Mechanical Engineering Department
College of Engineering
University of Bisha

قسم الهندسة الميكانيكية
كلية الهندسة
جامعة بيشة

Solved Problem (1)

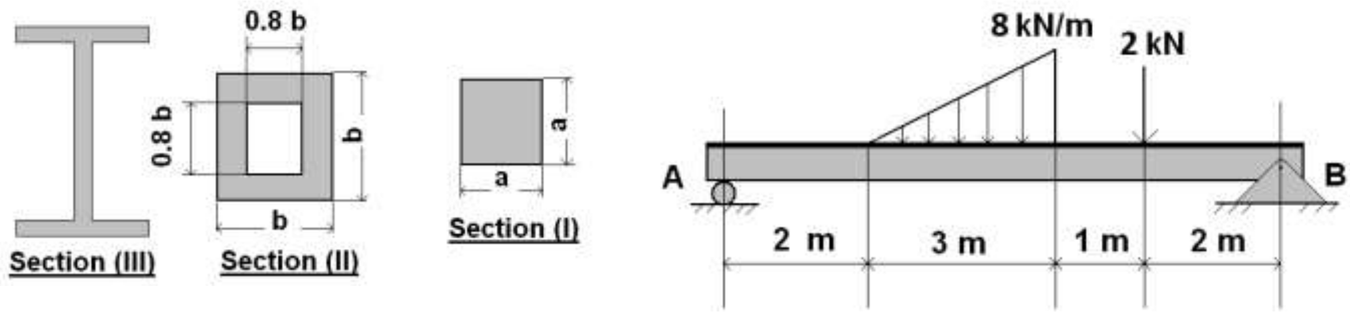

Figure (1)

For the shown steel loaded simply supported beam (Figure 1) get the following:

1. Beam reactions and bending moment diagram. Identify the value and the position of the maximum bending value in (N.mm).

2. Design the given steel loaded simply supported beam (Figure 2) on static flexural stress by getting the suitable cross-section among the following three cross-sections:
(a) Solid square section (I) , (b) Box square section (II) and (c) Standard I-beam section (III) in both x-x and y-y positions.

3. Get the factor of safety (n) for each standard cross-section.

4. If each 1 kg of the used structural steel (steel density 7.8 g/cm^3) beam costs **20 SAR**, calculate the cost of each designed cross-section beam and show **how much did you save in SAR** by selecting the lighter one?

(Use design factor (n_d) = 3 and material's yield strength =390 MPa)

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Problem (1)

$\sum M_A = 0$

$8 B_y - 2*6 - 12*4 = 0$

$B_y = 7.5\ kN \uparrow$

$\sum M_B = 0$

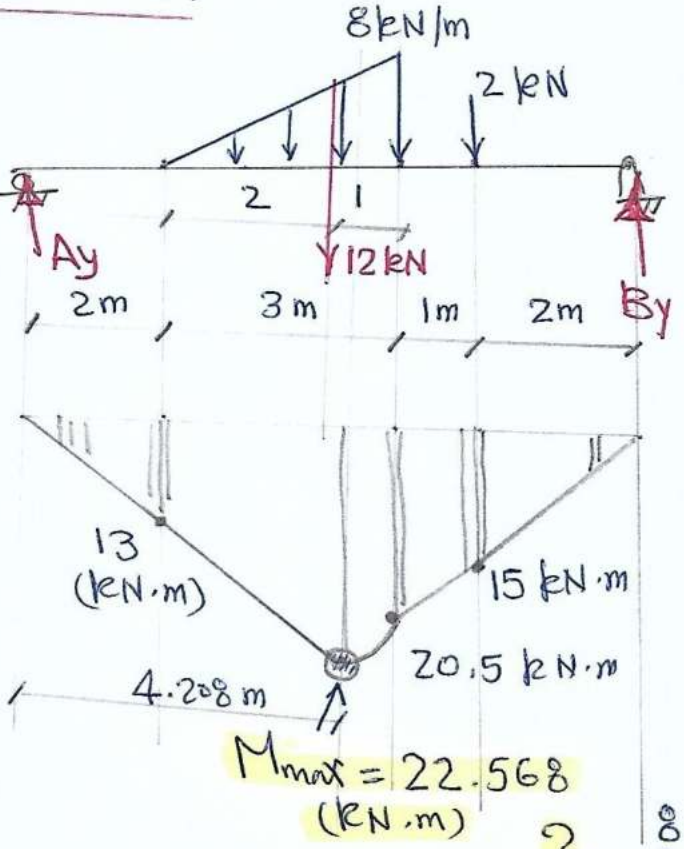
$8 A_y - 12*4 - 2*2 = 0$

$A_y = 6.5\ kN \uparrow$

check $\sum F_y = 0$

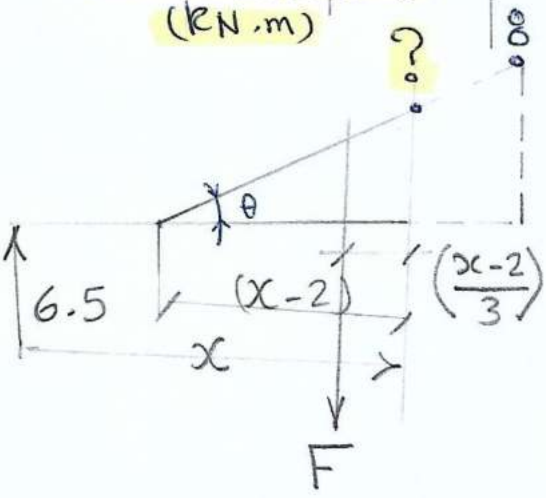
$A_y + B_y - 12 - 2 = 6.5 + 7.5 - 12 - 2$

$\sum F_y = 0$ o.K.



