin\lambda}{r_3}\right] \quad -(4)$

Now $\theta_4 = 180^\circ - \lambda - \beta \quad -(5)$

$\theta_3 = \psi - \beta \quad -(6)$

Crossed Position

The four-bar can be assembled in crossed position also, as shown.



Here, link 2 is in the same position but links 3 and 4 are crossed. We divide into triangles as before. Note that s and β are the same as before. Also, equations for λ and ψ are the same. So, we use equations ① – ④ as before. However, θ_3 and θ_4 are different.

$$\theta_4 = 180^\circ - \beta + \lambda$$

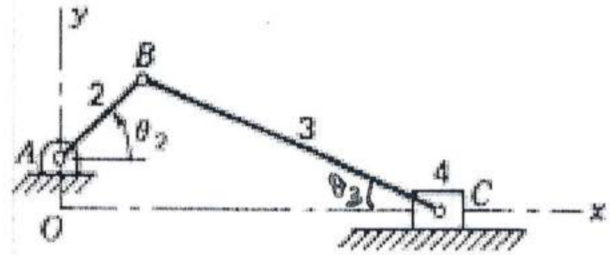
$$\theta_3 = 360^\circ - \psi - \beta$$

For the dimensions of the offset slider-crank linkage given below and value of θ_2, find the values of θ_3 and the position of slider, OC.

$\overline{OA} = 1$. $\overline{AB} = 2.5$, $\overline{BC} = 7$

$\theta_2 = 30^\circ$.

$\overline{OC} = ?$ $\theta_3 = ?$



SOLUTION: From fig

$$\overline{BC} \sin\theta_3 = \overline{OA} + \overline{AB} \sin\theta_2$$

$$7 \sin\theta_3 = 1 + 2.5 \sin 30^\circ$$

$$\theta_3 = \sin^{-1}\left[\frac{1 + 2.5 \sin 30^\circ}{7}\right] = 18.7^\circ$$

Then,

$$\overline{OC} = \overline{AB} \cos\theta_2 + \overline{BC} \cos\theta_3$$

$$= 2.5 \cos 30^\circ + 7 \cos 18.7^\circ = \underline{8.79}$$

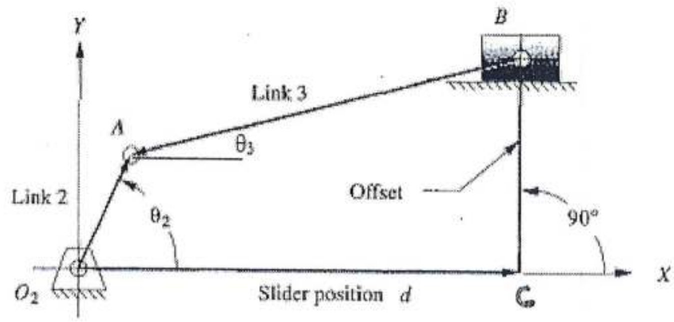
2. For the dimensions of the offset slider-crank linkage given below and value of θ_2, find the values of θ_3 and d.

$r_2 = O_2A = 1.4$

$r_3 = AB = 4$

$r_1 = BC = 1$ (fixed)

$\theta_2 = 45^\circ$



SOLUTION

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 = r_1$$

$$1.4 \sin 45^\circ + 4 \sin\theta_3 = 1$$

$$\therefore \theta_3 = \sin^{-1}\left[\frac{1 - 1.4 \sin 45^\circ}{4}\right] = 0.14^\circ$$

$$d = r_2 \cos\theta_2 + r_3 \cos\theta_3$$

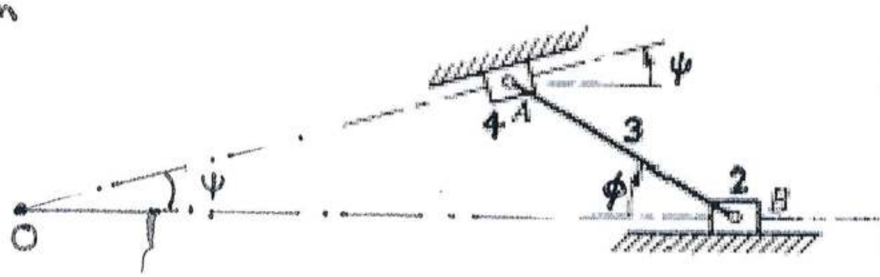
$$= 1.4 \cos 45^\circ + 4 \cos 0.14^\circ = 4.99$$

For the dimensions of the linkage and angle φ given below, find the position of sliders 2 and 4.

