DESIGN OF SHAFTS

Shaft is a mechanical part mainly used for power transmission as; crank shafts, propeller shafts, gear shafts,....etc.

Shaft design includes primarily the determination of its correct diameter to ensure satisfactory strength and rigidity when the shaft is transmitting power under various operating and loading conditions. Shafts are usually circular in cross-section, and may be either hollow or solid.

5.1- Design of Shafts

5.1.1- Shafts from ductile materials (as steels)

The design of shafts from ductile materials is performed based on the maximum shear stress (τ_{max}) theory.

$$\tau_{max} \le S_{S\ allow.}$$

Where;

$S_{S\ allow.}$ = allowable shear strength

= S_{SY}/F.S. *for static loading*

= S_{Se} /F.S. *for fatigue loading*

5.1.2- Shafts from brittle materials (as cast irons)

The design of shafts from brittle materials is performed based on the maximum normal stress (σ_{max}) theory.

Where;

$$\sigma_{max} \le S_{allow.}$$

Where,

$S_{allow.}$= allowable normal strength

= S y_{ield}/F.S. static loading

= Se /F.S. fatigue loading

Hint:- Because, most of the power transmitting shafts are made from ductile materials, we will perform our shaft design in the following analysis, based on the maximum shear stress theory.

a- <u>Design of Shafts subjected to Torsional Load only</u>

$$\tau_{xy} \le S_{s\,allow.}$$

<u>i.e. For solid shafts</u>

$$\tau_{xy} = \frac{16T}{\pi(d^3)} \le S_{s\,allow.}$$

<u>For hollow shafts</u>

$$\tau_{xy} = \frac{16Td_o}{\pi(d_o^4 - d_i^4)} \le S_{s\,allow.}$$

Where;

T= applied torque

d_o= shaft outer diameter

d_i = shaft inner diameter

$S_{s\,allow}$= allowable shear strength of the shaft material

Design of Shafts Subjected to Bending Static Loads only

$$\sigma_b = \frac{M_b y}{I} = \frac{32M_b}{\pi d^3} \le S_{allow.}$$

$$d = \left[\frac{32M_b}{\pi S_{allow}} \right]^{\frac{1}{3}}$$

i.e. For Solid shafts

For Hollow shafts

$$\sigma_b = \frac{32M_b d_o}{\pi(d_o^4 - d_i^4)} \le S_{allow.}$$

Design of Shafts Under Static Axial (Tensile or Compressive) Load only

For Solid shafts

$$\sigma_{axial} = \frac{F}{\frac{\pi}{4}(d^2)} \le S_{allow.}$$

For Hollow shafts

$$\sigma_{axial} = \frac{F}{\frac{\pi}{4}(d_o^2 - d_i^2)} \le S_{allow.}$$

Design of Shafts Subjected to Static Axial, Torsional and Bending Loads

ASME code equation for shafts gives:-

For Solid Shafts

$$d_o = \left[\frac{16}{\pi S_{s\,allow.}(1-\lambda^4)} \sqrt{(k_b M_b)^2 + (K_t M_t)^2} \right]^{1/3}$$

ASME code equation for ***Hollow Shafts***

$$d_o = \left[\frac{16}{\pi S_{s\,allow.}(1-\lambda^4)} \sqrt{\left(k_b M_b + \frac{\alpha F d_o (1+\lambda^2)}{8} \right)^2 + (K_t M_t)^2} \right]^{1/3}$$

Where;

M_b = bending moment

M_t = torsional moment

k_b= combined shock and fatigue factor applied to bending moment (from attached tables)

k_t= combined shock and fatigue factor applied to torsional moment (from tables)

λ= shaft diameters' ratio =d_i / d_o

α= column-action factor

For tensile loading $\alpha = 1$

For compressive loading

When $L/\rho \leq 115$

$$\alpha = \frac{1}{1-0.0044(\frac{L}{\rho})}$$

When $L/\rho > 115$

$$\alpha = \frac{S_y}{\pi^2 cE}(\frac{L}{\rho})^2$$

n = 1 for hinged ends
= 2.25 for fixed ends
= 1.6 for ends partly restrained, as in the bearings

ρ = Radius of gyration = $\sqrt{\frac{I}{A}}$

I = Moment of area , A = Cross sectional area

5.2- Allowable Stresses for Commercial Steel Shafts (ASME code)

$S_{s\ allow}$ = 55 MPa for shafts without key- ways

= 40 MPa for shafts with key-ways

ASME *code for steels purchased under definite specifications*

$S_{s\ allow}$ = 30 % Ssy

Such that

$S_{s\ allow}$ < 18 % $S_{s\ ut}$ for shafts with key-ways

$S_{s\ allow}$ < 25 % $S_{s\ ut}$ for shafts without key-ways

5.3- Design of Shafts for Torsional Rigidity

Angle of twist (θ) in radians

For Solid circular shafts

$$\theta = \frac{584TL}{Gd^4}$$

For Hollow circular shafts

$$\theta = \frac{584TL}{G(d_o^4 - d_i^4)}$$

Where:-

θ = angle of twist in degrees

G = modulus of rigidity of shaft material

Allowable value for θ_{allow} = 3^o for M/C tool shafts

5.4- Design of Shafts for Lateral Rigidity

The resulted deflection (**y**) in the shaft can be calculated starting from the following equation:-

$$\frac{d^2y}{dx^2} = \frac{M_b}{EI}$$

Where, y= bending deflection at a general position of length x.

By Integrating the above equation, one can get the deflection at any position (constants of integration depend on the end conditions), refer to **Chapter 1**.

5.5- Standard Recommendd Sizes for shafts' diameters

Up to 25 mm → 0.5 mm increaments

25 to 50 mm → 1 mm increments

50 to 100 mm → 2 mm increments

100 to 100 mm → 5 mm increments

Example

A steel bar had a pre-load of 25 kN and a fluctuation tensile load varying from 0 to 70 kN. Determine the diameter of the bar for an infinite life. Take theoretical stress concentration factor = 2.02 , ultimate strength = 690 MPa, yielding strength = 385 MPa, surface finish factor= 0.73, size factor= 0.85 and notch senestivity factor= 0.86. Assume factor of safety= 2.

Solution

Endulrance limit of bar Se, $S_e = K_a \times K_b \times K_c \times K_d \times K_e\, S_e = 114$ MPa

$F_{min.} = 0$, $F_{max} = 70$ kN , $F_m = F_a = 35$ kN

$$\frac{1}{F.S.} = \frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e}$$

$$\frac{1}{2} = \frac{70x10^3}{680\ A} + \frac{35x10^3}{114\ A}$$

The above equation gives shaft diameter = 32.25 mm (take it = 35mm)