B.M.D.

Get $M(x)$ for $2 \leq x \leq 5$



$$M(x) = 6.5x - F\frac{(x-2)}{3}$$

$F = \frac{1}{2}(x-2) * ?$ $\Rightarrow$ as θ for Both Triangle is equal

$$\tan\theta = \frac{?}{x-2} = \frac{8}{3} \Rightarrow ? = \frac{8}{3}(x-2)$$

$$\therefore F = \frac{1}{\cancel{2}}(x-2) * \frac{\cancel{8}^{4}}{3}(x-2) \Rightarrow F = \frac{4}{3}(x-2)^2$$

$$\therefore M(x) = 6.5x - \frac{4}{9}(x-2)^3 \quad \#$$

To get x at $M = M_{max}$

$$\frac{\partial M(x)}{\partial x} = 0$$

$$M(x) = 6.5x - \frac{4}{9}\left(x^3 - 6x^2 + 12x - 8\right)$$

$$\frac{\partial M(x)}{\partial x} = 6.5 - \frac{4}{9}\left(3x^2 - 12x + 12\right) = 0$$

$$-12x^2 + 48x + 10.5 = 0$$

$x = 4.208$ m ✓ Accepted or $x = -0.208$ m Refused

$$x = 4.208\,\text{m}$$

$$M_{max} = 6.5 * 4.208 - \frac{4}{9}(4.208-2)^3$$

$$M_{max} = 22.568 \ \text{kN.m}$$

@ $x = 4.208$ m

See B.M.D $\longrightarrow$ $\boxed{\text{P. } 1/6}$

3/6 Problem (1)

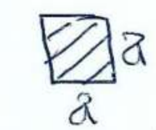
Use Flexural Formula

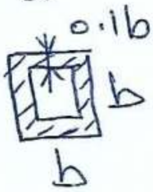
$$\boxed{\sigma = \frac{M_{max}}{Z} = \frac{\sigma_y}{n_d}}$$

σ_y — Yield strength given = 390 MPa

n_d — Design factor given = 3

Z — Sectional Modulus

For  $Z = \frac{a^3}{6}$

For  $Z = 0.081333\, b^3$

For I_{x-x} and I_{y-y} Go to standard Tables

as $M_{max} = 22{,}568{,}000$ N.mm

$$\therefore \frac{22{,}568{,}000}{Z} = \frac{390}{3} \Rightarrow Z = 173.6 * 10^3 \text{ m}$$

$$Z = 173.6 * 10^3 \text{ mm}^3$$

$$\therefore a = \sqrt[3]{6 * 173.6 * 10^3} = 101.4 \text{ mm}$$

$$b = \sqrt[3]{\frac{173.6 * 10^3}{0.081333}} = 128.8 \text{ mm}$$

For standard (multiplication of 5 mm)

$\boxed{4/6}$ Problem (1)

$a_{st} = 105 \text{ mm} \quad b_{st} = 130 \text{ mm}$

From table for $I_{y-y} \Rightarrow Z_{1st} = 175 * 10^3$ mr

" " For $I_{x-x} \Rightarrow Z_{1st} = 179 * 10^3$ m

To calculate n ... Factor of safety get Z_{1st}.

$$Z_{1st.\ 105 \times 105} = \frac{(105)^3}{6} = 192.9375 * 10^3 \text{ mm}^3$$

$$Z_{1st\ 130 \times 130,\ 13} = 0.081333(130)^3 = 178.689 * 10^3 \text{ mm}^3$$

$Z_{1st}\ I_{y-y} = 175 * 10^3 \text{ mm}^3$ fromtable

W460X74

$Z_{1st}\ I_{x-x} = 179 * 10^3 \text{ mm}^3$ fromtable

W250X18

Factor of safety $n \Rightarrow \frac{(M_{max})}{Z_{1st}} = \frac{\sigma_y}{n}$

$$n = \frac{\sigma_y \cdot Z_{1st}}{M_{max}}$$

$\boxed{5/6}$ Problem (1)

For 105 × 105 $\quad n = \dfrac{390 * 192.9375 * 10^{3}}{22,568,000} = 3.33$

For 130 × 130 (13) $\quad n = \dfrac{390 * 178.689 * 10^{3}}{22,568,000} = 3.09$

For I_{y-y} $\quad n = \dfrac{390 * 175000}{22,568,000} = 3.02$

For I_{x-x} $\quad n = \dfrac{390 * 179000}{22,568,000} = 3.09$

To calculate the Mass of and use the following equation

Mass = Volume * density = W in kg

Volume = area * Beam length

$$W\,(kg)_{105 \times 105} = \frac{\underbrace{10.5 * 10.5}_{\text{area } cm^2} * \underbrace{800}_{\text{length}} * \underbrace{7.8}_{\text{density } g/cm^3}}{1000} =$$

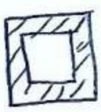
$$W\,(kg)_{130 \times 130} = \frac{\overbrace{(13 * 13) - (0.8 * 13)^{2}}^{\text{area}} * 800 * 7.8}{1000}$$

For I_{y-y} as W 460 × 74 → kg/meter

I_{x-x} as W 250 × 18 → kg/meter

6/6 Problem (1)

the following Table Summarizes the Results

Shape	$Z_{st} * 10^3$	n	Standard Dimension	Weight kg	Cost SAR
	192.94	3.33	105X105	688	13760
	178.69	3.09	130X130	380	7600
I Y-Y	175	3.02	W460X74	592	11840
I X-X	179	3.09	W250X18	144	2880

Saved money = 13760 − 2880 = 10880 SA
Per Beam

the Best cross-section is I_{x-x}
Then Box 
Then I_{y-y}
Then solid 

For aircraft and automobile applications