$\overline{AB} = 200$ mm

$\psi = 15^\circ$

$\phi = 45^\circ$



SOLUTION

∴ Extend axes of sliders to meet at fixed point O. Measure positions from O. So we want to find $\overline{OA}$ and $\overline{OB}$.

Apply Sine law on triangle OAB.

$$\frac{\overline{OA}}{\sin\phi} = \frac{\overline{AB}}{\sin\psi} \Rightarrow \frac{\overline{OA}}{\sin 45^\circ} = \frac{200}{\sin 15^\circ}$$

$$\overline{OA} = \frac{200 \sin 45^\circ}{\sin 15^\circ} = \underline{\underline{546}} \text{ mm}$$

The third angle in the triangle (at A) is

$$180^\circ - \phi - \psi = 180 - 45 - 15 = 120^\circ$$

$$\therefore \frac{\overline{OB}}{\sin 120^\circ} = \frac{200}{\sin 15^\circ}$$

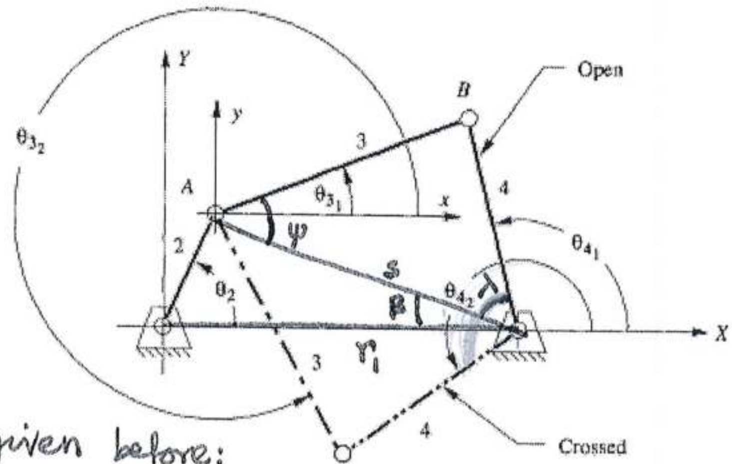
$$\overline{OB} = \frac{200 \sin 120^\circ}{\sin 15^\circ} = \underline{\underline{669}} \text{ mm}$$

1. For the dimensions of the four-bar linkage given below and value of θ_2, find the values of θ_3 and θ_4. Solve for both open and crossed configurations.

$r_1 = 6, \quad r_2 = 2$

$r_3 = 7, \quad r_4 = 9$

$\theta_2 = 30^\circ.$



Following the steps given before:

$$s = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta_2} = \sqrt{6^2 + 2^2 - 2\times6\times2\times\cos30^\circ} = 4.3835$$

$$\beta = \sin^{-1}\left[\frac{r_2 \sin\theta_2}{s}\right] = \sin^{-1}\left[\frac{2\sin30^\circ}{4.3835}\right] = 13.19^\circ$$

$$\lambda = \cos^{-1}\left[\frac{s^2 + r_4^2 - r_3^2}{2 s r_4}\right] = \cos^{-1}\left[\frac{4.3835^2 + 9^2 - 7^2}{2\times4.3835\times9}\right] = 49.53^\circ \text{ open config.}$$

For crossed configuration, λ is from s to r_4 in crossed position.

$$\psi = \sin^{-1}\left[\frac{r_4 \sin\lambda}{r_3}\right] = \sin^{-1}\left[\frac{9\sin49.53^\circ}{7}\right] = 78.0^\circ$$

Again, for crossed configuration, ψ is from s to r_3 in crossed position.

Now, for open configuration

$$\theta_{4_1} = 180^\circ - \beta - \lambda = 180 - 13.19 - 49.53^\circ = 117.3^\circ$$

$$\theta_{3_1} = \psi - \beta = 78.0 - 13.2 = 64.8^\circ$$

For crossed configuration

$$\theta_{4_2} = 180^\circ \mp \beta + \lambda = \overset{216.3^\circ}{\cancel{242.7}} \; \left(\text{or } \overset{216.3^\circ}{\cancel{242.7}} - 360 = -\overset{143.7^\circ}{\cancel{117.3}}\right)$$

$$\theta_{3_2} = -(\psi + \beta) \cancel{-360^\circ} = -91.2^\circ \; (\text{or } -91.2 + 360^\circ = 268.8^\circ)$$